GEODESY AND CARTOGRAPHY
Vol. 60, No 1, 2011, pp. 3-19
DOI No 10.2478/v10277-012-0014-9

© Polish Academy of Sciences

Effectiveness of chosen robust estimation methods compared to the level of network reliability

Mieczysław Kwaśniak

Department of Engineering Surveying and Detail Surveys
Faculty of Geodesy and Cartography
Warsaw University of Technology
Pl. Politechniki 1, PL-00 661 Warsaw, Poland
e-mail: m.kwasniak@gik.pw.edu.pl

Received: 10 November 2010/Accepted: 8 March 2011

Abstract: The work presents the results of studies on dependence of effectiveness of chosen robust estimation methods from the internal reliability level of a geodetic network. The studies use computer-simulated observation systems, so it was possible to analyse many variants differing from each other in a planned way. Four methods of robust estimation have been chosen for the studies, differing substantially in the approach to weight modifications. For comparative reasons, the effectiveness studies have also been conducted for the very popular method in surveying practice, of gross error detection basing on LS estimation results, the so called iterative data snooping. The studies show that there is a relation between the level of network internal reliability and the effectiveness of robust estimation methods. In most cases, in which the observation contaminated by a gross error was characterized by a low index of internal reliability, the robust estimation led to results being essentially far from expectations.

Keywords: adjustment, robust estimation, internal reliability, effectiveness of method

1. Introduction

Despite the fact that there exists a wide literature spectrum within this domain, the subject of gross error detection in geodetic networks is still being discussed. Generally, the existing methods of gross error detection can be divided into two groups. The first one is formed by methods, in which by use of statistical tests, the results of least squares adjustment (LS) are being analysed in many different ways (i.e. Pope, 1976; Ethrog, 1991; Nowak, 2002). It is often an iterative process, where in the successive iterations, the one observation suspected to be contaminated with a gross error is being eliminated from the network (i.e. Baarda, 1968). In some of the methods out of this group one takes the effort to accelerate the „cleaning" process of the observation system by assessing the correlation level of observations suspected to have gross errors. On this basis, one takes the decision to eliminate simultaneously more than one observation (i.e. Cross and Price, 1985; Ding and Coleman, 1996). The second group is constituted by methods, in which basing on criteria of the so-called robust estimation, one minimizes the influence of outlying observations on the final result of network adjustment. Such estimation is performed during an iterative process, where in each iteration one modifies the observation weights (e.g. Kadaj, 1984; Wiśniewski, 1993; Kamiński and Wiśniewski, 1994; Aduol, 1999). Unfortunately all these methods are sensible towards the weakness of geodetic network structure, especially the so called „masking effect" in case of multiple gross errors occurring within the network.

In recent years, many works appeared in the literature, in which for the detection of gross errors one was trying to use a priori knowledge, which is not connected with the adjustment process. The work (Kamiński, 2000) presents the utilization of Bayesian estimation for the adjustment of geodetic networks, where the measurements could be contaminated by gross errors. For this purpose one uses the Bayesian formula, as well as the rules of robust estimation. In turn, in (Xu, 2005) a robust method of outlier detection is presented, in which the estimation of the so-called subjective breaking point was based on a priori distribution for the signs of outliers. According to the author, this method allows to obtain a correct estimation result even when more than 50% contaminated observations exist in the network. Also in the work (Gui et al., 2007) the knowledge of a priori information on the unknown parameters has been assumed. Considering this assumption and assuming the independence of observations, basing on the principle of Bayesian statistical inference one gave the formulae for a posteriori probability calculation that the observation or several observations contain gross errors.

In most of publications related to gross error detection, the presented solutions are illustrated with a specific numerical example, without assessment of their actual effectiveness. In a few publications one can find the results of comparative analyses of chosen methods. Results of effectiveness assessment of several robust estimation methods on the examples of a simulated trilateral, triangular and angle-linear networks were presented (e.g. Hekimoglu and Berber, 2003). An insufficient effectiveness of these methods has been stated, even when a single gross error exists in the network. Also the effectiveness of classical statistic tests conducted on LS results has been compared with the effectiveness of chosen robust estimation methods (Knight and Wang, 2009). It has been stated that in case of a single gross error, the classical tests gave slightly better results than the robust estimation. However, as the number of gross errors were growing, the robust estimation turned out to be more effective.

In the present study the author took up the effort to find, if the internal reliability influences the results of robust estimation in case of the appearance of a single gross error in the network.

2. Chosen robust estimation methods

To obtain estimation results free of the influence of gross errors occurring in the observation system it is necessary to whether identify and eliminate the so-called outlying observations, or use such estimation method, in which the outliers will not influence the obtained results. Among many approaches to robust estimator construction, the highest popularity during the late years gained methods based on the so-called M-estimation. The theoretical basis of M-estimation has been proposed in (Huber, 1964). M-estimators are the most general ones; they constitute a generalization of maximum likelihood estimators (hence the name M) and are characterized by a high efficiency.

In M-estimation the objective function has the form

$$\Psi[\mathbf{v}(\mathbf{x})] = \sum_{i=1}^{n} \rho_i[v_i(\mathbf{x})] = \sum_{i=1}^{n} \rho_i(v_i) \tag{1}$$

where $\rho_i(v_i)$ is a component of the objective function, $\mathbf{v}(\mathbf{x})$ is a vector of residuals, $\mathbf{x}$ is a vector of estimated parameters and n is a number of observations. Depending on the shape of the objective function (convex or concave) one is looking for a global minimum or maximum of this function. To unify the approach of different methods one can transform the optimization criterion using one of the formulae (Wiśniewski, 1991)

$$\min_{\mathbf{x}} \Psi[\mathbf{v}(\mathbf{x})] = -\max_{\mathbf{x}}\{-\Psi[\mathbf{v}(\mathbf{x})] \tag{2}$$

$$\max_{\mathbf{x}} \Psi[\mathbf{v}(\mathbf{x})] = -\min_{\mathbf{x}}\{-\Psi[\mathbf{v}(\mathbf{x})]\} \tag{3}$$

Properties of M-estimation methods are determined by analysing the properties of the component of the objective function. Useful are also the following functions (Kamiński and Wiśniewski, 1992):

- influence function $\phi(v) = \mathrm{d}\rho(v)/\mathrm{d}(v)$ which determines the impact of change in individual observation on the result of estimation;
- weight function $w(v) = \mathrm{d}\rho(v)/\mathrm{d}(v^2)$ which determines the impact of each observation on the estimation result.

