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Evaluation of resisdence time measurements on heat exchangers for the determination of dispersive Peclet numbers

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Abstract The recently developed special unity Mach number dispersion model prescribes the corrections to heat transfer coefficients which are simple functions of the dispersive Peclet numbers. They can be determined through the residence time measurements. An evaluation method is described in which the measured input and response concentration profiles are numerically Laplace transformed and evaluated in the frequency domain. A characteristic mean Peclet number is defined. The method is also applied to the parabolic dispersion model and the cascade model. A calculated example of a tube bundle with maldistribution and backflow demonstrates the suitability of the evaluation method.

Keywords: Heat exchanger; Dispersion model; Transient tracer experiment; Evaluation method

Nomenclature

A	–	area, m^2
a	–	constant of polynomial function (a_0, a_1, a_2 and a_3)
C	–	propagation velocity of thermal disturbances, m/s
c_p	–	specific heat at constant pressure, J/kg K
F	–	transfer function
j	–	counter

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k	–	counter
L	–	flow length, m
M	–	dispersive thermal Mach number, $\mathrm{M} = w/C$
NTU	–	number of transfer units, $NTU = \alpha A \big/ \dot{W}$
NTU_d	–	effective number of transfer units, corrected for axial dispersion, $NTU_d = \alpha_d A/\dot{W}$
n	–	number of completely mixed zones in the cascade model
Pe	–	dispersive Peclet number, $\mathrm{Pe} = wL\rho c_p/\lambda_d$
$\dot{q}_x$	–	axial dispersive energy flux, $\mathrm{W/m^2}$
s	–	Laplace variable
T	–	dimensionless fluid temperature outside the heat exchanger
t	–	dimensionless fluid temperature inside the heat exchanger
t^*	–	dimensionless wall temperature
V	–	volume of fluid inside the flow channel, $\mathrm{m^3}$
$\dot{V}$	–	volumetric flow rate, $\mathrm{m^3/s}$
$\dot{W}$	–	heat capacity rate, $\dot{W} = wA_c\rho c_p$, W/K
w	–	mean flow velocity, m/s
x	–	dimensionless flow length, $0 \leq x \leq 1$
z	–	dimensionless time coordinate

Greek symbols

α	–	heat transfer coefficient, $\mathrm{W/(m^2\ K)}$
α_d	–	heat transfer coefficient, corrected for axial dispersion, $\mathrm{W/(m^2\ K)}$
λ_d	–	apparent thermal conductivity, caused by axial dispersion, W/(m K)
φ	–	dimensionless axial dispersive energy flux, $\varphi = \dot{q}_x L/(\lambda_d\ \Delta\vartheta)$
σ	–	integration boundary in eq (10), positive real number
ρ	–	density, $\mathrm{kg/m^3}$
$\Delta\vartheta$	–	characteristic temperature difference of the considered problem, K
τ	–	time, s
$\Delta\tau$	–	time impulse width
τ_r	–	residence time, s

Subscripts

d	–	dispersive
i	–	counter (0 or 1)
p	–	parabolic
w	–	wall
0	–	inlet
1	–	outlet or first stream
2	–	second stream
3	–	third stream
$\sim$	–	Laplace transform
$-$	–	mean value

1 Introduction

For the thermal design and rating of heat exchangers charts and formulas are available for numerous flow arrangements which are based on the assumption of nondispersive plug flow [1,2]. However, in most industrial heat exchangers the real flow pattern deviates from this ideal plug flow in form of backmixing, maldistribution, recirculation and three dimensional flows. These deviations cause a reduction of the usual plug flow mean temperature difference, which effect can approximately be taken into account with axial dispersion models [3]. The recently [4,5] proposed unity Mach number dispersion model is subject to this publication.

