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# Monotonicity of the Selling Price of Information with Risk Aversion in Two Action Decision Problems

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**Abstract**

Various approaches have been introduced over the years to evaluate information in the expected utility framework. This paper analyzes the relationship between the degree of risk aversion and the selling price of information in a lottery setting with two actions. We show that the initial decision on the lottery as well as the attitude of the decision maker towards risk as a function of the initial wealth level are critical to characterizing this relationship. When the initial decision is to reject, a non-decreasingly risk averse decision maker asks for a higher selling price as he gets less risk averse. Conversely, when the initial decision is to accept, non-increasingly risk averse decision makers ask a higher selling price as they get more risk averse if information is collected on bounded lotteries. We also show that the assumption of the lower bound for lotteries can be relaxed for the quadratic utility family.

**Keywords:** decision analysis, value of information, selling price, risk aversion, buying price

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# 1 Introduction

It is well known that information has value only when it offers the possibility of a decision change. Decision makers may seek information to see whether the best course of action they could take without information is still preferable when uncertainty is reduced. The uncertainty-reducing benefit of information may at first lead to a premature conclusion that risk averse decision makers assign a higher value to information. However, it has been proven in the expected utility framework that information is not necessarily preferred by decision makers that are less tolerant towards risk. In fact, the value of information may sometimes behave counterintuitively with respect to changes in some particular attributes of the decision environment such as the initial wealth, risk aversion and action flexibility (see Gould 1974 and Hilton 1981). If we leave the well-structured expected utility framework and explore how decision makers evaluate information in empirical settings, behavior of preferences towards information become even more difficult to predict. We encounter many instances in which decision makers reveal irrational and sometimes inconsistent preferences towards information. For example, they may acquire information even when it has no impact on their decisions (see Bastardi and Shafir 1998, Tykocinski and Ruffe 2003). Moreover, decision makers may overvalue information in strategic environments (see Gehrig *et al.* 2003), whereas this tendency might change in non-strategic environments (see Branthwaite 1975, Rötheli 2001, and Sakalaki and Kazi 2007).

In pricing uncertain prospects, approaches such as the buying price and the selling price are widely used in literature. An extensive comparative analysis of these two approaches can be found in the seminal paper by La Valle (1968) and the recent study by Lewandowski (2013). Information can also be classfied as an uncertain prospect for two reasons. First, decision makers do not know what information will be conveyed before buying or selling information. Second, unless decision makers deal with perfect information, uncertainty is not fully reduced even after information is revealed. As shown in Hazen and Sounderpandian (1999), selling and buying price approaches do not agree in ranking various information alternatives. In a two-action lottery setting considered here, Bakir and Klutke (2011) shows that quite restrictive conditions should be imposed on the behavior of the value of information to ensure that these approaches agree. Selling price, which is the main focus of this paper, assigns a monetary value to information by measuring the wealth increase that would make the decision indifferent between making the decision with and without information at the original wealth level. In fact, selling price approach converts the expected utility increase to monetary units.

It has been shown that attitude towards decision alternatives prior to information gathering has a significant effect on the value of information. In particular, choices made before information is revealed makes the value of information behavior more predictable as the degree of risk aversion changes. For example, in the context of the simplest of decision problems with two actions where the decision maker chooses between a sure outcome and a risky prospect (i.e., a lottery), we observe such an effect. In these decision problems, if the decision maker accepts the lottery, then the terminal wealth is the initial wealth plus the outcome of the lottery. Conversely, a decision to reject the lottery renders a sure outcome. In Mehrez (1985), we see that a risk neutral decision maker pays more for perfect information if the initial decision is to reject the lottery. Likewise, a simple example discussed in Eeckhoudt and Godfroid (2000) suggests that an uninformed decision made by a risk neutral newsvendor determines whether more should be paid for acquiring information. In a slightly more complex case where both actions generate random outcomes, Delquié (2008) shows that the value of certain information alternatives is maximized when the decision maker is initially indifferent between rejecting and accepting the lottery. This result is observed in other studies as well (see Fatti *et al.* 1987 for the case of a risk neutral decision maker and Bickel 2008 where a risk averse decision maker is considered).

This paper is concerned with the relationship between risk aversion and the selling price of information. This study is novel is the sense that none of the earlier studies that discussed this relationship offers a comparative analysis evaluating information using its selling price. However, from a technical point of view, results introduced here are an extension of findings presented in Abbas *et al.* (2013) where authors explore the relationship between risk aversion and the buying price of information. In Abbas *et al.* (2013), if the initial decision made by the decision maker is to reject the lottery without information acquisition, it is shown that less risk averse decision makers are willing to pay more for information. We show in this paper that the same is true for the selling price of information if the decision maker's utility function exhibits non-decreasing degree of risk aversion. In the case when the initial decision is to accept the lottery, we show that the behavior of the selling price is more predictable than that of the buying price. In particular, if an accept decision is made without information on a lottery whose potential loss to the decision maker is finite, then selling price is increasing in the degree of risk aversion for a non-increasingly risk averse decision maker.

The discussion of this paper continues in the following order. Section 2 provides some notation and definitions. Our main results are presented in Section 3, where the discussion is ordered based on the initial decision made on the lottery. First, we analyze the case with the reject decision and then consider the case with the accept decision. Section 4 presents some concluding remarks.

