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Assessment of costs in cut-off grades optimization by using grid search method

Introduction

Cut-off grade for an ore deposit is used to classify the material as ore or waste. For a metallic ore deposit, the material over the determined cut-off grade is considered as ore and can be mined whilst the material below the cut-off grade is considered as waste and, depending upon the mining method used, either left *in-situ* or sent to the waste dumps.

In order to get maximum profit from a mineral deposit, optimum cut-off grades must be applied. Optimum cut-off grades can be achieved only by a decreasing order of cut-off grades policy (Lane 1964; Dowd 1976; Cetin and Dowd 2002). However, the determination of an optimum cut-off grades schedule that gives the maximum discounted profit is a very complex process. In order to achieve this, one must reshuffle the grade tonnage distribution of the mineral deposit after each step of mining.

Kumral (2012) used a Mixed integer programming model for block sequencing and ore-waste discrimination together without using cut-off grades. He compared the results with a fixed cut-off grade policy.

Kumral (2013) and Moosavi, Gholamnejad, Ataee-pour, Khorram (2014) focused on grade control tools by minimizing economic loss. They argued that maximum profit can be achieved without using optimum cut-off grades. However, since block sequencing together with optimum cut-off grades was not yet accomplished, a comparison cannot be made.

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The objective of the work described in the paper is to find a feasible method of determining optimal production sequences of cut-off grades by using a grid search.

The main contribution of this work to the cut-off grade optimization is introducing a more realistic mining cost by using a probabilistic depletion rate and the addition of a rehabilitation cost. The work described in this paper is the first application of cut-off grades optimization in this context.

The use of grid search method in cut-off grades optimization is very popular since it was introduced by Lane (1988).

1. Optimization of cut-off grades by the grid search method

The grid search method is based on the concept of dividing the search area into equal size grids and searching for the optimum among the grid points. Lane (1988) was the first to propose the technique for optimizing the cut-off grades. He suggested using a primary grid of 9×9 cells, which yields 100 grid points. As a second step, he proposed a secondary grid of 6×6 cells, which yields 49 grid points, covering the four original cells that surround the maximum point. Finally, a third grid of 6×6 cells covering the four cells that surround the maximum point in the secondary grid was proposed. The process involves the calculation of 198 grid points. Lane claimed that the process of using three subsequent grids gives an accuracy of one in $9 \times 3 \times 3$, in other words 1 in 81, which is close to 1%. In fact, there is no obvious proof or evidence that using three subsequent grids, instead of one, improves the search process. The optimum point is not necessarily near the maximum point in the first grid and, by discarding the search area outside the four cells around the maximum point in the first grid, there is a danger of missing the global optimum target.

Asad (2005) applied Lane's grid search method to a copper and gold deposit. Cetin and Dowd (2013) extended the grid search method and applied it to mineral deposits that contain three mined minerals.

The following expressions are derived for the calculations used in the grid search method.

Assume that the grade-tonnage distribution of a mineral deposit consists of W grade cells for mineral 1. Hence, there would be $W + 1$ grade limits. The representation of the corresponding grade for the different cells would be:

$$[g_1(1), g_1(2)], [g_1(2), g_1(3)], \dots, [g_1(W-1), g_1(W)], [g_1(W), g_1(W+1)]$$

If the lower grade limit $g(w)$ for a given cell $[g(w), g(w+1)]$ is the cut-off grade representing interval p , the amount of the material above the cut-off grade, the amount of the material below the cut-off grade, the average grade above the cut-off grade can be found by using the following equations:

$$T_{ore}(p) = \sum_{w=1}^W T_p \quad (1)$$

where

$$T_p = \begin{cases} T_{(w)} & \text{if } W \geq w \geq p \text{ for } \forall w \in [1, W] \\ 0 & \text{otherwise} \end{cases}$$

