ARCHIVES OF ELECTRICAL ENGINEERING VOL. 65(4), pp. 773-787 (2016)

DOI 10.1515/aee-2016-0054

Nonlinear behavior analysis of split-winding dry-type transformer using a new star model and a coupled field-circuit approach

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(Received: 06.01.2016, revised: 15.09.2016)

Abstract: Regarding the importance of short circuit and inrush current simulations in the split-winding transformer, a novel nonlinear equivalent circuit is introduced in this paper for nonlinear simulation of this transformer. The equivalent circuit is extended using the nonlinear inductances. Employing a numerical method, leakage and magnetizing inductances in the split-winding transformer are extracted and the nonlinear model inductances are estimated using these inductances. The introduced model is validated and using this nonlinear model, inrush and short-circuit currents are calculated. It has been seen that the introduced model is valid and suitable for simulations of the split-winding transformer due to various loading conditions. Finally, the effects of nonlinearity of the model inductances are discussed in the following.

Key words: coupled field circuit, electromagnetic modeling, split-winding dry-type transformer, nonlinear equivalent circuit, finite element

1. Introduction

Nowadays, traction systems and smelting plants are growing very fast and so converter systems become more important in electric power networks. The key part of a converter system is the transformer that is usually manufactured from four split windings [1] (Fig. 1).

It is important to study the steady-state and transient behaviors of this split-winding transformer, when it is exposed to different loading conditions. For this reason, it is convenient to introduce a comprehensive equivalent circuit that can be used in all the possible loading conditions such as no-load. In the past years, many equivalent circuits are introduced for different types of transformers [2-13]. The general equivalent circuit models for multi-winding transformers have been proposed in some papers [2, 3]. Three-winding transformer star models matching leakage inductances have been presented in [4, 5]. Polygonal transformer models for different structures of windings are introduced in [5-8].

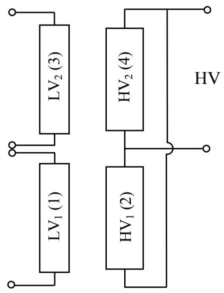

Fig. 1. Schematic view of split-winding transformer

Employing the transformer models, the performance of n-winding transformers has been analyzed and simulated in different power systems [9, 10]. The conventional star models cannot accurately simulate the transformers behavior while the windings are more than three [11]. Additionally, parameter estimation in the polygonal models is difficult. Thus, using the duality concept, new equivalent circuits had been introduced for a multi-winding transformer with split windings [11, 12]. References [11] introduces a star model for short-circuit calculations in a traction transformer with split windings [11]. This model that matches with leakage inductance measurements has acceptable accuracy in short-circuit current calculations. Unfortunately, it cannot be applicable for inrush current calculations. In spite of short-circuit calculations, assuming the inductances to be linear, causes a considerable error in simulation of different loading conditions such as no-loads. Some models have been introduced for nonlinear behavior analysis and inrush current calculations in different transformers [5, 14, 15].

As there is a need for a comprehensive study on inrush and short-circuit current calculations in split-winding transformers, it is essential to introduce a suitable nonlinear equivalent circuit. Thus here, a novel star equivalent circuit is extended for nonlinear simulation of the split-winding transformer. This nonlinear model is used for short-circuit and inrush current calculations and its results have been verified against the experimental data and the coupled field-circuit method [16-18]. The proposed model may be useful for both transformer designers and system analysts in no-load and short-circuit simulations.

Additionally, the effects of the model's nonlinearity on short-circuit and inrush current calculations are discussed in this paper. Finally, with the aim of having a complete no-load performance analysis in a split-winding transformer, the calculated inrush currents have been applied for electromagnetic force calculations. Consequently, novel schemes of this paper against previous works can be listed as below:

- extracting a new nonlinear model for a split-winding transformer,
- inrush current and short-circuit calculations in a split-winding transformer,
- inrush current force computations in a split-winding transformer,
- coupled field circuit method applied to both short-circuit and no-load calculations.

2. Linear star equivalent circuit

As HV windings in Fig. 1 are connected in parallel, three-winding transformer equivalent circuits may be employed to analyze the performance of the split-winding transformer. Three-winding equivalent circuits have been introduced in [4, 5]. Although the split-winding transformer of interest is used as a three winding transformer, but usually it is essential to model it with four distinct windings [11]. As the mentioned models [4, 5] are inappropriate to be used for a four-winding transformer, a comprehensive equivalent circuit is needed for the split-winding transformer. The general polygonal and star equivalent circuits of n-winding transformers are presented in [2, 3]. The conventional star models cannot entirely analyze the four-winding transformer's behavior and the conventional polygonal models are intricate. Thus in the following, a star model has been proposed for the four-winding transformer.

