Saleh's Model of AM/AM and AM/PM Conversions is not a Model without Memory – Another Proof

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Abstract—**The so-called Saleh's representation for description of the AM/AM and AM/PM conversions, occurring in communication power amplifiers, consists of two expressions that describe them as functions of a real-valued baseband signal modulating the carrier amplitude. It is a common view that this description forms a model without memory. We show here that the above belief is not correct; just the opposite is true. To prove this, we take into account an equivalent description of the Saleh's model called the quadrature model of bandpass nonlinearities and express it in a form of a nonlinear operator. Afterwards, we check whether this operator possesses a zero memory. To this end, we use an appropriate theorem of the nonlinear systems theory. Finally, as a result of this investigation, we observe that the memory of the above operator is nonzero.**

Kyeywords—**Saleh's model of AM/AM and AM/PM conversions, power amplifiers, memory effects, in-phase and quadrature representation of nonlinear devices**

I. INTRODUCTION

POWER amplifiers (PAs) used in wireless communications, as for example in satellite ones [1], exhibit nonlinear distortions in the operating regimes in which they work. These kinds of distortions manifest by occurrence of the so-called AM/AM and AM/PM conversions. More precisely, they are caused by the nonlinearities of amplifiers working in the transition area between their linear and nonlinear regions of operation, and also, slightly, in the nonlinear region.

There were presented in the literature many ways of modeling the AM/AM and AM/PM conversions occurring in the PAs. Here, we mention only a few of them [1-3], maybe most representative ones. An up-to-date survey of them was presented in a tutorial [4].

Saleh developed in [3] a simple but accurate model for description of the AM/AM and AM/PM conversions. It is one of the most frequently used and cited models. Note also that this model is assumed in the literature to belong to the group of memoryless ones. In other words, it does not incorporate memory effects.

Let us now describe briefly the Saleh's model in what follows, similarly as in [5]. We will do this after [3]. So, to this end, assume that the following bandpass input signal

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$$x(t) = r(t)\cos\left(\omega_c t + \psi(t)\right) \qquad (1)$$

is applied to a PA. In (1), $\omega_c = 2\pi f_c$ with f_c meaning the carrier frequency, t denotes a time variable, and $j = \sqrt{-1}$. Moreover, the bandpass input signal $x(t)$ contains a slowly varying real-valued baseband signal $r(t)$ that modulates the carrier amplitude. Furthermore, the carrier phase changes with time according to a function $\psi(t)$. This function, similarly as $r(t)$, represents also a slowly varying baseband signal.

Then, after [3] (for more details see also, for example, [6]), the PA output signal can be expressed as

$$y(t) = A(r(t))\cos\left(\omega_c t + \psi(t) + \Phi(r(t))\right). \qquad (2)$$

In (2), $A(r(t))$ and $\psi(t) + \Phi(r(t))$ denote the carrier amplitude and its phase, respectively, at the amplifier output. The function $A(r(t))$ is assumed to be a nonlinear function of $r(t)$. That is $A(r(t)) \neq a \cdot r(t)$ holds here, where a stands for a real-valued constant. Moreover, it is assumed that an additional phase component $\Phi(r(t))$ in (2) does not equal zero and depends upon the slowly varying baseband signal $r(t)$. This means that a kind of the amplitude modulation expressed by the nonlinear characteristic $A(r(t))$ as well as the phase modulation expressed by another function $\Phi(r(t))$ occur here, and both are caused by the signal $r(t)$. Therefore, we refer to $A(r(t))$ as the AM/AM characteristic and to $\Phi(r(t))$ as the AM/PM characteristic.

In [3], Saleh developed a model, which approximates the above characteristics by the following functions

$$A(r(t)) = \frac{a_1 r(t)}{1 + a_2 (r(t))^2}, \quad \Phi(r(t)) = \frac{b_1 (r(t))^2}{1 + b_2 (r(t))^2}, \quad (3)$$

where the coefficients a_1 and a_2 as well as b_1 and b_2 assume real values and need adjustment to the measured data for a given amplifier.

