Application of the entropy information for the optimization of an additional measurements location in thermal systems

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Abstract   For the optimal location of an additional surplus measurements in the design of redundant measurements system, from data reconciliation point of view, of thermal processes, an information entropy has been applied. The relative entropy – Kullback-Leibler divergence, has been used. As a criterion of the optimal location of an additional surplus measurements in a system of measurements data, the minimum of the entropy information of reconciled measurements data has been assumed. Hence, the objective function in the described optimization task is maximum of the relative entropy – Kullback-Leibler divergence concerning sets of raw and reconciled measurements data. Simulation calculation with application of data reconciliation algorithm and Monte Carlo method concerning the influence of installation of the additional surplus measurements on decrease of entropy information of measurements after data validation have been carried out. The example calculations concerned the cross high-pressure heat regeneration system with cascade flow of condensate installed in 153 MW power unit equipped with cooler of steam are presented. Calculations for all variants of configurations of an additional surplus measurements in the analyzed thermal system have been done. Usefulness of the proposed Kullback-Leibler divergence as a objective function has been demonstrated.

Keywords: Data reconciliation; Thermal processes; Information entropy; Optimal set of measurements

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Nomenclature

A	–	area of a heat transfer, m^2
b	–	width of class of a distributive series
c	–	coefficient
D_{KL}	–	Kullback–Leibler divergence, bit
D_M	–	Mahalanobis distance
d	–	assumed accuracy of estimation of searched value
$\dot{m}$	–	flow rate of substance, kg/s
h	–	specific enthalpy, kJ/kg
k	–	average heat transfer coefficient, $W/(m^2K)$ or current number of measurement
L	–	number of configurations of installation of an additional measurements data
M_0	–	modal value
m	–	number of a measurement variables
n_0	–	size of a modal value class
n_{+1}	–	size of class succeeding the modal value class
n_{-1}	–	size of class preceding the modal value class
n_{ss}	–	size of sample
p	–	pressure, Pa
$\dot{Q}$	–	heat flux, W
q	–	number of considered additional surplus measurements
r	–	number of a conditional equation
$\mathbf{S}_X$	–	variance-covariance matrix of raw measurement data
$\mathbf{S}_{\hat{X}}$	–	variance-covariance matrix of reconciled measurements data
s	–	specific entropy, kJ/(kg K) or estimator of the standard deviation
t	–	temperature, °C
$t_{1-\alpha/2}$	–	value of quantile of t-distribution
t'	–	saturation temperature, °C
u	–	number of a unmeasured variables
$\mathbf{u}_{\alpha/2}$	–	quantile of the standardized normal distribution
v	–	specific volume, m^3/kg
$\mathbf{x}$	–	raw measurement data
$\hat{\mathbf{x}}$	–	reconciled measurement data
x_0	–	lower limit of a modal value class
$\hat{y}$	–	reconciled unmeasured variable
ΔT	–	difference of a temperature, K

Greek symbols

α	–	significance level
η_{HE}	–	efficiency of heat exchanger concerning heat losses to the environment
σ	–	standard uncertainty of raw measurement data
$\hat{\sigma}$	–	standard uncertainty of reconciled measurement data
σ_V	–	standard deviation of a measurement correction
ν	–	number of degree of freedom
μ	–	expected value
Σ	–	variance-covariance matrix

Subscripts

fch	–	fluid stream in heat in exchanger
fth	–	fluid stream turn over in heat in exchanger
fw	–	feed water
HE	–	heat exchanger
m	–	average
st	–	steam
x	–	measurement variable
Δp	–	pressure drop

1   Introduction

A value of measurement result as well as its uncertainty is necessary to present the complete result of the measurement [1]. Measurement uncertainty is a parameter characterizing possible range of the value of this measurement. In many cases existing redundancy in measurements system of thermal process, from data reconciliation point of view, can't assure the required level of accuracy and uncertainty of calculated parameters of thermal process. Increase of the accuracy of thermal process parameters (for example energy efficiency) calculated using credible measurements data can be obtained in the following way:

- replacing of the existing measurements equipment,

- change of location of the measurements points in the measurements system,

- installation of the additional measurements in the measurements system.