2 Unity Mach number dispersion model

The most general model is the hyperbolic dispersion model which allows finite propagation velocities C of the thermal disturbances. The governing energy equation can be expressed as [6]

$$\begin{gathered}\frac{\partial t}{\partial z}+\frac{\partial t}{\partial x}+\frac{\mathrm{M}^2}{\mathrm{Pe}}\left(\frac{\partial^2 t}{\partial z^2}+2\frac{\partial^2 t}{\partial z \partial x}+\frac{\partial^2 t}{\partial x^2}\right)-\frac{1}{\mathrm{Pe}}\frac{\partial^2 t}{\partial x^2}\\ +NTU\,(t-t^*)+\frac{NTU\,\mathrm{M}^2}{\mathrm{Pe}}\left[\frac{\partial\,(t-t^*)}{\partial z}+\frac{\partial\,(t-t^*)}{\partial x}\right]=0\,.\end{gathered} \tag{1}$$

With suitable values of the propagation velocity, C, or the dispersive Mach number, $\mathrm{M}=w/C$, one could adapt the model to special flow patterns deviating from the ideal plug flow. As pointed out earlier [5] low Mach numbers (high propagation velocities), $0 \le \mathrm{M}^2 < 1$, can preferably describe pure axial mixing, conduction and diffusion, with limiting case $\mathrm{M}=0$ ($C=\infty$) for the Fourier type heat conduction. High Mach numbers, $\mathrm{M}^2>1$, can consider situations of pure maldistribution as occur in tube bundles or plate heat exchangers. For an infinite propagation velocity, $C^2=\infty$, the dispersive Mach number $\mathrm{M}=0$, and Eq. (1) leads to the original parabolic dispersion model. The special case of $\mathrm{M}^2=1$ in Eq. (1) represents the basis of the new unity Mach number dispersion model [4,5]. For $\mathrm{M}^2=1$ and steady state conditions in counterflow, parallel flow and pure cross-flow heat exchangers the solution to the system of energy equations leads to simple corrections of the true mean heat transfer coefficients:

$$\frac{1}{\alpha_d A}=\frac{1}{\alpha A}+\frac{1}{\dot{W}\mathrm{Pe}}\,. \tag{2}$$

This equation can approximately be applied to other flow arrangements with sufficient accuracy [5]. The degree of axial dispersion is expressed with the dispersive Peclet number Pe. $\mathrm{Pe} = \infty$ means ideal plug flow without dispersion. For the application of Eq. (2) appropriate values of Pe have to be known for the heat exchanger flow channel under consideration.

In previous investigations the Peclet numbers, Pe_p, for the parabolic dispersion model were experimentally determined. First through steady state measurements evaluated with extended Wilson plot techniques [7–10], later with residence time measurements using a tracer [11]. Balzereit compared measured outlet responses to calculated ones, as is also recommended by earlier researchers [12], who compared different evaluation methods in and outside the time domain, i.e., a transfer function fitting (Laplace transform). They did not consider closed heat exchanger systems (no dispersion in the fore and aft sections) but infinitely long open systems with parabolic dispersion ($\mathrm{M} = 0$).

In the following an alternative evaluation method for tracer experiments on heat exchangers (closed systems) is described, which can be applied to the unity Mach number dispersion model and other models. The measured profiles are evaluated in the frequency domain.

3 Evaluation of tracer experiments

The tracer experiment can be described in the same way as the adiabatic process. For $\mathrm{M}^2 = 1$ and $NTU = 0$ Eq. (1) simplifies to

$$\frac{\partial t}{\partial z} + \frac{\partial t}{\partial x} + \frac{1}{\mathrm{Pe}}\left(\frac{\partial^2 t}{\partial z^2} + 2\frac{\partial^2 t}{\partial z \partial x}\right) = 0 \, . \tag{3}$$

