A new technique to design planar dipole antennas by using Bezier curve and Particle Swarm Optimization

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Abstract: This research presents a new technique which includes the principle of a Bezier curve and Particle Swarm Optimization (PSO) together, in order to design the planar dipole antenna for the two different targets. This technique can improve the characteristics of the antennas by modifying copper textures on the antennas with a Bezier curve. However, the time to process an algorithm will be increased due to the expansion of the solution space in optimization process. So as to solve this problem, the suitable initial parameters need to be set. Therefore this research initialized parameters with reference antenna parameters (a reference antenna operates on 2.4 GHz for IEEE 802.11 b/g/n WLAN standards) which resulted in the proposed designs, rapidly converted into the goals. The goal of the first design is to reduce the size of the antenna. As a result, the first antenna is reduced in the substrate size from areas of 5850 mm^2 to 2987 mm^2 (48.93% approximately) and can also operates at 2.4 GHz (2.37 GHz to 2.51 GHz). The antenna with dual band application is presented in the second design. The second antenna is operated at 2.4 GHz (2.40 GHz to 2.49 GHz) and 5 GHz (5.10 GHz to 5.45 GHz) for IEEE 802.11 a/b/g/n WLAN standards.

Key words: planar dipole antenna, Bezier curve, particle swarm optimization, small antenna, dual band antenna

1. Introduction

Recently, wireless communications play an important role in transferring analog and digital data. Many researchers focus on development of wireless communication devices, such as an antenna. An antenna is a device to convert electrical signals into electromagnetic signals in a transmitter and covert electromagnetic signals into electrical signals in a receiver. Although the current antennas work for a wireless communication system, but researchers are still interested in its improvement, such as size minimization, multiband applications and so on.

In general, the frequency at which an antenna operates depends on its size and the shape of an antenna. At low frequency, the antenna which is of a basic geometry shape (such as dipole and rectangular patch antenna) tends to be large and the antenna often operates at a specific band. For this reason, researchers have been trying to design several antennas by modifying their shape with various methods [1-7]. For example, D.H. Werner modified the antenna shape with the fractal geometry in order to develop the antenna for multi-band application [8]. Ho sung Choo created the small wire antenna by modifying its shape, that resulted in reducing size and increasing bandwidth efficiency [9]. In terms of the optimization these seem to expand solution space of optimization, which leads to chances of searching for better answers, [10] however there are some limitations, for instance complex shapes of antennas make it take longer time to calculate and calculation of antenna characteristics has errors from numerical estimation.

Characteristics of the antenna are calculated by using numerical estimation. Whenever the shape of antenna is complex or very bendable it will affect calculation of antenna characteristics. If the antenna is designed by a Bezier curve which is a smooth curve, a numerical error will be decreased. However, it still takes a long time to process all steps of the algorithm. The optimization is required so as to counter these problems (between a numerical error and a waste of the processing time). A few years ago, there were various applications using the optimization to solve the problems [11-16]. Normally, the objective of each application is to reduce processing time of their algorithms. Particle swarm optimization (PSO) is one of the optimizations is fast and simple to implement.

This research presents the new techniques which use the principle of a Bezier curve and PSO in order to reduce numerical errors and the processing time of the algorithm, respectively. The overview technique is to substitute straight-copper strips on a substrate of a reference antenna by Bezier curves after that control points of the Bezier curve and the other design parameters are optimized to be suitable for PSO. As a result, the size of the antenna in the first design is reduced by 48.93% from original and the frequency band maintains unchanged at 2.4 GHz (2.37 GHz to 2.51 GHz). The goal of the second design is to increase the new band by placing a rectangular parasitic element on a bottom side of the model. The substrate size and the parasitic element size are optimized. As a result, this design increases a new frequency band at 5.25 GHz. It demonstrate that the antenna can operate in two frequency bands of 2.4 GHz (2.40 GHz to 2.49 GHz) and 5 GHz (5.10 GHz to 5.45 GHz).

