www.czasopisma.pan.pl www.journals.pan.pl

Arch. Min. Sci., Vol. 58 (2013), No 4, p. 1279–1287

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2013-0088

KRYSTIAN KALINOWSKI*, ROMAN KAULA*

VERIFICATION OF FLOTATION KINETICS MODEL FOR TRIANGULAR DISTRIBUTION OF DENSITY FUNCTION OF FLOTABILITY OF COAL PARTICLES

WERYFIKACJA MODELU KINETYKI FLOTACJI DLA TRÓJKĄTNEGO ROZKŁADU FUNKCJI GĘSTOŚCI FLOTOWALNOŚCI ZIAREN WĘGLA

Parameters of flotation kinetics model with a gamma and a triangular distribution have been determined based on the batch coal flotation experiments. Analyses were carried out at different values of the intensity of aeration air. The results of example analyses are presented in tables and graphs form. It follows from the carried out study on the mathematical models of flotability coefficient distribution of particles of examined coal samples that the triangular distribution model is statistical equivalent to the gamma flotability distribution. The assumption of the triangular distribution enables determination of the density function of distribution of fraction in the industrial flotation machines. Knowledge of this function will allow better evaluation of the phenomena occurring in the process of enrichment. It may be one of the basic information used in the supervisory control system.

Keywords: models of the flotation kinetics, coefficient of flotation velocity, distribution density function of the fraction, industrial flotation machine

Na podstawie badań eksperymentalnych flotacji cyklicznej węgla wyznaczono parametry modelu kinetyki flotacji z rozkładem gamma i trójkątnym. Analizy przeprowadzono przy różnych wartościach natężenia powietrza do aeracji. Wyniki przykładowych analiz przedstawiono tabelarycznie i w postaci wykresów. Z przeprowadzonych badań nad modelami matematycznymi rozkładu współczynnika flotowalności ziaren badanych próbek węgla wynika, że model o rozkładzie trójkątnym statystyczne jest równoważny rozkładowi flotowalności gamma. Przyjęcie rozkładu trójkątnego umożliwi wyznaczenie funkcji gęstości rozkładu frakcji we flotownikach przemysłowych. Znajomość tej funkcji pozwoli na lepszą ocenę zjawisk zachodzących w procesie wzbogacania. Może stanowić jedną z podstawowych informacji wykorzystywanych w nadrzędnym systemie sterowania.

Słowa kluczowe: modele kinetyki flotacji, współczynnik prędkości flotacji, funkcja gęstości rozkładu frakcji, flotowniki przemysłowe

---

* SILESIAN UNIVERSITY OF TECHNOLOGY, FACULTY OF MINING AND GEOLOGY, UL. AKADEMICKA 2, 44-100 GLIWICE, POLAND

1. Introduction

A density function of a distribution of a fraction due to the particles flotability is widely used in theory and simulation technique of a coal flotation process. The characteristics of this type may be used in the design of the flotation process systems as well as systems to optimize flotation in industrial settings. The density function of the distribution of the fraction is calculated based on the kinetics experiments to measure the recovery of material as a function of time in the laboratory flotation machines (in the batch coal flotation process). A commonly used approach is to assume the functional form of the mathematical model of the concentrate separation for the various functions of the distribution (Brożek & Młynarczykowska, 2009; Chudacek & Fischer, 1991; Kalinowski & Tumidajski, 1995).

The main point in this problem is select from accepted set of models the best model using the commonly used methods of verification. This problem was discussed in a lot of works (Brożek & Młynarczykowska, 2009; Imaizumi & Inoue, 1963; Woodburn & Loveday, 1965; Zuniga, 1935). In these works the authors have accepted that a feed material is characterized by continuous or discrete distribution of particles flotability.

The most commonly accepted mathematical models of the flotation kinetics are:
- model of multi-fractional and fractional one,
- model of the gamma flotability distribution.

The previous research results shows (Kalinowski & Kaula, 1994) that assumption of particles flotability distribution $f(k)$, in the form of gamma distribution in the flotation kinetics model accurately this process describes. At the same time, this model must take into account the transport delay (Kalinowski & Kaula, 1995b). Model of the mass separation of the concentrate $m(t)$ at time t has the following form:

$$m(t) = M \cdot \left(1 - \left(1 + k_0 \cdot (t - \tau)\right)^{-n}\right) \tag{1}$$

and corresponding the density function of the flotability distribution given by the formula:

$$f(k) = \frac{1}{k_0 \Gamma(n)} \left(\frac{k}{k_0}\right)^{\mathrm{n}-1} e^{\left(-\frac{k}{k_0}\right)} \tag{2}$$

where:

M — mass of flotable feed,
k_0, n — parameters of the gamma distribution,
$G(n)$ — gamma function,
k — the flotation velocity coefficient.

