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Reliability analysis of the results of RTN GNSS surveys of building structures using indirect methods of measurement

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Abstract: This paper presents accuracy characteristics of determining the position of corners of building structures with RTN GNSS surveying, using indirect methods of measurement. The studies included the following methods: a point on a straight line, intersection of straight lines and distance-distance intersection. The research experiment analyzed the coordinates of the corners of building structures obtained from the surveys and the mean errors of their position as well as mutual relationships of check measurements, or tie distances. The accuracy analysis also took into account base errors determined in real time. Statistical analysis of these parameters was carried out, as a result of which a distance-distance intersection method was very well rated. For other methods, the results were diversified. The article also emphasizes a need to search for other solutions to modernize the indirect methods of measurement in such a way that their use in RTN GNSS surveys would give results most probable when compared to the real ones.

Keywords: RTN GNSS, surveys of building structures, indirect methods of measurement

1. Introduction

Spatial determination of surface structures, such as building structures, is implemented using a variety of measurement techniques using, among others, the so-called offsets. Each such object is a field detail which is extremely important both in terms of creating a base map and the cadastre. The most commonly used measurement method for such objects (including building structures) includes tacheometric method of surveying. In recent years, however, real-time RTN GNSS surveys are applied more and more frequently, which use ASG EUPOS or other reference station networks included in the National Geodetic and Cartographic Resource database. The application of such a measurement method to determine the location of a building structure has numerous advantages, but also disadvantages and limitations. Undoubtedly, the

greatest advantage of using the real-time measurement method is a fast performance of surveys and no need to seek or establish a control (with the exception of control points) to which a building should be measured. A counter argument against the use of RTN GNSS method in surveying large structures is the horizon, which is partially or substantially obscured (which results from the nature of things), and thus there is a potentially lower accuracy of determining the position of each measured element relative to the classical surveying method, i.e. tacheometry.

One of the solutions which could enhance reliability of the coordinates determined in such conditions is to use a special technique proposed by (Bakuła, 2013). This technique, presented in (Bakuła, 2013), allows for a reliable determination of coordinates, even in very difficult conditions of observation (e.g., for the points located along the edge of a forest, or located completely in a forest, near buildings or near transmission power lines). Coordinates of the point are determined by three GNSS receivers arranged in a line on a special base (separated by a distance equal to 0.5 m), ensuring a reliable control of GNSS surveys.

Due to the specific nature of a building (high building structure for RTN GNSS surveying method), the surveys also use known, indirect methods of measurement, which ultimately result in the achievement of the intended purpose. However, a fundamental question arises here, whether the application of indirect methods used in the RTN GNSS surveying to determine the position of building structures meets all the accuracy requirements? Accuracy parameters to be met by a measured building (detail of the 1st class of accuracy) is governed by the Regulation (MIA, 2011). Errors that affect the final results of the measurements are estimated by both the accuracy of the surveying technology itself (GNSS surveys) and the reliability of the observers work. The reasons for the above-mentioned errors, their effect on the measurement results and their possible elimination or reduction have been discussed in (Kowalczyk, 2011). Analysis of the results obtained in real-time measurements and a need to carry out check measurements in adverse conditions of observation have also been discussed in the work by (Pelc-Mieczkowska, 2012). This trend of implementing real-time measurements in extremely difficult conditions is also continued in (Bakuła et al., 2009). It is true that their object of research (forest areas) has a completely different character, however, the most important conclusion from these studies is the ability to perform RTN GNSS measurements in difficult conditions with far-reaching precautions, control of measurement results and additional independent RTN GNSS solutions based on repetitive, independent re-initialization of uncertainty.

In addition to the errors generated by GNSS and observer's factors, geometric structure applied in indirect measurement methods used in RTN GNSS technology affects the final accuracy of positioning a measured building. A very detailed analysis of the accuracy of various indirect methods of measurement applied in real time surveys has been presented in (Beluch and Krzyżek, 2005). According to the criterion of accuracy of the point position – the corners of building structures – the authors suggested the following indirect measurement methods: a point on a straight line, intersection of straight lines and distance-distance intersection.

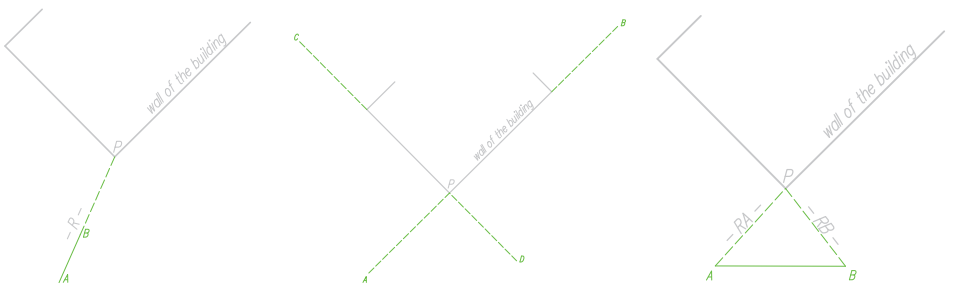
In each of the proposed indirect method of measurement, geometric structure conditioned by various factors were analyzed. An essential factor in the studied structures was taking setting out error into consideration (of the base point or of the corner of a building) in the following methods: a point on a straight line, intersection of straight lines. None of the methods, however, studied one of the most important elements, which has a significant effect on the mean error of positioning an object which is difficult to access (a building), namely the accuracy of determining the base points with the RTN GNSS technology. Therefore, the conclusions proposed by (Beluch and Krzyżek., 2005) are correct, but only in the context of the internal consistency of the study of geometric structure of the applied indirect methods of measurement.

While performing RTN GNSS surveys using indirect methods of measurement, one should be fully aware of the accuracy of the obtained final results. The measurement result of each field detail is characterized by a mean error of positioning relative to the control points. When calculating the mean error of the field detail positioning, an important factor will be appropriate assumptions regarding the errors, or lack of errors, of the reference points. If the final location of a building subject to real time measurements is determined by using indirect methods based on the so-called base points, then the mean error of positioning its corner will also depend on the accuracy of determining the base points. The complexity of this problem has been presented in (Kowalczyk, 2011). The author discusses, among others, the problem of a necessity to measure ground control points, which in the case of the 2000 system, increases the accuracy of fitting measurement results into the existing control.

