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THE HOT-WIRE ANEMOMETER

ANEMOMETR Z GRZANYM WŁÓKNEM

This study analyses the behaviour of a hot-wire anemometer incorporated into a bridge circuit in the function of the voltage U_z supplying the bridge circuit and hence the current I_w flowing through the hot wire. The dependence of differential voltage ΔU on I_w and the value of derivative $d(\Delta U)/dI_w$ as a function of the current supplying the hot-wire element were determined. These data enable the determination of working conditions of the constant-resistance (i.e. the constant-temperature) hot-wire anemometer.

Keywords: thermal anemometer, hot-wire anemometer, constant-resistance (constant-temperature) anemometer

W artykule analizowano zachowanie się grzanego włókna włączonego w mostek w zależności od napięcia U_z zasilania mostka, a co za tym idzie od prądu I_w płynącego przez grzane włókno. Wyznaczono napięcie różnicowe $\Delta U(I_w)$ oraz wielkość pochodnej $d(\Delta U)/dI_w$ jako funkcje prądu zasilania włókna. Te dane pozwalają na wyznaczenie warunków pracy grzanego włókna jako anemometru stałorezystancyjnego (stałotemperaturowego).

Słowa kluczowe: anemometr cieplny, anemometr z grzanym włóknem, anemometr stałorezys-tancyjny (stałotemperaturowy)

1. Introduction

The hot-wire anemometer is one of the most frequently used instruments for investigation of flows. In such anemometer, a hot-wire element of several to several tens of micrometres in diameter is heated by an electric current flowing through it and the measure of the velocity of analysed flow is expressed by means of the voltage measured across the heated hot-wire element,

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the intensity of current supplying the wire or its resistance. The system solutions vary depending on which quantity is chosen as the measure of flow velocity.

This study provides an analysis of the so-called anemometric bridge circuit. It consists of two branches – $(R_a + R_w)$ and $(R_b + R_c)$ – connected in parallel and supplied by voltage U_z. Schematic diagram of the bridge circuit is presented in Fig. 1. The output voltage ΔU is the voltage across the diagonal of the bridge which is the difference between the voltages occurring at terminals U_B and U_A.
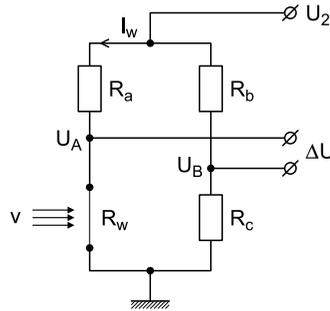

Fig. 1. The bridge circuit incorporating the hot-wire element

Thus, the voltage across the diagonal of the bridge ΔU equals

$$\Delta U = U_B - U_A = U_z \left[\frac{R_c}{R_b + R_c} - \frac{R_w}{R_w + R_a} \right] = U_z \left[\frac{R_a R_c - R_b R_w}{(R_w + R_a)(R_b + R_c)} \right] \qquad (1)$$

It is obvious, that if $U_z = 0$ then $\Delta U = 0$ irrespective of the value of the resistance of the wire R_w. When U_z increases, the current supplying the wire I_w rises and the temperature of the wire T_w also rises entailing the increase in resistance R_w, which is in the function of the supplying current I_w, what can be designated as $R(I_w)$. Moreover, it can be noticed that the power dissipated in the heated hot-wire equal to $I_w^2 R_w$ is transferred to the medium flowing by, by means of the thermal conduction and convection. Assuming balanced situation, one gets an equation

$$I_w^2 R_w = [a + b(v)](T_w - T_g) \qquad (2)$$

where a — stands for the coefficient of thermal conductivity of the flowing medium, $b(v)$ — analogical coefficient related to convection, v — the velocity of medium, T_g — the temperature of hot wire element and T_w — the temperature of flowing medium. Linking the wire resistance with the temperature, we may write that

$$R_w = R_0[1 + \alpha(T_w - T_0)] \qquad (3)$$

and

$$R_g = R_0[1 + \alpha(T_g - T_0)] \qquad (4)$$

where R_w and R_g stand for the resistances of hot-wire element heated up to the temperature of T_w

and at the temperature of flowing medium T_g, respectively, R_0 – the wire resistance at the reference temperature, at which the temperature coefficient of the resistance of wire α was determined.