2. Theoretical background

The design of an antenna will involves both the principle of a Bezier curve and PSO. In order to clearly understand this research, the basic of related theories will be presented. A section of the Bezier curve presents a basic method to create the Bezier curve with 4 control points in the Cartesian coordinate. Moreover, the solution space expansion of PSO is presented to show relation between the technique and the design method of this research.

2.1. Bezier curve

According to the previous section, shapes of the modified antenna are mostly required as smooth shapes. The computer graphic design has several curves to form shapes; one of the smooth curves is a Bezier curve [17-20]. All of the points on the Bezier curve at nth order are created by $n + 1$ control points and a function of points on the Bezier curve as written in Equations 1 and 2

$$P_B(t) = \sum_{i=0}^{n} B_{i,n}(t)P_i ,\tag{1}$$

$$B_{i,n}(t) = \frac{n!}{i!(n-i)}u^i(1-t)^{n-i} ,\tag{2}$$

where $B_{i,n}(t)$ are Bernstein polynomial functions, $P_B(t)$ are points on the Bezier curve and $t \in [0,1]$ are used to adjust resolution of the Bezier curve.

Normally, a Bezier curve will be plotted on a Cartesian coordinate thus points on a Bezier curve can be rewritten in terms of $x(t)$, $y(t)$, $z(t)$ as shown in Equation (3)-(5)

$$x(t) = \sum_{i=0}^{n} B_{i,n}(t)P_{i,x} ,\tag{3}$$

$$y(t) = \sum_{i=0}^{n} B_{i,n}(t)P_{i,y} ,\tag{4}$$

$$z(t) = \sum_{i=0}^{n} B_{i,n}(t)P_{i,z} ,\tag{5}$$

where $x(t)$, $y(t)$, $z(t)$ are functions of points of a Bezier curve on the X, Y, Z axis, respectively and $P_{i,x}$, $P_{i,y}$, $P_{i,z}$ are i^{th} order of control points on the X, Y, Z axis, respectively.

In order to clearly understand a Bezier curve, an example of a Bezier curve with 4 control points is plotted on a three dimension Cartesian coordinate system as shown in Fig. 1.
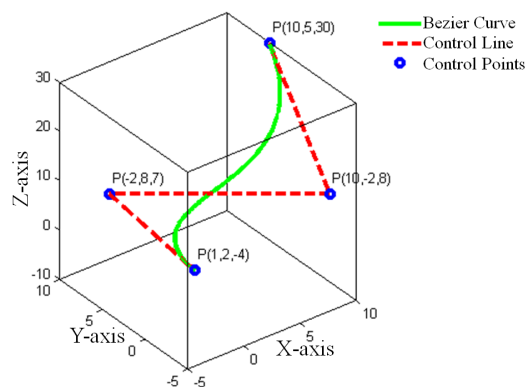

Fig. 1. Cubic Bezier curve plotting on the Cartesian coordinate

The Bezier curve as illustrated in Fig. 1, which is created by Equations (3)-(5) by substituting 4 control points ($\{P(X_0,Y_0,Z_0), P(X_1,Y_1,Z_1), P(X_2,Y_2,Z_2), P(X_3,Y_3,Z_3)\}$) into equations. A Bezier curve with 4 control points is called the Cubic Bezier curve. In this example Equations (3)(5) become

$$x(t) = (1-t)^3 X_0 + 3(1-t)^2 t X_1 + 3(1-t)t^2 X_2 + t^3 X_3 , \tag{6}$$

$$y(t) = (1-t)^3 Y_0 + 3(1-t)^2 t Y_1 + 3(1-t)t^2 Y_2 + t^3 Y_3 , \tag{7}$$

$$z(t) = (1-t)^3 Z_0 + 3(1-t)^2 t Z_1 + 3(1-t)t^2 Z_2 + t^3 Z_3 . \tag{8}$$

2.2. Solution space expansion of PSO by using Bezier curve

Nowadays, optimization techniques are widely used in engineering design. One of the several optimizations is PSO which is fast and easy to implement Furthermore, the principle of PSO is presented in many papers [21-24]. This section will not only mention about the basic of PSO but also indicate relation between the concept which is used in research and solution space expansion of PSO by using a Bezier curve. PSO is a Bio-inspired algorithm. The bee behavior is a good example. The bees randomly start flying in the garden (a solution space) so as to look for flowers. Now, the best flower has just been found by an only one bee after that the other bees will try to move close to a better position. Consequently, they will increase the chances of searching for the better flower next time.