Regardless of various indirect factors affecting the accuracy of determining the base points using RTN GNSS surveying, the quality of satellite signals reaching the antenna of the GNSS receiver will have an undoubtedly significant effect on their determination. The use of one of the systems, GPS, GLONASS, GALILLEO, or their combined use, strongly affect the quality of the obtained observations. A need to implement the third satellite system GALILEO and the modernization of the existing systems was already introduced in the year 2000 by (Gunter, 2000). A necessity of combining different GNSS systems is also emphasized by (Angrisano et al., 2013). They prove that such an action is the most desirable, as the multi-constellation system ensures a better satellite availability compared to GPS only, thereby providing for a greater accuracy, continuity and integrity of positioning. In their research, (Angrisano et al., 2013) combine GPS and GLONASS systems for positioning a single point, and their results are evaluated in a variety of configurations. Today, GPS and GLONASS systems are integrated most frequently in measurements, and GALILEO is still under development.

2. Research methods and research experiment

Figures 1-3 present field situation for the above-mentioned three indirect methods of measurement using RTN GNSS technology, which serve as research methods to the research experiment used later. For each method (Fig. 1-3), an assessment of the accuracy of positioning a point (the corner of a building) was conducted, with the assumption of falsity of direct references points, or base points. Various geometric structures of the above methods and their effect on the mean error of a field detail have already been discussed in (Beluch and Krzyżek, 2005) and are not explored in the current study. For the situations shown in Figures 1 and 2, a hypothetical situation of the location of the base points was illustrated, and for Figure 3 the most optimal geometry of the structure, in which the intersected angle is close to 180º, was adopted.



a b c

Legend:
A, B, C, D – base points measured with RTN GNSS surveying technology,
R – linear measurement of the section B-P with an open frame tape measure,
P – a point (the corner of a building) determined indirectly,
RA, RB – linear measurement of the sections A-P and B-P with an open frame tape measure.

Fig. 1. Positioning a building corner with RTN GNSS technology using the following methods: a) *a point on a straight line*, b) *intersection of straight lines*, c) *distance-distance intersection*

While considering the situation depicted in Figure 1a – the method of a point on a straight line – particular attention should be paid to the mean error of positioning the base points. Applying the rule of transmission of errors to the formula to determine the coordinates of the point *P* which is being defined, we shall get the mean error of positioning the point of the building corner:

$$m_P = \pm\sqrt{m_B^2 + m_R^2 + R^2 \cdot m_{A(AB)}^2} \tag{1}$$

where:

A_{BP} – azimuth of the *BP* side equal to the azimuth of the *AB* side,
m_B – mean error of positioning the base point *B*,

m_R – mean linear measurement error (open frame tape measure)
R – value of linear measurement.

The assumption that the azimuth of the *BP* side is equal to the azimuth of the *AB* side does not take into account the falsity of setting out point *B* on the straight line *AP*, which will be justified later in the article. The accuracy of positioning the base point *B* with RTN GNSS technology will have the biggest influence on the mean error of positioning the corner of a building. It is well known that the accuracy of real time measurements for the horizontal coordinates X and Y is at the level of 1-3 cm. This is confirmed by the analyzes of various concepts of scientific research, including the experiment conducted by (Krzyżek and Kudrys, 2011). Only solutions which ensure the mean error of a base point obtained at the level of ±0.01m give satisfactory accuracy results for the determined point (the corner of a building). The distance between the base point (*B*) and the point which is determined is also important. Each additional meter of the value *R* can generate the final mean error m_p by up to a few centimeters, which is very important with the 1st class of accuracy.

Another method shown in Figure 1b – intersection of straight lines – is a variant which can be easiest and fastest implemented in the field. In this method, to determine the mean error of positioning the point *P* (the corner of a building), the Gaussian model ought to be applied to the X and Y coordinates, determined from the intersection of the straight lines *AB* and *CD*.

$$X_P = \frac{Y_C - Y_A + \lambda \cdot X_A - \mu \cdot X_C}{\lambda - \mu} \tag{2}$$

$$Y_P = Y_A + \lambda \cdot (X_P - X_A)\ lub\ Y_P = Y_C + \mu \cdot (X_P - X_C)$$

where:

$\lambda = tgA_{(AB)}$ and $\mu = tgA_{(CD)}$

To simplify, we shall assume equal accuracy of the position of the base points *A, B, C, D*, which is $m_A = m_B = m_C = m_A$ by denoting them by m_N and the coefficient errors $m_\lambda = m_\mu$ denoting them by m_α. However, it should be strongly emphasized that this assumption is purely hypothetical, since, in fact, each of the base points has a slightly different accuracy of its location. Nevertheless, most often these are not significant differences in values, so this assumption is justified in practical terms. Then we get the formula (3) for the mean error m_p of positioning the point *P*, which is the corner of the building:

$$m_P = \pm\sqrt{\left(\frac{2+\lambda^2+\mu^2}{(\lambda-\mu)^2} + (1 + 2 \cdot \lambda^2)\right) \cdot m_N^2 + \left(\frac{i^2+j^2}{(\lambda-\mu)^4} + (X_P - X_A)^2\right) \cdot m_\alpha^2} \tag{3}$$

where:

$$i = [X_A \cdot (\lambda - \mu)] - (Y_C - Y_A + \lambda \cdot X_A - \mu \cdot X_C)$$

$$j = [-X_C \cdot (\lambda - \mu)] - (Y_C - Y_A + \lambda \cdot X_A - \mu \cdot X_C)$$

$$m_\alpha = \sqrt{\frac{2 \cdot [(X_B - X_A)^2 + (Y_B - Y_A)^2]}{(X_B - X_A)^4} \cdot m_N^2}$$

On the basis of the formula (3), it is possible to state unambiguously that the accuracy of determining base points will be the main factor generating the final error of the positioning of the point *P* (corner of the building). In this case, even the upper limit, that is, ±0.03m, of a potential mean error of the base point measured with RTN GNSS technology, will be fully satisfactory in terms of accuracy of determining the position of the point *P*. Thus, apparently it seems that this method is relatively accurate and can be used in the case of positioning the details which are difficult to access by the RTN GNSS method – the corners of buildings. The remainder of this article will, however, present other factors affecting the final qualification of this method in the context of the discussed subject matter.