In the paper the analysis of installation of an additional surplus measurements (from data reconciliation point of view) in the measurements system has been carried out. The investigated problem belongs to the group of optimization tasks. For its solution a different shape of the objective function has been applied [2]. In [3] and [4] optimization task as a minimization of the sum of a variances of a reconciled measurements data has been defined. This approach is possible only in the case, when the measurements are concerning the same physical quantity, for example when investigated problems concerns only the mass fluxes. Optimization of a redundant measurements system with application of minimization of the complex relative uncertainty of the selected parameter of thermal process – heat power of steam boiler, as a objective function in [5] has been presented. Installation

of additional measurement derives requires capital investments. Hence, the other criterion can be minimization of the capital investments for the additional measurements which simultaneously fulfills the assumption limiting the uncertainty of individual measurements. The next criterion could be the maximization of the increase of measurement uncertainty simultaneously limiting the capital investment for installation of additional measurement derives in the thermal process. As a rule however, the increase of measurement accuracy in the monitoring of thermal process and increase of the accuracy of calculation of a coefficient of technical operation, as a result of the capital investment has been expected.

In this paper a scalar quantity derived from the information theory – the entropy of information has been proposed as the objective function. In the optimization task the relative entropy – Kulback-Leibler divergence for the two multivariate normal probability distribution of the sets of raw and reconciled measurements data has been applied.

2 Principles of data reconciliation

Data reconciliation is the procedure of optimally adjusting measured and preliminary estimated unknown variables in such way that the adjusted values satisfy the laws of conservation and other constraints [2,6–8]. In general it is formulated by the following constrained weighted least-squares optimization problem:

$$\min\left\{\sum_{i=1}^{m}\left(\frac{\hat{x}_i - x_i}{\sigma_i}\right)^2\right\} , \tag{1}$$

subject to

$$g_l\left(\hat{x}_i, \hat{y}_j\right) = 0 \quad \text{for} \quad l = 1, ..., r . \tag{2}$$

Equation (2) defines the set of model constrains. These constrains are generally the mass and energy balances. In the so called generalized data reconciliations algorithm another constrains can be included as a part of the reconciliation problem [7,9].

Procedure of the data reconciliation in thermal analysis permits to achieve the following aims [2,6,8]:

- calculation of the most reliable values of thermal measurements,

- unique solution of the most probable unknown quantities in thermal processes,

- assessment of the accuracy of the corrected results of measurements and of calculated unknown quantities,

- reduction of uncertainty of measured quantities,

- control of fulfilling of the assumed measurements uncertainty.

3   Application of entropy information as a objective function in optimization task

As a result of application of the data reconciliation algorithm the reconciled measurements data of thermal process and reduction of its uncertainty have been achieved [2,6]. Decrease of the measurements uncertainty after data reconciliation in global way can be described by means of a sum of a weighted variances of the reconciled measurement variables. The inverse of input variances of raw measurements data were assumed as a weights of these variances. In the undetermined Lagrange multipliers the method of solution of the reconciliation task for the variance-covariance matrix of raw and reconciled measurements the following property occurs [10]:

$$Tr\left(\mathbf{S_{\hat{X}}S_{X}^{-1}}\right) = m + u - r \ .$$ (3)

For the diagonal form of the variance-covariance matrix of raw measurements data the following equation is fulfilled [10]:

$$\sum_{i=1}^{m} \frac{\hat{\sigma}_i^2}{\sigma_i^2} = Tr\left(\mathbf{S_{\hat{X}}S_{X}^{-1}}\right) \ .$$ (4)

Equations (3) and (4) show that sum of the weighted variances of measurement data for unambiguously defined data reconciliation problem (values m, u and r) is constant. Introduction of real measurement data in the place of unmeasured variable in the data reconciliation algorithm cause that sum of the weighted variances of measurements will include greater number of measurement variables. It can be concluded, that change of unmeasured quantity in the thermal process to the measured one, decrease all measurements uncertainty after data reconciliation.

Quality of a redundant system of measurements after introduction of new additional measurement should be evaluated by applying a criterion, which permits to measure the reliability increase of measurements – both values of measurements as well as its uncertainties. Because of the presented

property of the sum of the weighted variances of data measurements, this sum is not suitable to evaluate the quality of measurements system. It has been assumed that measurements system of thermal process represents a signals system of communication, which can be able to send the information about thermodynamic parameters of the thermal process. For this assumption the entropy of information can be applied as the assessment criterion of quality of measurements system. In information theory, entropy is a measure of uncertainty which is associated with a random variable. This concept was introduced by Shannon in [11].