Note now that (2) can be rewritten in an equivalent form as

$$y(t) = A(r(t))\cos(\Phi(r(t)))\cos(\omega_c t + \psi(t)) - \qquad (4)$$
$$- A(r(t))\sin(\Phi(r(t)))\sin(\omega_c t + \psi(t)) \ .$$

Next, it follows from (4) that the in-phase $p(t)$ and quadrature $q(t)$ components of the amplifier output signal $y(t)$ are given by

$$p(t) = P(r(t))\cos(\omega_c t + \psi(t)) \qquad (5)$$

and

$$q(t) = -Q(r(t))\sin(\omega_c t + \psi(t)) \ , \qquad (6)$$

respectively. Further, the form of the functions $P(r(t))$ and $Q(r(t))$ in (5) and (6), accordingly, follows from (4) and (3). That is we have

$$P(r(t)) = A(r(t))\cos(\Phi(r(t))) =$$
$$= \frac{a_1 r(t)}{1 + a_2(r(t))^2} \cdot \cos\left(\frac{b_1(r(t))^2}{1 + b_2(r(t))^2}\right) \qquad (7)$$

and

$$Q(r(t)) = A(r(t))\sin(\Phi(r(t))) =$$
$$= \frac{a_1 r(t)}{1 + a_2(r(t))^2} \cdot \sin\left(\frac{b_1(r(t))^2}{1 + b_2(r(t))^2}\right) \ . \qquad (8)$$

The expressions (4), (5), and (6) were the basis of a model developed in [2] and in other publications in the 1970s, which preceded Saleh's article [3]. In these papers, the functions $P(r(t))$ and $Q(r(t))$ were approximated in a different way from that in [3], for example, by using polynomials or Bessel functions. The resulting model was called the quadrature model of bandpass nonlinearities [2] or simply the quadrature model.

One thing is worth noting in the above model, namely that both its components, the in-phase and quadrature one, depend upon the AM/AM and AM/PM conversions. This is immediately seen by looking at the relations (5-8). However, when the AM/PM conversion becomes zero, "the quadrature path" in the aforementioned model vanishes.

In [3], Saleh linked also his model with the quadrature one, exploiting the relations (7) and (8). In the rest of this paper, we will use a form of the above model that was used by Benedetto et al. in [1].

II. Quadrature Model and Nonlinear Operator Describing It

In [1], Benedetto et al. used the quadrature model to describe behaviour of a nonlinear device (as, for example, a power amplifier working in its nonlinear region of operation) exhibiting the AM/AM and AM/PM conversions. This model is visualized here in Fig. 1; it is also used by the author of this paper in a complementing work [5].
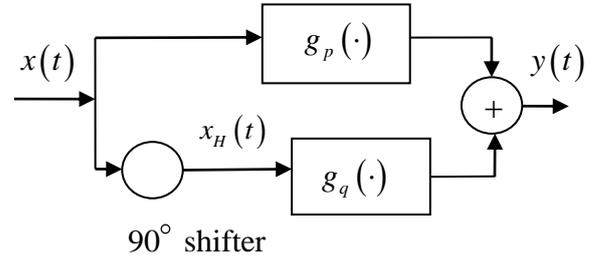
Fig. 1. The quadrature model of a nonlinear device exhibiting the AM/AM and AM/PM conversions.

The quadrature model of Fig. 1 consists of two parallel paths (branches). The upper one represents the model in-phase component, but the lower path its quadrature component, respectively. The first is described by a memoryless nonlinearity $g_p(\cdot)$, which can represented, for example, by a power series with real-valued coefficients. Similarly, the nonlinearity $g_q(\cdot)$ in the lower branch is assumed to be memoryless, too. So, it can be also modelled by a power series with real-valued coefficients, another one. However, the latter nonlinearity in Fig. 1 is preceded by a 90 degree shifter. And this makes the difference between the branches in Fig. 1.

Consider now the operation of shifting the signal $x(t)$ in Fig. 1 by 90 degrees to get another signal denoted as $x_H(t)$. This operation can be described in the time domain by the following convolution operation

$$x_H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \ . \qquad (9)$$

The relation (9) describes the operation performed by the so-called Hilbert transformer, in the time domain. For more details regarding this topic, see, for example, [7].