Combining (3) and (4) we get:

$$T_w - T_g = \left(\frac{R_w - R_g}{\alpha R_0} \right) \tag{5}$$

and substitution of this relationship into (2) leads to

$$I_w^2 R_w = [a + b(v)] \frac{R_w - R_g}{\alpha R_0} \tag{6}$$

Subsequently by making

$$\beta = \frac{\alpha R_0}{[a + b(v)]} \tag{7}$$

and substituting into equation (2) we get

$$I_w^2 R_w = \frac{1}{\beta}(R_w - R_g) \tag{8}$$

Bearing in mind that the heating coefficient of the anemometer's wire N is expressed by formula

$$N = \frac{R_w}{R_g} \tag{9}$$

one ends up finally with

$$I_w^2 = \frac{1}{\beta}(1 - \frac{1}{N}) \tag{10}$$

or

$$I_w^2 = \frac{a + b(v)}{\alpha R_0}(1 - \frac{1}{N}) \tag{11}$$

The equation (10) may be written in a dimensionless form as:

$$\frac{1}{N} = 1 - \beta I_w^2 \tag{12}$$

which says that for $I_w = 0$ the wire does not heat up, which means that $N = 1$, and analogically for increasing I_w the heating coefficient N rises. Fig. 2 presents the relationship (12) in the case of sensor made up of tungsten wire of 5 μm in diameter with resistance of $R_g = 4.82$ W.

The boundary value for $1/N = 0$ from equation (12) determines the current

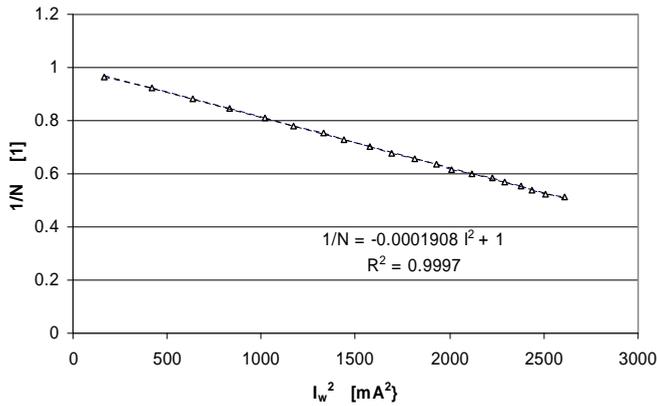$$I_w = \sqrt{\frac{1}{\beta}} \tag{13}$$

Fig. 2. Reciprocal dependence of heating coefficient N on the intensity of current supplying
the sensor under no-flow conditions, $R_g = 4.82$ W

at which the coefficient N becomes infinitely large. It is not equal to current flowing through
the wire, as it is degraded earlier as a result of the oxidation of metal which the wire element is
made of. It is thus more preferable to heat up the wire to reach $N = 2$. Then equation (12) gives
$\beta I_w^2 = 0,5$ and hence

$$\beta = \frac{0,5}{I_{w(N=2)}^2} \tag{14}$$

The voltage ΔU across the diagonal of the bridge circuit is used as the input voltage of the
amplifier which is in control of the voltage U_z supplying the bridge operating in classic circuit
of hot-wire anemometer.

2. The dependence of voltage U_z supplying the bridge and the offset voltage ΔU of the bridge on the current I_w heating up the wire

The dependence of the voltage U_z supplying the bridge on the current flowing through the
wire of the sensor is determined by equation

$$U_z = I_w(R_a + R_w) \tag{15}$$

and according to (12) we get

$$R_w = \frac{R_g}{1 - \beta I_w^2} \tag{16}$$

Combining (15) and (16) we end up with:

$$U_z = I_w\left(R_a + \frac{R_g}{1 - \beta I_w^2}\right) \tag{17}$$

This formula indicates that given the constant current supplying the wire, the voltage U_z will depend on parameter β defined by formula (7), and hence the velocity of the medium v.

By rewriting equation (1) we have

$$\Delta U = U_z \left(\frac{R_c}{R_b + R_c} - \frac{R_w}{R_a + R_w} \right) \tag{18}$$