The solution space of PSO has an impact on searching efficiency. The bigger the solution space, the more chances to find the optimum value. To compare between the bee and an observed particle, the bee which is moving into the best position can be explained with the observed particle as shown in Fig. 2. Considering Fig. 2a, it demonstrates that the observed particle can find a good solution. However the solution space is too small to search for the best solution. If the solution space is expanded as shown in Fig. 2b, the moving direction of the bee can be changed. Then, the better solution may be found in the bigger solution space.

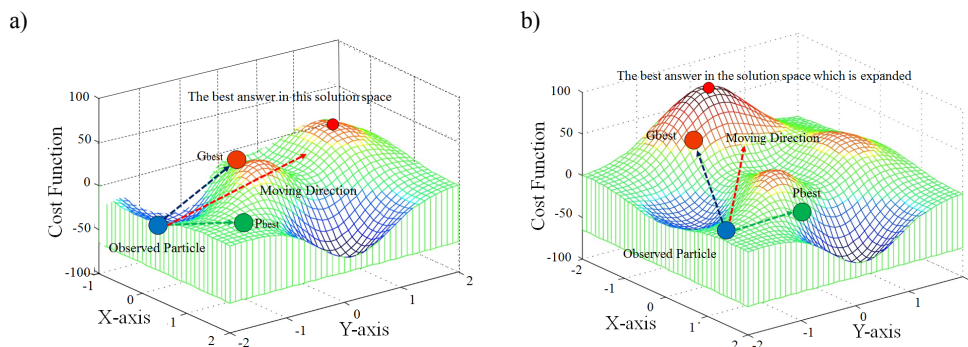a)   b)

Fig. 2. The difference of the solution space of PSO: (a) the original solution space; (b) the solution space which is expanded

In the real implementation, the design parameters of a cost function are positioned in the solution space; the position has an impact on the cost function. To clearly visualize, physical features of an antenna such as the size of an antenna and copper texture on the antenna are the position of the solution space. All of these features have an impact on the characteristics of the antenna such as return loss, a radiation pattern therefore the word "position" which is used in this research means the inputs of the cost function or physical features of an antenna.

The procedure of PSO starts from randomizing the positions of particles, after that each particle tries to move to the new positions with specific velocities which are calculated by Equation (9) and (10). All of the steps as shown in the PSO procedure.

PSO procedure
1: -*Assign the initial values with*
Random Positions(p) and Velocities(v) of each particle
-*Define Maximum Iteration to run process (m)*
for *k=1:m*
for *i=1:maximum particles*
2: *Evaluate particle (i) cost*
if *particle(i) cost is better than P_{best} (i) cost*
-*set position of particle (i) as P_{best}*
else
-*Keep P_{best}*
end
3:        -*Assign the best of all P_{best} positions is G_{best}*
-*Calculate velocity for each particle*
-*Update its data values (p, v) by equation (9), (10)*
end
end

To update new positions, specific velocities of each particle will be calculated by Equation (9) and then its positions are updated by Equation (10).

$$V_t = \omega V_{t-1} + C_1\alpha_1(P_{best} - P_{t-1}) + C_2\alpha_2(G_{best} - P_{t-1}), \qquad (9)$$

where V_t is the current velocity of particle i at iteration t, C_1 and C_2 are the learning factors usually to be set it of 2, ω is a inertial weight between 0.4 to 0.9, α_1 and α_2 are random numbers. The personal best position ($P_{best,i}$) is the best position of itself since it has been searched and the best position in solution space as called the global best position (G_{best}). After the velocity of each particle (V) is calculated, the particles can be updated to new positions by substituting the velocity into Equation (10).

$$P_t = P_{t-1} + V_t, \qquad (10)$$

where P_t and P_{t-1} are the current position and the previous position, respectively. The process continuously works until the condition becomes success or the maximum iterations.