Taking into account (9) and looking at Fig. 1, we see that the output signal $y(t)$ at the nonlinear device scheme of Fig. 1 is related with the input signal $x(t)$ by the relation

$$y(t) = g_p(x(t)) + g_q(x_H(t)) = \qquad (10)$$
$$= g_p(x(t)) + g_q\left(\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau\right) \ .$$

Further, (10) can be expressed in the operator form as

$$y(t) = N(x(t)) = N_1(x(t)) + N_2(x(t)) \qquad (11)$$

with

$$N_1(x(t)) = g_p(x(t)) = \\ = c_1 x(t) + c_2 (x(t))^2 + c_3 (x(t))^3 + ... \qquad (12)$$

and

$$N_2(x(t)) = g_q(x_H(t)) = \\ = d_1 x_H(t) + d_2 (x_H(t))^2 + d_3 (x_H(t))^3 + ... = \\ = \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau + \frac{d_2}{\pi^2} \left(\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \right)^2 + \\ + \frac{d_3}{\pi^3} \left(\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \right)^3 + ... \quad . \qquad (13)$$

By the way, note that (13) can be rewritten in the following form

$$N_2(x(t)) = \\ = \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau + \frac{d_2}{\pi^2} \left(\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \right)^2 + \\ + \frac{d_3}{\pi^3} \left(\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \right)^3 + ... = \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau + \\ + \frac{d_2}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x(\tau_1)}{t-\tau_1} \frac{x(\tau_2)}{t-\tau_2} d\tau_1 d\tau_2 + \\ + \frac{d_3}{\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x(\tau_1)}{t-\tau_1} \frac{x(\tau_2)}{t-\tau_2} \frac{x(\tau_3)}{t-\tau_3} d\tau_1 d\tau_2 d\tau_3 + ... \quad . \qquad (14)$$

And observe that the second series, equivalent to the first one in (14), is nothing else than a Volterra series [8]. Therefore, the operator $N_2(x(t))$ can be also called the Volterra operator [9] and, as such, it possesses memory [9]. The usage of the Volterra series to prove that the Saleh's model is not a model without memory is dealt with in [5], [10], and [11].

The operator $N(x(t)) = (Nx)(t)$ introduced above is defined by relations (11-13). (Note that $(Nx)(t)$ stands for another useful form of notation used for operators.) This operator is a sum of two other operators $N_1(x(t))$ and $N_2(x(t))$. The former is, according to (12), expanded in a

power series with real-valued coefficients c_1, c_2, c_3, and so on; we will see that it is memoryless. But the latter is a little bit more complicated; it is a composite operator. Its inner part means performing in the time domain the operation defined by the Hilbert transformer [7], but the outer one follows from the memoryless nonlinearity $g_q(\cdot)$, which is here (see (13)) expanded in a power series with real-valued coefficients d_1, d_2, d_3, and so on. Further, as we will see later, the operation performed by the Hilbert transformer is an operation with memory. So, altogether, in connection with the memoryless operation indicated by $g_q(\cdot)$, it makes the operator $N_2(x(t))$ to be an operator having memory. At this point, we remark that it is important to see that the operator $N_2(x(t))$, as indicated, works on the signal $x(t)$. In other words, it is an operator associated with the device input signal $x(t)$.

Remember now that the Saleh's model [3] and related ones like the quadrature model [3] or the quadrature model of bandpass nonlinearities [2] are assumed in the literature [4] to belong to the group of memoryless ones. That is they do not incorporate memory effects. In what follows, we will correct this common belief using the generic description for all the aforementioned models in form of the operator (or sum of operators), which was just developed above. We will do this by showing that this operator possesses nonzero memory. However, here, we will use another approach - different from that utilized in [5] - to achieve the aforementioned goal. In [5], a mathematical tool called the Volterra series [1] was used. But in this paper, we exploit some general results (theorems and lemmas) proved in the theory of operators. And with their use, we obtain the same result as the one got in [5]. Therefore, we can treat the derivations presented here as complementing the corresponding ones in [5] as well as strengthening them at the same time.