A way is to derive an equivalent circuit using the duality concept; in this method, some inductances will be assigned to the flux paths. For this reason, it is convenient to use trapezoidal leakage flux distributions of a traction transformer [11]. The flux path related to the leakage inductance test of a pair of concentric windings (windings 1 and 2 or windings 3 and 4) has predominantly axial component of magnetic field. The flux path related to the leakage inductance test of pancake windings (windings 1 and 3 or windings 2 and 4) has predominantly radial magnetic field. However, in some cases such as the leakage inductance test between windings 1 and 4 or windings 2 and 3, both axial and radial magnetic fluxes exist (unlike a two-winding transformer). Each flux path can be modelled with two inductances related to the windings [11]. Fig. 2a shows the magnetic circuits and the assigned inductances for the axial leakage flux paths. However, the extracted equivalent circuit cannot yet model the radial flux paths. To realize modeling both the axial and radial leakage flux paths, coupling between top and bottom windings should to be considered. Fig. 2b shows the final leakage model which is developed based on the mentioned strategy.

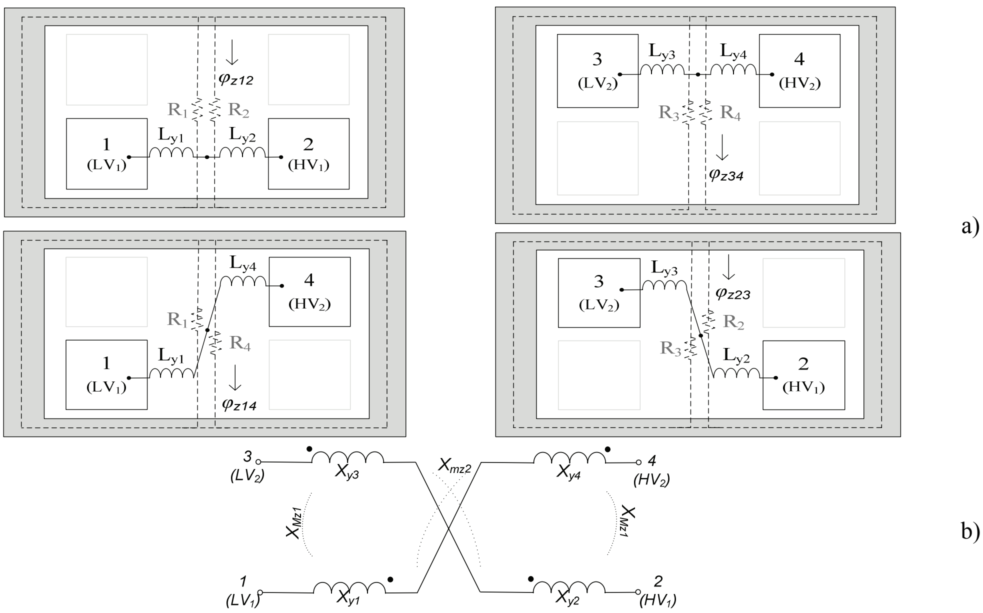

Fig. 2. a) Magnetic circuits represent the axial leakage flux paths and b) final Leakage model of a four-winding traction transformer

Assume the short-circuit test in Fig. 3 for the leakage inductance measurements [19]; by applying this leakage tests to the equivalent circuit of Fig. 2, all the six leakage reactances of the traction transformer can be calculated as functions of model reactances (for example for X_{l12}: terminal 2 is short circuited and the total reactance seen from terminal 1 is calculated from Fig. 2). Solving these six equations, the model reactances can be expressed as leakage reactances.

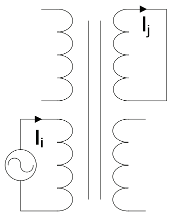

Fig. 3. Short-circuit test for measuring the leakage inductance between a pair of windings

So, X_{yi} and X_{Mzi} are computed as below, such that the leakage measurements are matched.

$$X_{y1} = \left(2X_{l12} + X_{l13} - X_{l23} + X_{l14} - X_{l24}\right)/4, \tag{1}$$

$$X_{y2} = \left(2X_{l12} - X_{l13} + X_{l23} - X_{l14} + X_{l24}\right)/4, \tag{2}$$

$$X_{y3} = \left(X_{l13} + X_{l23} - X_{l14} - X_{l24} + 2X_{l34}\right)/4, \tag{3}$$

$$X_{y4} = \left(-X_{l13} - X_{l23} + X_{l14} + X_{l24} + 2X_{l34}\right)/4, \tag{4}$$

$$X_{Mz1} = \left(X_{l12} - X_{l13} - X_{l24} + X_{l34}\right)/4, \tag{5}$$

$$X_{Mz2} = \left(X_{l12} - X_{l23} - X_{l14} + X_{l34}\right)/4, \tag{6}$$

where: X_{lij} is the leakage reactance between windings i and j. For a linear system, leakage reactances can be expressed as a function of design parameters [19].