- ↪ $T_{ore}(p)$ – amount of material above the cut-off grade for the p^{th} grade interval,
 $T_{(w)}$ – amount of material for the given grade limits,
 p – grade interval,

$$T_{waste}(p) = \sum_{w=1}^W T_p \quad (2)$$

where

$$T_p = \begin{cases} T_{(w)} & \text{if } 0 \leq w \leq p \text{ for } \forall w \in [1, W] \\ 0 & \text{otherwise} \end{cases}$$

- ↪ $T_{waste}(p)$ – amount of material below the cut-off grade for the p^{th} grade interval.

$$g_{avg}(p) = \frac{\sum_{w=1}^W T_p \cdot \left(\frac{g_w + g_{w+1}}{2} \right)}{\sum_{w=1}^W T_p} \quad (3)$$

where

$$\begin{bmatrix} T_p \\ g_w \\ g_{w+1} \end{bmatrix} = \begin{cases} \begin{bmatrix} T_{(w)} \\ g_1(w) \\ g_1(w+1) \end{bmatrix} & \text{if } W \geq w \geq p \text{ for } \forall w \in [1, W] \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \text{otherwise} \end{cases}$$

- ↗ $g_{avg}(p)$ – average grade above the cut-off grade for the p^{th} grade interval,
- $g(w)$ – lower grade limit of a given cell,
- $g(w + 1)$ – upper grade limit of a given cell.

The ore/material ratio can be found by using the Equation 4.

$$x(p) = \frac{T_{ore}(p)}{T_{ore}(p) + T_{waste}(p)} \quad (4)$$

- ↗ $x(p)$ – ore/material ratio for the p^{th} grade interval.

The average grades and the ore/material ratio values that were generated by these equations are for the grade intervals of a specified grade-tonnage distribution. If the grade intervals are chosen as grid points, the net present values for the three different limiting stages can be calculated by using the equations given below. However, if the grid points are assigned explicitly, the corresponding net present values for the grid points, the values of the ore/material ratio, the average grade above the cut-off grade, for each grid point, is found by interpolation. After finding these values for the grid points representing any pair of cut-off grade points, the net present values for the three different limiting stages can be calculated by using the following equations.

$$v_m = ((p - k) \cdot x \cdot y \cdot a) - x \cdot h - m - \frac{f + F}{M} \quad (5)$$

$$v_h = ((p - k) \cdot x \cdot y \cdot a) - x \cdot h - m - \frac{(f + F) \cdot x}{H} \quad (6)$$

$$v_k = ((p - k) \cdot x \cdot y \cdot a) - x \cdot h - m - \frac{(f + F) \cdot x \cdot y \cdot a}{K} \quad (7)$$

- ↗ p – price per unit of marketed product,
- k – variable refinery and/or marketing cost per unit of marketed product,
- x – ore/material ratio,
- y – yield during treatment (recovery),
- a – average grade,
- h – variable mineral processing cost,
- m – variable mining cost,
- f – fixed cost,
- F – opportunity cost,

- t – time unit of resource,
- v_m – profit for a limited mining rate,
- v_h – profit for a limited mineral processing rate,
- v_k – profit for a limited refinery or marketing rate,
- y – yield during treatment (recovery),
- a – average grade,
- M – maximum mining capacity,
- H – maximum refinery/marketing capacity,
- K – maximum refinery or marketing capacity.

Opportunity cost is the cost of the loss of potential gain from other mutually exclusive alternatives. In this case, opportunity cost is the cost of taking low grades when higher grades are still available. It is the loss of one period's interest on the present value of future cash flows of all ore remaining at that period.

After deciding on the profit for each stage, the highest possible net present value is found as:

$$v_{(\max)} = \text{maximum} \{ \text{minimum} (v_m, v_h, v_k) \} \quad (8)$$

As three stages restrict the process, for any grid point, the minimum among the three values gives the maximum possible profit and the maximum among them gives the optimum grid point. The cut-off grades that lie on the optimum grid point give the optimum cut-off grades.