3. Antenna modelling and cost function setting

This section will focus on antenna modeling and cost function setting. To initialize parameters of the designs, the reference model which is a planar antenna dipole has to be created in order to extract physical features to initialize parameters of the two designs. Parameters used in the simulation are as follows: The number of particles = 30, learning factors $C_1 = C_2 = 2$ and maximum iterations = 200, start optimizing these parameters to become suitable. All processes in this research are convenient to implement by using Ansoft HFSS and Matlab together.

3.1. The reference antenna

Because the designs initialize parameters by using the reference, the physical features of the reference have to be known. The reference is the planar dipole antenna which operates at IEEE 802.11 WLAN b/g/n standard (operates 2.40-2.4835 GHz) and consists of the two straight copper strips on the 1.6 mm-thick-FR4 substrate with relative permittivity of 4.4 and the total size is 65×90 mm^2. The structure of the reference antenna as shown in Fig. 3.
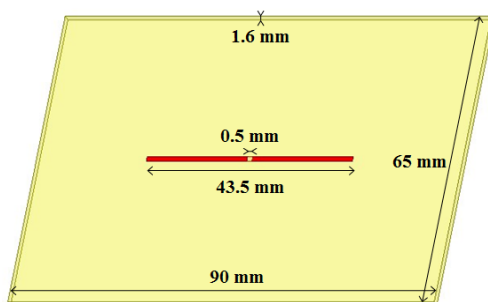

Fig. 3. The overall structure of the reference antenna

3.2. The minimized antenna design

The proposed antenna in this section will have two straight copper strips with the Bezier curves. From now, the antenna created in this section is called "Antenna1" and has the total size of $W_1 \times L_1$ mm^2. Each of the new copper strips, which likes the Bezier curve shape of 3rd order (Cubic Bezier curve), will be called "the Bezier copper strip (BCS)" and can control its shape with control points (4 points). The curve and width of the copper strip depends on the curvature of the Bezier curve which can be modified by the control points. Fig. 4 illustrates the total structure of Antenna1.

Fig. 4 illustrates Antenna1, which is on the XY-plane, can be modified in its structure by W_1, L_1 and 8 control points (4 points for each BCS). $P(X_{f1}, Y_{f1})$ and $P(X_{f2}, Y_{f2})$,which are starting points of each BCS, are defined to be constant of $P(0$ mm, -0.25 mm$)$ and $P(0$ mm, 0.25 mm$)$, respectively. The variables in the Y-axis are also set to be constant $\{Y_1, Y_2, Y_3, Y_4, Y_5, Y_6\}$ $= \{-20, -10, -5, 5, 10, 20\}$ so that two lines will not be overlapped with each other. Therefore, the other parameters which have an impact on the structure and characteristics of Antenna1 are X_1 to X_6, W_1, L_1. In this design uses them to be input parameters of optimization.
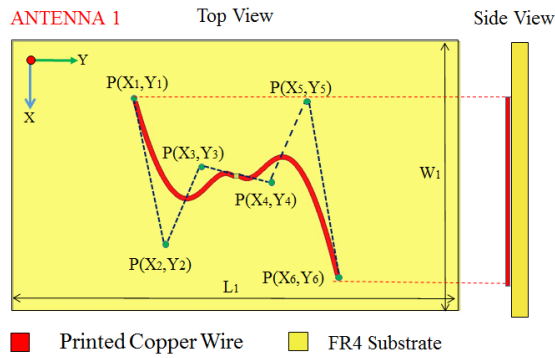
Fig. 4. The parameters are used to control physical shape of Antenna1

