Measurement of heat flux density and heat transfer coefficient

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Abstract The paper presents the solution to a problem of determining the heat flux density and the heat transfer coefficient, on the basis of temperature measurement at three locations in the flat sensor, with the assumption that the heat conductivity of the sensor material is temperature dependent. Three different methods for determining the heat flux and heat transfer coefficient, with their practical applications, are presented. The uncertainties in the determined values are also estimated.

Keywords: Heat flux; Heat transfer coefficient; Temperature measurement; Inverse problem

Nomenclature

a, b	–	coefficients
C, D	–	constants
f_i	–	i-th measured temperature, $^\circ$C
F_j	–	j-th equality limitation
L	–	thickness of the sensor (plate), m
M	–	number of control volumes

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$M1$	–	number of node, in which the first temperature sensor was mounted
N	–	number of temperature measuring points
q	–	heat flux density, W/m^2
S	–	sum of squares, K^2
T	–	temperature, $^\circ$C
T_f	–	exact temperatures, $^\circ$C
x	–	coordinate, m
x_i	–	location of the i-th temperature sensor
z_i	–	i-th determined parameter
h	–	heat transfer coefficient, $W/(m^2K)$
$\Delta x = L/(M-1)$	–	thickness of the control volume, m
k	–	thermal conductivity, $W/(m\ K)$

Greek symbols

| β_i | – | i-th Lagrange multiplicator |
| σ | – | standard deviation |

1 Introduction

In the model tests, conducted at temperature close to the ambient temperature, there are numerous measurement techniques used to determine the heat transfer coefficient [1–7]. The most popular ones are: the thin-film naphthalene mass-transfer technique [1–2], the electrochemical method [3–4] and methods using liquid crystals [5–7]. In tests conducted in higher temperatures, e.g. in steam boilers, industrial furnaces or in experimental tests of fluid boiling, most often the conductometric probes are used [8–9]. On the basis of temperature measurement conducted at points of various coordinates, the heat flux density and the heat transfer coefficient are determined.

In this paper, the problem of determining the heat flux q and heat transfer coefficient h, on the basis of temperature measurement at three locations in the flat plate is presented, with the assumption that the material of which the sensor is made is temperature dependent. Three different methods of determining the heat flux q and h, with their practical applications, are presented. The uncertainties in the determined values are also estimated.

2 Formulation of the problem

The measurement device presented in Fig. 1 was used for determination of the heat transfer coefficient on the surface of the solid body on which the liquid boiling takes place or over which the fluid flows [8–9]. The copper block is heated from the bottom with the use of the resistance heater made of chrome-nickel wire. The side surfaces of the copper block are thermally insulated, thus the heat flow is one dimensional. The heat transfer coefficient h on the surface of the block is determined from the simple formula:

$$h = \frac{q}{(T_s - T_f)} \; ,$$

(1)

where: h – heat transfer coefficient on the surface of the block, T_s – temperature of the surface of the block, T_f – bulk temperature of the fluid or gas, q – heat flux density.

In order to determine the heat flux density, the temperature of the block has to be measured by 2 different thermocouples, located at different positions. For the apparatus presented in Fig. 1, the temperature was measured at 2 points of the coordinates: $(L - x_3) = 2.54$ mm and $(L - x_2) = 7.44$ mm (Fig. 2). This allows to determine the heat flux density q. Considering that the heat flux density on the entire height of the block is constant, and assuming that the thermal conductivity k of the copper block is constant, the heat flux density can be easily calculated from the formula:

$$q = \frac{k \, (f_2 - f_3)}{x_3 - x_2} \; ,$$

(2)

where: $f_2 = f(x_2)$ – temperature measured at point x_2, $f_3 = f(x_3)$ – temperature measured at point x_3 (Fig. 2).