The last indirect method used in real time measurements of buildings is the distance-distance intersection method (Fig. 1c). Once again, applying the rule of transmission of errors to the formula to determine the X,Y coordinates of the point *P*, we shall obtain the formula (4) for the mean error of positioning the corner of a building

$$m_P = \pm \sqrt{m_{A/B}^2 + \left(\frac{m_R \cdot \sqrt{2}}{sin(\alpha + \beta)}\right)^2} \tag{4}$$

where:
$m_{A/B}$ – mean error of positioning the base point *A* or *B*,
m_R – mean linear measurement error (open frame tape measure),
α – angle of the vertex at point A calculated from the cosine theorem,
β – angle of the vertex at point B calculated from the cosine theorem

In the formula (4), the main elements that determine the final value of the mean error of the corner of a building m_p is the accuracy of a linear measurement m_R and the errors of determining the base points m_A or m_B in real time. A linear measurement error has a slightly greater effect on the mean error of positioning the point *P* than base point errors. Nevertheless, even for potentially large values of the above-mentioned errors, for example ±0.03m, the final mean error of the determined detail is relatively low, and therefore completely acceptable for the 1st class of accuracy.

For all the presented indirect methods of measurement used in surveying to determine the corners of buildings with the use of RTN GNSS surveying technology, a summary of the values of mean errors of the points determined depending on the errors of individual observations have been drawn (Table 1).

Table 1. Mean error of positioning the corner of a building for various indirect methods of measurement based on observation errors.

Method of measurement	[m]	[m]		
		0.01	0.02	0.03
		[m]		
Intersection of straight lines	0.01	0.020	0.020	0.020
	0.02	0.041	0.041	0.041
	0.03	0.061	0.061	0.061
	0.04	0.082	0.082	0.082
	0.05	0.102	0.102	0.102
A point on a straight line		R=5/6/7 [m]		
		[m]		
		0.01	0.02	0.03
	0.01	0.072/0.086/0.100	0.074/0.088/0.102	0.078/0.091/0.104
	0.02	0.143/...	0.144/...	0.146/...
Distance-distance intersection		[m]		
		0.01	0.02	0.03
	0.01	0.017	0.030	0.044
	0.02	0.025	0.034	0.047
	0.03	0.033	0.041	0.052
	0.04	0.042	0.049	0.058
	0.05	0.052	0.057	0.066

While analyzing the results presented in (Table 1) it could be stated that the accuracy classification of individual indirect methods differs from those presented in Chapter 2 of (Beluch and Krzyżek, 2005). The best results in terms of accuracy of positioning the corner of a building (Tab. 1) can be obtained by distance-distance intersection followed by intersection of straight lines. On the other hand, the method of a point on a straight line, although theoretically possible to apply in this type of measurement, very quickly reaches the limiting mean error of a field detail of the 1st class of accuracy. In order to accurately interpret the results contained in (Table 1), a mutual relationship of the X, Y coordinates of the corners of buildings for different methods must be analyzed as well. To have an even more reliable assessment of the suitability of the various indirect methods to survey buildings, the measurement results

(X and Y coordinates) using RTN GNSS technology should be referred to the results obtained from the tacheometric surveying. The tacheometric method of surveying can then provide some kind of reference for other results of building measurements due to its well-known accuracy and universality of use in surveying.

For this purpose, a research experiment was conducted on the selected test structure – a single-family building with a total of 20 points to be determined (corners of the building), located in the municipality of Jerzmanowice-Przeginia in the district of Krakow (Fig. 2).

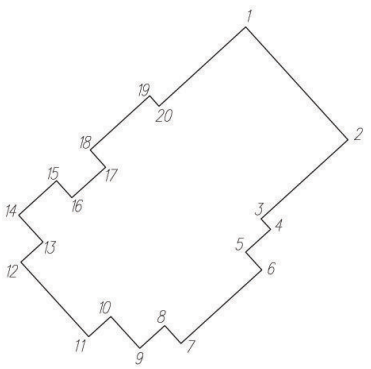


Fig. 2. The test structure – a single-family building

All corners of the building 1-20 (Fig. 2) were measured with RTN GNSS surveying technology using three indirect methods: a point on a straight line, intersection of straight lines and distance-distance intersection. The surveys were performed using Topcon HiperPro receiver, in real time, in relation to the reference station network ASG EUPOS, using NAWGEO service. The accuracy of the carried out surveys with the use of the NAWGEO service can be found at (www.asgeupos.pl), and in (Doskocz and Uradziński, 2010), as well as in (Krzyżek and Skorupa, 2012). As a result of the survey, for each indirect method X, Y coordinates were obtained in the PL-2000 system (the mean value of 30 epochs). Then, for the purposes of the research, a control was established on the structure, whose coordinates X, Y were determined in the PL-2000 system with the static GNSS surveys tied to the reference station network ASG EUPOS. Each measurement session lasted 110 minutes. Using the POZGEOD service of the ASG EUPOS system, the coordinates of the measurement control points were aligned to obtain a mean error of positioning m_p the points at the level of a few millimeters. The measurement control was a reference for tacheometric method of surveying buildings. High accuracy of positioning the measurement control points allowed for the adoption of the tacheometric survey results as a benchmark for the X and Y coordinates, which were obtained from RTN GNSS surveying. In this way, a homogeneous character of the coordinates of the control and of the corners of the building, determined in real time and with tacheometric method of surveying, were

obtained. For all the test points of the building, check measurements in the form of tie distances were performed as well.

The following Tables 2-4 and Figures 3-5 represent the following measurement results:

– Table 2 and Figure 3 – differences in the values of the coordinates dX, dY and the vector dL (Tab. 2), and only of the vector dL (Fig. 3), between the indirect methods of measurement, using RTN GNSS technology and Tacheometry (each method with each one),
– Table 3 and Figure 4 – differences in the check measurements (tie distances) dR of the building between the indirect methods of measurement using RTN GNSS technology and Tacheometry (each method with each one),
– Table 4 and Figure 5 – mean errors of positioning m_p the corners of buildings in individual indirect methods of measurement using RTN GNSS technology and Tacheometry.