2. Mining cost

The cost of excavating takes place in the life of a mine is considered as a variable mining cost and must be applied to all the drilling, blasting, excavating, loading, hauling and dumping.

The production or production amount is the part of material that is excavated and sent to mineral processing facilities.

The depletion is the amount of material depleted regardless of whether it is excavated or left *in-situ*.

The depletion rate is the rate of a mineral deposit which is to be depleted annually. Part of depletion is to be sent to the concentrator as ore, part of it is mined out and is dumped as waste material and part of it is left *in-situ*.

Each mineral deposit has special, unique characteristics. Since mineral deposits are not manmade, artificial matters, no two of mineral deposits are the same. Therefore, they are to be treated cautiously.

In cut-off grades determination, all parts of a mineral deposit are considered to be accessed immediately. However, if there are no access constraints, the variable mining costs should be applied to the production rates, not to the depletion rates.

It is well known that some of the blocks are left *in-situ*. It is always preferable to leave any portion of a mineral deposit that is not considered as ore, unexcavated. Unneeded excavation and dumping will bring up a further rehabilitation cost. The parts of a deposit that have an average grade less than the cut-off grade are left *in-situ* as much as possible.

The problem can be written in short as whether the variable mining cost is to be applied to the depletion rate of the deposit (total excavation) or to the production rate of the deposit (total amount of ore sent to the concentration units).

The depletion or depletion rate does not mean the portion or the rate of a deposit to be excavated. It means the portion or the rate of a deposit that is depleted, regardless of whether it is excavated or left *in-situ*.

Therefore, the application of the variable mining cost to the depletion rate or to the production rate is not realistic. The true rate lies somewhere in between. The true or applicable approach is that, the variable mining cost should be applied somewhere in between the depletion rate and the production rate. In order to solve this problem, the portion of depletion that is to be left *in-situ* each year must be known.

On the other hand, the application of variable mining cost to the depletion rate is more realistic for open pit mines and the application of variable mining cost to the production rate is more realistic for underground mines.

Kenneth F Lane is considered a pioneer of optimum cut-off grades theory. However, there are some points he did not mention. Should variable mining cost be applied to the depletion rate or to the production rate. As in the cut-off grades theory, the amount of material that has a grade lower than the determined cut-off grade should be left *in-situ* if there are no access constraints. As a result, the variable mining cost should be applied to the production rate.

On the other hand, if there are access constraints, the variable mining cost should be applied to the depletion rate.

As it is known from practice, depending on the characteristics of the mineral deposits, some of the blocks that have an average grade less than the determined cut-off grade are left *in-situ*, some of them are excavated and dumped as waste material. Naturally, variable mining costs should be applied to the blocks of a mineral deposit that are actually excavated.

The main problem arises from the fact that the part of a mineral deposit that is to be left *in-situ* cannot be known before a complete mine scheduling takes place, and mine scheduling cannot be done before cut-off grades determination. They must be done simultaneously. However, it is a large scale optimization problem that has yet to be solved. Instead, a probabilistic approach can be used in this context. That is, the part of the depletion rate of a mineral deposit in a time period which is to be left *in-situ* can be determined probabilistically. After a probabilistically determined partial depletion cost is to

be added to the algorithm, the basic profit algorithm per unit of a resource can be written as:

$$v = (p - k) \cdot x \cdot y \cdot a - x \cdot h - z \cdot m - (f + F) \cdot t \quad (9)$$

- v – profit,
 p – price per unit of marketed product,
 k – variable refinery and/or marketing cost per unit of marketed product,
 x – ore/material ratio,
 z – (ore + material to be dumped)/material ratio,
 y – yield from treatment (recovery),
 a – average grade,
 h – variable mineral processing cost,
 m – variable mining cost,
 f – fixed cost,
 F – opportunity cost,
 t – time per unit of resource.

The determination of the portion of the depletion that is to be dumped after excavation can be searched for by means of statistical tools. For open pit mines, exponential distribution is considered as a good tool for this subject. Some blocks can be left *in-situ* only for last years of an open pit mine, and the exponential approach suits this case very well.