For the device presented in Fig. 1 $x_3 - x_2 = 7.44 - 2.54 = 4.9$ mm. In addition, to check the determined value of q, the temperature is measured at the point of the coordinate $(L - x_1) = 12.3$ mm. This is easy to arrange, as the temperature distribution along the height of the block is linear. Additionally in order to improve the accuracy of calculations, the third measuring point can be used for determination of the values of q and h. Since the number of the measuring data points is greater than the number of the unknown parameters, the problem becomes overdetermined and the values of q and h can be determined more precisely.

The overdetermined inverse heat conduction problems are also encountered in experimental determination of metal thermal conductivity. Figure 1b depicts a simple device used to measure thermal conductivity [11].
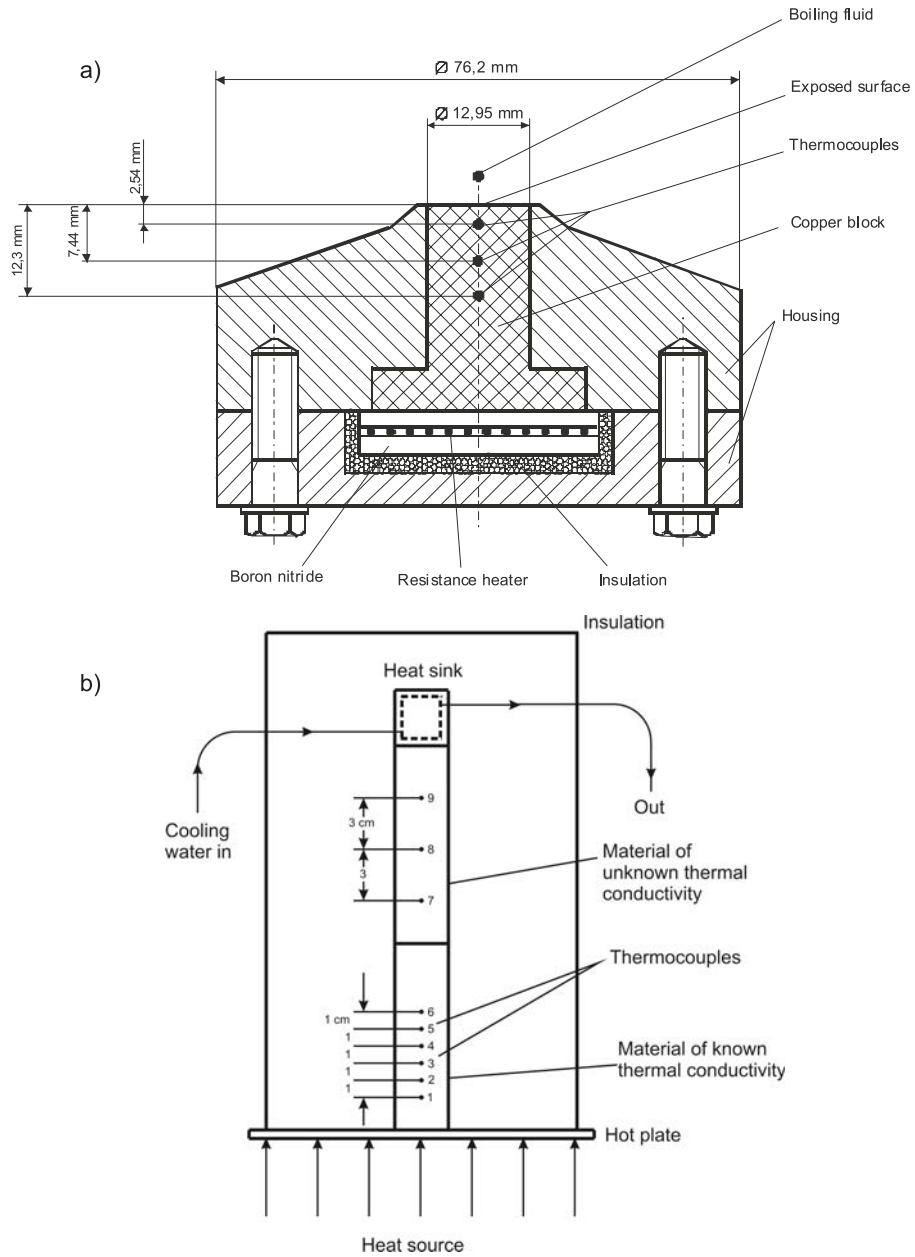
Figure 1. Schematics of devices for measuring heat transfer coefficient (a) and for measuring thermal conductivity of metals (b).