The M-estimation methods elaborated so far present different approaches to the definition of modified values of weights assigned to observations. Constant coefficients, being a critical value for the weight function occur in many of them. The proper choice of values of those coefficients requires a good recognition of data set distribution and may have a significant impact on the results of the estimation (Xu, 1993).

There are two ways to implement M-estimation. The first one is the use of nonlinear algorithms, and the other is a modified method of LS. The second way constitutes a group of methods which use the iteratively re-weighted LS algorithm, but differ in a weight function. A special case of a method from this group is LS estimation

$$\rho^{LS}(v) = pv^2 \tag{4}$$

$$\varphi^{LS}(v) = 2pv \tag{5}$$

$$w^{LS}(v) = p \tag{6}$$

where v is a residual and p is a weight of observation.

The LS estimation is not robust, because the influence function is not bounded and the outlying observations could dominate conforming observations, which fit the selected network model.

Below are presented the characteristic functions for the methods chosen for studies discussed in this paper.

2.1. An Alternative to the Method of Least Absolute Deviations (ALAD)

This method has been proposed in (Kadaj, 1988; Wiśniewski, 1993) as an alternative to the method of least absolute deviations. Basing on the general dependence of

$$\lim_{c \to 0} \sqrt{x^2 + c^2} = |x| \tag{7}$$

where x is an argument and c is the sufficiently small constant value ($c \to 0$), one accepted a component of the objective function in following form

$$\rho^{ALAD}(v) = p\sqrt{v^2 + c^2} \tag{8}$$

where v is the residual and p is the initial weight of observation.

The resulting influence function $\varphi^{ALAD}(v)$ and the weight function $w^{ALAD}(v)$ have the following form

$$\varphi^{ALAD}(v) = p\frac{v}{\sqrt{v^2 + c^2}} \tag{9}$$

$$w^{ALAD}(v) = p\frac{1}{2\sqrt{v^2 + c^2}} \tag{10}$$

2.2. Huber Method (HU)

In the method proposed by Huber (1964), the component of the objective function is a spline having the following form

$$\rho^{HU}(v) = \begin{cases} \frac{1}{2}pv^2 & \text{for } |v| \le c \\ p\left(c\,|v| - \frac{c^2}{2}\right) & \text{for } |v| > c \end{cases} \tag{11}$$

where p is the initial value of observation weight and c is a constant value representing the permitted value for residuals ($v \in \langle -c; c \rangle$).

The influence function $\varphi^{HU}(v)$ as well as weight function $w^{HU}(v)$ have the following forms

$$\varphi^{HU}(v) = \begin{cases} pv & \text{for } |v| \le c \\ pc \cdot \mathrm{sgn}(v) & \text{for } |v| > c \end{cases} \tag{12}$$

$$w^{HU}(v) = \left\{ \begin{array}{ll} \frac{1}{2}p & \text{for } |v| \leq c \\ \frac{1}{2}p\frac{c}{|v|} & \text{for } |v| > c \end{array} \right\} \tag{13}$$

2.3. *A Choice Rule of Alternative (CRA)*

The method presented by Kadaj (1984) has been named "A Choice Rule of Alternative". One searches the maximum of global objective function. To unify the approach with other methods selected for this paper, in which one searches the minimum of global objective function, we will consider here the transformation (3). Then the characteristic functions of this method will take following forms:

- component of the objective function

$$\rho^{CRA}(v) = -\exp\left(-\frac{v^2}{2\sigma^2}\right) \tag{14}$$

where σ^2 is the variance of observation, being defined before adjustment;

- influence function

$$\varphi^{CRA}(v) = pv \cdot \exp\left(-p\frac{v^2}{2}\right) \quad \text{where} \quad p = \sigma^{-2} \tag{15}$$

- weight function

$$w^{CRA}(v) = \frac{1}{2}p \cdot \exp\left(-p\frac{v^2}{2}\right) \tag{16}$$

2.4. *Aduol Method (AD)*

Aduol (1999) presented the method of robust estimation based on the following model of observation contaminated by a gross error

$$\tilde{y} = y + b + \varepsilon \tag{17}$$

where y is the true value of observation; b is the gross error, ε is a random error of measurement. Into such observation one introduces the notion of mean squared error m

$$m^2 = \sigma^2 + b^2 \tag{18}$$

where σ^2 is an observation variance. Considering that

$$E(v) = E(b + \varepsilon) = b + E(\varepsilon) = b \tag{19}$$

where $E(\cdot)$ means the expected value, one proposes the calculation of observation weights in subsequent iterations of LS estimation according to the formula

$$w_i = \frac{\sigma_0^2}{m_i^2} = \frac{\sigma_0^2}{\sigma^2 + v^2} \tag{20}$$

Calculated on this basis the component of objective function and the influence function have the following forms

$$\rho^{AD}(v) = \sigma_0^2 \cdot \ln\left(\sigma^2 + v^2\right) \tag{21}$$

$$\varphi^{AD}(v) = \frac{2v\sigma_0^2}{\sigma^2 + v^2} \tag{22}$$

Out of the formulae presented above one can read the basic assumption of robust methods: the observation having the greater residual should have a lower weight.