The probability density function of an exponential distribution is used to find the portion of the depletion rate over the production rate that is to be left *in-situ*. As a result, inverse

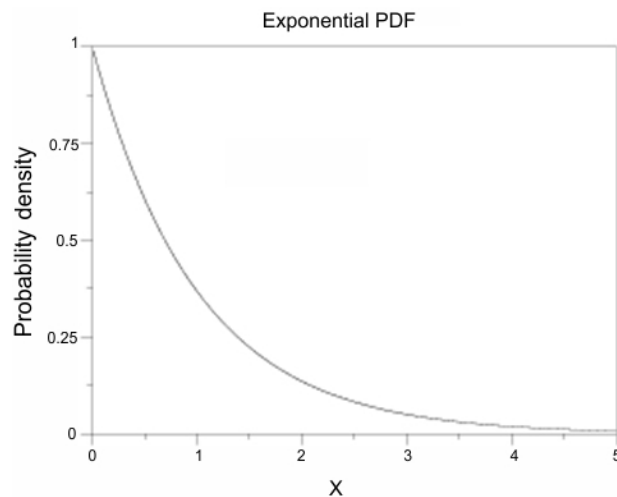


Fig. 1. Probability density function of an exponential distribution

Rys. 1. Funkcja gęstości prawdopodobieństwa rozkładu wykładniczego

probability density function is to be applied as the portion of the depletion rate over the production rate that is to be excavated and dumped. Therefore, by using the inverse probability density function of an exponential distribution, the portion of depletion actually excavated can be determined.

The probability density function (pdf) of an exponential distribution is given in Formula 10 and Figure 1.

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (10)$$

3. Rehabilitation cost

Another cost that is to be added to the cut-off grades algorithm is the reclamation or rehabilitation cost originating from the fact that the blocks that are excavated but will be dumped as waste materials incur some additional cost of rehabilitation. Gholamnejad (2008, 2009) mentioned the rehabilitation cost and included it in Lane's algorithm. He added the rehabilitation cost into Lane's formulations of the net present value determination.

Rehabilitation is the treatment of disturbed land for the purpose of establishing a stable environment. It includes removing hazardous materials, reshaping the land, restoring top soil revegetation, etc. It can be carried out both during mining operations and after the operations have ended.

The aim of rehabilitation is to ensure mining activities will be designed to minimize or mitigate adverse environmental and social impacts and create a self-sustaining ecosystem, but not necessarily the one that existed before mining began.

4. Optimization of cut-off grades with rehabilitation cost by using a grid search

The grid method used in this research is a single grid. The selection of grid points in the grid is done explicitly, rather than using the grade intervals for the given grade-tonnage distribution. The number of grid points are not fixed but rather left to the user of the program.

The rehabilitation cost is added to the algorithm explicitly, i.e., the user must enter the data. The inverse probability density function of exponential distribution is used for the part of depletion applied to the variable mining cost.

After the inclusion of different depletion rates and the rehabilitation cost to the algorithm, the basic profit algorithm per unit of a resource becomes:

$$v = (p - k) \cdot x \cdot y \cdot a - x \cdot (h - r) - z \cdot (m + r) - (f + F) \cdot t \quad (11)$$

- ↩ v – net present value,
 p – price per unit of marketed product,
 k – variable refinery and/or marketing cost per unit of marketed product,
 x – ore/material ratio,
 z – (ore + material to be dumped)/material ratio,
 y – yield from treatment (recovery),
 a – average grade,
 h – variable mineral processing cost,
 m – variable mining cost,
 r – variable rehabilitation
 f – fixed cost,
 F – opportunity cost,
 t – time per unit of resource.

5. Case study

A case study has been included here to illustrate the application of the software for determining optimum cut-off grades.

The case study is an open pit copper mine. The grade tonnage distribution for the deposit is shown in Table 1 and the technical and economic data is given in Table 2. The computation results showing the complete cut-off grade policy is given in Table 3.