It consists of a hot plate as a heat source. In contact with the hot plate is a 25 mm diameter rod made of stainless steel of known thermal conductivity that has thermocouples attached for obtaining temperature readings. Resting on the stainless steel is a 25 mm diameter aluminium rod of unknown thermal conductivity that also contains thermocouples [11]. However, it is necessary that the thermal conductivity of stainless steel must be known. The least squares method may be used to determine the heat flow in the axial direction, thermal conductivities of upper and lower rod, surface temperatures of both rods at the contact, and thermal resistance of the contact between two rods.

In this paper, a more general problem of determining the values of q and h, is considered when the number of the measurement data N is equal or greater than 2 and the thermal conductivity k is temperature dependant.

Infinitely long plate or rod, which is thermally insulated on the side surface (Fig. 2) is heated with the heat flux q. On the top surface, $x = L$ the heat is absorbed by a liquid having temperature T_f. On the basis of temperature measurement in $N \geq 2$ locations (in this case $N = 3$), the heat flux q and the heat transfer coefficient h were determined. The thermal conductivity k of the sensor material, that is the plate or the rod, depends on temperature T.
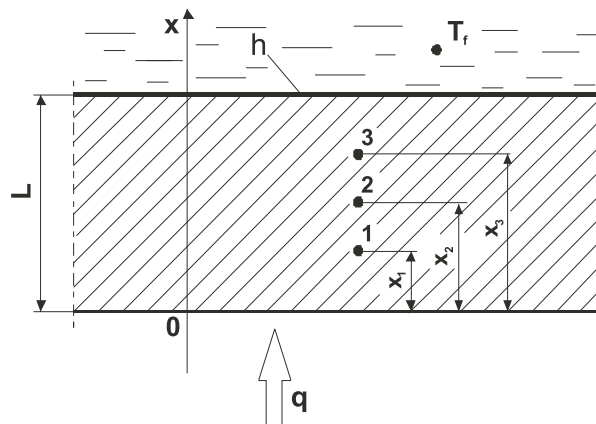

Figure 2. Locations of thermocouples in the sensor.

The steady-state heat conduction equation has the following form:

$$\frac{dq}{dx} = 0 \ , \tag{3}$$

where

$$q = -k\left(T\right)\frac{dT}{dx} \ .$$ (4)

The temperatures at N internal locations are known from measurement:

$$T\Big|_{x=x_1} = f_1 \ ,$$

$$T\Big|_{x=x_2} = f_2 \ ,$$ (5)

$$\cdots\cdots\cdots$$

$$T\Big|_{x=x_N} = f_N \ .$$

The heat flux density q and the heat transfer coefficient h, appearing in Eq. (1) are sought from expressions:

$$-k\frac{dT}{dx}\Big|_{x=0} = q \ ,$$ (6)

$$-k\frac{dT}{dx}\Big|_{x=L} = h\left(T\Big|_{x=L} - T_f\right) \ .$$ (7)

The number of unknowns is 2, thus it is lower or equal to the number of the measured data $N \geq 2$. Thus, the problem is overdetermined and will be solved using the weighed least squares method:

$$S = \sum_{i=1}^{N}\frac{(T_i - f_i)^2}{\sigma_i^2} \rightarrow \min \ ,$$ (8)

where σ_i is the standard temperature deviation f_i measured at the point x_i. Because of the weight coefficients $w_i = \sigma_i^{-2}$ in Eq. (8), the thermocouples (x_i, f_i), which have the biggest great uncertainty are mostly ignored and do not worsen the quality of the approximation. The problem of the least squares (8) can be solved using several methods, depending on the method used for solving Eq. (3); that is, it is solved differently when the exact analytical solution exists and differently when the temperature distribution $T(x)$ is determined numerically.