In probability theory and information theory also the concept of a relative entropy (also information divergence) called the Kullback–Leibler divergence has been introduced [12]. The Kullback–Leibler divergence, is a non-symmetric measure of the difference between two probability distributions. Typically one of them $(N0)$ represents the true distribution of data, observations, or a precisely calculated theoretical distribution. The other distribution $(N1)$ typically represents the theory, model, description or approximation of data. The Kullback–Leibler divergence (in nats) from multivariate normal distribution $N0\left(\mu_0, \Sigma_0\right)$ to $N1\left(\mu_1, \Sigma_1\right)$, for non-singular matrices Σ_0 and Σ_1, is:

$$D_{KL}\left(N0||N1\right) = \frac{1}{2}\left[\ln\left(\frac{\det\Sigma_1}{\det\Sigma_0}\right) + Tr\left(\Sigma_1^{-1}\Sigma_0\right) + D_M^2\left(\mu_0, \mu_1\right) - m\right],$$

(5)

where:

$$D_M\left(\mu_0, \mu_1\right) = \sqrt{\left(\mu_1 - \mu_0\right)^T \Sigma_1^{-1}\left(\mu_1 - \mu_0\right)}.$$

(6)

In the assessment of increase of measurement data reliability, in principle, we are not interested in the absolute value of information entropy, but only in the decrease of this entropy from state of raw measurements data to state of reconciled measurements. Hence, assuming that distribution $N1$ concerns the raw measurements data and distribution, $N0$ concerns reconciled measurements data, the calculation of decrease of information entropy of measurements data after application of data reconciliation algorithm can be determined by use of formula (5). In this case the Kullback-Leibler divergence can constitute the criterion of the assessment of increase of a measurement data reliability in a redundant system of measurements of thermal systems.

Introduction to the Eq. (5) of the variance-covariance matrices property describing by the Eq. (3) and knowing that the Mahalanobis squared

distance between raw and reconciled measurements data (6), the Kullback–Leibler divergence (in bits) has the following form:

$$
D_{KL}\left(\hat{N}\,\middle\|\,N\right) = \frac{1}{2\ln 2}\left\{\ln\left[\prod_{i=1}^{m}\left(\frac{\sigma_i}{\hat{\sigma}_i}\right)^2\right] + \sum_{i=1}^{m}\left(\frac{\hat{x}_i - x_i}{\sigma_i}\right)^2 + u - r\right\}.
\tag{7}
$$

Minimum of the entropy information of reconciled measurements value, that is the maximum of the Kullback–Leibler divergence (7) from reconciled measurements to the raw measurements data, can be the criterion of optimal location of additional measurements in the measurements system of the thermal systems. Objective function of such described optimization task can be defined as follows:

$$
\left.\begin{array}{c}
D_{KL}\left(\hat{N}\,\middle\|\,N\right)_k = \max\left\{D_{KL}\left(\hat{N}\,\middle\|\,N\right)_{kj}\right\} \\
\text{for} \\
k = 1,...,q\,, \\
j = 1,...,L\left(k\right)
\end{array}\right\}.
\tag{8}
$$

4   Number of configurations of the additional measurements installation

Value of the Kullback–Leibler divergence (7) constitutes the assumed criterion of the assessment of uncertainty measurement decrease after data reconciliation. The criterion is dependent on the number of surplus measurements quantities, as well as dependent on its location in the analyzed thermal system. The procedure of optimization calculations requires first of all the identification of potential places for installation of an additional measurements in a thermal system, considering the technical constrains. These potential places determines the maximum number of available surplus measurements in the system. The determined number of surplus measurements (lower than possible maximum) can be installed in measurements system in different configurations. Solution of the optimization task (8) requires determination of the Kullback–Leibler divergence for all available configurations of installation of these measurements in the analyzed thermal system. Number of these installation configurations results from the binomial coefficient. For any set containing n-elements, the number of considered

k-elements subsets is given by the formula:

$$L\left(k\right) = C_n^k = \left(\begin{array}{c} n \\ k \end{array}\right) = \left(\frac{n!}{k!\left(n-k\right)!}\right), \text{ where } k \leq n, \quad n,k \in N^+ . \quad (9)$$