The temperatures can be regarded as tracer concentrations. Inside the exchanger flow channel the temperature is denoted with t. In front and rear the channel cross-section where no dispersion occurs the temperatures are denoted with T. At the inlet ($i = 0$, $x = 0$) and the outlet ($i = 1$, $x = 1$) step changes in temperature take place which can be expressed as

$$t_i\left(z\right) + \frac{2}{\mathrm{Pe}}\frac{\partial t_i\left(z\right)}{\partial z} = T_i\left(z\right) + \frac{1}{\mathrm{Pe}}\frac{\partial T_i\left(z\right)}{\partial z} \, . \tag{4}$$

The derivation of this equation is given in the appendix. At the inlet ($i = 0$) Eq. (4) is one of two boundary conditions. Before the experiment ($\tau \leq 0$) all temperatures are uniform. The dimensionless temperatures are defined such that at the beginning of the measurements all temperatures (or concentrations) become zero. This definition is useful for the Laplace transform solution. At the time $\tau = 0$ the tracer is injected in front of the inlet cross-section causing the input temperature (concentration) profile $T_0(\tau)$ to be measured. The function $T_0(\tau)$ is the second inlet boundary condition for the solution to Eq. (3). Immediately following the outlet cross-section the response profile $T_1(\tau)$ is measured as well, and the comparison of inlet and outlet profiles yields the desired Peclet numbers.

Independent of the used evaluation method, the mean residence time in the flow channel, $\tau_r = V/\dot{V}$, has to be determined for the calculation of the dimensionless time $z = \tau/\tau_r = \tau\dot{V}/V$. The mean residence time can be calculated from the first moments of the measured inlet and outlet profiles [11,12] according to

$$\frac{V}{\dot{V}} = \tau_r = \frac{\int_0^\infty T_1 \tau \, d\tau}{\int_0^\infty T_1 \, d\tau} - \frac{\int_0^\infty T_0 \tau \, d\tau}{\int_0^\infty T_0 \, d\tau} . \tag{5}$$

Due to the conservation of energy (if T = temperature) or mass (if T = concentration)

$$\int_0^\infty T_1 \, d\tau = \int_0^\infty T_0 \, d\tau . \tag{6}$$

The Eq. (3) is solved using the Laplace transforms

$$\tilde{t}\,(s) = \int_0^\infty t \exp\left(-sz\right) dz; \quad \tilde{T}\,(s) = \int_0^\infty T \exp\left(-sz\right) dz \tag{7}$$

leading to the transfer function

$$F\,(s) = \frac{\tilde{T}_1\,(s)}{\tilde{T}_0\,(s)} = \frac{\tilde{t}_1\,(s)}{\tilde{t}_0\,(s)} = \exp\left(-s\frac{\mathrm{Pe} + s}{\mathrm{Pe} + 2s}\right) . \tag{8}$$

The real outlet response $T_1(\tau)$ could be calculated with the aid of numerical retransformation only for special functions of $T_0(\tau)$, e.g., the Dirac impulse. Alternatively the measured profiles $T_0(\tau)$ and $T_1(\tau)$ can be evaluated in the frequency domain. The measured temperatures as functions of the dimensionless time, z, have to be numerically transformed according to

Eq. (7), leading to the experimental transfer function $F(s)$ in Eq. (8). This equation can be solved for Pe yielding Pe(s) for any value of s.

If the channel flow obeys the unity Mach number dispersion model, Eq. (8), yields the constant Peclet number for any value of s, $\mathrm{Pe}(s) = const$. A plot of s/Pe versus s would be a straight line through the origin, its slope being equal to $1/\mathrm{Pe}$. In reality the model will not fit exactly and the resulting Peclet number from Eq. (8) varies with s, $\mathrm{Pe}(s) \neq const$. A characteristic mean value of Pe has to be found. Intuitively the Peclet number for $s = 0$ appears to be the characteristic mean value as all parts of the profiles $T_0(z)$ and $T_1(z)$ get the same weight. Unfortunately the value $\mathrm{Pe}(s = 0)$ cannot be calculated directly from Eq. (8). This problem is solved in the following way.