Industrial flotation machines are flow machines, and thus can specify the recovery of the concentrates and tailings from each flotation cell (on the basis of measurements of quality parameters in steady state). Recovery of the concentrate in this state from i-th flotation cell can be presented as the following formula:

$$W_{Ki} = \int_0^\infty \frac{\frac{k}{D_0}}{\left(1+\frac{k}{D_0}\right)^i} f\left(k\right) dk \tag{3}$$

whereas the tailings can be presented by the formula:

$$W_{Oi} = \int_0^\infty \frac{1}{\left(1+\frac{k}{D_0}\right)^i} f\left(k\right) dk \tag{4}$$

where: D_o — coefficient which characterize the outflow of the particles from the tailings zone.

Assuming that the function $f(k)$ is the gamma distribution, the analytical solution of the integrals (3-4) is not known.

The authors of this study proposed such the form of the function $f(k)$ that the integrals (3-4) will have an analytical solution. This will determine the function parameters of the distribution in the industrial flotation machines.

The function $f(k)$ composed of straight line sections in the form $ak + b$ it has the advantage that the solution of the integrals (5-6) can be presented through analytical formulas.

$$\int_d^g \frac{\frac{k}{D_0}}{\left(1+\frac{k}{D_0}\right)^i} \left(ak+b\right) dk \tag{5}$$

$$\int_d^g \frac{1}{\left(1+\frac{k}{D_0}\right)^i} \left(ak+b\right) dk \tag{6}$$

where:

a, b — parameters of straight line,
d, g — the lower and the upper limit of integration.

It is therefore necessary to use a continuous model of the distribution of particles flotability $f(k)$ consisting of straight line sections under which the precision of model of the flotation kinetics $m(t)$ is comparable to the accuracy of the kinetics determined for the model with the gamma distribution (1).

In a preliminary research the distribution of the flotability was assumed in the shape of a quadrangle (Fig. 1).

Identification of the model parameters of kinetics of coal flotation for different data series were carried out taking into account the distribution of particles flotability $f(k)$ as in Fig. 1. The identified parameters of distribution k_1, k_2, k_3, k_4 adopted such values that the function $f(k)$ from Fig. 1 has transformed into form the distribution of a triangular (Fig. 2) for each considered case.

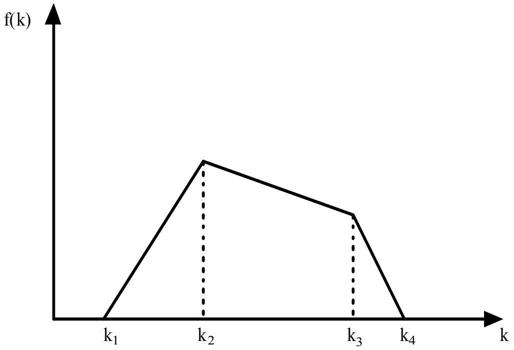

Fig. 1. Distribution of the coal particles flotability $f(k)$ in the shape of a quadrangle

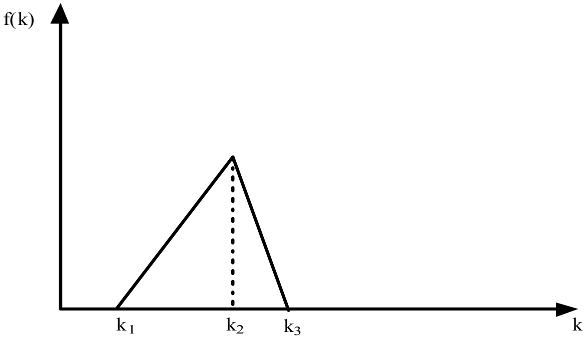

Fig. 2. Distribution of the coal particles flotability $f(k)$ in the shape of a triangle