III. CONSIDERATION OF LINEAR PART OF OPERATOR N

To understand better the verification problem of whether a nonlinear operator possesses memory or does not have it, let us start with checking this on the linear part of the operator N defined in the previous section. So, consider now the linear part of (11), which will be given by

$$y_l(t) = N_l(x(t)) = N_{1l}(x(t)) + N_{2l}(x(t)) \qquad (15)$$

with

$$N_{1l}(x(t)) = g_{pl}(x(t)) = c_1 x(t) \qquad (16)$$

and

$$N_{2l}(x(t)) = g_{ql}(x_H(t)) = d_1 x_H(t) = \\ = \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \qquad , \qquad (17)$$

where the added index "l" means a linear part of.

In the next step, we invoke a theorem regarding representation of linear systems published by Sandberg in [12]. It has been shown in [13] that this theorem can be used in classification of linear systems or linear operators with respect to the memory held. Without going into details, the aforementioned theorem [12] and the discussion in [13] referred to as say the following:

Let N_l be a linear, causal, and time-invariant operator (representation, map). Further, take a "windowing" operator W_σ given by

$$(W_\sigma x)(t) = w_\sigma(t) x(t) \quad (18)$$

with the function $w_\sigma(t)$ having the following form

$$w_\sigma(t) = \begin{cases} 1 & \text{for } |t| \le \sigma \\ 0 & \text{for } |t| > \sigma \end{cases}, \quad (19)$$

where σ means any positive number. Now, under the above assumptions, if

$$(N_l x)(t) = (N_l W_\sigma x)(t) \quad (20)$$

holds for every positive number σ; then the operator N_l possesses no memory. That is it is a memoryless one.

In what follows, we will use the above statement to verify whether the following operators N_l, N_{1l}, and N_{2l} occurring in (15) have memory or are memoryless. And let us start with the operator N_{1l}. Obviously, we take a time while of verification or observation, say t_v, lying in the range of activity of the windowing function $w_\sigma(t)$. That is we choose $t_v \le \sigma$. Note that for this time while we obtain $(N_{1l} x)(t_v) = a_1 x(t_v)$ for every $\sigma > 0$ and $t_v \le \sigma$. In the next step, see that for the same values of $\sigma > 0$ and $t_v \le \sigma$, we get $(N_{1l} W_\sigma x)(t_v) = a_1 x(t_v)$ because then $(W_\sigma x)(t_v) = x(t_v)$ holds. So, the condition (20), i.e. $(N_{1l} x)(t_v) = (N_{1l} W_\sigma x)(t_v)$, is fulfilled in this case. Finally, this means that the operator N_{1l} is memoryless.

Now, consider the operator N_{2l}. Obviously, as before, we choose a time while of verification $t_v \le \sigma$. That is such time while which lies in the range of activity of the windowing function $w_\sigma(t)$. Observe now that we get

$$(N_{2l} x)(t_v) = \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t_v - \tau} d\tau \quad (21)$$

from (17). However, we have

$$(N_{2l} W_\sigma x)(t_v) = \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{w_\sigma(\tau) x(\tau)}{t_v - \tau} d\tau =$$
$$= \frac{d_1}{\pi} \int_{-\sigma}^{\sigma} \frac{x(\tau)}{t_v - \tau} d\tau \quad (22)$$

for the value of the composite operator $N_{2l} W_\sigma$ at the time while t_v by virtue of the function $w_\sigma(t)$ given by (19).

Comparison of the right-hand sides of (21) and (22) shows that they contain different expressions. Let us check whether their values differ for the time while t_v. To this end, let us evaluate their difference Δ_{2l} by taking into account (21) and (22). That is we will consider

$$\Delta_{2l} = (N_{2l} x)(t_v) - (N_{2l} W_\sigma x)(t_v) =$$
$$= \frac{d_1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t_v - \tau} d\tau - \frac{d_1}{\pi} \int_{-\sigma}^{\sigma} \frac{x(\tau)}{t_v - \tau} d\tau . \quad (23)$$

Next, observe that (23) can be rewritten as

$$\Delta_{2l} = (N_{2l} x)(t_v) - (N_{2l} W_\sigma x)(t_v) =$$
$$= \frac{d_1}{\pi} \int_{-\infty}^{-\sigma} \frac{x(\tau)}{t_v - \tau} d\tau + \frac{d_1}{\pi} \int_{\sigma}^{\infty} \frac{x(\tau)}{t_v - \tau} d\tau . \quad (24)$$