In order to minimize the substrate of the reference antenna and remain operated at the original frequency standard therefore the substrate of Antenna1 has to be smaller than the substrate of the reference antenna. The cost function for design Antenna1 as shown in Equation (11).

$$\text{Cost function} = \frac{\text{weight}}{\left| S_{11,\,2.45\,\text{GHz}} \right|} + \left(\frac{0.001}{S} \right), \tag{11}$$

where S is the substrate size of $L_1 \times W_1$, S_{11}, 2.45 GHz is the return loss at 2.4 GHz and

$$\text{weight} = \begin{cases} 0, & S_{11,\,2.45\,\text{GHz}} < -13 \\ 100, & \text{otherwise} \end{cases},$$

3.3. The dual band antenna design

In this section, the second antenna (Antenna2) design will be optimized. As the target of this design is more difficult than the previous design. Thus, this design should be increased in the design parameters. In Fig. 5, the structure of Antenna2 is almost the same as Antenna1 but there are differences in the two models. Antenna2 is added a rectangular parasitic element on a bottom side so as to expand the solution space of optimization. Therefore, Antenna2 has the new input parameters, W_2 and L_2, thus the total input parameters for optimization are X_1 to X_6, W_1, W_2, L_1, L_2. The overall structure of Antenna2 as shown in Fig. 5.

Antenna2 was designed for dual band applications (2.45 GHz and 5.25 GHz) therefore a number of observed frequencies should be 2.45 GHz and 5.25 GHz. The Equation (11) can be rewritten to become Equation (12).

$$\text{Cost function} = \frac{\text{weight}}{\left| S_{11,2.45\,\text{GHz}} + S_{11,5.25\,\text{GHz}} \right|}, \tag{12}$$

where $S_{11,\,2.45\,\text{GHz}}$ and $S_{11,\,5.25\,\text{GHz}}$ are the return loss at 2.4 GHz and 5.25 GHz, respectively and

$$\text{weight} = \begin{cases} 0, & S_{11,\,2.45\,\text{GHz}} \text{ and } S_{11,\,5.25\,\text{GHz}} < -13 \\ 100, & \text{otherwise} \end{cases}.$$
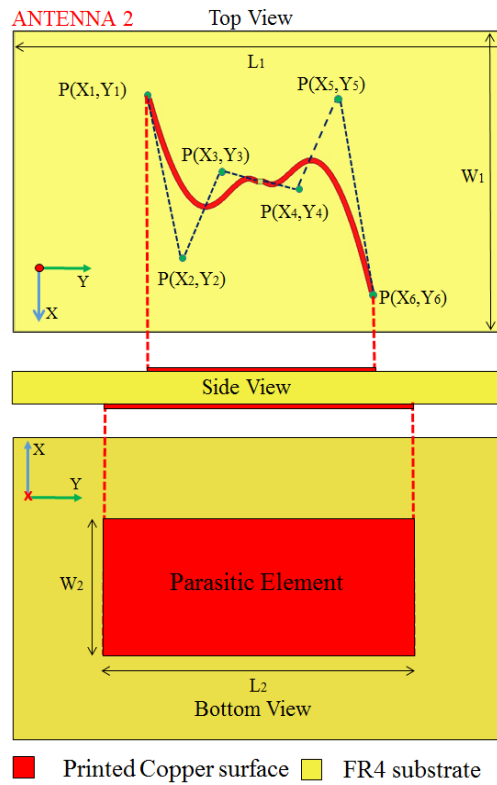
Fig. 5. The parameters are used to control physical shape of Antenna2

4. The experimental results

After the optimization process, the suitable parameters of both antennas as shown in Table 1. Fig. 6 and 7 show the overall structure of Antenna1 and Antenna2, respectively.

The end-result of the BCS can be written in terms of polynomial functions by substituting the control points of the Bezier curve in Table 1 into Equations (6)-(8). The polynomial functions to create all BCS as shown in Equations (13)-(16).