A mathematical model of the flotation kinetics based on the triangular distribution may be written:

$$m(t) = M \cdot \left(1 - \left(\frac{2 \cdot e^{-k_3(t-\tau)} \cdot (k_2 - k_1) + 2 \cdot e^{-k_2(t-\tau)} \cdot (k_1 - k_3) + 2 \cdot e^{-k_1(t-\tau)} \cdot (k_3 - k_2)}{(t-\tau)^2 \cdot (k_1 - k_2) \cdot (k_1 - k_3) \cdot (k_3 - k_2)}\right)\right) \tag{7}$$

and corresponding the density function of the flotability distribution given by the formula:

$$f(k) = \begin{cases} 0 & \text{for} \quad k \le k_1 \\ \dfrac{2k}{(k_1 - k_2)(k_1 - k_3)} & \text{for} \quad k_1 \le k \le k_2 \\ \dfrac{2k}{(k_1 - k_3)(k_3 - k_2)} - \dfrac{2k_3}{(k_1 - k_3)(k_3 - k_2)} & \text{for} \quad k_2 \le k \le k_3 \\ 0 & \text{for} \quad k > k_3 \end{cases} \tag{8}$$

where: k_1, k_2, k_3 — triangular distribution parameters.

Comparison of mathematical models of the mass separation of the concentrate $m(t)$ at time t, based on the distribution of gamma and proposed triangular model were carried out based on the experiments of the concentrate separation in the batch coal flotation process.

2. Analysis of calculation results

Examples of the measurement data of the mass separation of the concentrate $m(t)$ at time t used to determine the parameters of mathematical models are presented in Table 1. Detailed description of the experiments of batch flotation on which the data was obtained for the calculation was presented in the work (Kalinowski & Kaula, 1995). The research was carried out for different values of the air flow rate.

TABLE 1

Results of measuring the mass of the concentrate $m(t)$ in % to weight of the feed, as a function of time t in s, at different values of the air flow rate Vp in 10^{-5} m^3/s

t	***m*(*t*)**		
	1/3*Vp*	**1/2*Vp***	***Vp***
10	29,70	31,00	45,60
20	54,40	58,60	73,20
30	70,40	72,80	81,70
50	81,20	82,40	85,70
70	85,60	85,70	87,00
90	87,70	87,10	87,50
120	88,90	88,20	88,10
210	90,20	89,20	88,70

In Table 2 and Table 3 the calculated parameters of the model (1) and model (7) are shown. These parameters were obtained by least squares method. To assess the degree of fit of the model to the measurement data the residual variance w_r was used. For this purpose, the measurement data are divided into two sets: a set of learning and set of checking. The learning set was used to determine the model parameters while based on the checking set was determined the residual variance. Method of determining this index is detailed described in (Kalinowski & Kaula, 1994).

TABLE 2

The calculated values of the kinetics parameters of the model (1) based on the experimental results from Table 1

	1/3*Vp*	**1/2*Vp***	***Vp***
M, %	89,93	88,97	88,50
k_0, s^{-1}	0,0121	0,0198	0,0741
n	5,25	4,06	2,39
τ, s	3,53	4,39	5,24
w_r	0,1380	0,1023	0,1337

TABLE 3

The calculated values of the kinetics parameters of the model (7) based on the experimental results from Table 1

	1/3*Vp*	**1/2*Vp***	***Vp***
M, %	90,49	89,33	88,35
k_1, s–1	0,00070	0,000001	0,000001
k_2, s^{-1}	0,0911	0,0830	0,1320
k_3, s^{-1}	0,0911	0,1420	0,2750
τ, s	3,37	4,14	4,27
w_r	0,0675	0,0593	0,1513

As you can see from the presented results (Table 2 and Table 3) values of residual variances determined for both kinetic models are small and not much different among themselves.

The results are shown graphically in Fig 3 to Fig. 5. The graphs shows the measured data (Table 1) and the time courses of the mass separation of the concentrate for analysed kinetics models. The marks on the graphs are as follows: measurement data –D, the time course of the kinetics model with the triangular distribution –T, the time course of the kinetics model with gamma distribution –G.

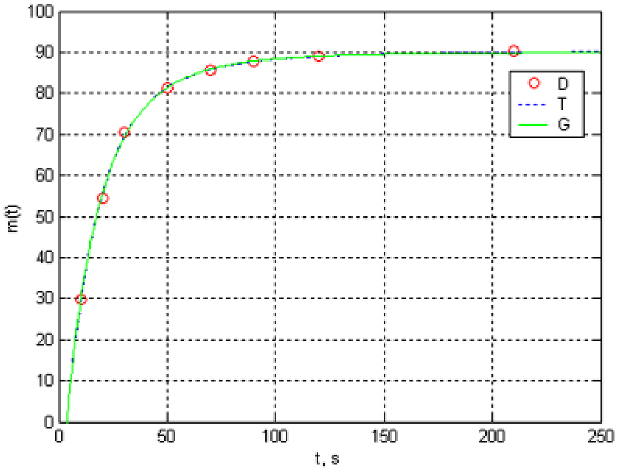

Fig. 3. Time courses of the concentrate separation for 1/3*Vp*

Comparing the estimated courses of the kinetics models of the mass separation of the concentrate $m(t)$ (described in formulas 1 and 7) we can state that the shapes of the curves are similar. It may be noted also that the fitting of the curves to the measured data is very accurate.

