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DYNAMIC CONTROL FOR GAS PIPELINE SYSTEMS

OPTYMALNE STEROWANIE SIECIĄ GAZOWĄ

An algorithm for optimal control of a gas network with any configuration based upon hierarchical control and decomposition of the network is described. Local problems are solved using a gradient technique. The subsystems are coordinated using „good coordination" method to find the overall optimum. Discrete state equation for the case in which output pressures are treated as elements of the control vector has been formulated. Results of investigations are included.

Keywords: flow control, large scale systems, networks, optimal control, transient analysis

W artykule omówiono algorytm optymalnego sterowania siecią gazową o dowolnej konfiguracji wykorzystujący teorię systemów hierarchicznych oraz zasady dekompozycji systemu na podsystemy. Lokalne problemy optymalizacji są rozwiązywane stosując metodę gradientową. Koordynacja rozwiązań lokalnych pozwala na znalezienie rozwiązania optymalnego dla całego systemu. Optymalizowany system opisano za pomocą dyskretnego równania stanu przyjmując, że elementami wektora sterowania są wartości ciśnienia wyjściowego elementów nierurowych. W artykule przedstawiono rezultaty badań algorytmu.

Słowa kluczowe: Sterowanie przepływem, wielkie systemy, sieci, optymalne sterowanie, stan nieustalony

1. Introduction

The growth of the complexity of gas transmission systems is accompanied by increasing opportunities for more efficient management. Gas transmission system operators, whose main task is the overall management of the system have, as the number of compressors increased, recognized the rising importance of fuel usage.

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Gas compressor stations form a major part of the operational plant on each transmission system. The compressors are driven mostly by gas turbines which use natural gas as fuel, taken directly from the transmission pipelines. Minimizing this fuel usage is a major objective in the control of gas transmission costs. The problem has received considerable attention over the past several years in the literature (Mallinson et al., 1993; Gill et al., 2002; Mahlke et al., 2010; Babonneau et al., 2012). Above all, the system must be operated so that gas is supplied where needed, in the quantities needed and at the appropriate pressure. The basic problem of running the transmission system is security of supply versus costs.

2. Objective function

This work is concerned with the minimization of operating costs for high pressure gas networks under transient conditions. The algorithm for steady-state optimization of large gas networks, based upon the generalized reduced gradient method was described in (Osiadacz, 2011).

Dynamic optimization requires the use of distributed-parameter models: a partial differential equation or a system of such equations. The form of these equations varies with the assumptions made as regards the conditions of operation of the gas pipeline. For such a problem the optimal parameters of the operation of the system (structure and pressures and flows) are functions of time.

The goal of optimization – to minimize the following expression

$$I_j = \int_{t_0}^{t_f} \sum_{j=1}^{M} A_j \cdot Q_j(t) \cdot \left\{ \left(\frac{p_{dj}(t)}{p_{sj}(t)} \right)^{R_j} - 1 \right\} \cdot dt \tag{1}$$

where:

M — the number of operating compressors

is based on the assumption (Francis, 1981) that the cost of running a compressor is proportional to the integral of horsepower over the control interval

$$I_j = \int_{t_0}^{t_f} A_j \cdot Q_j(t) \cdot \left\{ \left(\frac{p_{dj}(t)}{p_{sj}(t)} \right)^{R_j} - 1 \right\} \cdot dt \tag{2}$$

p_d — discharge pressure for j^{th} compressor [Pa],
p_s — suction pressure for j^{th} compressor [Pa],
Q_j — flow through j^{th} compressor [m^3/s],
A_j, R_j — constants for the j^{th} compressor, ($A_j = m_1 p_s/(\eta_m(m_1 - 1))$, $R_j = (m_1 - 1)/m_1$),
t_0, t_f — beginning and final time for function evaluation,
m_1 — isentropic exponent,
η_m — the compressor (overall) efficiency.