3   Solving the inverse problem

The density of the heat flux q and the heat transfer coefficient h will be determined using three different methods. In the first one, the $T(x)$ function will be determined analytically and in two next methods, the temperature distribution $T(x)$ will be determined discretely, as a result of the numerical solution.

3.1   Method I – Analytical determination of temperature distribution

Taking into consideration that the heat flux density is constant, $q = const.$, and assuming the linear dependency of the conductivity k on temperature

$$k\left(T\right) = a + bT \; , \tag{9}$$

where a and b are constants and temperature T is expressed in °C, Eq. (4) can be solved analytically.

After substituting (10) to (4) and after integration we obtain

$$qx = -\left(aT + \frac{1}{2}bT^2\right) + C \; , \tag{10}$$

where C is the integration constant. From condition:

$$T\Big|_{x=L} = \frac{q}{h} + T_f \tag{11}$$

we obtain

$$C = qL + a\left(\frac{q}{h} + T_f\right) + \frac{b}{2}\left(\frac{q}{h} + T_f\right)^2 \; . \tag{12}$$

After considering Eq. (13) in Eq. (11) and solving the quadratic equation with regard to T, we obtain

$$T\left(x\right) = -\frac{a}{b} + \sqrt{\left(\frac{a}{b}\right)^2 - \frac{2q}{b}\left(x - L\right) + \frac{2a}{b}\left(\frac{q}{h} + T_f\right) + \left(\frac{q}{h} + T_f\right)^2} \; . \tag{13}$$

The temperature distribution $T(x)$ is a non-linear function of q and h. The values of q and h, for which the sum S as determined from Eq. (8) reaches the minimum, will be determined using the Levenberg-Marquardt method [10,15].

For the thermal conductivity independent of temperature, that is when $k = a$, then from Eq. (11) we obtain

$$qx = -aT + C = -kT + C \ . \tag{14}$$

From which it results

$$T = -\frac{qx}{k} + D \ . \tag{15}$$

After substituting (16) to (8) we obtain

$$S = \sum_{i=1}^{N} \frac{\left(-\frac{qx_i}{k} + D - f_i\right)^2}{\sigma_i^2} \to \min \ . \tag{16}$$

The necessary conditions of the existence of the minimum of the sum of squares S are:

$$\frac{\partial S}{\partial q} = 0 \ ,$$
$$\frac{\partial S}{\partial D} = 0 \ , \tag{17}$$

from which the following system of equations is obtained:

$$\frac{q}{k^2} \sum_{i=1}^{N} \frac{x_i^2}{\sigma_i^2} - \frac{D}{k} \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2} = -\frac{1}{k} \sum_{i=1}^{N} \frac{f_i x_i}{\sigma_i^2} \ ,$$
$$-\frac{q}{k} \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2} + D \sum_{i=1}^{N} \frac{1}{\sigma_i^2} = \sum_{i=1}^{N} \frac{f_i}{\sigma_i^2} \ . \tag{18}$$

After solving the system of equations (17) q and D are obtained. The heat transfer coefficient h is calculated from Eq. (1), and the surface temperature T_s is calculated from formulation (16), taking $x = L$.

3.2   Numerical determination of temperature distribution

The control volume method was used to determine the temperature distribution. The division of the wall into control volumes is presented in Fig. 3. The problem of determining q and h will be solved using two different methods: the Lagrange multipliers method and the Levenberg-Marquardt method.
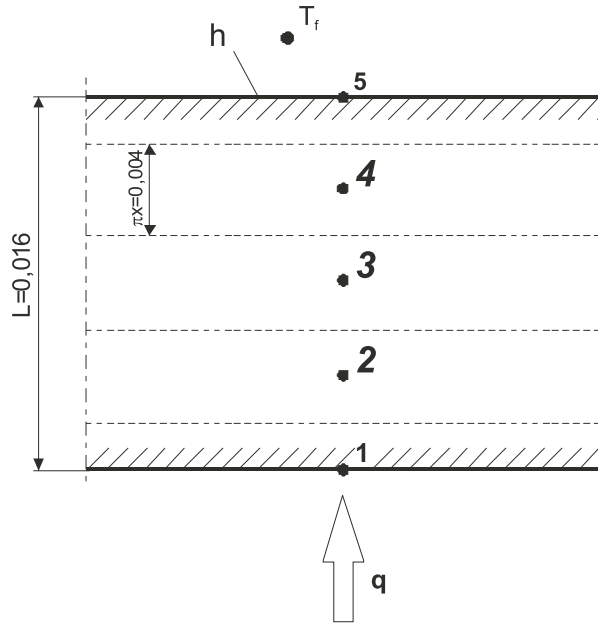
Figure 3. Division of the sensor into control volumes.