For the set of q-elements of all considered additional surplus measurements data, the number of all available configurations of their installation in the thermal system will be the sum of configurations $L(k)$ (9) results from the formula:

$$L = \sum_{k=1}^{q} C_q^k = \sum_{k=1}^{q} \left(\begin{array}{c} q \\ k \end{array}\right) = \sum_{k=1}^{q} \left(\frac{q!}{k!\left(q-k\right)!}\right). \quad (10)$$

5 Computational example

Example data reconciliation calculations concerning thermal processes in the high-pressure heat regeneration exchangers in the thermal system of selected power unit have been worked out to confirm the usefulness of the proposed criterion of optimization. Scheme of the analyzed cross high-pressure heat regeneration system (Ricard's system) with cascade flow of condensate is presented in Fig. 1. Sets of measured and non-measured variables based on the real measurements in the distribution control system of power unit have been defined. Possibility of installation of the additional surplus measurements in the analyzed thermal system were considered. The measurements variables ensuring minimum information for solution of mass and energy balances of investigated heat regeneration system have been showed in Fig. 1 in a circles. The thermal process variables with status of non-measured quantities but possible to calculate from mass and energy balances have been marked in Fig. 1 in a squares. These variables in the optimization calculations obtain the status of measured quantity.

Application of the data reconciliation requires the mathematical model of the considered thermal process. Such model is constructed as algebraic equations (so-called conditional equations). Besides the mass and energy balances, the following additional equations in the generalized method of data reconciliation are applied [9]: Peclet's equations for heat transfer in the heat exchangers, equation of over-cooling of condensate from the heat exchangers, Darcy's equations for the steam pressure drop in the steam pipeline from the steam bleedings to the exchangers. Particularly in the case of the considered cross heat regeneration system the conditional equations are as follows:
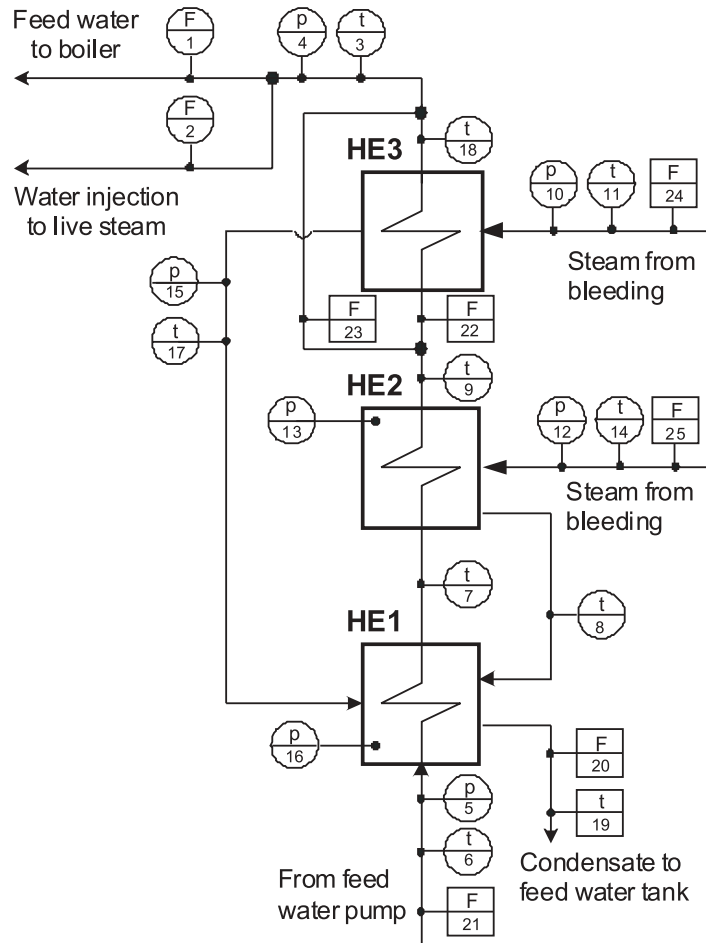
Figure 1. Scheme of the analyzed cross high-pressure heat regeneration system.