and considering that

$$I_w = \frac{U_z}{R_a + R_w} \tag{19}$$

we get

$$\Delta U = I_w (R_a + R_w) \left(\frac{R_c}{R_b + R_c} - \frac{R_w}{R_a + R_w} \right) = \frac{I_w}{R_b + R_c} \left(R_a R_c - \frac{R_b R_g}{1 - \beta I_w^2} \right) \tag{20}$$
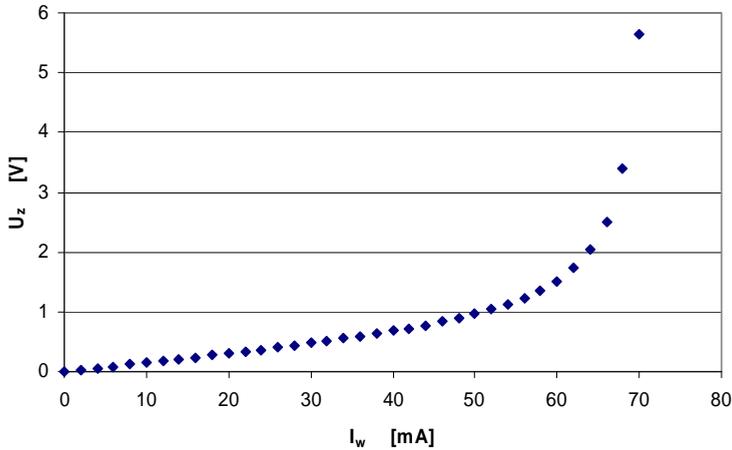

Fig. 3. Dependence of the bridge supplying voltage U_z on intensity of current I_w supplying the wire

The voltage ΔU is in the function of the voltage supplying the wire I_w, the conditions of cooling of the wire (by means of β) and parameters of the bridge. If $I_w = 0$, the voltage ΔU equals zero. If the current I_w increases slightly, the voltage ΔU rises, although for $I_w^2 = 1/\beta$ it converges towards $-\infty$. Hence a conclusion can be drawn that ΔU has an inflection point. This point can be determined equating the derivative of ΔU with respect to I_w to zero. Differentiating the voltage ΔU and equating the derivative to zero, an equation can be obtained for the value of I_w, at which the maximum of voltage ΔU occurs.

$$\frac{d(\Delta U)}{dI_w} = \frac{R_a R_c}{R_b + R_c} \left[1 - \frac{R_b R_g}{R_a R_c} \frac{1}{(1 - \beta I_w^2)} + \frac{R_b R_g}{R_a R_c} \frac{2\beta I_w^2}{(1 - \beta I_w^2)^2} \right] \tag{21}$$

Figure 4 presents the graph of $\Delta U(I_w)$ and $(d(\Delta U)/dI_w)$ as well as $U_z(I_w)$. The data refer to the sensor made of tungsten wire with a diameter of 5 mm and the resistance $R_g = 4.82$ W. The remaining resistances of the bridge circuit were $R_a = 10.0$ W, $R_b = 1.0$ kW and $R_c = 1.0$ kW. The value of parameter β for this wire equalled 0.00019015 [1/mA^2] at the temperature $T_g = 295.5$ K.
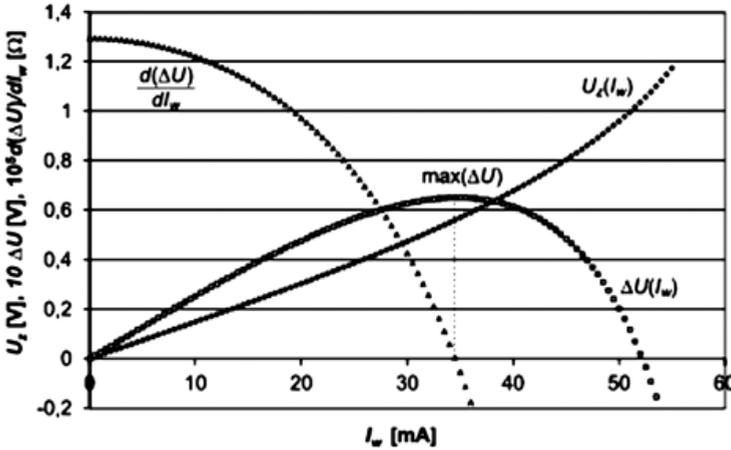

Fig. 4. Dependence of $U_z(I_w)$, $\Delta U(I_w)$ and $d(\Delta U)/dI_w$ on the intensity of current supplying the wire for $v = 0$

From Figure 4 it is obvious, that the voltage ΔU depending on the intensity of current supplying the wire I_w initially increases, but still slower and slower to reach the maximum at approx. 35 mA. The voltage ΔU then begins to decrease and for I_w around 52 mA becomes equal to zero. For higher intensities of current I_w, the voltage ΔU across the diagonal of the bridge will be negative.