3.2.1   Method II – The Lagrange multiplipliers method

Assuming that the plate $0 \leq x \leq L$ is divided into $M > N$ control volumes and N temperature measurement points are situated in nodes $M1, ..., MN = M1 + N - 1$ of the coordinates $x_1, ..., x_N$, the temperature distribution will be searched firstly in the $x_1 \leq x \leq x_N$ zone and subsequently, using the extrapolation algorithm, the temperatures in zones $0 \leq x \leq x_1$ and $x_N \leq x \leq L$ will be determined. Next, on the basis of the determined temperature distribution, the heat transfer coefficient h and the heat flux density q will be calculated. Searched temperatures in nodes $M1, ..., MN$ should additionally be in compliance with the heat conduction equation (3), which means that for the used control volume method, the heat balance equations for every control volume in the $x_1 \leq x \leq x_N$ zone:

$$F_i = \frac{k\left(T_{i-1}\right) + k\left(T_i\right)}{2} \frac{T_{i-1} - T_i}{\Delta x} + \frac{k\left(T_i\right) + k\left(T_{i+1}\right)}{2} \frac{T_{i+1} - T_i}{\Delta x} = 0 \ , \quad (19)$$

$$i = M1 + 1, ..., M1 + N - 2 \ .$$

Thus, the stated conditions constitute the optimisation problem (8) with equality constraints given by Eq. (18). This problem will be solved using the Lagrange multipliers method, according to which, the minimised function assumed the following form:

$$S = \sum_{i=M1}^{M1+N-1} \frac{(T_i - f_i)^2}{\sigma_i^2} + \sum_{i=M1+1}^{M1+N-2} \beta_i F_i \to \min . \qquad (20)$$

The minimised function S, Eq. (19), is non linear with respect to the searched temperatures $T_{M1}, ..., T_{M1+N-1}$. One of the methods, which can be applied to solve such a problem is the Gauss-Newton method [8,10].

Another method is the determination of the set of normal equations

$$\frac{\partial S}{\partial T_i} = 0 , \qquad i = M1, ..., M1 + N - 1 , \qquad (21)$$

which, together with the equality constraints equations

$$F_j = 0 , \qquad j = M1 + 1, ..., M1 + N - 2 , \qquad (22)$$

provide a set of non-linear algebraic equations, which can be solved using the Newton-Raphson method. As a solution, the set of N temperatures T_i and a set of $(N-2)$ Lagrange multipliers are obtained. For three measuring points presented in Fig. 3 ($M1 = 2$, $N = 3$) the sum in Eq. (19) is given by:

$$S = \sum_{i=2}^{4} \frac{(T_i - f_i)^2}{\sigma_i^2} + \qquad (23)$$

$$+\beta_1 \left[\frac{k(T_2) + k(T_3)}{2} \frac{T_2 - T_3}{\Delta x} + \frac{k(T_4) + k(T_3)}{2} \frac{T_4 - T_3}{\Delta x} \right] \to \min .$$