- energy balance of the heat exchanger HE1:

$$\left[\dot{m}_{24}h\left(p_{16}, t_{17}\right) + \dot{m}_{25}h\left(p_{13}, t_8\right) - \dot{m}_{20}h\left(p_{16}, t_{19}\right)\right]\eta_{HE} - \\ -\dot{m}_{fw}\left[h\left(p_{fw}, t_7\right) - h\left(p_{fw}, t_6\right)\right] = 0, \qquad (11)$$

- energy balance of the heat exchanger HE2:

$$\dot{m}_{25}\left[h\left(p_{13}, t_{14}\right) - h\left(p_{13}, t_8\right)\right]\eta_{HE} - \dot{m}_{21}\left[h\left(p_{fw}, t_9\right) - h\left(p_{fw}, t_7\right)\right] = 0, \qquad (12)$$

- energy balance of the heat exchanger (steam attemperator) HE3:

$$\dot{m}_{24}\big[h\left(p_{10},t_{11}\right) - h\left(p_{15},t_{17}\right)\big]\eta_{HE} - \dot{m}_{21}\big[h\left(p_{fw},t_3\right) - h\left(p_{fw},t_9\right)\big] = 0 \,, \tag{13}$$

- energy balance of the feed water mixing point behind steam attemperator:

$$\dot{m}_{22}h\left(p_{fw},t_{18}\right) + \dot{m}_{23}h\left(p_{fw},t_9\right) - \left(\dot{m}_1 + \dot{m}_2\right)h\left(p_3,t_4\right) = 0 \,, \tag{14}$$

- mass balance of the feed water mixing point behind steam attemperator:

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_{22} - \dot{m}_{23} = 0 \,, \tag{15}$$

- mass balance of the feed water:

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_{21} = 0 \,, \tag{16}$$

- mass balance of condensate from heat exchanger HE1:

$$\dot{m}_{24} + \dot{m}_{25} - \dot{m}_{20} = 0 \,. \tag{17}$$

Equations describing subcooling of the condensate from the heat exchanger HE1 and HE2 have the following form:

$$\Delta t_{con\,HE} - \big[t'\left(p_{HE}\right) - t_{con}\big] = 0 \,, \tag{18}$$

whereas the equations for pressure drop of steam to the heat exchanger HE1, HE2 and HE3 have the form:

$$p_{stb} - p_{HE} - c_{\Delta p\,HE}v\left(p_{stb},t_{stb}\right)\dot{F}^2_{st\,HE} = 0 \,. \tag{19}$$

In the generalized data the reconciliation method heat transfer in the heat exchanger is simply expressed by the Peclet's equation. For the considered heat exchangers this equation has the following form:

$$\dot{m}_{fw}\big[h\left(p_{fw},t_{HEout}\right) - h\left(p_{fw},t_{HEin}\right)\big] = k_{HE}A_{HE}\Delta T_{m\,HE} \,. \tag{20}$$

For the analyzed system of heat exchangers system the average temperature difference ΔT_{mHE} has been calculated as a thermodynamic temperature difference of fluid streams flowing through the heat exchanger:

$$\Delta T_{mHE} = T_{m\,fth} - T_{m\,fch} = \left(\frac{\Delta h}{\Delta s}\right)_{fth} - \left(\frac{\Delta h}{\Delta s}\right)_{fch} \,. \tag{21}$$

The specific enthalpy and entropy increase of the feed water in the heat exchangers results from the formulas:

$$\Delta h_{fch} = h\left(p_{fw}, t_{HEout}\right) - h\left(p_{fw}, t_{HEin}\right) \ , \tag{22}$$

$$\Delta s_{fch} = s\left(p_{fw}, t_{HEout}\right) - s\left(p_{fw}, t_{HEin}\right) \ . \tag{23}$$

The specific enthalpy decrease and entropy increase of the hot fluid streams in exchanger HE1 (steam and condensate) results from the formulas:

$$\Delta h_{fth\ HE1} = \frac{\dot{m}_{24} h\left(p_{15}, t_{17}\right) + \dot{m}_{25} h\left(p_{13}, t_8\right)}{\dot{m}_{24} + \dot{m}_{25}} - h\left(p_{16}, t_{19}\right) \ , \tag{24}$$

$$\Delta s_{fth\ HE1} = s\left(p_{16}, t_{19}\right) - \frac{\dot{m}_{24} s\left(p_{15}, t_{17}\right) + \dot{m}_{25} s\left(p_{13}, t_8\right)}{\dot{m}_{24} + \dot{m}_{25}} \ . \tag{25}$$

Equations concerning the decrease of specific enthalpy and the increase of specific entropy of the hot fluid stream for the heat exchangers HE2 and HE3 has the same form like Eqs. (22) and (23) for the feed water.