Note that the function $x(t)$ occurring in (24) means any function. So, let us choose for our purpose of testing a function $x(t)$, which is defined by

$$x_{test}(t) = \begin{cases} 1 & \text{for } |t| \le 3\sigma \\ 0 & \text{for } |t| > 3\sigma \end{cases} . \quad (25)$$

Substituting the function given by (25) into (24) gives

$$\Delta_{2l} = (N_{2l} x_{test})(t_v) - (N_{2l} W_\sigma x_{test})(t_v) =$$
$$= \frac{d_1}{\pi} \int_{-3\sigma}^{-\sigma} \frac{1}{t_v - \tau} d\tau + \frac{d_1}{\pi} \int_{\sigma}^{3\sigma} \frac{1}{t_v - \tau} d\tau . \quad (26)$$

Using the following

$$\int \frac{1}{t_v - \tau} d\tau = -\ln|\tau - t_v| \quad (27)$$

in (26), we get from (26)

$$\Delta_{2l} = \frac{d_1}{\pi}\Big[\ln\big|-3\sigma-t_v\big|-\ln\big|-3\sigma-t_v\big|+$$
$$+\ln\big|\sigma-t_v\big|-\ln\big|3\sigma-t_v\big|\Big]\ . \tag{28}$$

Obviously, there are such values of σ and t_v for which $\Delta_{2l} \neq 0$. This fact is sufficient to conclude that the operator N_{2l} possesses memory.

Finally in this section, consider the operator N_l that according to (11) is a sum of operators N_{1l}, and N_{2l} analysed just before. So, by virtue of this, we have

$$\Delta_l = (N_l x)(t_v) - (N_l W_\sigma x)(t_v) =$$
$$= (N_{1l} x)(t_v) - (N_{1l} W_\sigma x)(t_v) + \tag{29}$$
$$+ (N_{2l} x)(t_v) - (N_{2l} W_\sigma x)(t_v)$$

or in a more detailed form

$$\Delta_l = (N_l x)(t_v) - (N_l W_\sigma x)(t_v) =$$
$$= c_1 x(t_v) - c_1 x(t_v) + \tag{30}$$
$$+ \frac{d_1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t_v-\tau}\,d\tau - \frac{d_1}{\pi}\int_{-\sigma}^{\sigma}\frac{x(\tau)}{t_v-\tau}\,d\tau\ .$$

From (30), we get

$$\Delta_l = (N_l x)(t_v) - (N_l W_\sigma x)(t_v) =$$
$$= (N_{2l} x)(t_v) - (N_{2l} W_\sigma x)(t_v) =$$
$$= \frac{d_1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t_v-\tau}\,d\tau - \frac{d_1}{\pi}\int_{-\sigma}^{\sigma}\frac{x(\tau)}{t_v-\tau}\,d\tau = \tag{31}$$
$$= \Delta_{2l}\ .$$

But, as shown above, $\Delta_{2l} \neq 0$ holds for many combinations of values of σ and t_v. Therefore, the operator N_l possesses memory. In other words, this is not a memoryless operator.

IV. CONSIDERATION OF NONLINEAR OPERATOR N

Using the results obtained in the previous section and applying the same methodology, it can be shown that the nonlinear operator N possesses the memory like its linear part N_l. On the other hand, this seems to be obvious because the strictly nonlinear part of N considered here does not have any property to remove the memory from the linear part of N.

V. SUMMARY

This is the third paper in a series of three, together with [5] and [11], in which the problem of possessing memory by the Saleh's model of AM/AM and AM/PM conversions is discussed. In all of these works, we have proved that this model is not a memoryless one. Further, note that to show the above fact we have used three different approaches and exploited two different mathematical tools: the Volterra series [8], [9] and Sandbergs's theorem [12], [13]. In all of the three cases, we have arrived at the same result.

Finally, we draw the reader's attention to the fact that above conclusions regard also the models related with the Saleh's one, like the quadrature model [3] and the quadrature model of bandpass nonlinearities [2]. These are also models possessing memory.

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