3. Novel nonlinear model for split-winding transformer

Here the leakage and the magnetizing reactances in the split-winding transformer are computed. Afterwards, the star equivalent circuit is developed to extract a novel nonlinear model for the split-winding transformer.

3.1. Reactance calculations

Leakage reactances: If the nominal currents are applied to windings i and j, and currents in other windings are assigned to be zero, the leakage magnetic field can be evaluated by solving Poisson's equation [19]. For electromagnetic calculations in the split-winding transformer, a two-dimensional finite element model [19] is employed. After computing the magnetic vector potential (A_{lij}), leakage reactance of the windings referred to N_0 is computed as:

$$X_{lij} = \left(\frac{N_0}{N_i}\right)^2 \frac{4\pi f \times W_{lij}}{I_i^2} = \left(\frac{N_0}{N_i}\right)^2 \frac{4\pi f \times \int\limits_{vol} A_{lij} \cdot J\,\mathrm{d}v}{I_i^2}, \tag{7}$$

where: I_i and N_i are the current and the turns number of the winding i.

Magnetizing reactances: The magnetizing flux can be represented by four magnetizing inductances that are connected to the windings' terminals. If these magnetizing inductances are assumed to be linear, one can calculate them using the geometry specifications [4]. Assuming the magnetizing inductances as a lumped linear inductance is valid for short-circuit calculations. But as the magnetizing flux is much considerable in no-load conditions, the magnetizing inductance cannot be assumed as a linear inductance. It is not easy to compute the nonlinear magnetizing inductances separately. Thus, the magnetizing flux can be represented as a single lumped inductance that is connected to the star point [11]. Actually, the magnetizing inductance is nonlinear. If a current is applied to the primary (HV) windings and the secondary (LV) windings kept open-circuit, the lumped magnetizing flux can be calculated (as a function of passing current) using FE models [11, 19]. After computing the magnetic vector potential (A_m), magnetizing reactance (X_m) referred to the HV side can be computed as:

$$X_m = \frac{4\pi f \times W_m}{I_{\mathrm{HV}}^2} = \frac{4\pi f \times \int\limits_{\mathrm{vol}} A_m \cdot J\,\mathrm{d}v}{I_{\mathrm{HV}}^2}. \tag{8}$$

3.2. Nonlinearity of the reactances and the complete equivalent circuit

The ferromagnetic core is constructed of a nonlinear material that causes the inductances to be nonlinear and vary as a function their currents. To avoid complexity of nonlinear differential equations, it is convenient to sectionalize the λ-*i* curve and assume it to be linear in each section (Fig. 4) [20].

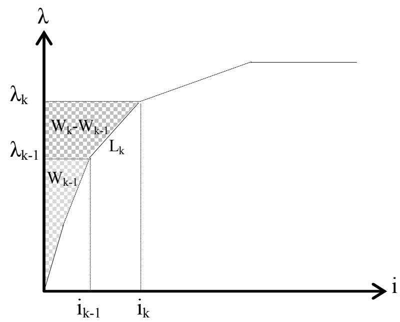

Fig. 4. Sectionalize λ-*i* curve

Define the inductance to be the gradient of the flux linkage ($L_k = \Delta\lambda_k/\Delta i_k = (\lambda_k - \lambda_{k-})/(i_k - i_{k-1})$) in any location on the curve. And consequently the energy related to the k-th linear section can be expressed as (area of the left-hand side of the curve in Fig. 4)

$$\Delta W_k = W_k - W_{k-1} = \int_{\lambda_{k-1}}^{\lambda_k} i\mathrm{d}\lambda = \frac{1}{2}(i_k + i_{k-1})(\lambda_k - \lambda_{k-1}) = \frac{1}{2}(i_k + i_{k-1})L_k(i_k - i_{k-1})$$
$$= \frac{1}{2}L_k(i_k^2 - i_{k-1}^2)\,, \tag{9}$$

where: W_k is the total energy related to i_k. Thus, it can be shown that the inductance (reactance) related to k-th linear region can be computed as

$$X_k = 2\pi f \times L_k = 4\pi f \times \frac{W_k - W_{k-1}}{i_k^2 - i_{k-1}^2}\,. \tag{10}$$

A typical cast-resin split-winding transformer [11] is considered and the calculations are done for this typical transformer. Fig. 5 shows the variation of the leakage reactances in the split-winding dry-type transformer of interest. The leakage reactance of concentric windings is constant as seen in Fig. 5a.

But unlike the concentric windings, the leakage reactances between pancake windings are varied with a variation in their current. It can be shown that the current passing through the pancake-winding reactances is very low in the short-circuit condition and these reactances can be assumed to be in the linear region during the short-circuit calculations. But behavior of these reactances is not linear in the no-load conditions.