The input values of measurements and their standard uncertainty for reconciliation calculations are shown in Tab. 1. Values of pseudo-measurements variables k_{HE}, $c_{\Delta pHE}$, Δt_{conHE} in the Eqs. (18), (19) and (20) and their uncertainty were calculated applying special measurements of the analyzed power unit and statistical methods [10]. Empirical functions for preliminary assessment of pseudo-measurement variables have been carried out. The standard error of a prediction has been used for estimating of uncertainty of these variables.

For optimization calculations the Monte Carlo method has been applied. The values of measurements for the data reconciliation and Kullback–Leibler divergence calculations from the normal probability distribution have been randomly generated. Statistical tools for determination of the size of sample have been used to determine the required number of random vectors of measurements values. The accuracy of assessment expressed by means of assumed in advance length of the confidence interval $2d$ for the given confidence level $p = 1-\alpha$ is applied as a criterion to determine the size of the sample. The size of sample depends then on the required accuracy of estimation and probability of acceptable risk of not found estimated parameter in the confidence interval. Minimum size of sample from population described by normal distribution and unknown mean value and standard deviation results from the inequality [13]:

$$n_{ss} \geq 1 + \left(\frac{t_{1-\alpha/2}s}{d}\right)^2 \ . \tag{26}$$

Table 1. List of measurements for the data reconciliation calculations.

No.	Basic measurements data	Value	Standard uncertainty
1	Flow of feed water – boiler inlet, t/h	402.1	10.0
2	Flow of water injection to live steam, t/h	27.2	1.0
3	Temperature of feed water – boiler inlet, °C	254.0	2.0
4	Pressure of feed water – boiler inlet, MPa	16.9	0.1
5	Pressure of feed water – inlet HE1, MPa	18.1	0.1
6	Temperature of feed water – inlet HE1, °C	189.5	2.0
7	Temperature of feed water – inlet HE2, °C	212.0	2.0
8	Temperature of condensate from HE2, °C	245.2	2.0
9	Temperature of feed water – inlet HE3, °C	250.0	2.0
10	Steam pressure to heat exchanger HE3, MPa	2.12	0.04
11	Steam temperature to heat exchanger HE3, °C	468.1	2.0
12	Steam pressure to heat exchanger HE2, MPa	3.92	0.08
13	Pressure in heat exchanger HE2, MPa	3.89	0.08
14	Steam temperature to heat exchanger HE2, °C	395.3	2.0
15	Steam pressure to heat exchanger HE1, MPa	1.99	0.04
16	Pressure in heat exchanger HE1, MPa	1.96	0.04
17	Steam temperature to heat exchanger HE1, °C	253.2	2.0
18	Temperature of feed water – outlet HE3, °C	257.2	2.0
	Surplus measurements data		
19	Temp. of condensate from heat exchanger HE1, °C	204.3	2.0
20	Flow of condensate from heat exchanger HE1, t/h	53.1	2.0
21	Flow of feed water – heat regeneration system inlet, t/h	429.3	15.0
22	Flow of feed water – heat exchanger HE3 inlet, t/h	269.1	12.0
23	Flow of feed water – heat exchanger HE3 bypass, t/h	160.2	10.0
24	Steam flow from bleeding to heat exch. HE3, t/h	18.3	0.6
25	Steam flow from bleeding to heat exch. HE2, t/h	34.8	1.2

For calculation of the quantities in the inequality (26) the results of calculation in the case of maximum set of measured variables have been used. Calculation of the pilot test of sample size $n_{ps} = 1000$ of random vectors of measurements has been carried out. The estimator of the standard deviation of the Kullback–Leibler divergence takes the value $s = 8.52$. For assumed confidence level $p = 0.99$ and degree of freedom $\nu = 999$, the value of quantile $t_{1-\alpha/2} = 2.8133$. As the accuracy of the Kullback–Leibler divergence 1% of calculated value resulting from the carried out pilot test has been assumed. The value of the minimum of sample size resulting from inequality (26) is $n_{ss} = 3909$. Finally for the next calculation the value $n_{ss} = 4000$ has been accepted.