2.5. Iterative data snooping (IDS)

This method is a commonly known iterative method based on the LS estimation results. Basing on the theory of data snooping (Baarda, 1968), for uncorrelated observations, the normalized residual u should fulfil the test

$$u = \frac{|v|}{\sigma_v} \leq k_\alpha \tag{23}$$

where v and σ_v are residual and its standard deviation, respectively, obtained from the LS estimation and k_α is a critical value taken from N(0,1) for the selected test relevance level α. The identification of gross errors within the observation system is based on putting of each observation to the test. In case of observations not fulfilling the criterion (23), the observation having the highest u is considered as the most suspected to be contaminated by a gross error. In the following iteration, the LS estimation is conducted without the suspected observation. Rejecting observations in subsequent iterations means that this method is not based on the estimation of a fixed objective function.

This method has been chosen for studies because of frequent result comparisons of proposed robust methods with the LS results acquired out of first iteration. The subsequent part of this study assumes that the observations are not correlated and the network model is linear or linearized.

3. Assumptions and criterions for evaluation the selected methods

Because of a restricted volume of this paper, the presentation of results obtained will be done on the example of a simple 5-point levelling test network (Fig. 1).

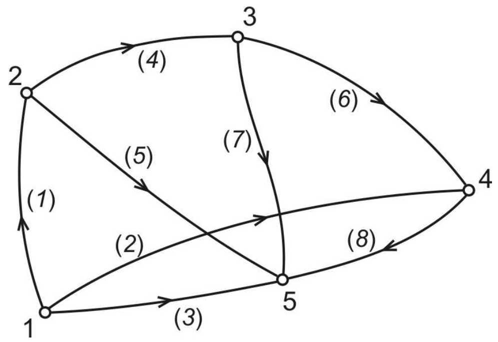

Fig. 1. Test levelling network

Table 1 shows 8 variants of exactitude relations ($\sigma_{h,i}$) (i = 1, 2, ..., 8) between observations in the network (further called: network variants), together with the calculated internal reliability indices (σ_V) for them. These indices are calculated as $\sigma_{V,i} = \sqrt{\{\mathbf{R}\}_{i,i}}$ where $\mathbf{R} = \mathbf{I} - \mathbf{A}_s(\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_s)^{-1}\mathbf{A}_s^{\mathrm{T}}$ and $\mathbf{A}_s$ is the design matrix of full rank in standardized system of equations.

Table 1. Network variants and internal reliability indices of observations

Obs. No.	Variant 1		Variant 2		Variant 3		Variant 4		Variant 5		Variant 6		Variant 7		Variant 8	
	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$	$\sigma_{h,i}$	$\sigma_{V,i}$
1	0.65	0.82	0.25	0.49	0.50	0.74	0.45	0.85	0.65	0.88	0.25	0.67	0.75	0.89	0.15	0.34
2	0.25	0.50	0.50	0.66	0.45	0.71	0.65	0.82	0.25	0.41	0.75	0.85	0.15	0.26	0.65	0.77
3	0.50	0.88	0.45	0.82	0.65	0.85	0.25	0.53	0.75	0.89	0.15	0.42	0.65	0.92	0.25	0.58
4	0.45	0.64	0.65	0.93	0.25	0.44	0.75	0.94	0.15	0.27	0.65	0.84	0.25	0.40	0.50	0.83
5	0.65	0.84	0.25	0.55	0.75	0.91	0.15	0.33	0.65	0.87	0.25	0.68	0.50	0.77	0.45	0.87
6	0.25	0.44	0.75	0.86	0.15	0.31	0.65	0.81	0.25	0.48	0.50	0.63	0.45	0.72	0.65	0.77
7	0.75	0.94	0.15	0.26	0.65	0.93	0.25	0.42	0.50	0.80	0.45	0.68	0.65	0.87	0.25	0.47
8	0.15	0.35	0.65	0.82	0.25	0.50	0.50	0.72	0.45	0.75	0.65	0.80	0.25	0.50	0.75	0.84

Figure 2 presents the variation scope of internal reliability indices for individual observations when combining all 8 variants.

Each of the network variants has been used to studies in 5 following versions of the random error vector

$$\boldsymbol{\varepsilon}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{\varepsilon}_1 = \boldsymbol{\sigma}_{\mathbf{h}} \begin{bmatrix} 1.042 \\ 0.627 \\ -1.873 \\ -1.628 \\ -0.134 \\ -0.933 \\ 0.300 \\ -0.307 \end{bmatrix} \quad \boldsymbol{\varepsilon}_2 = \boldsymbol{\sigma}_{\mathbf{h}} \begin{bmatrix} -0.423 \\ -0.203 \\ -1.297 \\ -0.852 \\ -2.051 \\ 0.748 \\ -0.593 \\ 0.775 \end{bmatrix} \quad \boldsymbol{\varepsilon}_3 = \boldsymbol{\sigma}_{\mathbf{h}} \begin{bmatrix} -0.680 \\ -0.381 \\ -0.188 \\ 0.912 \\ -1.229 \\ 0.111 \\ 1.198 \\ 1.217 \end{bmatrix} \quad \boldsymbol{\varepsilon}_4 = \boldsymbol{\sigma}_{\mathbf{h}} \begin{bmatrix} -0.280 \\ -0.861 \\ 0.752 \\ -1.743 \\ 1.546 \\ -0.196 \\ -0.871 \\ -0.719 \end{bmatrix}$$

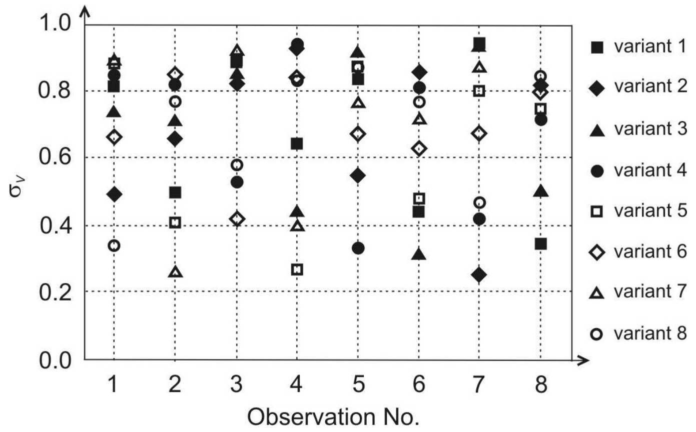

Fig. 2. Variation of internal reliability indices for observations in test network

where $\boldsymbol{\sigma_h} = \text{diag}(\sigma_1, \sigma_2, ..., \sigma_8)$ is the matrix of a priori average errors of observations.