3. Mathematical model of gas network under transient conditions

Transient flow through a pipe is described by the following linear diffusion equation (Osiadacz, 1996):

$$\frac{\partial p}{\partial t} = \frac{c^2 \cdot \partial^2 p}{A \cdot \lambda \cdot \partial x^2} \tag{3}$$

where:

$$\lambda = \lambda(p, M) = \frac{2 \cdot f_F \cdot c^2 \cdot |M|}{D \cdot A^2 \cdot p}$$

$$M = -\frac{1}{\lambda} \cdot \frac{\partial p}{\partial x} \quad ,$$

M — mass flow,
f_F — friction factor.

A mathematical model of the dynamic properties of a gas network has been elaborated using Eq. (3) and based upon the generalization of the idea of a node including grid points along pipes, multi-junctions and pipe-ends being treated as off-takes with demand equal to zero.

For the j^{th} node we obtain the following equation (Osiadacz, 1987) :

$$\left\{ \frac{dp_j}{dt} + \frac{1}{3} \cdot \left(\frac{dp_i}{dt} - \frac{dp_j}{dt} \right) \right\} = \sum_{i=1}^{k} S_{ij} \cdot \left(p_i - p_j \right) - M_j \tag{4}$$

where:

k — number of nodes incident to node j^{th},
$S_{ij} = 1/(\lambda_i \, \Delta x_i)$,
$V_{ji} = V_{ij} = A_i \, \Delta x_i / 2$,
M_j — (mass flow-load) at j^{th} node.

Using the trapezoidal rule integration of Eq. (4) between t and $t + \Delta t$, (where Δt is the time step) yields a linear equation in p^{n+1} of the form:

$$h_{jj} \cdot p_j^{n+1} = \sum_{i=1}^{k} h_{ji} \cdot p_j^{n+1} = r_j - M'_j \tag{5}$$

where:

$$h_{jj} = \frac{2}{3} \cdot \sum_{i=1}^{k} \frac{V_{ji}}{\Delta t \cdot c^2} + \sum_{i=1}^{k} S_{ji}$$

$$h_{ji} = \frac{V_{ji}}{3 \cdot \Delta t \cdot c^2} - S_{ij}$$

$$r_j = \sum_{i=1}^{k} \frac{V_{ji}}{\Delta t \cdot c^2} \cdot \left(\frac{2}{3} \cdot p_j^n + \frac{1}{3} \cdot p_i^n \right)$$

$$M'_j = \frac{1}{\Delta t} \cdot \int_t^{t+\Delta t} M_j \cdot dt$$

For the whole network, Eq. (5) can be written in the matrix form as

$$\underline{H} \cdot \underline{p}^{n+1} = \underline{R} \cdot \underline{p}^n - \underline{M} \tag{6}$$

dim $\underline{H} = n \times n$, **dim** $\underline{R} = n \times n$, **dim** $\underline{M} = n \times 1$

n — number of nodes,

Matrices $\underline{H}$ and $\underline{R}$ are sparse and symmetric. Adding in sources, compressors (compressor stations), regulators and valves, and assuming the flow through each unit is a positive demand at the inlet node and a negative demand at the outlet, Eq. (6) can be written as

$$\underline{H} \cdot \underline{p}^{n+1} = -\underline{K} \cdot \underline{f}^n + \underline{R} \cdot \underline{p}^n - \underline{M} \tag{7}$$

dim $\underline{K} = n \times u \; (u - number\ of\ units)$

$$k_{ij} = \begin{cases} +1, & if\ the\ j^{th}\ unit\ has\ its\ inlet\ at\ node\ i \\ -1, & if\ the\ j^{th}\ unit\ has\ its\ outlet\ at\ node\ i \\ 0, & otherwise \end{cases}$$

dim $\underline{f} = u \times 1$

f_i — the flow through i-th unit.