The optimization calculations for previously defined variants of configurations of the additional measurements in the analyzed thermal system have

been carried out. Number of the variants of measurement locations for each amount of a surplus measurements is calculated by means of formula (9). Column 2 of Tab. 2 presents the number of analyzed variants of additional measurement locations. Total number of analyzed measurement locations variants resulting from formula (10) amounted to $L = 127$.

The optimization calculations don't take into account these solutions of data reconciliation tasks that not fulfilled the laws of conservation and statistical test of control of the assumed uncertainty of measurements using variance-covariance matrix of measurements corrections according to VDI 2048 [8]. In the applied statistical method for each measured quantity the formulated null hypothesis H_{0i} (i-th measurement fulfills assumed accuracy) is not rejected, if value of test of a statistics Z meets the following inequality:

$$Z_i = \frac{|\hat{x}_i - x_i|}{\sqrt{\max\left(\sigma_{Vi}^2, \frac{\sigma_i^2}{10}\right)}} \leq u_{\alpha/2} \; . \tag{27}$$

Analysis of the distribution of the Kullback–Leibler divergence values obtained from the Monte Carlo calculations shows that this distribution is not normal distribution, especially for the small number of surplus of measurements data [14]. For this reason the modal value of obtained Kullback–Leibler divergence distribution has been used in the next analysis. The modal value represents the value that occurs most frequently in a data set or a probability distribution. For determined continuous distributive series of values of the Kullback–Leibler divergence the modal value of these series can by calculated from the following formula:

$$M_0 = x_0 + b\frac{n_0 - n_{-1}}{2n_0 - (n_{-1} + n_{+1})} \; . \tag{28}$$

Results of optimization calculations for all variants of configurations of an additional surplus measurements in the analyzed thermal system have been gathered in Tab. 2.

6   Conclusions

The worked-out computational example concerning the high-pressure heat regeneration system confirmed usefulness of the proposed Kullback-Leibler divergence as the objective function in optimization of location of additional surplus measurements in thermal systems. According to expectations the

Table 2. Results of simulation calculations.

Number of surplus measurements data , k	Number of configuration of installation of surplus measurements data, $L(k)$	Sets of optimal configuration of surplus measurements data (number of measurements according to the Tab. 1)	Value of the Kullback–Leibler divergence,bit (modal value)
1	2	3	4
k = 1	$L(1) = C_7^1 = 7$	$M_{k=1}^{opt} = \{24\}$	22.27
k = 2	$L(2) = C_7^2 = 21$	$M_{k=2}^{opt} = \{19,24\}$	26.43
k = 3	$L(3) = C_7^3 = 35$	$M_{k=3}^{opt} = \{19,22,25\}$	28.65
k = 4	$L(4) = C_7^4 = 35$	$M_{k=4}^{opt} = \{19,21,23,24\}$	30.89
k = 5	$L(5) = C_7^5 = 21$	$M_{k=5}^{opt} = \{19,21,23,24,25\}$	35.66
k = 6	$L(6) = C_7^6 = 7$	$M_{k=6}^{opt} = \{20,21,22,23,24,25\}$	35.78
k = 7	$L(7) = C_7^7 = 1$	$M_{k=7}^{opt} = \{19,20,21,22,23,24,25\}$	37.93

optimal value of the Kullback–Leibler divergence (column 4 in the Tab. 2) increases together with the increase of number of additional surplus measurements in the analyzed thermal system. Thus it can be concluded that application of the additional surplus measurements in the thermal systems causes decrease of the information entropy of the reconciled measurements system. The Kullback–Leibler divergence can be the measure of quality of the redundant measurements system from data reconciliation point of view. It should be however emphasized that the results of these optimization calculations closely depend on the standard uncertainty of raw measurements data assumed to the data reconciliation calculations.

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