Considering that in each of the network variant, each observation was disturbed by a gross error, 320 estimations in total were performed for each method.

The gross error size g_i introduced into each observation has been established as 1.5 times bigger than the minimum detectable gross error being proposed in (Baarda, 1968)

$$g_i = 1.5 \cdot \Delta_{min,i} = 1.5 \cdot \frac{4.1 \cdot \sigma_{h,i}}{\sigma_{V,i}} \tag{24}$$

For the studies reported it has been assumed that the LS estimation results obtained for the observation system affected only by random errors $(\boldsymbol{\varepsilon}_0, \boldsymbol{\varepsilon}_1, ..., \boldsymbol{\varepsilon}_4)$ within the accepted range will constitute a reference (*ref*) for the results obtained by the use of selected method (*met*) for systems contaminated by a gross error.

To evaluate the distance from the solution vector $\mathbf{x}^{met}$ to the expected solution vector $\mathbf{x}^{ref}$ two indices were introduced

1. average deviation Δx_{av} calculated as

$$\Delta x_{av} = \sqrt{\frac{\sum_{i=1}^{u} (x_i^{met} - x_i^{ref})^2}{u}}, \text{with } u - \text{number of rows of the solution vector } \mathbf{x} \tag{25}$$

2. maximum deviation Δx_{max} calculated as

$$\Delta x_{max} = \max_i \left(\left| x_i^{met} - x_i^{ref} \right| \right), \quad i = 1, 2, ..., u \tag{26}$$

An additional index used in the studies is the compensation level of the gross error g in the residual v^{met} for the actually disturbed observation, calculated as

$$g\% = \left| \frac{v^{met} - v^{ref}}{g} \right| \cdot 100\% \tag{27}$$

4. Illustration of obtained results

Because of the big size of tables containing numerical settings, the obtained results are presented in a graphical form. For each method investigated the relations between the indices Δx_{av}, Δx_{max}, as well as $g\%$ and the reliability index σ_V of the observation disturbed by a gross error are illustrated.

ALAD method

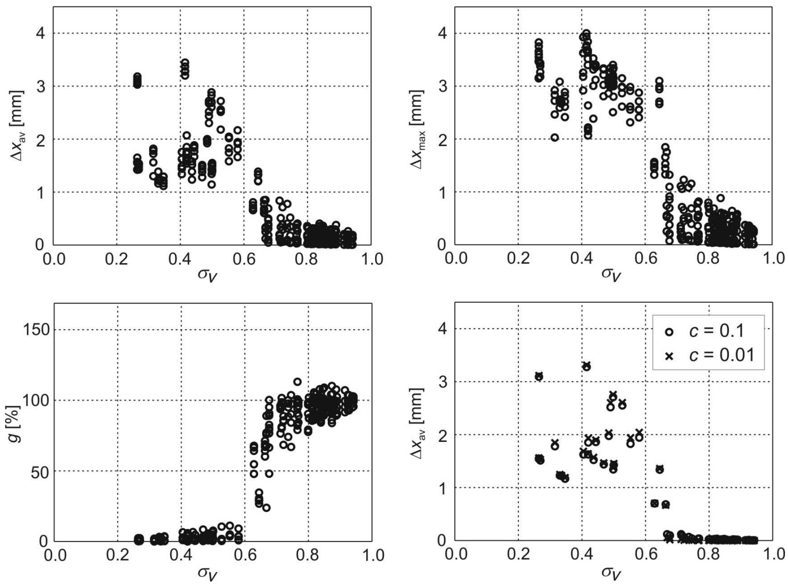

Fig. 3. ALAD method – dependence of Δx_{av}, Δx_{max} and $g\%$ indices from the reliability level of disturbed observation

The graphs presented in Figure 3 show that the reliability index of disturbed observation has a significant influence on the effectiveness of ALAD method. Only when it reaches the value of about 0.7, the effectiveness of this method can be assumed satisfactory. The fourth graph (bottom right in Figure 3) illustrates the dependence of the index Δx_{av} on the index of internal reliability of disturbed observation σ_V for $\boldsymbol{\varepsilon}_0$ network variant. This graph shows that different values of the constant c, which should be close to 0, give insignificant differences in estimation results.

HU method

As it results from graphs showed in Figure 4, the HU method behaves very similarly to the ALAD method. Only when the reliability index of disturbed observation reaches the value of about 0.7, the effectiveness of this method can be assumed satisfactory. Similarly as for method ALAD, different values of the constant c give small differences in estimation results (see bottom right graph in Figure 4).

CRA method

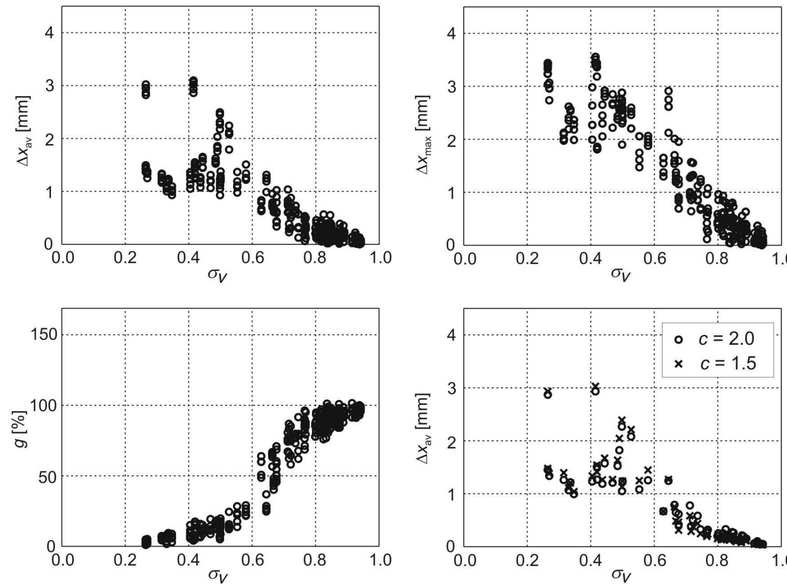

Fig. 4. HU method – dependence of Δx_{av}, Δx_{max} and $g\%$ indices from the reliability level of disturbed observation

As it results from Figure 5, the CRA method behaves in a different way than both methods discussed so far. The effectiveness of this method can be assumed satisfactory even from the reliability index value for disturbed observation of about 0.5, though before reaching the value of 0.7 there is a probability of wrong identification of outliers. This could be the reason of especially unfavourable random errors dispersion within the network.