Considered distributions of the particles flotability $f(k)$ are shown graphically in Fig. 6 to Fig. 8.

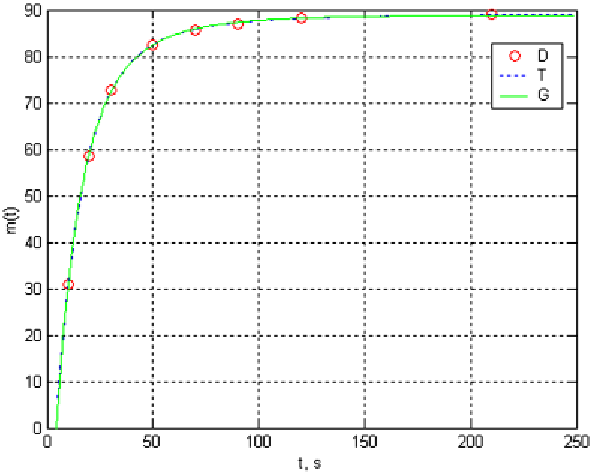

Fig. 4. Time courses of the concentrate separation for 1/2*Vp*

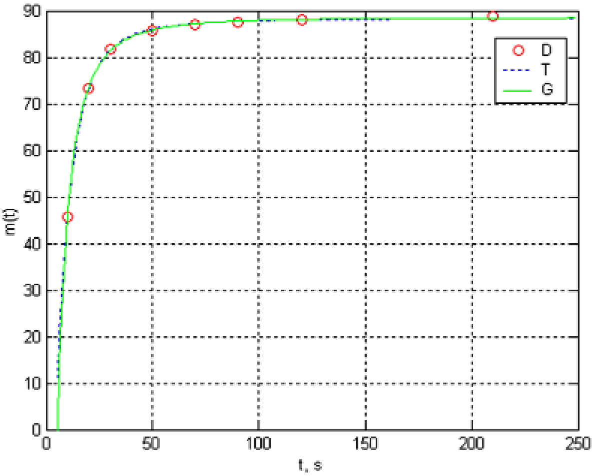

Fig. 5. Time courses of the concentrate separation for *Vp*

3. Conclusions

On the basis the research of mathematical models of the distribution of particle flotability coefficient of analysed coal samples we can state that the triangular distribution model is statistical equivalent to the distribution of the gamma flotability.

Using of the triangular distribution has the advantage that the flotability coefficient has limited value what is consistent in a physical sense (feed particles of coal cannot have unlimited flotability factor what assumes the gamma distribution).