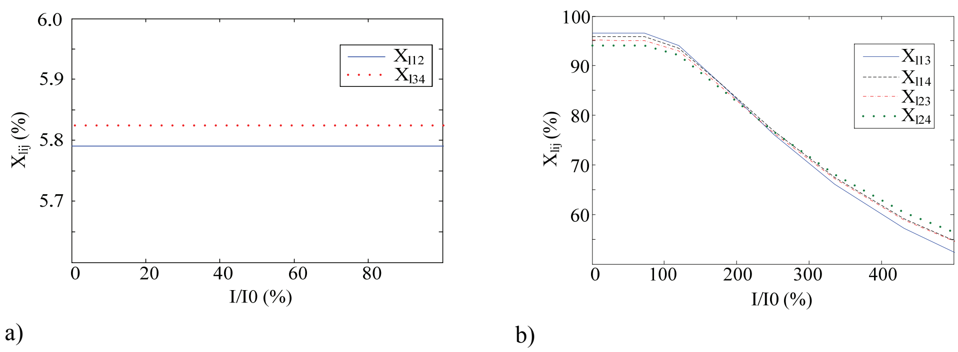

Fig. 5. Nonlinear reactances of a) concentric and b) pancake windings in the split-winding transformer

Using Fig. 5, the leakage model parameters can be calculated from (1)-(6). Fig. 6 shows the variation of the X_{y1} and X_{Mz1} in the nonlinear model.

The effect of nonlinear core can be seen in variation of the magnetizing reactance in Fig. 7. Following the ongoing sections, a complete equivalent circuit can be extracted as Fig. 8. The winding resistances are derived from the design parameters that are given in [11]. Note that all the reactances are nonlinear and they depend on their passing current.

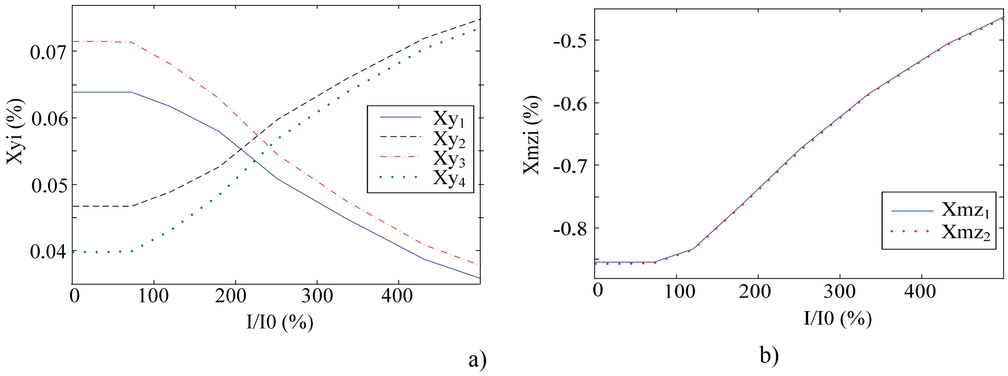

Fig. 6. Nonlinear leakage reactances of the nonlinear star model, a) Xy_i, b) Xmz_i

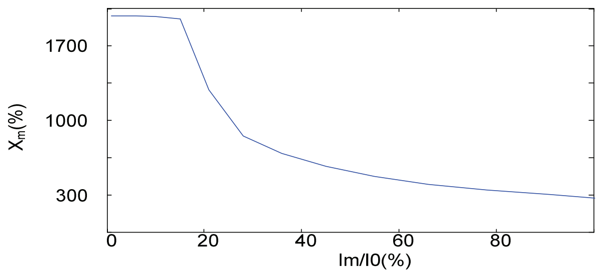

Fig. 7. Nonlinear magnetizing reactance (X_m)

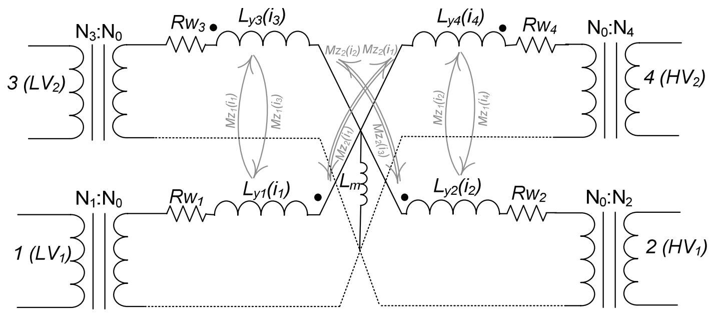

Fig. 8. Novel nonlinear equivalent circuit for split-winding transformer, including magnetizing reactances and windings resistances