Left BCS of Antenna1

$$
\begin{aligned}
x(t) &= -74.25(1-t)^2 t - 69.27(1-t)t^2 + 21.54t^3 \\
y(t) &= -0.25(1-t)^3 - 15t(1-t)^2 - 30(1-t)t^2 - 20t
\end{aligned}
\tag{13}
$$

Right BCS of Antenna1

$$
\begin{aligned}
x(t) &= -(1-t)^2 t - 12.06(1-t)t^2 - 13.02t^3 \\
y(t) &= 0.25(1-t)^3 + 15t(1-t)^2 + 30(1-t)t^2 + 20t^3
\end{aligned}
\tag{14}
$$

Table 1. The suitable parameters of Antenna1 and Antenna2

Parameter	Control parameters	Antenna1	Antenna2
Control points	X_1	21.54	5.15
	X_2	−23.09	−12.60
	X_3	−24.75	−2.32
	X_4	−0.335	19.11
	X_5	−4.022	17.18
	X_6	−13.02	−16.15
Size of Substrate (mm)	L_1	53.26	91.25
	W_1	51.10	54.73
Size of Parasitic (mm)	L_2	–	43.60
	W_2	–	6.68
Total size (mm^2)	–	2987	3978

Left BCS of Antenna2

$$x(t) = -6.96(1-t)^2 t - 37.8(1-t)t^2 + 5.15t^3$$
$$y(t) = -0.25(1-t)^3 - 15t(1-t)^2 - 30(1-t)t^2 - 20t^3 \qquad (15)$$

Right BCS of Antenna2

$$x(t) = 57.33(1-t)^2 t + 51.54(1-t)t^2 - 16.15t^3$$
$$y(t) = 0.25(1-t)^3 + 15t(1-t)^2 + 30(1-t)t^2 + 20t^3 \qquad (16)$$

where t = {0, 0.005, 0.010, 0.015,..., 1}.

Considering Table 1, it shows all of the control parameters after optimization process (the control points, the size of substrate and the parasitic element). The details illustrate that the substrate of Antenna1 is reduced to be of 2987 mm^2 which is smaller than the reference size (5850 mm^2). Moreover, Table 1 also shows the size of Antenna2 which is 3978 mm^2. It demonstrates that both antennas are different owing to the difference of design goals and the design parameters. Fig. 6 shows Antenna1, which has only the copper texture on a top side whereas Antenna2 consists of the copper texture on both sides. A top side has the copper texture which is the two BCS and a bottom side has the rectangular parasitic element as shown in Fig. 7.

The next point will focus on operating bands of the proposed antennas. The indicator which indicates that the proposed antenna can operate in the observed frequency is return loss of the proposed antenna. If the return loss at the observed frequency is lower than -10 dB ($S_{11} \leq$ -10 dB), the proposed antenna can operate at the observed frequency.

Figs. 8 and 9 show the return loss and the impedance of three different antennas (the reference antenna, Antenna1 and Antenna2). The details show that the reference antenna and Antenna1 operate in the same standard. The reference antenna operates between 2.2 to 2.5 GHz and Antenna1 operates between 2.37 to 2.51 GHz.
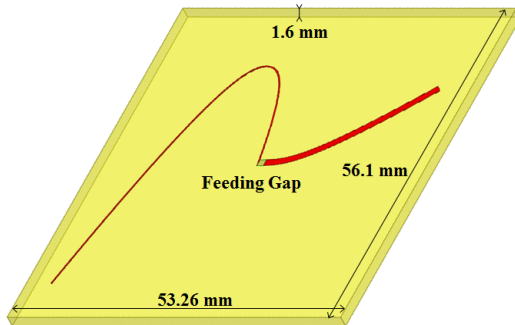
Fig. 6. The structure of Antenna1;
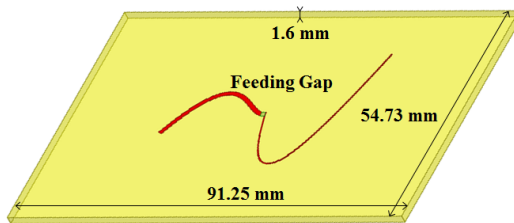Minimized Antenna Application