For a clear definition of the accuracy assessment of indirect methods of measurement presented in Tables 2-4 and in Figures 3-5, a statistical analysis of the test sample was also performed (Tabl. 2-4). Due to the limited volume of the article, Tables 2-4 present the results of dX, dY, dL, dR, and m_p for a few records only (while maintaining statistical analysis for all 20 test points). In order to simplify the editing of the performed analysis, conventional notation of individual methods and measurement results was made, according to the following designations of the methods: tacheometric surveying – *T*, a point on a straight line – *NP*, intersection of straight lines – *PP*, distance-distance intersection – *WL*, measurement with an open frame tape measure – *R*.

For the vectors dL results contained in Table 2, the null hypothesis H_0 was defined: *the average value of μ, for the vectors dL in various combinations of the methods – T-PP, T-NP, T-WL, PP-NP, PP-WL, NP-WL – is equal to a fixed value of*

$$H_0 : \mu = \mu_0 \tag{5}$$

For the null hypothesis, alternative hypothesis H_1 was specified: *the average value of μ, for the vectors dL in various combinations of the methods – T-PP, T-NP, T-WL, PP-NP, PP-WL, NP-WL – is less than the predetermined value of* μ_0

$$H_1 : \mu \leq \mu_0 \tag{6}$$

The adopted fixed value of $\mu_0 = 0.10\,m$ requires an explanation. It results from the boundary value of the mean error of positioning a field detail of the 1st class of accuracy. Such reasoning may raise some controversy as to being too tolerant in terms of accuracy, however, there are no specific legal standards today relating to this type of comparative parameters. Hence, it seems to be justified to adopt the value of the mean error of positioning a field detail of the 1st class of accuracy as a fixed value of μ_0 to the statistical analysis.

Table 2. Differences in the values of the coordinates dX, dY and the vector dL between the indirect methods of measurement using RTN GNSS technology and Tacheometry

Differences in the values of the coordinates																		
No. of the corner of the building	T-PP			T-NP			T-WL			PP-NP			PP-WL			NP-WL		
	dX	dY	dL	dX	dY	dL	dX	dY	dL	dX	dY	dL	dX	dY	dL	dX	dY	dL
	[mm]																	
1	13	-41	43	22	37	43	-5	6	8	9	78	79	-18	47	50	-27	-31	41
2	70	-88	112	27	40	48	9	-6	11	-43	127	134	-61	82	102	-18	-46	49
3	76	-209	222	-14	4	15	-11	0	11	-90	213	231	-87	209	226	3	-4	5
....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
20	-28	-131	134	-5	-6	8	9	68	69	23	125	127	37	199	202	14	74	75
average value – μ			101			30			17			107			106			32
average deviation – δ			11			3			3			13			13			4
test model – T			0.114			-22.508			-25.946			0.525			0.446			-16.070
for significance level of 5%, quantile k = n-1 : 19																		
critical value of T-Student distribution – t : 2.0930																		
hypothesis verification			*H0*			*H1*			*H1*			*H0*			*H0*			*H1*

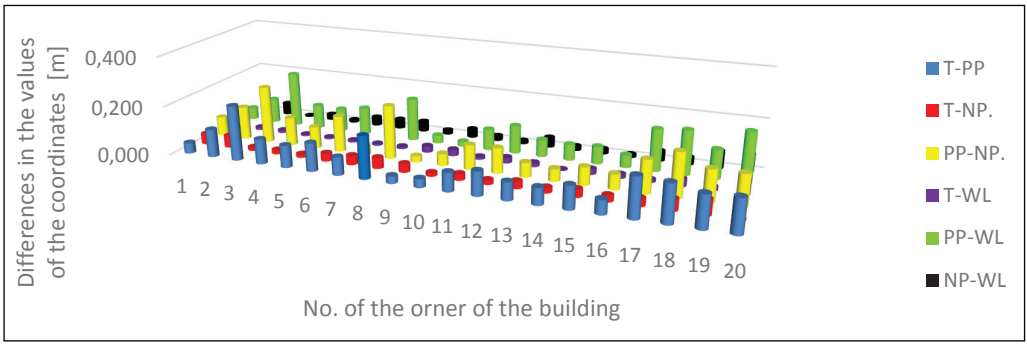


Fig. 3. Differences in the values of the vectors dL between the indirect methods of measurement using RTN GNSS technology and Tacheometry

In turn, for the results representing check measurements (tie distances – Tab. 3) the following null hypothesis H_0 was defined: *the average value of μ, for the differences in tie distances dR of the building in various combinations of the methods – T-PP, T-NP, T-WL, T-R, PP-NP, PP-WL, PP-R, NP-WL, NP-R, R-WL – is equal to the predetermined value of* $\mu_0 = 0.04\,m$ (formula 5). For such a null hypothesis, the alternative hypothesis H_1 is as follows: *the average value of μ, for the differences in tie distances dR of the building in various combinations of the methods – T-PP, T-NP, T-WL, T-R, PP-NP, PP-WL, PP-R, NP-WL, NP-R, R-WL – is less than the predetermined value of* μ_0 (formula 6). In this case, the fixed value of $\mu_0 = 0.04\,m$ results from the adoption of the limiting error (the quadruple value resulting from the regulation MIA, 2011) of the accuracy of tie distance measurement with a linear measuring tape. For the purpose of the studies, a linear measurement was assumed with an open frame tape measure with an accuracy of ±0.01 m.

Table 3. Differences in check measurements (tie distances) dR of the building between the indirect methods of measurement using RTN GNSS technology and Tacheometry

Differences in tie distances – dR										
Side	T-PP [m]	T-NP [m]	T-R [m]	T-WL [m]	PP-NP [m]	PP-R [m]	PP-WL [m]	NP-R [m]	NP-WL [m]	R-WL [m]
1_2	-0.074	-0.002	0.010	-0.018	0.072	0.084	0.055	0.012	-0.017	-0.028
2_3	0.085	0.054	0.004	0.009	-0.031	-0.081	-0.076	-0.050	-0.045	0.005
3_4	0.176	-0.021	-0.003	-0.003	-0.197	-0.178	-0.179	0.019	0.018	0.000
....	....	....	....	....	....	....	....	....	....	....
20_1	0.094	0.050	-0.027	-0.055	-0.044	-0.121	-0.149	-0.077	-0.106	-0.028
average val. – ***μ***	-0.010	-0.005	-0.003	-0.005	0.004	0.007	0.004	0.002	0.000	-0.003

Table 3.