AD method

The graphs presented in Figure 6 show that the AD method behaves similarly to the CRA method. In both methods no constant coefficients are used. The effectiveness of this method can be considered satisfactory for the reliability index value of disturbed observation being larger than 0.5.

IDS method

As already mentioned in the method description, the decision to terminate the iterative estimation process by this method was based on the fulfilment of the criterion (23) for each observation within the network. The graphs presented in Figure 7 show that the utilized assessment indices of estimation results (Δx_{av}, Δx_{max}, and $g\%$) are significantly better than for the analysed robust methods. Only in 7 cases (for 320 considered), the IDS method (k_α = 2.5) wrongly detected the disturbed observation. The reason for these failures is not a low reliability index of disturbed observations but far more an unfavourable dispersion of random errors within the network. However, the effectiveness of this method depends on the accepted critical value k_α. To compare,

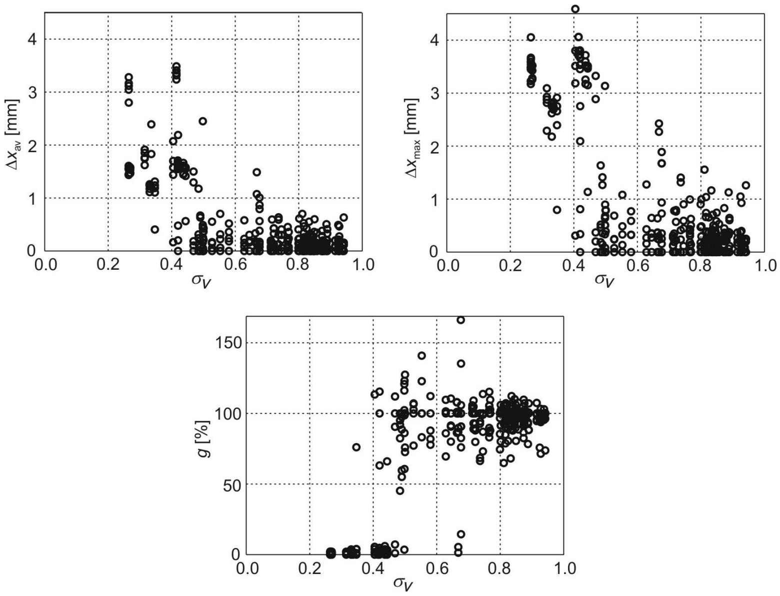

Fig. 5. CRA method – dependence of Δx_{av}, Δx_{max} and $g\%$ indices from the reliability level of disturbed observation

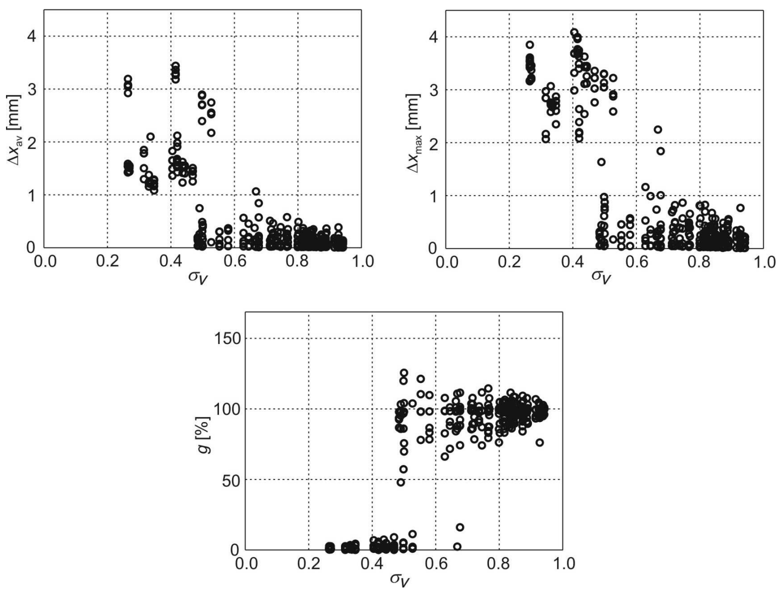

Fig. 6. AD method – dependence of Δx_{av}, Δx_{max} and $g\%$ indices from the reliability level of disturbed observation

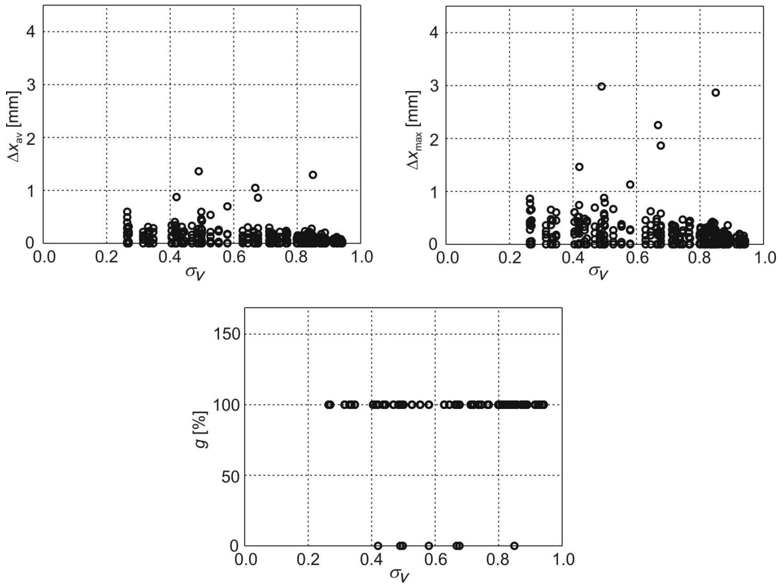

Fig. 7. IDS method (k_α = 2.5) – dependence of Δx_{av}, Δx_{max} and $g\%$ indices from the reliability level of disturbed observation