Equation (7) can be written in the form:

$$\underline{p}^{n+1} = \underline{H}^{-1} \cdot \underline{R} \cdot \underline{p}^n - \underline{H}^{-1} \cdot \underline{K} \cdot \underline{f}^n - \underline{H}^{-1} \cdot \underline{M} \tag{8}$$

or

$$\underline{x}(k+1) = \underline{A}(k) \cdot \underline{x}(k) + \underline{B}(k) \cdot \underline{m}(k) + \underline{C}(k) \cdot \underline{z}(k) \tag{9}$$

where:

$\underline{x} = \underline{p}$ (state vector)

$\underline{m} = \underline{f}$ (control vector)

$\underline{z} = \underline{M}$ (input vector)

$\underline{A} = \underline{H}^{-1} \cdot \underline{R}$, $\underline{B} = -\underline{H}^{-1} \cdot \underline{K}$, $\underline{C} = -\underline{H}^{-1}$

Equation (9) is a discrete state equation for a dynamic gas network with the assumption that nodal pressures are elements of the state vector and flows through units are elements of the control vector.

Writing Eq (7) in the form

$$\underline{H}_1 \cdot \underline{p}_1^{n+1} + \underline{H}_2 \cdot \underline{p}_2^{n+1} + \underline{K} \cdot \underline{f}^n = \underline{R}_1 \cdot \underline{p}_1^n + \underline{R}_2 \cdot \underline{p}_2^n - \underline{M}^n \tag{10}$$

where:

$\underline{p}_1$ — the vector of non-outlet node pressures

$\underline{p}_2$ — the vector of outlet node pressures

$\mathbf{dim}\ \underline{p}_1 = (n-u)\times 1,\ \mathbf{dim}\ \underline{p}_2 = u\times 1$

finally, we have:

$$\left[\underline{H}_1 \mid \underline{K}\right] = \begin{bmatrix} \underline{p}_1^{n+1} \\ \hdashline \underline{f}^{n+1} \end{bmatrix} = \left[\underline{R}_1 \mid \underline{0}\right] \cdot \begin{bmatrix} \underline{p}_1^{n} \\ \hdashline \underline{f}^{n} \end{bmatrix} + \left(\underline{R}_2 - \underline{H}_2\right)\cdot \underline{p}_2^n - \underline{M}^n \tag{11}$$

$$\left[\underline{H}_1 \mid \underline{K}\right] = \underline{A}_1,\ \left[\underline{R}_1 \mid \underline{0}\right] = \underline{B}_1,\ \left(\underline{R}_2 - \underline{H}_2\right) = \underline{C}_1$$
$$\mathbf{dim}\ \underline{A}_1 = n\times n,\ \mathbf{dim}\ \underline{B}_1 = n\times n,\ \mathbf{dim}\ \underline{C}_1 = n\times u$$

or

$$\underline{x}(k+1) = \underline{A}(k)\cdot\underline{x}(k) + \underline{B}(k)\cdot\underline{m}(k) + \underline{C}(k)\cdot\underline{z}(k) \tag{12}$$

where $\underline{x}$ is partitioned:

$$\underline{x} = \begin{bmatrix} \underline{p}_1 \\ \hdashline \underline{f} \end{bmatrix},\ \underline{m} = \underline{p}_2,\ \underline{z} = \underline{M} + \underline{K}\cdot\underline{f}$$

$$\underline{A} = \underline{A}_1^{-1}\cdot\underline{B}_1,\ \underline{B} = \underline{A}_1^{-1}\cdot\underline{C}_1,\ \underline{C} = -\underline{A}_1^{-1}$$

Equation (12) is a discrete state equation for the case in which output pressures are treated as elements of the control vector. Elements of the state vector are non-outlet node pressures and flows through units.

4. Operational constraints

- envelope
  The operating regime of the centrifugal compressors used on the transmission system can be expressed by what is known as an envelope. The envelope is defined by four constraints which enclose an area in which the compressor can properly run. The constraints are defined as:
  - „surge": this is the point at which flow through the compressor becomes so low that reversal of flow can occur which can be damaging to the compressor.
  - „choke": at the opposite end of the diagram, a compressor can reach choke.
  - „maximum and minimum speed": obviously a compressor can run up to some given maximum speed consistent with machine safety and equally there is a minimum speed line.
- linepack (pressures at selected nodes of the network cannot drop below a certain value).