average deviation – δ	0.023	0.008	0.003	0.004	0.025	0.024	0.025	0.010	0.009	0.004
test model – T	-2.130	-5.375	-16.865	-10.356	-1.434	-1.399	-1.439	-3.896	-4.434	-10.696
for significance level of 5%, quantile k = n-1 : 19										
critical value of T- Student distribution – t : 2.0930										
hypothesis verification	*H(1)*	*H(1)*	*H(1)*	*H(1)*	*H(0)*	*H(0)*	*H(0)*	*H(1)*	*H(1)*	*H(1)*

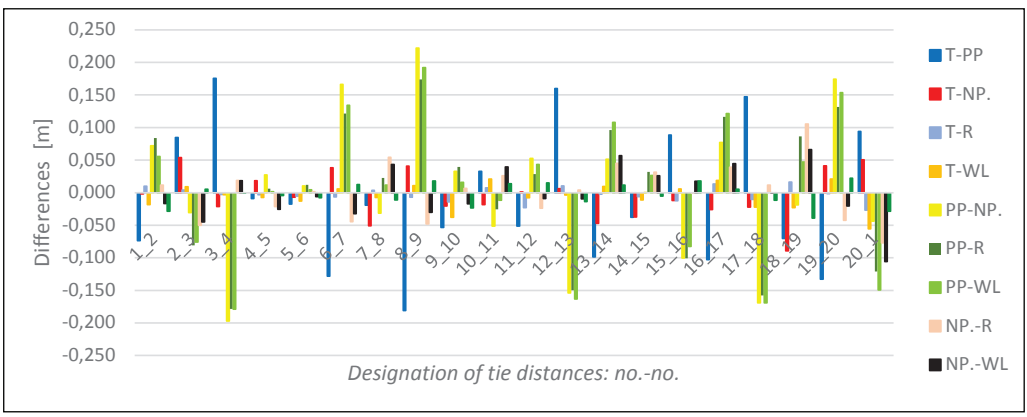


Fig. 4. Differences in check measurements (tie distances) dR of the building between the indirect methods of measurement using RTN GNSS technology and Tacheometry

The last attribute which is analyzed statistically are mean errors of positioning m_p corners of buildings (Tab. 4). Similarly, as in the case of adopting a fixed value for the data in (Tab. 2), also in this case the mean error of positioning a field detail of the 1st class of accuracy, which is the corner of a building, is the basis for adopting $\mu_0 = 0.10\,m$. Therefore, the null hypothesis H_0 is as follows: *the average value μ, for mean errors of positioning m_p corners of buildings in various methods – T, PP, NP, WL, – is equal to the predetermined value of* $\mu_0 = 0.10\,m$ (formula 5). On the other hand, the alternative hypothesis H_1 was defined as: *the average value μ, for mean errors of positioning m_p corners of buildings in various methods – T, PP, NP, WL, – is less than the predetermined value of* μ_0 (formula 6).

Table 4. Mean errors of positioning m_p corners of buildings in various indirect methods of measurement using RTN GNSS technology and Tacheometry

m_p				
No.	T [m]	PP [m]	NP [m]	WL [m]
1	0.023	0.029	0.056	0.019
2	0.017	0.044	0.194	0.019
3	0.023	0.040	0.069	0.029
....	....	....	....	....
20	0.018	0.028	0.096	0.031
average value – $\boldsymbol{\mu}$	0.019	0.035	0.097	0.023
average deviation – $\boldsymbol{\delta}$	0.001	0.001	0.009	0.001
test model – T	-131.036	-49.966	-0.325	-74.494
for significance level of 5%, quantile k = n-1 : 19				
critical value of T- Student distribution – t : 2.0930				
hypothesis verification	*H(1)*	*H(1)*	*H(0)*	*H(1)*

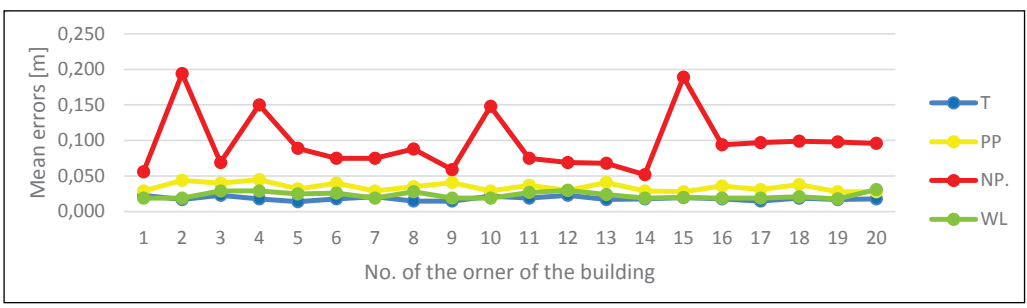


Fig. 5. Mean errors of positioning m_p corners of buildings in various indirect methods of measurement using RTN GNSS technology and Tacheometry

The basic assumption was to prove alternative hypotheses for all three of the analyzed values: the vectors dL, differences in tie distances dR and mean errors of positioning m_p corners of buildings. For this objective, the average value μ was calculated for the above mentioned attributes in the various combinations of the methods. Then the mean value of the standard deviation δ was calculated from the following formula:

$$\delta(\mu) = \frac{\hat{\delta}}{\sqrt{n}} \tag{7}$$

where:

$\hat{\delta}$ – standard deviation for the vectors dL, differences in tie distances dR and mean errors of positioning m_p corners of buildings in the various combinations of the calculation methods,

n – number of the vectors dL, differences in tie distances dR and mean errors of positioning m_p corners of buildings in the various combinations of the calculation methods.

In order to verify these hypotheses, one of the models of the *T*-test of a single mean value was selected (formula 8). This formula is provided for the test sample of the number <30.

$$T = \frac{\mu - \mu_0}{\delta(\mu)} \tag{8}$$

The statistical study assumed a significance level of the value of $\alpha = 5\%$ and $k = n - 1$degrees of freedom. For such adopted values, the variable of the *T*-test of a single mean value assumes *T-Student* distribution for which two-tailed critical region with respect to the quantile $t(\alpha,k)$ as constructed.

The obtained results (Tables 2-4) in the context of the conducted statistical analysis lead to the following subject conclusions:

Differences in the values of the coordinates dX, dY and the vector dL between the indirect methods of measurement using RTN GNSS technology and Tacheometry.