The normal equation set (20) assumes the form:

$$\frac{2(T_2 - f_2)}{\sigma_2^2} + \beta_1 \left[\frac{\partial k(T_2)}{\partial T_2} \frac{T_2 - T_3}{2(\Delta x)} + \frac{k(T_2) + k(T_3)}{2(\Delta x)} \right] = 0 ,$$

$$\frac{2(T_3 - f_3)}{\sigma_3^2} + \beta_1 \left[\frac{\partial k(T_3)}{\partial T_3} \frac{T_2 - 2T_3 + T_4}{2(\Delta x)} - \frac{k(T_2) + 2k(T_3) + k(T_4)}{2(\Delta x)} \right] = 0 ,$$

$$\qquad (24)$$

$$\frac{2(T_4 - f_4)}{\sigma_4^2} + \beta_1 \left[\frac{\partial k(T_4)}{\partial T_4} \frac{T_4 - T_3}{2(\Delta x)} + \frac{k(T_4) + k(T_3)}{2(\Delta x)} \right] = 0 .$$

In this case we have only one constraint Eq. (21), i.e. the heat balance equation for the node 3 (Fig. 3)

$$\frac{k\,(T_2) + k\,(T_3)}{2}\frac{T_2 - T_3}{\Delta x} + \frac{k\,(T_4) + k\,(T_3)}{2}\frac{T_4 - T_3}{\Delta x} = 0 \ . \qquad (25)$$

By solving the set of Eqs. (24)–(25) using the Newton-Raphson method, the temperatures T_2, T_3, T_4 and the multiplier β_1 were obtained. In order to determine the temperature field and subsequently q and h, the extrapolation of the temperature distribution from domain $x_1 \leq x \leq x_N$ (Fig. 2) towards the edges $x = 0$ and $x = L$ was performed. For this case, from the heat balance equation for node 2 (Fig. 3), one obtains:

$$\frac{k\,(T_1) + k\,(T_2)}{2}\frac{T_1 - T_2}{\Delta x} + \frac{k\,(T_2) + k\,(T_3)}{2}\frac{T_3 - T_2}{\Delta x} = 0 \ . \qquad (26)$$

Thus, using a simple iteration method, the T_1 is determined

$$T_1^{(k+1)} = T_2 + \frac{k\,(T_2) + k\,(T_3)}{k\left(T_1^{(k)}\right) + k\,(T_2)}\,(T_2 - T_3), \quad k = 0, 1, 2, \dots , \qquad (27)$$

and $T_1^{(0)} = T_2$ can be assumed as a first approximation in (26). After a few iterations the solution, satisfying the condition

$$\left|\frac{T_1^{(k+1)} - T_1^{(k)}}{T_1^{(k+1)}}\right| < \varepsilon \ , \qquad (28)$$

can be obtained, where ε is the assumed tolerance of the calculations.

The T_5 can be determined in a similar way. From the heat balance equation for node 4, the following equation is obtained:

$$\frac{k\,(T_3) + k\,(T_4)}{2}\frac{T_3 - T_4}{\Delta x} + \frac{k\,(T_5) + k\,(T_4)}{2}\frac{T_5 - T_4}{\Delta x} = 0 \ , \qquad (29)$$

from which, using the simple iteration method, the T_5 temperature is determined

$$T_5^{(k+1)} = T_4 + \frac{k\,(T_3) + k\,(T_4)}{k\left(T_5^{(k)}\right) + k\,(T_4)}\,(T_4 - T_3), \quad k = 0, 1, 2, \dots , \qquad (30)$$

where $T_5^{(0)} = T_4$ can be assumed as a first approximation in (29).

Knowing temperatures in all 5 nodes (Fig. 3), the heat flux density q

$$q = \frac{k\left(T_1\right) + k\left(T_2\right)}{2} \frac{T_1 - T_2}{\Delta x} \tag{31}$$

and the heat transfer coefficient h can be determined

$$h = \frac{k\left(T_4\right) + k\left(T_5\right)}{2} \frac{T_4 - T_5}{\Delta x} \frac{1}{T_5 - T_{cz}} \ . \tag{32}$$