4. Coupled field-circuit approach for electromagnetic modeling

In spite of equivalent circuits, windings currents can be computed using a field calculation method that is coupled with an external circuit; this is called as coupled field-circuit [16-18]. Analytical and numerical methods have been introduced for magnetic field calculations

[11, 21-24]; but an accurate method for this purpose is finite element method [11, 24-26]. A Coupled field-circuit approach is an accurate, however time-consuming method. In this paper a coupled field-circuit is presented to verify the accuracy of the introduced nonlinear equivalent circuit. According to Fig. 9, the basis of this method is stemmed from the coupling effect that exists between electromagnetic fields of interest and some external circuits. The equations of induced voltages in the external circuit can be described in a matrix form as shown in (11).

$$\{V\}=\{E\}+[R]\{I\}+[L]\left\{\frac{\mathrm{d}I}{\mathrm{d}t}\right\}, \tag{11}$$

where: R and L are the matrices of the external resistors and inductances; accordingly, I, V and E are the vectors of the winding current, internal voltage and exciting voltage.

Exciting voltage (E) in (11) can be described as integration of the magnetic vector potential; this is shown in (12).

$$E(t)=\frac{2\pi rN}{S_w}\int_{S_w}\frac{\mathrm{d}A}{\mathrm{d}t}ds, \tag{12}$$

where: r is the radius, N is the turn number and S_w is the winding's cross-section.

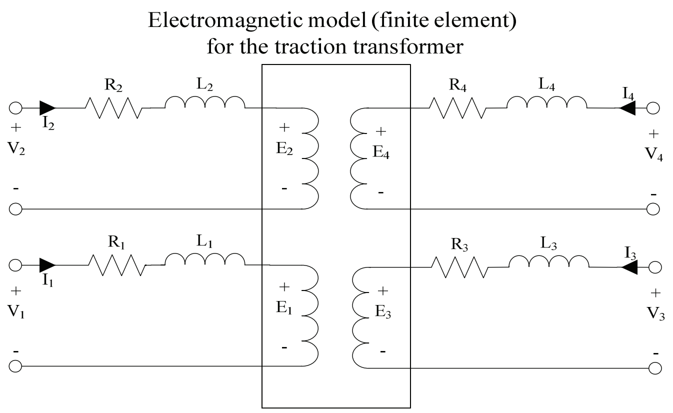

Fig. 9. Schematic view of the coupled field-circuit approach

Combining (11) and (12), nonlinear equations describing the coupled field-circuit method can be extracted as shown in (14) [17].

$$\begin{bmatrix}0 & 0\\ G & L\end{bmatrix}\begin{Bmatrix}\dot{A}_R\\ \dot{I}\end{Bmatrix}+\begin{bmatrix}K & D\\ 0 & R\end{bmatrix}\begin{Bmatrix}A_R\\ I\end{Bmatrix}=\begin{Bmatrix}0\\ V\end{Bmatrix}. \tag{13}$$

Using a numerical integration, these equations are rewritten as (14) [17].

$$\begin{bmatrix} K & D \\ D^T & -L-\frac{R\Delta t}{2} \end{bmatrix} \begin{Bmatrix} A \\ I \end{Bmatrix}_{n+1} = \begin{bmatrix} -K & D \\ D^T & L-\frac{R\Delta t}{2} \end{bmatrix} \begin{Bmatrix} A \\ -I \end{Bmatrix}_{n} + \begin{Bmatrix} 0 \\ -\frac{V_n + V_{n+1}}{2}\Delta t \end{Bmatrix}. \tag{14}$$

In this paper, a coupled field-circuit program is written based on the procedure shown in Fig. 10. In this procedure, windings currents in each instant are determined in such a way to satisfy the equation (11).

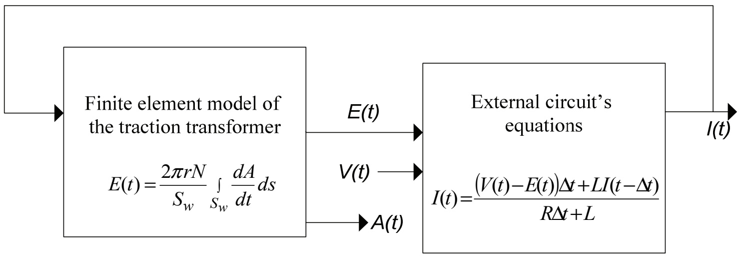

Fig. 10. The coupled field-circuit method

5. Short-circuit and inrush current calculations by means of the nonlinear equivalent circuit

Various short-circuit and no-load conditions can be considered in the split-winding transformer. These important loading conditions are listed in Table 1. The proposed nonlinear star model and coupled field circuit method are employed to simulate these loading conditions; results of the presented methods are compared with each other.