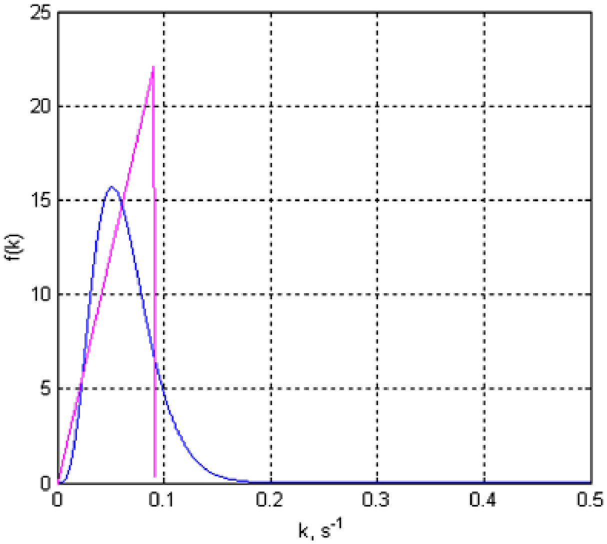

Fig. 6. Density functions of the triangular and gamma flotability for 1/3*Vp*

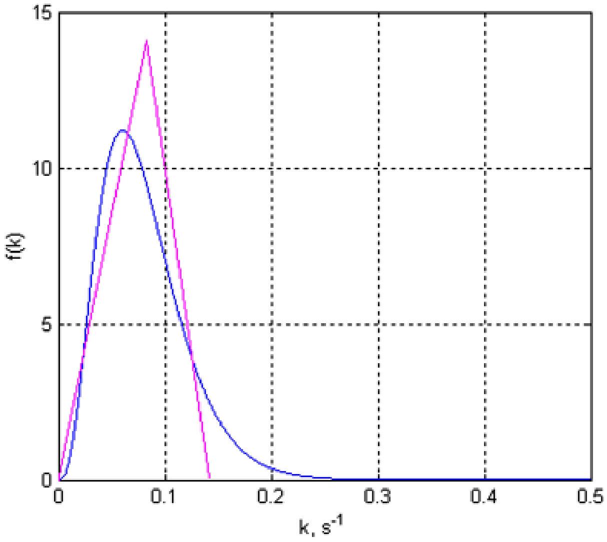

Fig. 7. Density functions of the triangular and gamma flotability for 1/2*Vp*

It may be said that adoption of the triangular distribution will allow determination of the density function of the distribution of fraction in industrial flotation machines. Knowledge of this feature will allow a better assessment of the phenomena occurring in the enrichment process. It can be one of the basic information used in the supervisory control system.

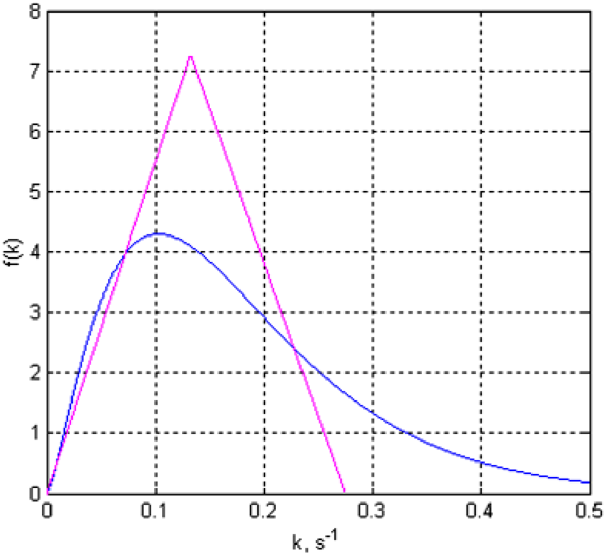

Fig. 8. Density functions of the triangular and gamma flotability for *Vp*

References

Brożek M., Młynarczykowska A., 2009. *Kinetyka flotacji*. Wydawnictwo IGSMiE PAN, Kraków.

Fischera M.A., Chudacek M.W., 1991. *Batch cell flotation models*. A Review. Minerals Engineering, 5(1), p. 41-54.

Imaizumi T., Inoue T., 1963. *Kinetic considerations of froth flotation*. Proc. 6th International Mineral Processing Congress (Cannes) 1963, p. 581-593.

Kalinowski K., Kaula R., 1996. *A Verification Method Of Coal Flotation Kinetic Models*. New Trends in Coal Preparation Technologies and Equipment: Proceedings of the 12th International Coal Preparation Congress. Cracow, May 1994, published by Gordon and Brench Publishers, p. 737-740.

Kalinowski K., Kaula R., 1995. *Wpływ natężenia powietrza aeracyjnego na rozkład flotowalności w eksperymentach flotacji cyklicznej węgla*. Zeszyty Naukowe Politechniki Śląskiej, seria Górnictwo, z. 225, s. 111-118.

Kalinowski K., Kaula R., 1995. *Transport delay in a mathematical models of batch coal flotation kinetics*. Arch. Min. Sci., Vol. 40, p. 339-349.

Kalinowski K., Tumidajski T., 1995. *Przegląd Modeli matematycznych kinetyki flotacji cyklicznej*. Zeszyty Naukowe AGH, Górnictwo, z. 3.

Woodburn E.T., Loveday B.K., 1965. *The effect of variable residence time on the performance of a flotation systems*. Journal of South African Institute of Mining and Metallurgy, vol. 65, p. 612-628.

Zuniga H.G., 1935. *Flotation recovery as an exponential function of its rate*. Boletin de Minero, Sociedad Nacional de Mineria, Santiago, Chile, vol. 47, p. 83-86.

Received: 11 March 2013