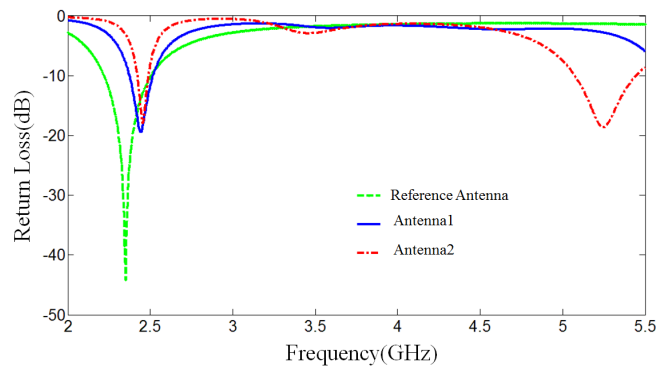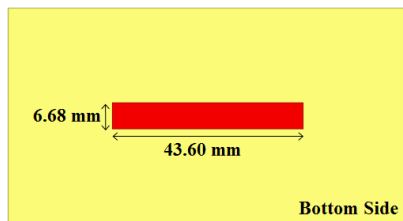Fig. 7. The structure of Antenna2;
Dual band Antenna Application

Fig. 8. The return loss of the antennas

It illustrates that the both Antenna1 and Antenna2 can operate on IEEE 802.11 b/g/n WLAN standard. Antenna2 operates in two frequency bands of 2.40-2.49 GHz and 5.10-5.45 GHz which cover IEEE 802.11 a/b/g/n WLAN standard and the resonant frequencies are of 2.45 and 5.25 GHz.
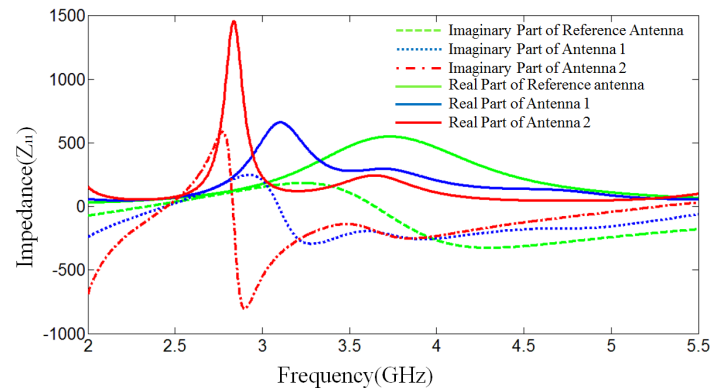
Fig. 9. The real and imaginary impedance of antennas

So as to clearly demonstrate the difference of two resonant modes, the current distribution of Antenna2 as shown in Fig. 10. It illustrates that there is difference of two resonant modes; that is to say, the mainly prevailing surface current is along the left BCS for the first resonant frequency at 2.45 GHz while the second resonant frequency at 5.25 GHz, the mostly prevailing surface current distributes along the right BCS close to the power feeding point. It demonstrates that each of the BCS responds to different modes of resonant frequencies
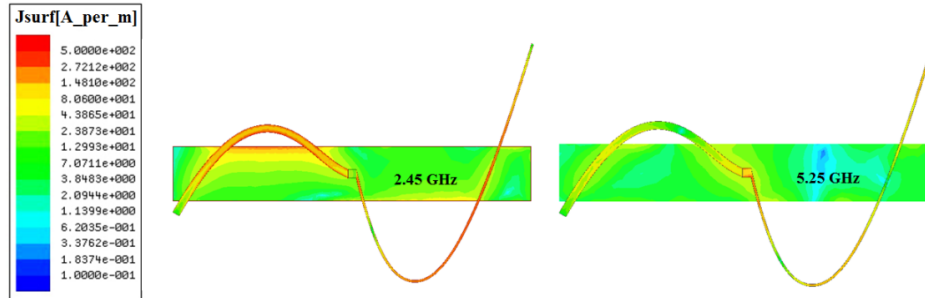