3.2.2   Method III – The implementation of the Levenberg-Marquardt method

The practical implementation of the method described in Section 3.1 can be difficult for two-dimensional cases, because the extrapolation outside the inverse zone requires that the temperature should be measured inside the body, along the closed curve. If the temperature measurement points are distributed within the analysed zone, not within the closed curve, then more appropriate would be to apply the least squares method, described below. The values of q and h, are assumed to be unknowns, as in the first method and the minimum of the function (8) is searched for using the Levenberg-Marquardt method [10,15]. The temperature distribution at the k-th iteration step, for the given values of $q^{(k)}$ and $h^{(k)}$ is determined using the control volume method, from the following set of equations:

$$q^{(k)} + \frac{k\left(T_1\right) + k\left(T_2\right)}{2\left(\Delta x\right)}\left(T_2 - T_1\right) = 0 \ ,$$

$$\frac{k\left(T_{i-1}\right) + k\left(T_i\right)}{2} \frac{T_{i-1} - T_i}{\Delta x} + \frac{k\left(T_i\right) + k\left(T_{i+1}\right)}{2} \frac{T_{i+1} - T_i}{\Delta x} = 0, \ i = 2, ..., M-1,$$

$$\frac{k\left(T_{M-1}\right) + k\left(T_M\right)}{2} \frac{T_{M-1} - T_M}{\Delta x} + h^{(k)}\left(T_f - T_M\right) = 0 \ , \tag{33}$$

where M is the number of control volumes. For the division presented in Fig. 3 we have $M = 5$.

For every iteration step, the non-linear set of algebraic equations is solved using the Gauss-Seidel method. The values of h and q are determined using the Levenberg-Marquardt method [10,15], in such a way, that the temperatures $T_{M1}, ..., T_{M1+N-1}$, determined from the set of equations (32) satisfy the condition:

$$S = \sum_{i=M1}^{M1+N-1} \frac{\left(T_i - f_i\right)^2}{\sigma_i^2} \to \min \ . \tag{34}$$

The advantage of this method of determining q and h on the basis of measured temperatures $f_{M1}, ..., f_{M1+N-1}$ is the possibility to consider any type of dependency of heat conductivity k on T, not only linear Eq. (10) and high precision of q and h determination. Contrary to methods I and II, the temperature distribution from the zone of the direct solution is not extrapolated to the zone of the inverse solution. The described method can be used for the two- and three-dimensional problems for the interval approximation of changes of q and h at the boundary. For this case, the number of determined components of q and h can be significant.

4 Calculating uncertainty of measurement of q and h

Assuming that $z_1 = q$ and $z_2 = h$ only depend on the precision of the temperature measurement $f_{M1}, ..., f_{M1+N-1}$ and assuming that there are no errors in x_i coordinates of the thermocouples mounting points, the σ_{z_i} standard deviation can be calculated in accordance to the error propagation rule [13–15]

$$\sigma_{z_i} = \sqrt{\sum_{j=M1}^{M1+N-1} \left(\frac{\partial z_i}{\partial f_j}\right)^2 \sigma_{f_j}^2} , \qquad (35)$$

where σ_{f_j} is standard deviation of f_j, which is the measure of temperature measurement uncertainty. Partial derivatives $\partial z_i / \partial f_j$ will be approximated by central differences

$$\frac{\partial z_i}{\partial f_j} = \frac{z_i \left(f_{M1}, f_{M1+1}, ..., f_{M1+j} + \delta, ..., f_{M1+N-1}\right)}{2\delta} - \qquad (36)$$
$$\frac{z_i \left(f_{M1}, f_{M1+1}, ..., f_{M1+j} - \delta, ..., f_{M1+N-1}\right)}{2\delta} ,$$

where δ is a small positive number.

5   Example of calculations

All three, described above methods of measuring q and h will be tested for the probe used for measuring the heat flux density [8] of the thickness 0.016 m. The thermal conductivity of the material of the probe is given by Eq. (10), with $a = 14.65$ W/(mK) and $b = 0.0144$ W/(mK °C). The temperature probes (thermocouples) were installed in 3 locations: $x_1 = 0.004$ m, $x_2 = 0.008$ m and $x_3 = 0.012$ m.