The experimental function $s/\mathrm{Pe}(s)$ versus s is a weekly curved line which can be expressed as a polynomial

$$\frac{s}{Pe} = a_0 + a_1 s + a_2 s^2 + a_3 s^3 \,. \tag{9}$$

Dividing Eq. (9) by s yields $1/\mathrm{Pe}(s)$. Forming the integral mean value of $1/\mathrm{Pe}(s)$ along s between $s = -\sigma$ and $s = +\sigma$ and shrinking the range of integration to zero yields the mean value $\overline{\mathrm{Pe}}$ at the point $s = 0$:

$$\frac{1}{\overline{\overline{\mathrm{Pe}}}} = \lim_{\sigma \to 0} \left(\frac{1}{2\sigma} \int_{-\sigma}^{+\sigma} \frac{ds}{\mathrm{Pe}} \right) = a_1 \,. \tag{10}$$

This equation is independent of the degree of polynomial Eq. (9) and of the value of a_0. If $s/\mathrm{Pe}(s)$ is linear in the range $-s_1 \leq s \leq +s_1$, only two Peclet numbers have to be calculated and

$$\frac{1}{\overline{\overline{\mathrm{Pe}}}} = \frac{1}{2} \left[\frac{1}{\mathrm{Pe}\,(-s_1)} + \frac{1}{\mathrm{Pe}\,(+s_1)} \right] \,. \tag{11}$$

If the polynomial Eq. (9) is of the third degree, at least four points are required. With the values $\mathrm{Pe}(-s_1)$, $\mathrm{Pe}(-s_1/2)$, $\mathrm{Pe}(+s_1/2)$ and $\mathrm{Pe}(+s_1)$ the following formula for $a_1 = 1/\overline{\mathrm{Pe}}$ is derived:

$$\frac{1}{\overline{\overline{\mathrm{Pe}}}} = \frac{2}{3} \left[\frac{1}{\mathrm{Pe}\left(-\frac{s_1}{2}\right)} + \frac{1}{\mathrm{Pe}\left(+\frac{s_1}{2}\right)} \right] - \frac{1}{6} \left[\frac{1}{\mathrm{Pe}\,(-s_1)} + \frac{1}{\mathrm{Pe}\,(+s_1)} \right], \tag{12}$$

which is recommended for the evaluation. The value of s_1 can arbitrarily be selected. Alternatively the coefficient a_1 could be determined through a least square estimation of more than four points with arbitrarily selected values of s.

The described evaluation method is not restricted to the unity Mach number dispersion model. It can be applied to other one-dimensional flow models for which the transfer functions $F(s)$ is available. One has merely to replace $F(s)$ in Eq. (8) by the function under consideration.

For the parabolic dispersion model with the related Peclet number Pe_p Balzereit [11] gave the solution, which is rearranged here to

$$\begin{aligned}\frac{1}{F(s)} = \frac{1}{2}\left(1+\frac{1+\frac{2s}{\mathrm{Pe}_p}}{\sqrt{1+\frac{4s}{\mathrm{Pe}_p}}}\right)\exp\left[-\frac{\mathrm{Pe}_p}{2}\left(1-\sqrt{1+\frac{4s}{\mathrm{Pe}_p}}\right)\right] \\ +\frac{1}{2}\left(1-\frac{1+\frac{2s}{\mathrm{Pe}_p}}{\sqrt{1+\frac{4s}{\mathrm{Pe}_p}}}\right)\exp\left[-\frac{\mathrm{Pe}_p}{2}\left(1+\sqrt{1+\frac{4s}{\mathrm{Pe}_p}}\right)\right] .\end{aligned} \tag{13}$$

Numerical methods have to be applied to find $\mathrm{Pe}_p(s)$ from given s and measured $F(s)$.