Fig. 10. The current distribution of Antenna2 in difference frequencies

The radiation pattern of the two proposed antennas was shown in Fig. 11. It illustrates that the radiation patterns of Antenna1 and Antenna2 which operate at 2.4 GHz, are close to radiation patterns of the dipole antenna. It can be seen that the radiation pattern in the XZ-plane of both antennas are similar to an omnidirectional pattern and they have the butterfly pattern in the XY-plane.

Although the patterns of Antenna1 and Antenna2 are almost the same patterns, however Antenna2 has the radiation gain higher than Antenna1 because Antenna1 is bigger than Antenna2. Antenna2 is used for dual band application. Fig. 11c shows the radiation of Antenna2 at 5.25 GHz, the patterns in the XZ-plane are nearly close to an omnidirectional pattern and quad directional pattern in the XY-plane.
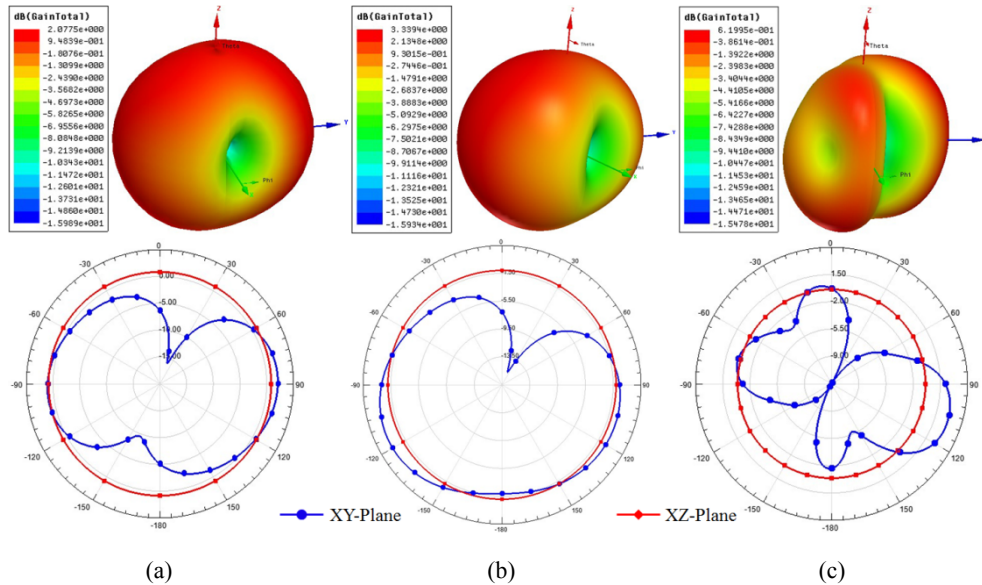
(a)   (b)   (c)

Fig. 10. 2D and 3D far-field gain of Antenna1 and Antenna2 (a) Far-field gain of Antenna1 operates at 2.45 GHz (b) and (c) Far-field gain of Antenna2 operates at 2.45 GHz and 5.25 GHz, respectively

5. Conclusions

The technique as presented in this research can reduce the size of Antenna1 by 48% from the reference antenna whereas it still remains operated in the original standard (IEEE 802.11 b/g/n WLAN standard). Furthermore Antenna2, which is designed for dual-band application, can operate in dual band frequencies of 2.40-2.49 GHz and 5.10-5.45 GHz which cover IEEE 802.11 WLAN a/b/g/n standard. These results demonstrate that both Antenna1 and Antenna2 have features dependent on the cost function of the optimization process. The shape of an antenna was represented by the Bezier curve. As the result of the search on the space expansion, the probabilities of finding better solution will be increased. Moreover, this technique can apply to several applications by modifying the cost function and may help to create new features of various antennas.

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