The course of these three curves depends on conditions of hot-wire cooling. Figure 5 presents the courses of $1/N$ in function of the squared intensity of current supplying the wire for five selected flow velocities v. Table I summarizes the values of the $\beta(v)$ parameter and the coefficients of correlation R^2 given these flow velocities.

TABLE 1

Values of parameters of lines $1/N = b - \beta I_w^2$ for five different flow velocities

Designation of the curve	The flow velocity	β parameter [1/mA^2]	b parameter [1]	R^2 [1]
a	$v = 9.54$ cm/s	−0.000223	0.9831	0.9989
b	$v = 27.84$ cm/s	−0.000206	0.9895	0.9995
c	$v = 50.72$ cm/s	−0.000189	0.9904	0.9997
d	$v = 78.18$ cm/s	−0.000176	0.9895	0.9996
e	$v = 123.9$ cm/s	−0.000163	0.9911	0.9995

Based on the table it is obvious, that the b parameter, which should theoretically be equal to unity, is very close to this value (0.98870 on average) and the R^2 parameter is close to 0.999 or even better.
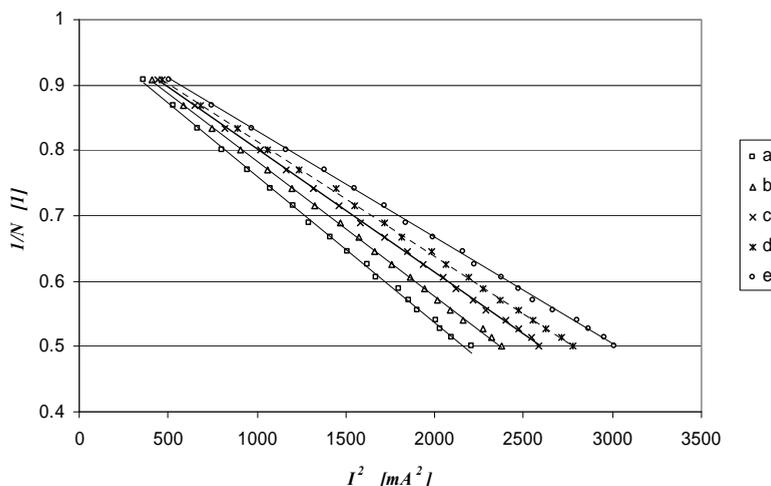
Fig. 5. The reciprocal dependence of heating coefficient of the anemometer's how-wire element
on squared intensity of current heating the wire

Given the values of $\beta(v)$, the voltages $\Delta U(I_w)$ for the above mentioned values of flow veloci-
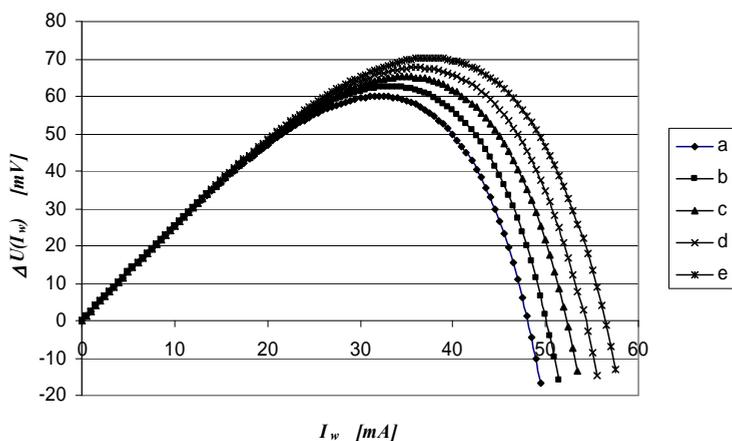ties were calculated according to formula (20). Obtained results are presented in Fig. 6.

Fig. 6. Dependence of the voltage across the diagonal of the bridge circuit shown in Fig. 1 on the current
supplying the hot-wire for different values of air flow velocity

Based on Figure 6 it is obvious, that in the initial range of current supplying the wire the volt-
age across the diagonal of the bridge circuit changes linearly and does not depend on the velocity
of flow around the wire. For current intensities above approx. 20 mA, the curves separate from
each other and visible maximums show up, the positions of which depend on the velocity of air

flow. Further increase of the heating current I_w causes the voltage across the bridge diagonal to drop and, finally, with subsequent increase of current intensity I_w the voltage becomes negative. Naturally, the presented curves apply to sensors made of tungsten wires with 5 μm in diameter only.