The assumptions adopted in the statistical analysis against the obtained results of calculations (Tab. 2) lead to two conclusions.

The first conclusion concerns the relationship between two indirect methods of measurement using RTN GNSS surveying technology – *NP* and *WL* – relating to the tacheometric method of surveying *T* and mutual relationships: *the average value of μ, for the vectors dL in various combinations of the methods – T-NP, T-WL, NP-WL – is statistically significant, resulting in the rejection of the hypothesis H_0 in favor of the hypothesis H_1*. This means that the coordinates in the methods of a point on a straight line *NP* and of the distance-distance intersection *WL* are at a similar level of the values in mutual relationships and in relation to the tacheometric method *T*. For the combination of the methods *T-NP,* the differences in the values of the coordinates dX (Tab. 2) vary within the range of -0.014 m to +0.027 m, with a mean value of 0.008m, dY (Tab. 2) vary from -0.062 m to +0.040 m, with a mean value of 0.002 m, and the vector dL from 0.008 m to 0.062 m (Tab. 2 and Fig. 3). On the other hand, the relationship of the coordinate differences in the methods of *T-WL* are as follows: dX (Tab. 2) vary within the range from -0.014 m to +0.009 m, with a mean value of -0.001 m, dY (Tab. 2) vary from -0.024 m to +0068 m with a mean value of 0.007 m, and the vector dL from 0.003 m to 0.069 m (Tab. 2 and Fig. 3). As it can be seen, only the differences in the values of the coordinates dY for the relationship of the methods (*T-NP* and *T-WL*) have a significant range of values, 0.102 m and 0.092 m,

respectively. However, with the mean value of the vector dL at the level of ±0.030 m (for the methods *T-NP*) and ±0.017 m (for the methods *T-WL*), as well as the adopted assumption in the statistical analysis, it can be assumed that the obtained results in the relationships of both methods to the reference method (*T-NP* and *T-WL*) are convergent. The same conclusions can be drawn from the comparison of the obtained results (Tab. 2 and Fig. 3) in the relationships directly between two indirect methods of measurement *NP-WL*.

The second conclusion concerns the relationship of the third indirect method of measurement using RTN GNSS surveying – *PP* – relating to the tacheometric method of surveying *T* and other indirect methods of measurement *NP* and *WL: the average value of μ, for the vectors dL in various combinations of the methods -T-PP, PP-NP, PP-WL is statistically insignificant, resulting in a lack of rejection of the hypothesis* H_0 *in favor of the hypothesis* H_1. For the method of intersection of straight lines *PP,* the differences in the values of the coordinates dX, dY and the vector dL in all combinations of the methods *T-PP, PP-NP, PP-WL* are at a similar level, and so: dX (Tab. 2) vary within the range of approximately ±0.20 m with mean values of -0.01 m (*T-PP*), 0.01 m (*PP-WL*) and 0.02 (*PP-NP*), dY (Tab. 2) vary within the range from approximately ±0.25 m with mean values of -0.07 m (*T-PP*) and 0.07 m (*PP-NP, PP-WL*), and the vector dL is within the range of approximately 0.25 m (Tab. 2 and Fig. 3) with the mean values of 0.10 m (*T-PP*) and 0.11 m in other combinations of the methods. Given the relatively high mean value of the vectors dL (0.10 m and 0.11 m) as well as a large spread of the results of the obtained differences in the values of the coordinates dX and dY, and the assumptions made for the statistical analysis, it can be assumed that the obtained measurement results in the method of intersection of straight lines *PP* are divergent.

Based on the conclusions of convergence or divergence of the obtained measurement results in the context of the statistical analysis of the various indirect methods of measurement, it is still not possible to clearly and comprehensively assess the suitability of a given method in the measurements of the corners of buildings with the use of RTN GNSS surveying technology. Some other parameters that affect the accuracy of the measurement results should still be considered.

Differences in check measurements (tie distances) dR of a building between the indirect methods of measurement using RTN GNSS technology and Tacheometry.

The assumptions adopted in the statistical analysis against the obtained results of calculations (Tab. 3) lead to two conclusions.

The first conclusion concerns the relationship between two indirect methods of measurement using RTN GNSS technology – *NP* and *WL* – relating to the tacheometric method of surveying *T* and the measurement with an open frame tape measure, as well as their mutual relationships: *the average value of μ, for the differences in tie distances dR of a building in various combinations of the methods – T-PP, T-NP, T-WL, T-R, NP-WL, NP-R, R-WL – is statistically significant, resulting in the rejection of the hypothesis* H_0 *in favor of the hypothesis* H_1. In this case, for the measurement results, a reference parameter was adopted in the form of a direct linear

measurement of tie distances with an open frame tape measure *R*. The spread of the obtained differences in the measured tie distances *dR* (Fig. 4) and summarized in the various methods are as follows: *T-PP* from -0.181 m to 0.176 m, *T-NP* from -0.089 m, 0.054 m, *T-R* from -0.027 m to 0.016 m, *T-WL* from -0.055 m to 0.021 m, *NP-R* from -0.077 m to 0.105 m, *NP-WL* from -0.106 m to 0.066 m, *R-WL* from -0.039 m to 0.022 m. Based on the presented ranges, it can be pre-assumed that the best results are obtained with the distance-distance intersection method and, of course, Tacheometry, for which these ranges are at the level of 6 and 4 centimeters, respectively. The results from the other methods are definitely more divergent from the reference measurements and reach the range of even several centimeters. Nevertheless, if we consider the mean values of the obtained differences in tie distances in the analyzed combinations of the methods (Tab. 3), it is noticeable that they are at a very low level, not exceeding ±0.01 m. Based on the data in (Tab. 3), once again it is possible to talk about the convergence of the measurement results in the combinations of the methods *T-NP, T-WL, T-R, NP-WL, NP-R, R-WL*. Given the negative accuracy evaluation of the method *PP,* its positive relationship with the tacheometric method of surveying *T-PP* comes as a surprise. However, when we take a closer look at the test values of the model (*T*) and the critical value of T-Student distribution for the combination of *T-PP* (Tab. 3), it is immediately apparent that these values (absolute values) are relatively similar. If the significance level of 10% was adopted, then the combination of the methods *T-PP* would not be included in the positively evaluated conclusions.