Table 1. Grade-tonnage distribution for the copper deposit

Tabela 1. Rozkład tonażowy dla złoża miedzi

| Grade (%) | Tonnage (*million tons) | Grade (%) | Tonnage (*million tons) |
|-----------|-------------------------|-----------|-------------------------|
| 0.00–0.10 | 0.12 | 1.00–1.10 | 22.17 |
| 0.10–0.20 | 0.57 | 1.10–1.20 | 20.19 |
| 0.20–0.30 | 0.69 | 1.20–1.30 | 18.76 |
| 0.30–0.40 | 1.11 | 1.30–1.40 | 7.22 |
| 0.40–0.50 | 3.44 | 1.40–1.50 | 3.11 |
| 0.50–0.60 | 5.10 | 1.50–1.60 | 2.24 |
| 0.60–0.70 | 6.68 | 1.60–1.70 | 2.12 |
| 0.70–0.80 | 11.12 | 1.70–1.80 | 1.78 |
| 0.80–0.90 | 17.44 | 1.80–1.90 | 0.61 |
| 0.90–1.00 | 21.83 | 1.90–2.00 | 0.16 |

Table 2. Technical and economic data for the copper deposit

Tabela 2. Dane techniczne i ekonomiczne dla złoża miedzi

| Description | Value |
|---|------------|
| Lower limit of cut-off grades (%) | 0.0 |
| Upper limit of cut-off grades (%) | 2.0 |
| Interval between cut-off grade decisions (%) | 0.001 |
| Mining Capacity (tons per year) | 13 000 000 |
| Mineral Processing Capacity (tons per year) | 10 000 000 |
| Marketing and/or refining capacity (tons per year) | 130 000 |
| Selling price for copper (dollars per ton) | 5 660 |
| Variable mining cost of material mined (dollars per ton) | 2.4 |
| Variable rehabilitation cost of material mined (dollars per ton) | 0.8 |
| Variable concentration cost of material processed (dollars per ton) | 9.6 |
| Variable marketing and/or refining cost (dollars per ton) | 1 500 |
| Recovery rate (%) | 92 |
| Discount rate (%) | 10 |

Table 3. Output file. The total discounted profit is USD 2,059,078,086

Tabela 3. Plik wyjściowy. Całkowity zdyskontowany zysk wynosi 2 059 078 086 USD

| Year | Cut-off (%) | Disc. Profit (\$) | Profit (\$) | Depletion (tons) | Production (tons) | Refinery (tons) |
|------|-------------|-------------------|-------------|------------------|-------------------|-----------------|
| 1 | 0.796 | 271,309,694 | 298,440,664 | 12,404,002 | 10,000,000 | 102,575 |
| 2 | 0.770 | 244,723,801 | 296,115,799 | 12,107,534 | 10,000,000 | 101,845 |
| 3 | 0.744 | 220,636,508 | 293,667,193 | 11,824,908 | 10,000,000 | 101,093 |
| 4 | 0.715 | 198,620,600 | 290,800,420 | 11,524,842 | 10,000,000 | 100,231 |
| 5 | 0.684 | 178,982,342 | 288,252,851 | 11,281,879 | 10,000,000 | 99,478 |
| 6 | 0.652 | 161,512,469 | 286,129,191 | 11,099,121 | 10,000,000 | 98,862 |
| 7 | 0.616 | 145,525,456 | 283,587,945 | 10,900,468 | 10,000,000 | 98,137 |
| 8 | 0.578 | 131,149,854 | 281,131,360 | 10,725,584 | 10,000,000 | 97,445 |
| 9 | 0.537 | 118,163,677 | 278,623,769 | 10,563,822 | 10,000,000 | 96,749 |
| 10 | 0.493 | 106,374,672 | 275,908,503 | 10,404,146 | 10,000,000 | 96,004 |
| 11 | 0.444 | 95,895,115 | 273,599,953 | 10,281,040 | 10,000,000 | 95,378 |
| 12 | 0.392 | 86,429,041 | 271,251,354 | 10,166,682 | 10,000,000 | 94,748 |
| 13 | 0.335 | 78,279,729 | 270,242,855 | 10,122,226 | 10,000,000 | 94,48 |
| 14 | 0.273 | 21,475,128 | 81,551,762 | 3,053,745 | 3,028,856 | 28,537 |