Table 1. Loading conditions (no-load and short-circuit) in the split-winding transformer

	Case I (fully open-circuit)	**Case II (fully short-circuit)**	**Case III (single short-circuit)**	**Case IV (single short-circuit)**
LV_1	open circuited	short circuited	short circuited	open circuited
LV_2	open circuited	short circuited	open circuited	short circuited

Figure 11 shows the inrush currents when both LVs are open circuited (case I) and Fig. 12 shows the windings currents when both LVs are short-circuited (case II).

Figure 13 shows transient currents while one LV winding is short circuited and the other is open. As shown, in the cases I and II, the inrush and short-circuit currents are presented respectively. But in cases III and IV, the short-circuit currents are presented in the short circuited low voltage winding and the related concentric high voltage winding.

In these cases, the other high voltage winding that is concentric with the open circuited winding, carries a much lower inrush current. Consequently, Table 2 shows the steady-state currents (rms) calculated in mentioned loading conditions that are calculated by means of the presented star model.

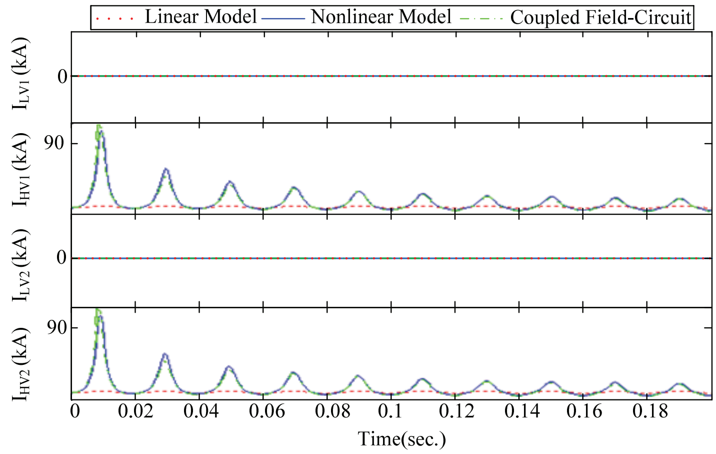

Fig. 11. Inrush currents, while both LV windings are open circuited (case I)

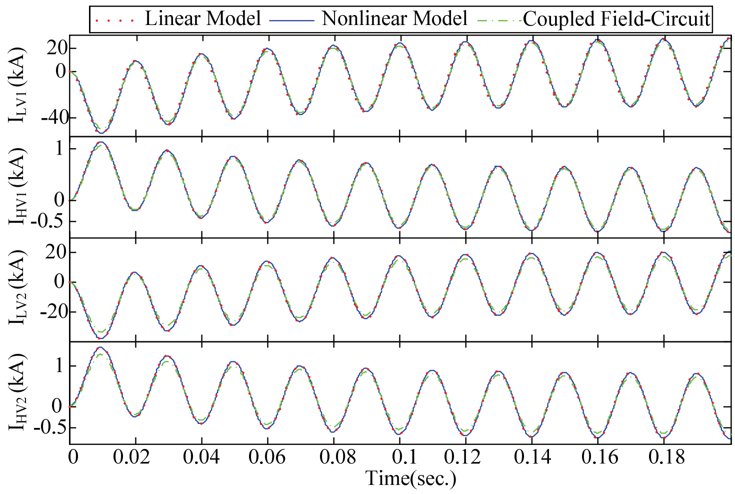

Fig. 12. Short-circuit currents, while both LV windings are short circuited (case II)

As it is clear, the linear model has enough accuracy in short-circuit current calculations. In spite of short-circuit computations, the linear star model causes significant inaccuracy while the inrush current is calculated. As the nonlinearity of the inductances is taken into the account in the model, both short-circuit and inrush currents can be calculated with a good precision. So, the introduced nonlinear equivalent circuit is suitable for analysing the performance of the split-winding transformer under various loading conditions such as no-load and short-circuits. The estimated short-circuit and inrush currents can be used for force computation in the split-winding transformers. Additionally, one can see that the introduced nonlinear equivalent circuit may be useful for power system analysts and can be used for network analysis in presence of split-winding transformers.