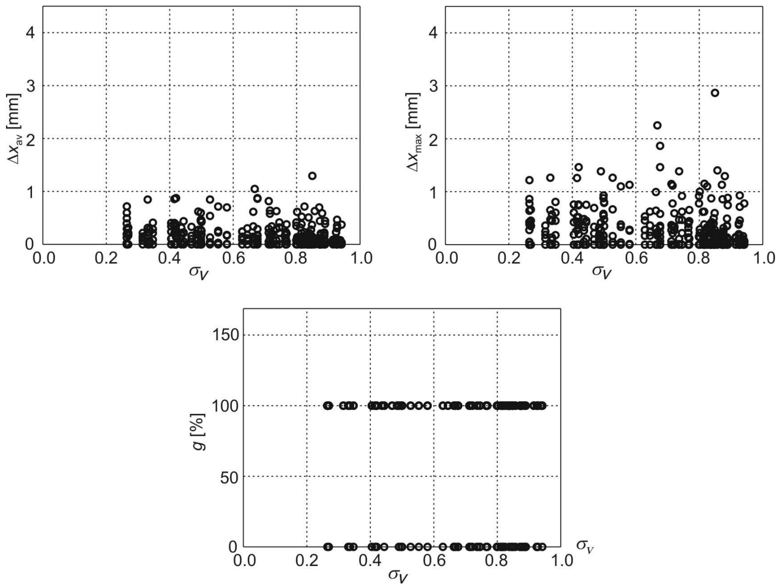

Fig. 8. IDS method (k_α = 2) – dependence of Δx_{av}, Δx_{max} and $g\%$ indices from the reliability level of disturbed observation

Figure 8 shows the graphs of Δx_{av}, Δx_{max} and $g\%$ indices for the IDS method at $k_\alpha = 2.0$. In this case, the method 62 times wrongly detected observations contaminated by a gross error.

The results presented above concern one specific levelling network structure. To confirm resulting conclusions, below are presented results of additional study. The design matrix **A** has been substituted by 5 different matrices having identical dimensions but randomly set element values. Furthermore, only the variant of faultless observations ($\boldsymbol{\varepsilon}_0$) with a single gross error, having its size defined according to (24) was used.

$$\mathbf{A}_1 = \begin{bmatrix} 0.75 & 0.00 & 0.68 & 0.00 & 0.46 \\ -0.70 & -0.60 & 0.34 & 0.00 & 0.88 \\ 0.82 & 0.98 & 0.53 & 0.00 & 0.99 \\ 0.00 & 0.00 & 0.00 & 0.89 & -0.73 \\ -0.60 & 0.96 & -0.80 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.40 & -0.94 & -0.49 \\ 0.00 & -0.80 & 0.37 & 0.00 & -0.60 \\ 0.74 & -0.30 & -0.50 & -0.32 & 0.00 \end{bmatrix} \mathbf{A}_2 = \begin{bmatrix} 0.00 & 0.77 & 0.00 & 0.77 & 0.41 \\ 0.00 & 0.00 & -0.51 & 0.76 & 0.94 \\ -0.54 & -0.72 & 0.82 & 0.46 & 0.74 \\ -0.41 & 0.00 & 0.35 & -0.83 & 0.00 \\ 0.00 & -0.57 & 0.00 & 0.00 & -0.68 \\ 0.33 & 0.48 & 0.00 & 0.48 & 0.00 \\ 0.65 & -0.47 & 0.35 & -0.45 & -0.75 \\ 0.61 & 0.55 & -0.73 & 0.00 & -0.87 \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} 0.00 & -1.76 & 1.86 & -1.56 & 0.00 \\ -1.82 & -0.73 & -1.79 & 0.00 & 0.00 \\ -0.69 & -1.84 & 0.00 & 1.21 & -1.17 \\ 0.00 & -1.93 & -1.52 & -1.96 & 0.00 \\ -0.69 & 0.00 & 1.69 & 0.00 & 1.29 \\ 0.00 & -0.74 & 1.22 & -1.24 & -0.68 \\ 0.66 & 0.81 & 1.14 & -0.86 & 0.74 \\ 0.71 & 0.00 & 0.00 & 1.76 & 1.64 \end{bmatrix} \mathbf{A}_4 = \begin{bmatrix} 1.81 & 0.00 & 1.10 & 2.73 & -1.42 \\ -2.82 & 0.98 & 0.00 & -1.89 & 2.67 \\ -1.11 & -2.10 & 1.03 & 0.00 & 0.00 \\ 0.00 & -1.84 & 2.78 & 2.76 & 2.07 \\ 2.13 & -1.58 & 0.00 & 2.01 & 0.00 \\ -1.07 & 1.72 & -1.13 & 1.87 & 0.00 \\ 2.62 & 0.00 & 1.23 & 0.00 & 1.62 \\ 0.98 & -1.03 & 1.74 & 0.00 & -2.60 \end{bmatrix}$$

$$\mathbf{A}_5 = \begin{bmatrix} 0.00 & -2.30 & 2.60 & -2.86 & 0.00 \\ -2.05 & -1.99 & -3.99 & -2.14 & -2.38 \\ 0.00 & 2.90 & -2.20 & 0.00 & -2.00 \\ 2.30 & 0.00 & 0.00 & -3.00 & 3.71 \\ 1.62 & 1.57 & -2.40 & -3.10 & 0.00 \\ 3.87 & 2.99 & -3.70 & 2.46 & -1.70 \\ 2.71 & -1.40 & 0.00 & -2.80 & 2.85 \\ -2.40 & 0.00 & 0.00 & 0.00 & 1.53 \end{bmatrix}$$

The graphs presented in Figure 9 confirm that the correctness of robust estimation results depends on the level of internal reliability of observation contaminated by a gross error.