The structure of the high pressure gas network was represented by a directed graph $G_g = (V, E)$ which consists of a set of nodes V and another set E whose elements are called branches. For network analysis, it was necessary to select the following nodes and branches:

⇒ supply nodes; sources and supplying storages $V_z \subset V$,

⇒ pressure nodes; nodes at which constraints on the pressures were imposed $V_w \subset V$,
⇒ units; $E_p \subset E$; $E_p \equiv (V_s, V_d)$; V_s — suction nodes, V_d — discharge nodes,
⇒ control nodes; $V_c \subset \{V_z \cup V_d\}$,
⇒ units of the system; $U \equiv V_z \cup E_p$.

Finally, inequality constraints were imposed on:

- maximum source flow

$$q_j \le q_j^{\mathbf{max}}$$

where: $j \in V_z$,

- maximum compressor ratio

$$\frac{p_{dj}}{p_{sj}} \le \varepsilon_j^{\mathbf{max}}$$

- minimum compressor ratio

$$\frac{p_{dj}}{p_{sj}} \ge \varepsilon_j^{\mathbf{min}}$$

where: $j \in E_p$,

- minimum pressure at selected nodes

$$p_j \ge p_j^{\mathbf{min}}$$

where: $j \in V_w$,

- maximum horsepower of the compressor

$$W_j = A_j Q_j \left\{ \left[\frac{p_{d_j}}{p_{s_j}} \right]^{R_j} - 1 \right\} \le W_j^{\mathbf{max}}$$

where: $j \in V_w$,

- range of the pressure variations at control nodes

$$p_{z_j}^{\mathbf{min}} \le p_{z_j} \le p_{z_j}^{\mathbf{max}}$$

where: $j \in V_c$,

The operational constraints together with state equation (Eq. (12)) form a complete set of constraints imposed on the high pressure gas network.

5. Method of solution

To solve the above problem, an hierarchical method has been used. An algorithm for the optimal control of a gas network with any configuration based upon hierarchical control and decomposition of the network has been developed. Local problems were solved using a gradient technique.

Implicit in all of hierarchical system theory is the idea that it is generally easier to deal with several low order systems than with one system of high order. Spatial decomposition according to which the network is divided into physically small subsystems was employed assuming that each subsystem has to contain at least one operating compressor.

After decomposition each subsystem is interconnected with some other subsystems using simple permutations:

$$\underline{\alpha}_i(k) = \sum_{j=1}^{N} \underline{L}_{ij} \cdot \underline{\beta}_j(k) \tag{13}$$

$\underline{\alpha}_i$ — input vector into i^{th} subsystem. (nodal pressures at nodes incident to the other subsystems)

$\underline{L}$ — matrix of interconnections

$\underline{\beta}_i$ — output vector from j^{th} subsystem. (nodal pressures at nodes of j^{th} subsystem incident to the i^{th} subsystem)

N — number of subsystems.

Vector $\underline{\beta}$ is calculated from the following matrix equation:

$$\underline{\beta}_i(k) = \sum_{j=1}^{N} \underline{G}_{ij} \cdot \underline{x}_j(k) \tag{14}$$

$\underline{G}$ — matrix of interconnections.

Discretization of the cost function yields:

$$I = \sum_{k=k_0}^{k_f-1} \Phi_1\left[\underline{x}(k), \underline{m}(k), k\right] \tag{15}$$

where:

$\underline{x}$ — state vector (non-outlet node pressures and flow through units),

$\underline{m}$ — control vector (outlet node pressures),

k_f — desired final stage.

Constraints are taken by introducing an Augmented Lagrangian function.