The Eqs. (11) and (12) are also valid for Pe_p. Under steady state conditions of balanced counterflow and zero heat transfer coefficients (adiabatic case) the comparison of both dispersion models leads to the relationship

$$\mathrm{Pe} = \frac{\mathrm{Pe}_p^2}{\mathrm{Pe}_p - 1 + \exp(-\mathrm{Pe}_p)} , \tag{14}$$

which is also valid for parallel flow if $NTU_1 + NTU_2 \to 0$ (adiabatic case; NTU_1 and NTU_2 for stream 1 and stream 2 in the exchanger). Separate calculations have shown that Eq. (14) is also fulfilled for the Peclet numbers $\overline{\mathrm{Pe}}$ and $\overline{\mathrm{Pe}}_p$ from the tracer experiment. This will be demonstrated later in this paper.

The evaluation method can also be applied to the cascade model with n consecutive completely mixed zones in the flow channel. The transfer function is

$$F(s) = \left(1+\frac{s}{n}\right)^{-n} . \tag{15}$$

Equating the transfer functions of Eqs. (15) and (8) yields for the limiting case $s \to 0$ the relationship

$$\overline{Pe} = 2\bar{n} , \tag{16}$$

which confirms the findings for steady state conditions [5], $\mathrm{Pe} = 2n$ for counterflow with $NTU_1 - NTU_2 \to 0$ and for parallel flow with $NTU_1 + NTU_2 \to 0$ (NTU_1 and NTU_2 for stream 1 and stream 2 in the exchanger).

4 Calculated example

A tube bundle with 7 tubes is considered as shown in Fig. 1. Heat is transferred from the fluid inside the tubes to the tube wall of uniform dimensionless temperature, $t_w = 0$. The fluid cools down from the dimensionless inlet temperature, $T_0 = 1$ to the dimensionless outlet temperature, T_1. Maldistribution and backflow take place in the bundle. Three streams are considered. Stream 1 in 4 tubes with velocity w_1. Stream 2 in 2 tubes with velocity $w_1/2$. Stream 3 backflow in 1 tube with velocity $w_1/2$. The temperature distribution is qualitatively shown in Fig. 2.

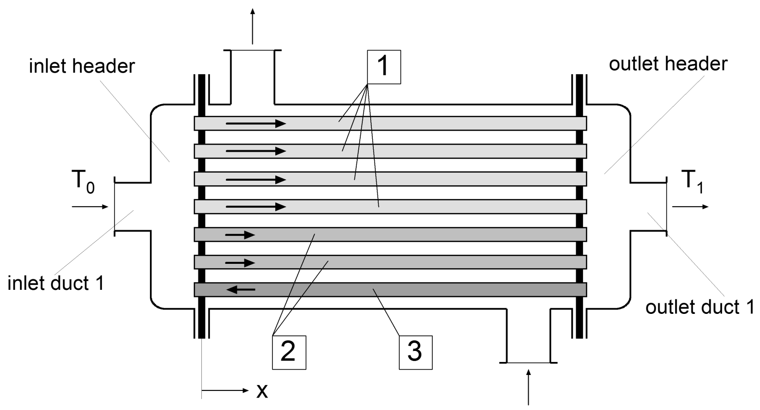

Figure 1: Tube bundle with maldistribution and backflow.

For a uniform heat transfer coefficient, α, the dimensionless outlet temperature

$$\frac{1}{T_1} = \frac{1}{1+\frac{\dot W_2}{\dot W_1}-\frac{\dot W_3}{\dot W_1}}\left[\frac{\left(1+\frac{\dot W_2}{\dot W_1}\right)^2}{\exp\left(-NTU_1\right)+\frac{\dot W_2}{\dot W_1}\exp\left(-NTU_2\right)}-\frac{\dot W_3}{\dot W_1}\exp\left(-NTU_3\right)\right],$$

$$NTU_1 = \frac{\alpha A_1}{\dot W_1}, \quad NTU_2 = NTU_1\frac{A_2}{A_1}\frac{\dot W_1}{\dot W_2}, \quad NTU_3 = NTU_1\frac{A_3}{A_1}\frac{\dot W_1}{\dot W_3}. \tag{17}$$