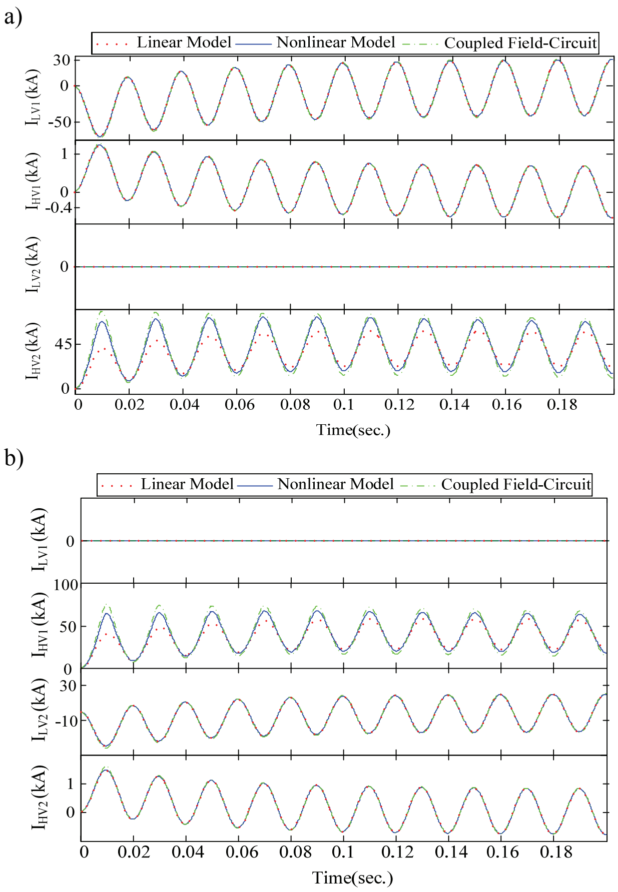

Fig. 13. Inrush and short-circuit currents while a) LV1 or b) LV2 winding is short-circuited (cases III, IV)

Table 2. Steady-state windings currents (rms) in different loading conditions

Loading condition			Case I	Case II	Case III	Case IV
nonlinear equivalent circuit	Extracted currents (A) based on 2D FE method	LV_1	0	26.78 k	0	25.92 k
		HV_1	0.89	566	12.49	559
		LV_2	0	0	15.31 k	14.82 k
		HV_2	0.88	14.82	563	559
	Extracted currents (A) based on 3D FE method	LV_1	0	26.40 k	0	28.19 k
		HV_1	0.892	546	22.7	606
		LV_2	0	0	15.47 k	16.52 k
		HV_2	0.906	26.7	559	624
	Extracted currents (A) based on measured inductances	LV_1	0	26.56 k	0	24.02 k
		HV_1	0.927	548	19.88	513
		LV_2	0	0	15.39 k	13.92 k
		HV_2	0.955	28	559	530
Measured currents (A)		LV_1	0	27.11 k	0	–
		HV_1	0.933	–	–	–
		LV_2	0	0	15.68 k	–
		HV_2	0.933	–	–	–

6. Magnetic flux and force computations in split-winding transformer due to short-circuit and no-load conditions

Consequently, the computed inrush and short-circuit currents can be used for electromagnetic field and force calculations. Here, finite element models had been used for electromagnetic modeling of the split-winding transformer; detail information about finite element models, core modeling and modeling parameters have been presented in [11, 19]. Fig. 14 shows the computed magnetic flux densities related to the inrush and the short-circuit currents.

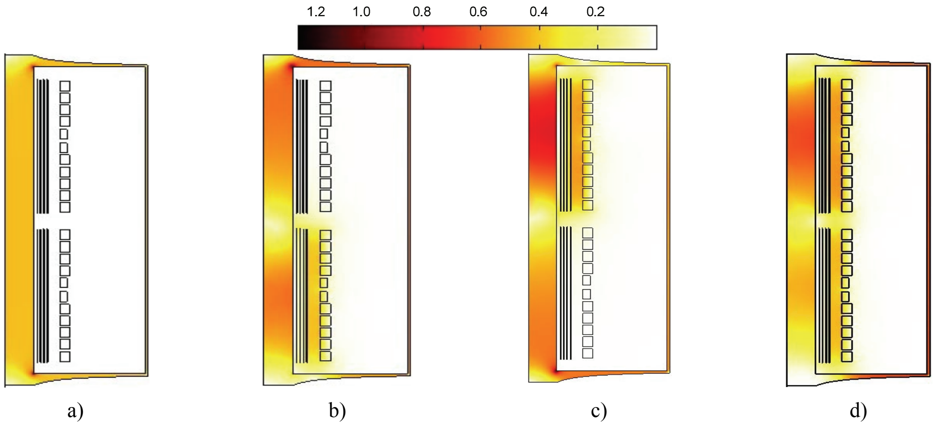

Fig. 14. Magnetic flux density in different loading conditions: cases a) I, b) II, c) III and d) IV