Thus

$$I = \sum_{k=k_0}^{k_f-1} \Phi\left[\underline{x}(k), \underline{m}(k), \underline{x}_1(k), k\right] \tag{16}$$

where:

$$\Phi = \Phi_1 + \underline{\mu}(k)^T \cdot \underline{u}\left(\underline{x}_1(k)\right) + \frac{c(k)}{2} \cdot \underline{u}\left(\underline{x}_1(k)\right)^T \cdot \underline{S}(k) \cdot \underline{u}\left(\underline{x}_1(k)\right) \tag{17}$$

$\underline{\mu}$ — Lagrange multiplier

$\underline{u}(\underline{x}_1(k))$ — contains the constraints that are active at $\underline{x}_1$

$c(k)$ — penalty factor

The Hamiltonian for the integrated system is:

$$H(k) = \Phi\left[\underline{x}(k), \underline{m}(k), \underline{x}_1(k), k\right] + \underline{\lambda}^T(k+1) \cdot \underline{f}\left[\underline{x}(k), \underline{m}(k), \underline{z}(k), k\right] \tag{18}$$

In terms of the subsystems, the Hamiltonian may be written as (Singh et al., 1978)

$$H(k) = \sum_{i=1}^{N} \Bigg(\Phi_i\left[\underline{x}_i(k), \underline{m}_i(k), \underline{x}_{1i}(k), k\right] + \underline{\lambda}^T(k+1) \cdot \underline{f}_i\left[\underline{x}_i(k), \underline{m}_i(k), \underline{z}_i(k), k\right] + \\ + \underline{\pi}_i^T(k) \cdot \left(\sum_{j=1}^{N} \underline{L}_{ij} \cdot \underline{\beta}_j(k) - \underline{\alpha}_i(k) \right) \Bigg) \tag{19}$$

where:

$\underline{\lambda}_i$, $\underline{\pi}_i$ — multipliers,

Thus for the i^{th} subsystem we have:

$$H_i(k) = \Phi_i\left[\underline{x}_i(k), \underline{m}_i(k), \underline{x}_{1i}(k), k\right] + \underline{\lambda}^T(k+1) \cdot \underline{f}_i\left[\underline{x}_i(k), \underline{m}_i(k), \underline{z}_i(k), k\right] + \\ + \underline{K}_i^T(k) \cdot \underline{\beta}_i(k) - \underline{\pi}_i^T(k) \cdot \underline{\alpha}_i(k) \tag{20}$$

Then the optimization problem for the i^{th} subsystem is the following: minimize with respect to $\underline{m}_i$

$$I_i = \sum_{i=1}^{k_f - 1} \left[\Phi_i\left(\underline{x}_i(k)\right) + \underline{K}_i^T(k) \cdot \underline{\beta}_i(k) - \underline{\pi}_i^T(k) \cdot \underline{\alpha}_i(k) \right] \tag{21}$$

subject to the constraints.

6. Local optimization

The computation process is as follows.

- Guess or determine $\underline{m}_i(k)$, $k = k_0,\ldots,_f$ and then determine $\underline{x}_i(k+1)$ from

$$\underline{x}_i = \underline{f}_i\left[\underline{x}_i(k), \underline{m}_i(k), \underline{z}_i(k), k\right] \tag{22}$$

$$\underline{x}_i(k_0) = \underline{x}_{i,0} \tag{23}$$

- Solve the adjoint equation, Eq.(24) backwards from stage k_f with the terminal condition of Eq.(25) to stage k_0.
- Calculate

$$\frac{\partial H_i(k)}{\partial \underline{m}_i(k)} = \frac{\partial \Phi_i(k)}{\partial \underline{m}_i(k)} + \left[\frac{\partial \underline{f}_i(k)}{\partial \underline{m}_i(k)} \right]^T \cdot \underline{\lambda}_i(k+1) \tag{24}$$

which will not normally be zero.

The perturbation in the cost function with perturbations in $\underline{x}_i(k)$ and $\underline{m}_i(k)$ may be obtained by rewriting Eq. (23) as (Sage et al., 1977)

$$I_i = \sum_{k=k_0}^{k=k_f-1} \left[H_i(k) - \underline{\lambda}_i^T(k_f) \cdot \underline{f}_i(k) \right] = \sum_{k=k_0}^{k=k_f-1} \left[H_i(k) - \underline{\lambda}_i^T(k+1) \cdot \underline{f}_i(k+1) \right] =$$
$$= \underline{\lambda}_i^T(k_0) \cdot \underline{x}_i(k_0) - \underline{\lambda}_i^T(k_f) \cdot \underline{x}_i(k_f) + \sum_{k=k_0}^{k=k_f-1} \left[H_i(k) - \underline{\lambda}_i^T(k) \cdot \underline{x}_i(k) \right] \tag{25}$$