The same steady state process can be expressed with the dispersion model

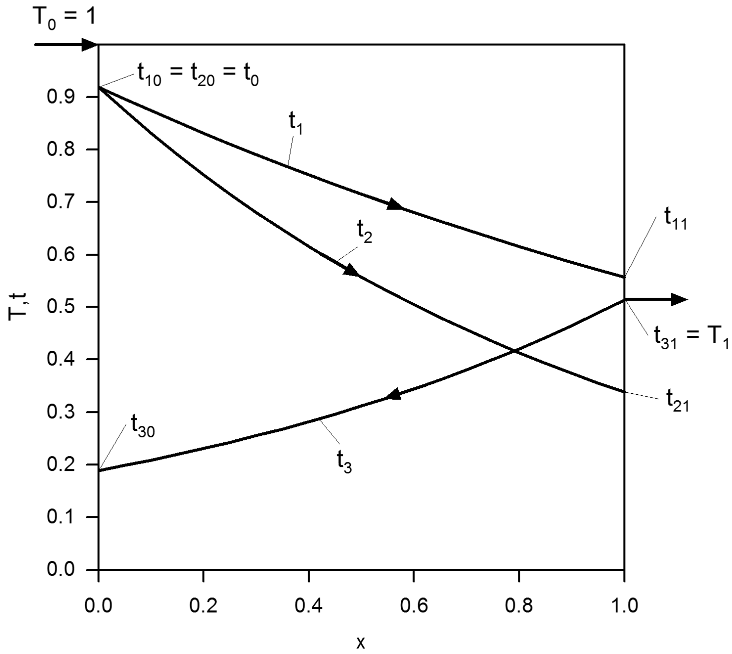

Figure 2: Temperature distribution.

yielding

$$\frac{1}{T_1} = \exp\left(NTU_d\right) = \exp\left(\frac{1}{\frac{1}{NTU} + \frac{1}{\mathrm{Pe}}}\right) ,$$
$$NTU = \frac{\alpha\left(A_1 + A_2 + A_3\right)}{\dot{W}_1 + \dot{W}_2 - \dot{W}_3} = NTU_1 \frac{1 + \frac{A_2}{A_1} + \frac{A_3}{A_1}}{1 + \frac{\dot{W}_2}{\dot{W}_1} - \frac{\dot{W}_3}{\dot{W}_1}} . \tag{18}$$

Equating T_1 of Eqs. (17) and (18) and solving for Pe gives the Peclet number for the steady state heat transfer process.

In this example $\dot{W}_2/\dot{W}_1 = \dot{V}_2/\dot{V}_1 = 1/4$, $\dot{W}_3/\dot{W}_1 = \dot{V}_3/\dot{V}_1 = 1/8$, $A_2/A_1 = V_2/V_1 = 1/2$, $A_3/A_1 = V_3/V_1 = 1/4$. With these data the outlet temperature and the Peclet number can be calculated for different values of NTU_1. For the comparison with the Peclet number from the tracer experiment the adiabatic limiting value of Pe is formed resulting in

$$\lim_{NTU_1 \to 0} \mathrm{Pe}\left(NTU_1\right) = \frac{245}{73} . \tag{19}$$

In the tracer experiment a rectangular impulse $T_0 \Delta\tau (\Delta\tau \to 0)$ is given as the input profile. The height T_0 is fixed to $T_0 = 1$. For simplification the volume of inlet and outlet headers is assumed to be zero. The residence time of stream 1 is given as $\tau_{r1} = V_1/\dot{V}_1 = 0.1$ s. The residence times of stream 2 and 3 are $\tau_{r2} = 0.1$ s $(V_2/V_1)(\dot{V}_1/\dot{V}_2) = 0.2$ s and $\tau_{r3} = 0.1$ s $(V_3/V_1)(\dot{V}_1/\dot{V}_3) = 0.2$ s. The mean residence time of the bundle $\tau_r = V/\dot{V} = \tau_{r1}(1 + V_2/V_1 + V_3/V_1)/(1 + \dot{V}_2/\dot{V}_1 - \dot{V}_3/\dot{V}_1) = 7/45$ s. The mean residence time has been calculated from the given data. In a real experiment it is determined from the measured profiles using Eq. (5).