A total of 2001 different cut-off grades were searched for the optimum. The mining operation terminates in 13 years and 4 months and total production is 133,028,856 tons. The total discounted profit is USD 2,059,078,086.

The cut-off grades, and as a result the depletion rates, are lowered progressively throughout the life of the mine. Cut-off grades start with 0.796% and end up with 0.273% in less than 16 years.

The results show that the cut-off grades, and as a result the depletion rates, are lowered progressively throughout the life of the mine. Due to the time value of money, the optimum can only be achieved with a decreasing order of cut-off grades schedule. These results are concordant with the optimum cut-off grades theory.

The depletion amount actually excavated is always less than the depletion rate and more than the production rate. By using the inverse probability density function of an exponential distribution, the portion of depletion actually excavated is used for mining cost calculation and related rehabilitation cost. Total depletion amount is 146,459,999 tons and total actually excavated depletion amount is 146,415,534

The rate of actually excavated depletion of the case study is shown in Table 4.

The sequence of optimum cut of grades given in Table 3 is found by using actually excavated depletion rates shown in Table 4 and the related rehabilitation cost. This approach is

Table 4. Actually excavated depletion rate

Tabela 4. Aktualny wskaźnik zużycia urobku

| Year | Depletion (tons) | Excavated depletion (tons) | Production (tons) |
|------|------------------|----------------------------|-------------------|
| 1 | 12,404,002 | 12,404,000 | 10,000,000 |
| 2 | 12,107,534 | 12,107,529 | 10,000,000 |
| 3 | 11,824,908 | 11,824,897 | 10,000,000 |
| 4 | 11,524,842 | 11,524,817 | 10,000,000 |
| 5 | 11,281,879 | 11,281,821 | 10,000,000 |
| 6 | 11,099,121 | 11,098,985 | 10,000,000 |
| 7 | 10,900,468 | 10,900,166 | 10,000,000 |
| 8 | 10,725,584 | 10,724,922 | 10,000,000 |
| 9 | 10,563,822 | 10,562,424 | 10,000,000 |
| 10 | 10,404,146 | 10,401,423 | 10,000,000 |
| 11 | 10,281,040 | 10,275,893 | 10,000,000 |
| 12 | 10,166,682 | 10,158,383 | 10,000,000 |
| 13 | 10,122,226 | 10,105,685 | 10,000,000 |
| 14 | 3,053,745 | 3,044,589 | 3,028,856 |

first used in cut-off grades optimization. Therefore, this paper makes previous approaches obsolete.

Conclusions

The objective of the work described in this paper is to find a feasible method for determining optimal production sequences of cut-off grades. A complete, detailed mine production schedule that includes mining and other access constraints is beyond the scope of this work. For this reason no mining or other access constraints are included in the formulation of the problem. The orebody is completely defined by grade-tonnage distributions and it is assumed that any parcels of ore selected for production have the same characteristics as the specified grade-tonnage curves and are immediately accessible.

The computer program is based on the grid search method. The grid search method described here is based on Lane's grid search method but with some important differences in the size and the method of the selection of the grid.

The work described in this paper is the first application of the grid search method to exponentially distributed depletion rates and rehabilitation cost. The inclusion of rehabilitation cost is known in the mining sector. But the use of exponential distribution in the determination of the depletion rates, which the variable mining cost is to be applied to, is the first application to the mining sector. This paper shows that Lane's basic algorithm is to be used cautiously in that the variable mining cost must not to be applied to the depletion rates directly, and a cost for rehabilitation must be added.