Thus we obtain:

$$\Delta I_i = \underline{\lambda}_i^T(k_0) \cdot \Delta\underline{x}_i(k_0) - \underline{\lambda}_i^T(k_f) \cdot \Delta\underline{x}_i(k_f) +$$
$$+ \sum_{k=k_0}^{k=k_f} \left\{ \left[\frac{\partial H_i(k)}{\partial \underline{x}_i(k)} - \underline{\lambda}_i^T(k) \right]^T \cdot \Delta\underline{x}_i(k) + \left[\frac{\partial H_i(k)}{\partial \underline{m}_i(k)} \right]^T \cdot \Delta\underline{m}_i(k) \right\} \tag{26}$$

Finally, we have:

$$\Delta I_i = \sum_{k=k_0}^{k=k_f} \left[\frac{\partial H_i(k)}{\partial \underline{m}_i(k)} \right]^T \cdot \Delta\underline{m}_i(k) \tag{27}$$

7. Coordination problem

To find the overall optimum the coordination variables ($\underline{\pi}$) for optimization are introduced. The subproblems treat the coordination variables as known inputs which remain fixed until the coordinator supplies new values. The following theorem relates the subproblems to the integrated problem.

Theorem – If a solution exists to the integrated problem and to each of the subproblems, then there exists a $\underline{\pi}^*(k)$ such that solutions satisfying the necessary conditions for the subproblems also satisfy the necessary conditions for the integrated problem.

The purpose of the coordination variables is to ensure satisfaction of the interconnection constraints, for only when these constraints are satisfied can it be claimed that the subproblem solutions also solve the overall problem.

It can be seen that the main problem of coordinating subproblem solutions so that they solve the integrated problem is in the determination of $\underline{\pi}^*$. Since adjusting $\underline{\pi}$ adjusts the subproblem objective functions or goals this is a goal coordination method.

If $\underline{\Pi} \neq \underline{\Pi}^*$, then the interconnection constraints are not satisfied, so that it is rational to examine the effect of $\underline{\Pi}$ on the interconnection error, which will be defined as:

$$\begin{bmatrix} \vdots \\ \sum_{j=1}^{N} \underline{L}_{ij} \cdot \underline{\beta}_j^* \left(\underline{\Pi}, k\right) - \alpha_i^* \left(\underline{\Pi}, k\right) \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \underline{e}_i \left(\underline{\Pi}, k\right) \\ \vdots \end{bmatrix} \qquad \text{(error vector)}$$

For updating $\underline{\Pi}(k)$ to minimize $\underline{e}_i(\underline{\Pi}, k)$ a gradient procedure to produce a new $\underline{\Pi}$ is applied:

$$\underline{\pi}_j^{n+1}\left(k\right) = \underline{\pi}_j^{n}\left(k\right) + k_j^n \cdot \frac{\partial H\left(k\right)}{\partial \Pi_j\left(k\right)} \tag{28}$$

Thus, the original problem has been exchanged for $N(j = 1, 2, ..., N)$ subproblems.

8. Test network

The correctness of the elaborated algorithm for optimization was checked using gas transmission grid shown in Fig. 1. The algorithm has been tested using network containing 23 nodes, 13 pipes, 3 compressor stations, 2 storage supply nodes and 1 source. There are also 15 demands in the network (Fig. 4). The period of optimization was 24 h and the discretization steps were Δt = 7200s, and Δx = 16000 m.

The loads at nodes 2, 6, 8, 9, 10, 13, 14, 15, 16, 17, 18, 21, 22 and 23 varied in accordance with the curves are presented in Fig. 2. The inequality constraints are imposed on:

pressure at selected nodes: $p_6 \leq 3.5$ MPa, $p_{16} \leq 4.1$ MPa, $p_{23} \leq 4.62$ MPa, 3.45 MPa $\leq p_1 \leq$ 6.72 MPa, 3.45 MPa $\leq p_4 \leq$ 7.36 MPa, 3.45 MPa $\leq p_7 \leq$ 5.83 MPa, 3.45 MPa $\leq p_{20} \leq$ 6.91 MPa. Selected results of optimization are given in Fig. 3, 4, 5, 6 and 7.