The input impulse travels through the channels 1 and 2 and arrives at the outlet header after the residence times 0.1 s and 0.2 s, respectively. In the outlet header they mix with the other stream and leave the header with the lowered temperature. At the same time they enter the backflow channel and arrive at the inlet header after the residence time $\tau_{r3} = 0.2$ s. There they mix with the main inlet flow stream and travel again through channels 1 and 2, and so on. In this way an infinite number of single outlet impulses of width $\Delta\tau$ and rapidly decreasing height are created. This is shown in Fig. 3.

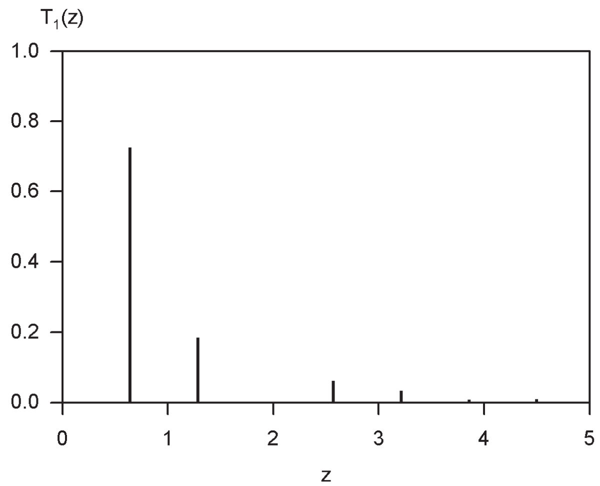

Figure 3: Outlet impulses of calculated example.

According to Eq. (6) the sum of all outlet impulses is equal to the inlet impulse. This is described and confirmed with

$$\sum_{k=1}^{\infty}\sum_{j=1}^{k+1}\frac{9}{50^k}\binom{k}{j-1}4^{(k+1-j)} = \sum_{k=1}^{\infty}\frac{9}{50^k}(4+1)^k = 1\,. \qquad (20)$$

According to Eq. (5) the mean residence time of the bundle can be calculated from the "measured" response

$$\frac{V}{\dot{V}} = \sum_{k=1}^{\infty}\sum_{j=1}^{k+1} \frac{9}{50^k} \binom{k}{j-1} 4^{(k+1-j)} \frac{1}{10} (3k+j-3) = \frac{7}{45}\ \mathrm{s}\,. \tag{21}$$

The summation gives the exact value from the given data.

Finally the transfer function $F(s)$ according to Eq. (8) can be calculated for given values of s:

$$F(s) = \sum_{k=1}^{\infty}\sum_{j=1}^{k+1} \frac{9}{50^k} \binom{k}{j-1} 4^{(k+1-j)} \exp\left[-s\frac{9}{14}(3k+j-3)\right]. \tag{22}$$

Selecting the value $s_1 = 0.1$ and calculating Pe for $-s_1$, $-s_1/2$, $+s_1/2$, and $+s_1$ yields from Eq. (12) the characteristic mean Peclet number $\overline{\mathrm{Pe}}$. The results of the calculation are given in Tab. 1. The resulting value $\overline{\mathrm{Pe}}$ agrees with the steady state value of Eq. (19): $245/73 = 3.3562$.