The reader (should such show up) may find it weird, that the curve a ($v = 9.54$ cm/s) in Figure 6 finds itself to the left from the analogical curve presented in Figure 4 obtained for v equal to zero. In both cases, the position of the wire was the same (the horizontal position). This is probably caused by the "chimney draft" effect arising around the heated wire, which does not occur under conditions of existing flow.

3. Conclusion

Analysing the $\Delta U(I_w)$ curve shown in Figure 4 and analogical ones shown in Figure 6 it is clearly obvious, that if the hot-wire element is to operate in a constant-resistance (i.e. constant-temperature) anemometer circuit, its working point should find itself in the negative part of the $\Delta U(I_w)$ characteristics, and thus must pass the maximum $\Delta U(I_w)$ early. But especially in the initial part of this characteristic, the increase ΔU results in increased current I_w and thus increased heating of the wire. Therefore we conclude, that should the sensor be able to operate as the CTA, it must be delivered an impulse in order to place its working point in decreasing part of the $\Delta U(I_w)$ characteristics, and more specifically, in its negative part. This can be achieved by several means and the most simple one is by adding the voltage ΔU by the voltage U_{off} which is selected such that $\Delta U(I_w) + \Delta U >$ approx. 60 mV, that is, such that the current I_w flowing through the hot wire exceeded the intensity of 35 mA, but this depends on selected parameters of the bridge circuit and, above all, on the value of heating coefficient. This situation is illustrated in Fig. 7.
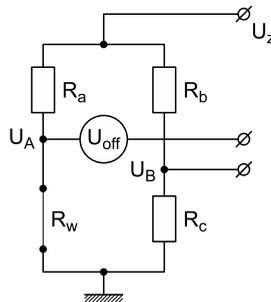

Fig. 7. The voltage $\Delta U = (U_B - U_A)$ is added by U_{off}

The voltage ΔU will be the input voltage of balanced voltage amplifier constituting the control unit of the power amplifier supplying the bridge current with the voltage U_z. This situation is illustrated in Fig. 8.

Selection of the offset voltage U_{off} is hindered by the fact, that the A_1 and A_2 amplifiers may both have their own offsets δ_U which may have arisen during the process of their production. These are difficult to be measured and will inevitably sum up with U_{off}. Their common regulation

is possible by means of the δ_U terminal. It often may by the case, that the range of regulation of δ_U is so wide, that the introduction of U_{off} is unnecessary and redundant. This can be verified by measuring the voltage U_a across the resistor R_a, which depends on the intensity of current flowing through the hot wire element R_w.
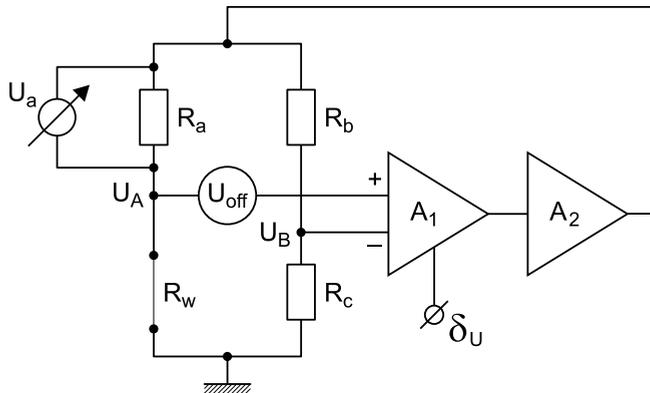

Fig. 8. The block scheme of the constant-resistance (i.e. constant temperature) hot-wire anemometer

The anemometer must be set to specific operational conditions. First, the value of R_g is measured at given temperature T_g and under no-flow conditions ($v = 0$). Then, knowing the value of heating coefficient N at which the anemometric sensor is to be calibrated, the value of R_c at which the bridge circuit is to operate with given heating coefficient N is calculated. The value of resistor R_c is calculated according to formula

$$R_c = \frac{N R_a R_g}{R_b} \qquad (22)$$

expressing the condition for balance within the bridge circuit. Following such setup of the bridge, the anemometer can be introduced into operation. The voltage U_a across the resistor R_a is the measure of the current supplying the hot-wire element under given operational conditions (known temperature T_g, known heating coefficient N, known diameter of the wire and its length). Following the sensor calibration for a given medium and in a specified flow velocity range, it may be used for measurement of unknown velocity of this medium.

Here, it is also worth emphasizing that the courses shown in Fig. 4 and 6 depend strongly on the value of heating coefficient N. It will each time influence the intensity of current $I_w(v,N)$.