TABLE 1

Pipe data

Sending node	Receiving node	Length (m)	Diameter (mm)
1	**2**	**3**	**4**
1	2	32991	884
2	3	34279	884
4	5	88675	884

1	2	3	4
5	6	30095	584
7	8	27037	884
9	8	13197	439
8	10	10461	584
8	11	24784	884
11	12	1770	732
12	13	1770	732
14	13	19795	584
13	15	14001	732
15	16	19312	732
1	17	64856	884
17	18	53591	884
18	19	13679	884
20	21	13358	884
21	22	28163	884
22	23	6437	732

TABLE 2

Unit data

Compressor stations:			
Nodes	P_{min} (MPa)	P_{max} (MPa)	ε_{max}
3-4	3.45	7.36	1.8
5-7	3.45	5.83	1.8
19-20	3.45	6.91	1.8
Supply nodes:			
Node	P_{min} (MPa)	P_{max} (MPa)	Q_{max} (m^3/s)
1	3.45	6.72	985
Storage supply nodes:			
Node	Q(m^3/s) = const.		
9	2.6		
14	32.5		

9. Conclusions

To deal with hierarchical algorithm the network was split into three subsystems. Investigations have shown that the developed algorithm works properly. The validity of the solution was checked by composing solutions with those from simulation programs run with appropriate control values, it was verified that the equations of gas flow were satisfied. Maximum discrepancy does not exceed 10%.