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**ASSESSMENT OF COSTS IN CUT-OFF GRADES OPTIMIZATION
BY USING GRID SEARCH METHOD**

Abstract

The optimization of cut-off grades is a fundamental issue for metallic ore deposits. The cut-off grade is used to classify the material as ore or waste. Due to the time value of money, in order to achieve the maximum net present value, an optimum schedules of cut-off grades must be used. The depletion rate is the rate of depletion of a mineral deposit. Variable mining costs are to be applied to the really excavated material, as some of the depletion can be left *in-situ*. Due to access constraints, some of the blocks that have an average grade less than the determined cut-off grade are left *in-situ*, some of them are excavated and dumped as waste material. Naturally, variable mining costs should be applied to the blocks of a mineral deposit that are actually excavated. The probability density function of an exponential distribution is used to find the portion of the depletion rate over the production rate that is to be left *in-situ*. As a result, inverse probability density function is to be applied as the portion of the depletion rate over the production rate that is to be excavated and dumped. The parts of a mineral deposit that are excavated but will be dumped as waste material incur some additional cost of rehabilitation that is to be included in the algorithm of the cut-off grades optimization. This paper describes the general problem of cut-off grades optimization and outlines the further extension of the method including various depletion rates and variable rehabilitation cost. The author introduces the general background of the use of grid search in cut-off grades optimization by using various depletion rates and variable rehabilitation cost. The software developed in this subject is checked by means of a case study.

Key words: cut-off grades, optimization, grid search, rehabilitation cost, depletion rate

**OCENA KOSZTÓW W OPTIMALIZACJI WARTOŚCI BRZEŻNYCH
ZA POMOCĄ METODY WYSZUKIWANIA W SIECI****Streszczenie**

Optymalizacja wartości brzeżnych jest sprawą podstawową dla złóż rud metali. Wartość brzeżna służy do klasyfikacji materiału jako rudy lub odpadu. Ze względu na wartość pieniądza w czasie, aby osiągnąć maksymalną wartość bieżącą netto, należy zastosować optymalne harmonogramy wartości brzeżnej. Wskaźniki zubożenia to stopa szczypania złoża mineralnego. Zmienne koszty wydobycia mają zastosowanie dla rzeczywistego wydobycia surowca, ponieważ część zubożonego surowca można pozostawić na miejscu. Z powodu ograniczeń dostępu niektóre bloki, które mają średnią ocenę mniejszą niż określona wartość brzeżna, pozostawia się na miejscu, a niektóre z nich są wydobywane i składowane jako odpady. Naturalnie, zmienne koszty wydobycia powinny być stosowane do bloków złoża mineralnego, które są faktycznie wydobywane. Funkcja gęstości prawdopodobieństwa rozkładu wykładniczego jest stosowana do znalezienia części wskaźnika zubożenia w stosunku do wielkości produkcji, która ma pozostać na miejscu. W rezultacie odwrotna funkcja gęstości prawdopodobieństwa może być zastosowana jako część wskaźnika zubożenia w stosunku do wielkości produkcji, która ma zostać wydobyta i składowana. Część złóż mineralnych, które są wydobyte, ale zostają składowane jako odpady, pociąga za sobą dodatkowe koszty rekultywacji, które należy uwzględnić w algorytmie optymalizacji wartości granicznych. W artykule opisano ogólny problem optymalizacji wartości brzeżnych i przedstawiono dalsze rozszerzenie metody, w tym różne wskaźniki zubożenia i zmienny koszt rekultywacji. Autor wprowadza ogólne tło zastosowania przeszukiwania siatki w optymalizacji ocen granicznych za pomocą różnych wskaźników zubożenia i zmiennego kosztu rekultywacji. Oprogramowanie zastosowane w tym przypadku jest sprawdzane za pomocą studium przypadku.

Słowa kluczowe: wartości brzeżne, optymalizacja, wyszukiwanie w sieci, koszty rehabilitacji, wskaźnik zubożenia