The second conclusion concerns the relationship of the indirect measurement method using RTN GNSS technology – *PP* – relating to other indirect methods of measurement *NP* and *WL,* as well as to the measurements with the open frame tape measure *R*: *the average value of μ, for the differences in tie distances dR of a building in various combinations of the methods – PP-NP, PP-R, PP-WL – is statistically insignificant, resulting in a lack of rejection of the hypothesis* H_0 *in favor of the hypothesis* H_1. For various combinations of the methods, ranges of the obtained differences in tie distances *dR* are very significant and they are at the level of ±0.35 m to ±0.42 m (Fig. 4). Although small values assume average values of the differences in tie distances *dR*, from 0.004 m to 0.007 m, their large spread and assumptions of the statistical analysis indicate a limited degree of confidence in the application of the method *PP* in the measurements of buildings using RTN GNSS technology.

Prior accuracy analyzes of indirect methods of measurement using RTN GNSS technology to measure the corners of buildings emphasizes great advantages of the methods of a point on a straight line *NP* and distance-distance intersection *WL,* while strongly limits the possibilities of using the method of intersection of straight lines *PP*. However, in order to have a complete picture of the obtained measurement results, one must still analyze the results in the context of mean errors of positioning the determined corners of a building.

✓ ***Mean errors of positioning*** m_p ***corners of buildings in various indirect methods of measurement using RTN GNSS technology and Tacheometry.***

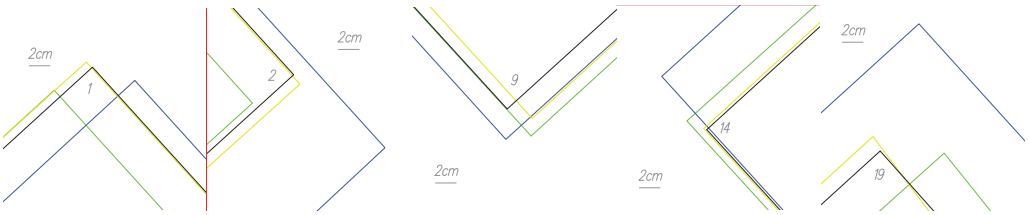
The assumptions adopted in the statistical analysis against the obtained results of calculations (Tab. 4) lead to two conclusions.

The first conclusion concerns two indirect methods of measurement using RTN GNSS technology – *PP* and *WL* – as well as the tacheometric method of surveying *T*: *the average value of μ, for the mean errors of positioning m_p corners of buildings in various methods – T, PP, WL – is statistically significant, resulting in the rejection of the hypothesis H_0 in favor of the hypothesis H_1*. The average value of the mean error of positioning points for the methods *T* and *WL* is at a similar level and amounts to ±0.019 m and ±0.023 m, respectively, and their ranges fall within ±0.015 m to ±0.023 m (for the method *T*) and from ±0.019 m to ±0.031 m (for the method *WL*) – Figure 5. These are very satisfactory results in terms of measurement accuracy of details of the 1st class of accuracy. A big surprise, however, is the occurrence of the method of intersection of straight lines *PP* and the absence of a point on a straight line method *NP* in the positively evaluated conclusions. In the method of intersection of straight lines *PP,* the average value of the mean error of positioning is ±0.035 m, and its range includes the values of ±0.028 m to ±0.045 m (Fig. 5). Although the method *PP* had somewhat weaker results than those obtained from the methods *T* and *WL*, we can talk, with some approximation, about the convergence of the mean errors of all three methods: *T, WL, PP.*

The second conclusion relates to one indirect method of measurement using RTN GNSS technology – *NP*: *the average value of μ, for the mean errors of positioning m_p corners of buildings in the method NP is statistically insignificant, resulting in a lack of rejection of the hypothesis H_0 in favor of the hypothesis H_1*. The average value of the mean error of positioning a corner of a building determined with a point on a straight line method *NP* stands at ±0.097 m, and so it is at the level of a limiting error provided for the details of the 1st class of accuracy. The mean error of positioning of a corner of a building ranges from ±0.052 m to ±0.194 m (Fig. 5).

Evaluation of the usefulness of each of the measurement methods using RTN GNSS surveying in the measurements of building structures in the context of mean errors of positioning specific points brings conclusions which are a little different from the previous ones in the relationship of the methods *PP* and *NP*. In this case, a point on a straight line method *NP* would not be recommended for the implementation of measurements of buildings, in contrast to the intersection of straight lines *PP*, which is characterized by high accuracy of positioning determined points.

For better visualization of the positioning of the corners of a building in the relationships of the particular measurement methods, Figure 6 presents fragments of sections of the building walls for the measured corner of the building using all methods. Due to a large area of the figure for all 20 determined points, only 5 examples of the corners of the building were presented.



Legend:

Tacheometric method of surveying
The method of a point on a straight line
The method of intersection of straight lines
The method of distance-distance intersection

Fig. 6. Interrelationships of individual corners of the building determined with various methods of measurement

Looking in detail at the interrelationships of individual corners of the building walls determined by various methods, a significant deviation of the position of points determined by the method of intersection of straight lines, as compared with the other methods, is immediately noticeable. On the other hand, the closest position is represented by the results of distance-distance intersection method and then of a point on a straight line, relative to the reference method, i.e. Tacheometry. As it follows from the analysis of the statistical test sample, a big surprise, especially in the context of the results visualized in Figure 6 and contained in Table 2, are significant values of mean errors of positioning corners of the building with the method of a point on a straight line, in contrast to the method of intersection of straight lines (Tab. 4). The complexity of the problem derives from the formulas implementing mean errors of positioning the determined points in both methods. In the formula determining the accuracy of the position of the point m_p by the method of a point on a straight line (formula 1), the mean error of the base point m_B plays a key role in the final accuracy of determining the corner of a building. It is (m_B) is not only a similar attribute to the value of the mean error from the linear measurement with an open frame tape measure m_R, but it is also a component of the attribute $m^2_{A(AB)}$, which comes in correlation of the product with the square value from the linear measurement, e.g. with an open frame tape measure. It is this factor which significantly affects a rapid increase in the value of the mean error m_p of the position of the corner of a building, with the base point errors greater than ±0.01 m. Only achieving a mean error of the base point m_p at the level of ±0.005 m would allow for a substantial increase in the value of the linear measurement while retaining the accuracy requirements for determining a detail of the 1st class of accuracy. On the other hand, (formula 4) describing the mean error of positioning the corner of a building carried out by the method of intersection of straight lines, actually depends only on the accuracy of determining the base points. In this formula, although there are various calculation relationships, they are always correlated only with respect to the data relating to the position of the base points

(homogeneity of the attributes) instead of to the other observations. In the method of intersection of straight lines, this factor generates a slower increase in the value of the mean error of positioning a point which is being determined (the corner of a building), compared to the accuracy analysis of point positioning in the method of a point on a straight line.