The "exact measurement data" will be calculated assuming: $q = 274\,800$ W/m², $h = 2400$ W/m²K and $T_f = 15\,°C$. The temperature distribution over the thickness of the plate and the "non-exact measurement data" were presented in Tab. 1. The results of the calculations were presented in Tabs. 1 and 2.

Table 1. Exact temperature distribution over the thickness of the measurement plate and the results of calculations.

x [m]	Exact temperatures $T_{i,e}$ [°C]	Measuring data f_i [°C]	$w_i = \sigma_i^{-2}$ [°C²]	Method I T_i [°C]	Method II T_i [°C]	Method III T_i [°C]
0.000	370.43			370.99	370.99	370.99
0.001	356.61			357.17		
0.002	342.65			343.21		
0.003	328.54			329.10		
0.004	314.29	315.30	1	314.84	314.84	314.84
0.005	299.88			300.43		
0.006	285.31			285.86		
0.007	270.58			271.12		
0.008	255.68	252.70	0.25	256.22	256.22	256.22
0.009	240.60			241.13		
0.010	225.33			225.87		
0.011	209.88			210.41		
0.012	194.23	195.70	0.4444	194.76	194.76	194.76
0.013	178.38			178.90		
0.014	162.31			162.83		
0.015	146.02			146.54		
0.016	129.50			130.01	130.01	130.01
Sum S				3.702	3.702	3.702

Table 2. Determined values of heat flux q and heat transfer coefficient k and other results of calculations.

	Exact data	Method I	Method II	Method III
q [W/m²]	274 800	274 973.09	274 973.09	274 964.40
h [W/m²K]	2 400	2 390.76	2 390.76	2 390.55
σ_q [W/m²]		4 024.669	4 022.730	4 027.383
σ_α [W/m²K]		82.650	82.815	82.681
$T_1 - T_{1,e}$ [°C]		0.55	0.55	0.55
$T_2 - T_{2,e}$ [°C]		0.54	0.54	0.54
$T_3 - T_{3,e}$ [°C]		0.53	0.53	0.53

The analysis of the obtained results shows that all of them are identical for all methods. The temperature distribution on the thickness of the sensor (plate), determined using the first of the described methods was presented in Fig. 4. The agreement of the temperature distribution, determined using the least squares method with the exact distribution presented in Tab. 1 is very high.
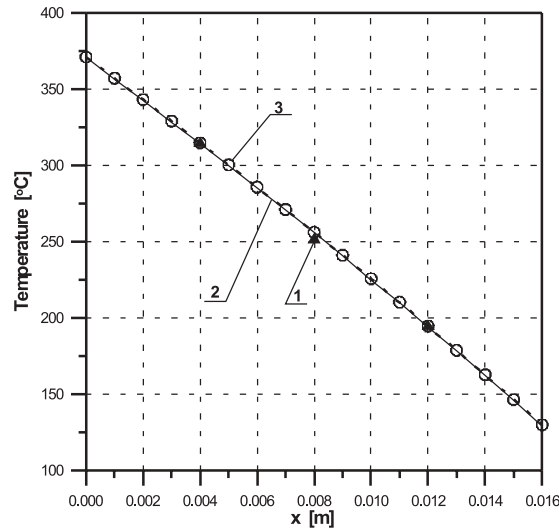

Figure 4. Temperature distribution in the probe: 1–experimental data, 2–exact temperature distribution (Tab. 1), 3–temperature distribution obtained by the least squares method, where temperature is calculated using analytical expression (14).

6   Conclusions

From the presented three methods of determination of heat flux density q and heat transfer coefficient h, the third method, in which the temperature distribution in the analysed zone is determined using the Gauss-Seidel method and the unknown parameters are determined using the Levenberg-Marquardt method was proven to be the most versatile. This method of identification of the boundary conditions can be successfully used for solving multi-dimensional, steady-state and dynamic problems; for much greater number of searched parameters than two. Additional advantage of this method is its good convergence, even for inaccurate approximation

of the initial values of the searched parameters. Standard deviations for the obtained values can be then determined using the Gauss rule of error propagation.

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