It could easily be verified that identical studies lead for a properly strong geodetic network structure would show satisfying effectiveness of single gross error detection by the use of robust estimation methods analysed in this paper. For example, to the levelling network presented in Figure 1 four observations (bold line in Fig. 10) were added and it has been assumed that all observations are equally precise.

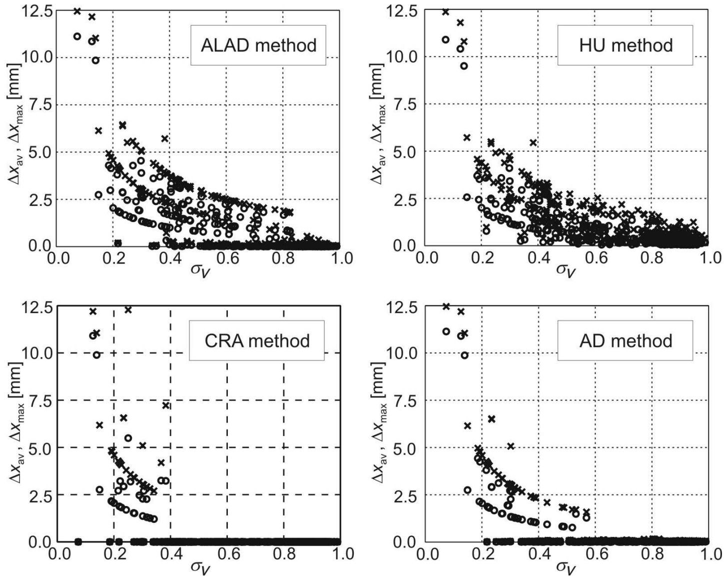

Fig. 9. Dependence of Δx_{av} (symbol ○), Δx_{max} (symbol ×) indices from the reliability level of disturbed observation

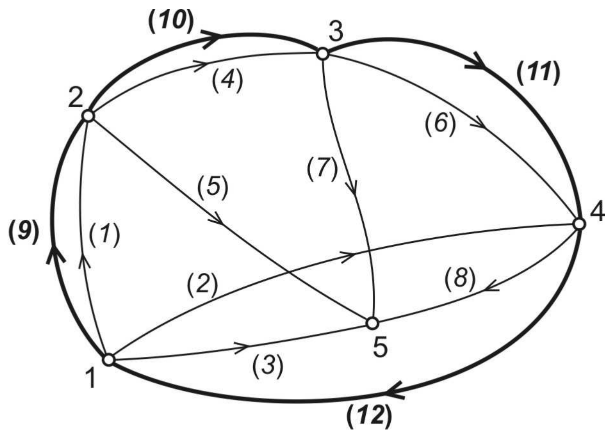

Fig. 10. Reinforced test levelling network

For such network, the internal reliability indices for all observations are included within the range <0.79, 0.83>. Figure 11 shows the graphs of indices Δx_{av} and $g\%$ obtained for the HU method (c = 2). For the remaining methods, a similar image of Δx_{av} and $g\%$ indices was obtained.

www.czasopisma.pan.pl PAN www.journals.pan.pl
POLSKA AKADEMIA NAUK

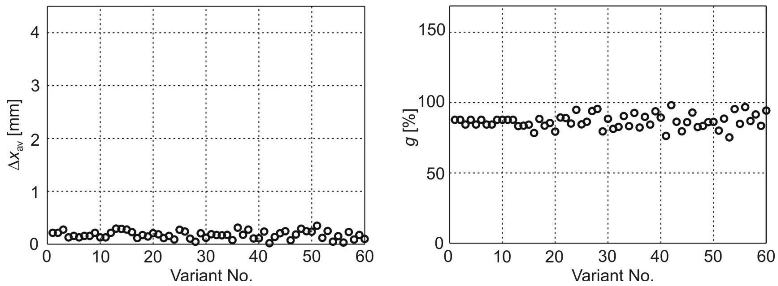

Fig. 11. Graphs of Δx_{av} and $g\%$ indices obtained for the HU method

5. Conclusions

The results of research presented in the paper entitle to formulate the following conclusions.

1. The effectiveness of gross error detection by the use of robust estimation methods depends on the level of internal reliability of a geodetic network. All analysed methods provided satisfactory results only at an appropriate high reliability level of observation disturbed by a gross error. For the ALAD and HU method, this level was $\sigma_V \geq 0.7$. The CRA and AD methods provided satisfactory results even at $\sigma_V \geq 0.5$. In case of a gross error occurring in an observation having a low reliability index, the robust estimation results could not be correct.
2. Basing on results showed in Figure 9, acquired for faultless observations disturbed by single gross error, it can be stated that:
   a) CRA and AD methods, where no constant parameters were being used, provided better results than HU and ALAD methods. For the latter, the change of the constant parameter value does not cause any significant changes in the effectiveness of a gross error detection;
   b) the highest effectiveness of a gross error detection has been stated for the CRA method and the lowest for the HU method.
3. From the dispersion of Δx_{av}, Δx_{max} and $g\%$ indices presented in Figures 3÷6 it results that the correctness of a gross error detection depends also on the dispersion of random errors within the network. In some cases, despite a high reliability index of disturbed observation, the estimation results were not correct. Opposite situations happened as well.
4. The additionally used IDS method showed a quite good detection effectiveness of the gross error being introduced into the observation. The effectiveness of this method depends also on the parameter k_α value:
   – when $k_\alpha = 2.5$, the IDS method detected correctly the disturbed observation in 97.3% variants of observation systems;
   – when $k_\alpha = 2.0$, the effectiveness of this method was only as much as 76.2%.
5. The results of research presented above confirm the conclusion in the paper (Prószyński, 1994) that for an effective detection of a single gross error, the geode-

tic network should be designed in such way that for each observation the relation $\sigma_V \geq 0.71$ was fulfilled.