3. Summary and conclusions

Based on the experimental research, it can be concluded that only the method of distance-distance intersection used in the RTN GNSS surveying to determine the position of the corners of a building structure gives the best results in every respect – capturing the most probable coordinates and their mean errors as well as a consistent geometric structure (consistency of tie distances). Other methods of a point on a straight line and of intersection of straight lines require some modifications in order to enhance their reliability in determining a field detail, such as a building structure, in real time. Results and their summary contained in Tables 2-4 and in Figures 3-5, as well as a graphical representation of the final effect of applying indirect methods to measure a building with the use of RTN GNSS technology (Fig. 6), suggest a search for even better solutions, resulting in a much greater accuracy of surveyed buildings in real time. Those solutions should be applied which lead to the consistency of tie distances of a building measured with RTN GNSS surveying technology using the indirect method of measurement, as compared to the direct linear measurement with an open frame tape. In this way, the geometric structure of a building will be consistent with the reality. However, the errors of setting out base points should be kept in mind as well (the method of intersection of straight lines or a point on a straight line), which should be eliminated in the process of implementing a consistency of the geometric structure of a building. Another modernization of the indirect methods, especially of the method of intersection of straight lines, would be the modification of their results (the coordinates X and Y), relative to the most probable position of the corners of a building structure. As the final result, we would obtain a building structure determined in real time, using the indirect methods of measurement, of the same position (the X, Y coordinates) and the geometric structure (consistency of tie distances) within the tolerance limits of measurement errors. These are the research issues which are currently being implemented by the author and they constitute the subject matter of the anticipated subsequent articles.

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References

Angrisano, A., Gaglione S. & Gioia C. (2013). Performance assessment of GPS/GLONASS single point positioning in an urban environment. *Acta Geodaetica et Geophysica*, Vol. 48, Issue 2, 149-161.

Bakuła, M. (2013). Study of Reliable Rapid and Ultrarapid Static GNSS Surveying for Determination of the Coordinates of Control Points in Obstructed Conditions. *Journal of Surveying Engineering*, Vol. 139, No. 4, 188-193.

Bakuła, M., Oszczak S. & Pelc-Mieczkowska R. (2009). Performance of RTK Positioning in Forest Conditions: Case Study. *Journal of Surveying Engineering*, Vol. 135, No. 3, 125-130.

Beluch, J. & Krzyżek R. (2005). GPS RTK technology used for indirect methods of topographical surveys. *Technical Sciences. Supplement/University of Warmia and Mazury in Olsztyn*, suppl. 2, 47-60.

Doskocz, A. & Uradziński M. (2010). Initial analysis of the accuracy of posiotion determination using ASG-EUPOS NAVGEO (RTK VRS). *Geomatics and Environmental Engineering*, Vol. 4, no. 4, 113-118.

Gunter, W. Hein, (2000). From GPS and GLONASS via EGNOS to Galileo – Positioning and Navigation in the Third Millennium. *GPS Solutions*, Vol. 3, Issue 4, 39-47.

Kowalczyk, K. (2011). Analysis of the errors generated during the measurement of situational details RTK GPS method. *Acta Scientiarum Polonorum Geodesia et Descriptio Terrarum*, 10(1), 5-22.

Kowalczyk, K. (2011). Evaluation of usefulness of measurements of inaccessible points using GPS RTK method with consideration of base errors. *Przegląd Geodezyjny*, 10, 7-12.

Krzyżek, R. & Kudrys J. (2011). Analysis of coordinates time series obtained using the NAWGEO service of the ASG-EUPOS system. *Geomatics and Environmental Engineering*, Vol. 5, no. 4, 39-46.

Krzyżek, R. & Skorupa B. (2012). Analysis of accuracy of determination of eccentric point coorditanes of the KRAW permanent geodetic station in RTK GPS measuring mode with the application of the NAWGEO service of the ASG EUPOS system. *Geomatics and Environmental Engineering*, Vol. 6, no. 4, 35-46.

MIA. (2011). *Regulation of Minister of Interior and Administration – in case of technical standards of performing detailed surveys and working out and sending results of these surveys to National Geodetic and Cartographic Database* (in Polish). Journal of Laws of 2011 No. 263, entry 1572. Warsaw: Government Legislation Centre.

Pelc-Mieczkowska, R. (2012). Analysis of GPS/RTK positioning reliability in hard observational conditions. *ZeszytyNaukowe Politechniki Rzeszowskiej*, z. 59 (1/II), 217-226.

Analiza wiarygodności wyników pomiaru budynków technologią RTN GNSS z wykorzystaniem pośrednich metod pomiaru

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Streszczenie

W pracy przedstawiono charakterystykę dokładnościową wyznaczenia położenia naroży budynków technologią RTN GNSS z wykorzystaniem pośrednich metod pomiarów. W badaniach uwzględniono metody: punktu na prostej, przecięcia prostych oraz wcięcia liniowego. W eksperymencie badawczym analizowano uzyskane z pomiarów współrzędne naroży budynków i ich średnie błędy położenia oraz wzajemne relacje pomiarów kontrolnych, czyli czołówek. Analiza dokładnościowa uwzględniała także błędy punktów bazowych wyznaczonych w czasie rzeczywistym. Przeprowadzono analizę statystyczną wspomnianych parametrów, w wyniku których bardzo dobrze oceniono metodę wcięcia liniowego. W stosunku do pozostałych metod wyniki są zróżnicowane. Zwrócono uwagę na konieczność poszukiwania innych rozwiązań zmierzających do zmodernizowania pośrednich metod pomiaru w taki sposób, aby ich zastosowanie w technologii RTN GNSS dawało rezultaty najbardziej prawdopodobne do rzeczywistych.