The evaluation method is also applied to the cascade model and the parabolic dispersion model using the transfer functions of Eqs. (15) and (13), respectively. The results are presented in Tab. 1 as well. The Eqs. (16) and (14) are confirmed for the mean values $\overline{\mathrm{Pe}}$, $\overline{\mathrm{Pe}}_p$ and $\bar{n}$, defined by Eq. (10).

Table 1: Calculated values of $\mathrm{Pe}(s)$, $2n(s)$ and $\mathrm{Pe}_p(s)$ from the "experimental" transfer function $F(s)$. $\mathrm{Pe}(s=0) = \overline{\mathrm{Pe}}$, $\mathrm{Pe}_p(s=0) = \overline{\mathrm{Pe}}_p$, $n(s=0) = \bar{n}$. $\overline{\mathrm{Pe}} = 2\bar{n}$. $\overline{\mathrm{Pe}}$ and $\overline{\mathrm{Pe}}_p$ fulfil Eq. (14).

s	–0.1	–0.05	+0.05	+0.1	0, Eq. (12)
$F(s)$, Eq. (22)	1.1088	1.0521	0.9519	0.9073	1.000
Pe, Eq. (8)	3.2958	3.3257	3.3871	3.4185	3.3562
$2n$, Eq. (15)	3.2298	3.2926	3.4206	3.4858	3.3562
Pe_p, Eq. (13)	1.6838	1.7417	1.8577	1.9159	1.7996

5 Conclusions

The described evaluation method in the frequency domain is applicable to the unity Mach number dispersion model, the parabolic dispersion model and the cascade model. The resulting newly defined characteristic mean values $\overline{\mathrm{Pe}}$, $\overline{\mathrm{Pe}}_p$ and $\bar{n}$ agree with the corresponding steady state adiabatic limiting values.

The evaluation method gives reliable results independent of the shape of the measured inlet and outlet profiles. In particular it can be applied to cases in which the usual method of comparing measured and calculated outlet profiles is impossible.

Appendix. Derivation of Eq. (4)

The energy equation Eq. (3) of [5] with $NTU = 0$ is used,

$$\frac{\partial t}{\partial z} + \frac{\partial t}{\partial x} + \frac{1}{\mathrm{Pe}}\frac{\partial \varphi}{\partial x} = 0 \,, \tag{23}$$

with the dimensionless axial energy flux, $\varphi = \dot{q}_x L/(\lambda_d\, \Delta\vartheta)$, according to Eq. (4) of [5] with $\mathrm{M} = 1$

$$\varphi + \frac{1}{\mathrm{Pe}}\frac{\partial \varphi}{\partial z} + \frac{1}{\mathrm{Pe}}\frac{\partial \varphi}{\partial x} = -\frac{\partial t}{\partial x} \,. \tag{24}$$

Substituting $\partial t/\partial x$ in Eq. (23) according to Eq. (24) yields

$$\varphi + \frac{1}{\mathrm{Pe}}\frac{\partial \varphi}{\partial z} = \frac{\partial t}{\partial z} \,. \tag{25}$$

The energy balances at the inlet ($x = 0$, $i = 0$) and outlet ($x = 1$, $i = 1$) according to Eqs. (17) and (18) of [5] give

$$\varphi_i = \mathrm{Pe}\,(T_i - t_i)\,. \tag{26}$$

Applying Eq. (25) to inlet and outlet and substituting φ_i according to Eq. (26) yields

$$t_i\,(z) + \frac{2}{\mathrm{Pe}}\frac{\partial t_i\,(z)}{\partial z} = T_i\,(z) + \frac{1}{\mathrm{Pe}}\frac{\partial T_i\,(z)}{\partial z} \,. \tag{4}$$

The Laplace transform of Eq. (4) reveals that

$$F\,(s) = \frac{\tilde{T}_1\,(s)}{\tilde{T}_0\,(s)} = \frac{\tilde{t}_1\,(s)}{\tilde{t}_0\,(s)} \,. \tag{27}$$

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