6. In the case of two or more gross errors appearing in the network, a significant decrease of efficiency of tested methods is expected. Studies within this scope are continued.

Acknowledgments

The paper presents the results of research carried out in 2009 within the statutory research in the Department of Engineering Surveying and Detail Surveys at the Warsaw University of Technology.

References

Aduol F.W.O., (1999): *Robust geodetic parameter estimation under least squares through weighting on the basis of the mean square error*, Technical Reports of the Department of Geodesy and Geomatics, University of Stuttgart, Report No 1999.6-1, pp. 5-15.

Baarda W., (1968): *A testing procedure for use in geodetic networks*, Netherlands Geodetic Commission, Publications on Geodesy, New Series, Vol. 2, No 5, Delft.

Cross P.A., Price D.R., (1985): *A strategy for the distinction between single and multiple gross errors in geodetic networks*, Manuscripta Geodaetica, Vol. 10(4), pp. 172-178.

Ding X., Coleman R., (1996): *Multiple outlier detection by evaluating redundancy contributions of observations*, Journal of Geodesy, Vol. 70, pp. 489-498.

Ethrog U., (1991): *Statistical test of significance for testing outlying observation*, Survey Review, Vol. 31, No 240, pp. 62-70.

Gui Q., Gong Y., Li G., Li B., (2007): *A Bayesian approach to the detection of gross errors based on posterior probability*, Journal of Geodesy, Vol. 81, pp. 651-659.

Hekimoglu S., Berber M., (2003): *Effectiveness of robust methods in heterogeneous linear models*, Journal of Geodesy, Vol. 76, pp. 706-713.

Huber P.J., (1964): *Robust estimation of a location parameter*, Ann. Math. Statist., No 35, pp. 73-101.

Kadaj R., (1984): *Die Methode der besten Alternative: ein Ausgleichungsprinzip für Beobachtungssysteme*, ZfV, Vol. 109, No 6, pp. 301-308.

Kadaj R., (1988): *Eine verallgemeinerte Klasse von Schätzverfahren mit praktichen Anwendungen*, ZfV, Vol. 113, No 4, pp. 157-166.

Kamiński W., (2000): *Robust Bayesian estimation in geodetic network* (in Polish), Wyd. UWM, seria: Rozprawy i Monografie 26, Olsztyn.

Kamiński W., Wiśniewski Z., (1992): *Analysis of chosen, resistant to gross errors, methods of adjustment of geodetic observations. Part II. Analysis* (in Polish), Geodesy and Cartography, Vol. XLI, No 3-4, pp. 183-195.

Kamiński W., Wiśniewski Z., (1994): *The method of growing rigor for the adjustment of geodetic observations contaminated by gross errors*, Manuscripta Geodaetica, Vol. 19(2), pp. 55–61.

Knight N.L., Wang J., (2009): *A Comparison of Outlier Detection Procedures and Robust Estimation Methods in GPS Positioning*, Journal of Navigation, Vol. 62, pp. 699-709.

Nowak J., (2002): *Research of reliability properties of photogrammetric techniques in aspect of engineering applications* (in Polish), PhD thesis, Warsaw University of Technology, Warsaw.

Pope A.J., (1976): *The statistics of residuals and the detection of outliers*, NOAA Technical Report NOS65 NGS1, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National

www.czasopisma.pan.pl PAN POLSKA AKADEMIA NAUK www.journals.pan.pl

Ocean Survey, Geodetic Research and Development Laboratory, National Geodetic Survey, Rockville, Maryland, 23 pp.

Prószyński W., (1994): *Criteria for internal reliability of linear least squares models*, Bulletin Géodésique, Vol. 68, pp. 162-167.

Wiśniewski Z., (1991): *Comparative categories in analysis of methods of geodetic observation adjustment*, Zeszyty Naukowe AGH, No 1423, Geodezja, Vol. 112, pp. 41-55.

Wiśniewski Z., (1993): *An Alternative to the method of least absolute deviations* (in Polish), Geodesy and Cartography, No XLII (3), pp. 199-214.

Xu P.L., (1993): *Consequences of constant parameters and confidence intervals of robust estimation*, Bollettino di Geodesia e Scienze Affini, Vol. 52, No 3, pp. 231-249.

Xu P.L., (2005): *Sign-constrained robust least squares, subjective breakdown point and the effect of weights of observations on robustness*, Journal of Geodesy, Vol. 79, pp. 146-159.

Skuteczność wybranych metod estymacji mocnej a poziom niezawodności sieci

Mieczysław Kwaśniak

Zakład Geodezji Inżynieryjnej i Pomiarów Szczegółowych
Wydział Geodezji i Kartografii
Politechnika Warszawska
Pl. Politechniki 1, PL-00 661 Warszawa
e-mail: m.kwasniak@gik.pw.edu.pl

Streszczenie

W pracy przedstawiono wyniki badań nad zależnością skuteczności wybranych metod estymacji odpornej od poziomu niezawodności wewnętrznej sieci geodezyjnej. W badaniach wykorzystano symulowane komputerowo układy obserwacyjne, dzięki czemu możliwe było przeanalizowanie wielu wariantów różniących się w zaplanowany sposób. Do badań wybrano cztery metody estymacji odpornej różniące się istotnie podejściem do modyfikacji wag. Dla porównania, badaniom skuteczności poddano również popularną w praktyce geodezyjnej metodę wykrywania błędów grubych bazującą na wynikach estymacji MNK, tzw. metodę Baardy. Z przeprowadzonych badań wynika, że istnieje związek pomiędzy poziomem niezawodności wewnętrznej sieci a skutecznością metod estymacji odpornej. W większości przypadków, w których obserwacja obarczona błędem grubym charakteryzowała się niskim wskaźnikiem niezawodności wewnętrznej, estymacja odporna prowadziła do uzyskania wyników istotnie odbiegających od oczekiwanych.