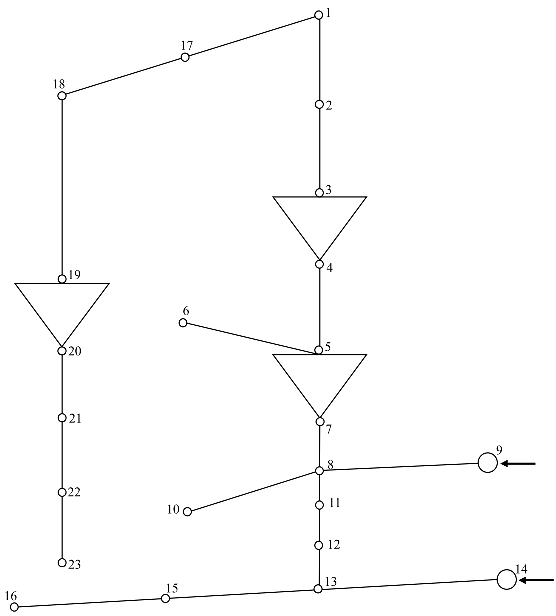

Fig. 1. Structure of the gas network

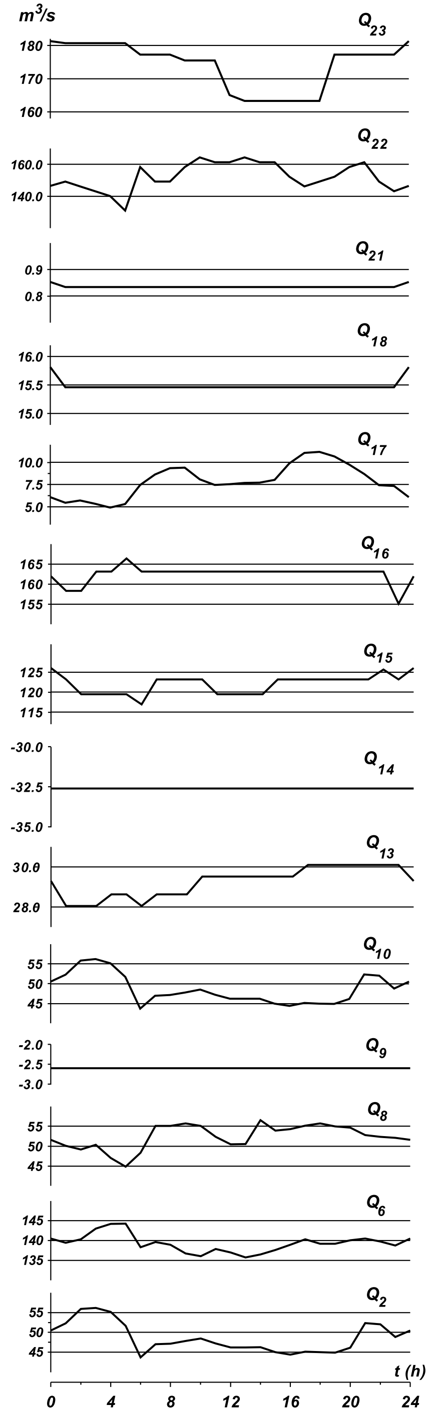

Fig. 2. Changes of load at selected nodes

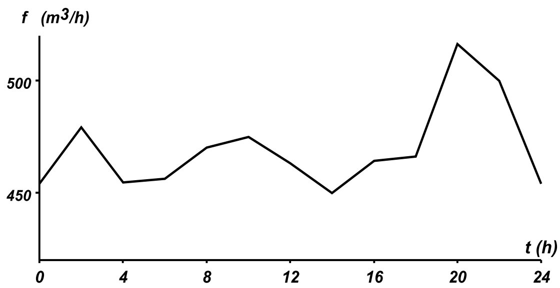

Fig. 3. The variations of flow through compressor station 3-4

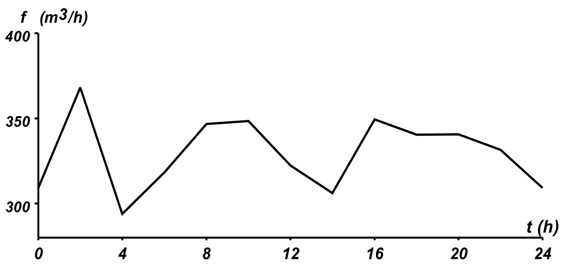

Fig. 4. The variations of flow through compressor station 5-7

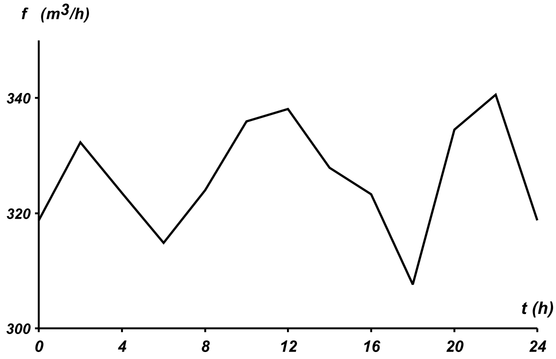

Fig. 5. The variations of flow through compressor station 19-20

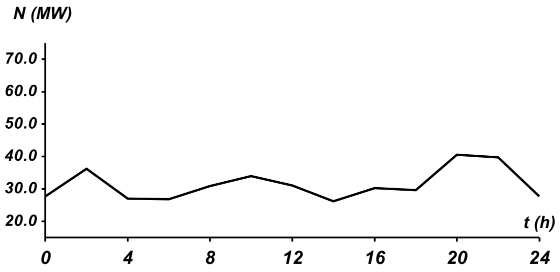

Fig. 6. The variations of total power of compressor stations

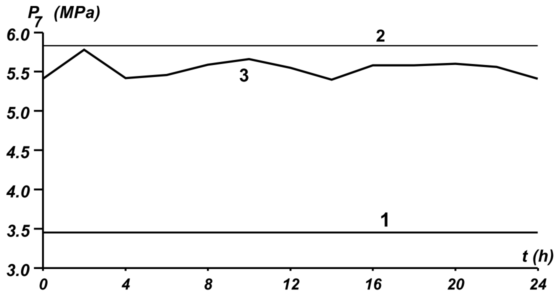

Fig. 7. The variations of pressure at node 7,
1 – lower constrain, 2 – upper constrain, 3 – optimal profile of the pressure at node 7

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