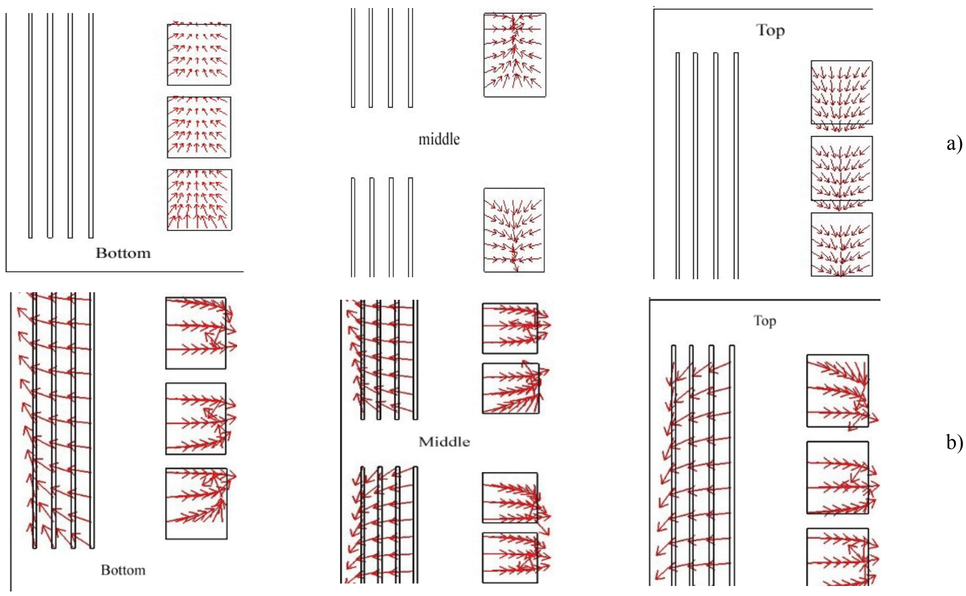

Fig. 15. Electromagnetic force distribution in bottom, middle and top of windings due to a) no-load and b) short-circuit conditions

While both LVs are short-circuited, the leakage flux density is much considerable in both bottom and top windings (Fig. 14a). Similarly, in the single short-circuit (Fig. 14b, c), the leakage flux density can just be seen in the short-circuited (top or bottom) windings. Consequently, the magnetizing flux density is seen in the no-load condition (Fig. 14d).

Finally, it is convenient to compute the electromagnetic forces deduced from the computed inrush and short-circuit currents. Using the magnetic flux densities (Fig. 14), local force density can be calculated by $\vec{f} = \vec{J} \times \vec{B}$ as shown in Fig. 15.

Consequently, total forces of the bottom high voltage windings can be computed as shown in Fig. 16.

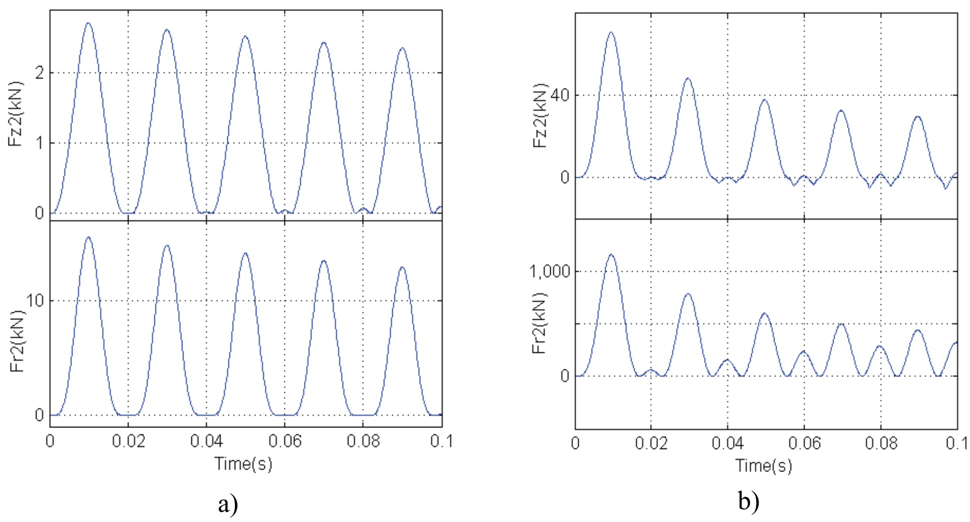

Fig. 16. a) Inrush and b) Short-circuit forces in bottom high voltage winding (Fz, Fr: axial and radial forces)

7. Conclusions

In this paper, a nonlinear equivalent circuit was presented for short-circuit and inrush current calculations in split-winding transformer. The effect of nonlinear inductances can be neglected in short-circuit current calculations. By considering the nonlinear inductances in the model, the introduced nonlinear equivalent circuit can be used for both short-circuit and no-load calculations. So, the introduced nonlinear equivalent circuit is a useful model for designers and experts to analyze the nonlinear behavior of the split-winding dry-type transformer in the whole of the system.

Finally, the simulated currents were applied and the magnetic forces are calculated in various loading conditions. It was seen that when both bottom and top windings are short-circuited (or open-circuited), the leakage (or magnetizing) flux is much considerable. But while only one winding is short-circuited, both leakage and magnetizing fluxes are presented and the electromagnetic forces are much greater.

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