## 2 Model formulation

We begin with a description of the problem. A decision maker makes a decision about a lottery $\Pi : \Omega \to \mathrm{E}, \mathrm{E} \subseteq \mathbb{R}$ with monetary outcomes in E that are either positive or negative. This decision is made with a continuous and monotonically increasing utility function $u : \mathbb{R} \to \mathbb{R}$ and an initial wealth level of $w$. The decision maker has two alternative courses of action available in action set $\mathcal{A}$: He could either reject or accept the lottery. If the decision maker decides to accept the lottery, his terminal utility is $u(w + \Pi)$ whereas rejecting the lottery leads to a terminal utility of $u(w)$. There is also a probability distribution (with a cumulative distribution function $F$) that governs the occurrence of outcomes in E.

The decision maker is presented with the opportunity to gather information on the lottery $\Pi$. The piece of information that is gathered will make it known to the decision maker whether one or more events on the lottery outcome occur or not. In this regard, information is not necessarily perfect; however the decision maker will reduce uncertainty by shrinking the set of possible outcomes of $\Pi$. More technically, information is gathered on a collection of disjoint events $\{A_1, \ldots, A_k\}$. They partition the entire outcome space into $k$ events (i.e., $\cup_{j=1}^{k} A_j = \mathrm{E}$) and generate an information alternative $\mathcal{I}$. By gathering information $\mathcal{I}$, the decision maker learns not only whether or not $A_1$ to $A_k$ occur, but also the occurrence of events that either complement or are formed as a union or intersection of an arbitrary number of events in $\{A_1, \ldots, A_k\}$. Accordingly, if the decision maker gathers $\mathcal{I}$ and learns that $A_j$, $1 \leq j \leq k$, occurs, then he will know that the set of outcomes is indeed $A_j \subset \mathrm{E}$. The probabilities of all other outcomes in $\mathrm{E} - A_j$ that was deemed possible before information acquisition become zero.

The major question that we address in this study is concerned with the price of $\mathcal{I}$. We assume that the decision maker uses the selling price approach for pricing $\mathcal{I}$. According to this approach, the price of information is the minimum monetary compensation that the decision maker asks to forgo the opportunity to make a decision in the light of information. In other words, this compensation will make the decision maker equally well off between making an informed and an uninformed decision on $\Pi$. We denote the selling price with $S(w, \mathcal{I}, u)$ highlighting its dependence on $\mathcal{I}$, $u$ and $w$. To simplify the notation though, we use a shorthand form $S_u = S(w, \mathcal{I}, u)$ throughout the paper unless an explicit notation is needed. $S_u$ satisfies

$$\max\{u(w + S_u), \mathbb{E}[u(w + \Pi + S_u)]\} = \sum_i P\{A_i\} \cdot \max\{\mathbb{E}[u(w + \Pi)|A_i], u(w)\}. \tag{1}$$

In (1), the right hand side of the equation is the expected utility of making the decision with information whereas the left hand side is the expected utility of making an uninformed decision after receiving $S_u$ (as compensation). Hence, we argue that $S_u$ balances the expected utility of making an informed and an uninformed decision. We define an optimal decision function $d_u : \mathbb{R} \times \mathcal{B} \to \mathbb{R}$ (where $\mathcal{B}$ is the collection of Borel sets in $\mathbb{R}$) to simplify the problem by updating the partition imposed by an information alternative. The optimal decision function is

$$d_u(w, A) = \begin{cases} +1 & \textit{if } \; \mathbb{E}[u(w + \Pi)|A] \geq u(w) \\ -1 & o.w. \end{cases}$$

The idea in defining the optimal decision function is clustering the outcomes of the lottery in two sets based on the decision made after acquiring an information alternative. In the perfect information case, these clusters can be formed easily based on the sign of an outcome (i.e., the decision maker chooses to accept if the outcome is positive, and to reject if the outcome is negative). However, in the case of arbitrary information alternatives, an outcome $\pi \in \mathbb{R}$ is placed into one of these clusters based on the decision made after observing the event in the information alternative that includes $\pi$. Using the optimal decision function, we could define $\Gamma_u(w,\mathcal{I}) = \{\ \pi \in \mathbb{R} : d_u(w,A) = +1$ for $A \in \{A_1,\dots,A_k\}$ and $\pi \in A\}$. In words, $\Gamma_u(w,\mathcal{I})$ is the union of the disjoint events that generate $\mathcal{I}$ on which the decision maker accepts the lottery. We should also have $\Gamma_u(w,\mathcal{I}) \cup \Gamma_u^c(w,\mathcal{I}) = \mathrm{E}$ because the decision maker's action set includes only two elements. The following example should make the clustering idea more clear.

**Example 1** Suppose that a decision maker with a utility function $u(w)$ should make a decision on a lottery $\mathit{\Pi}$ with five outcomes, $\mathrm{E} = \{\pi_1,\pi_2,\pi_3,\pi_4,\pi_5\}$. An information alternative $\mathcal{I}$ generated by the disjoint events $\{\pi_1,\pi_2\},\{\pi_3,\pi_4\}$, and $\{\pi_5\}$ is also available for purchasing. If the decisions conditional upon the occurrence of $\{\pi_1,\pi_2\}$ and $\{\pi_5\}$ are both accept and the decision conditional on $\{\pi_3,\pi_4\}$ is reject, then the cluster sets should be $\Gamma_u(w,\mathcal{I}) = \{\pi_1,\pi_2,\pi_5\}$, and $\Gamma_u^c(w,\mathcal{I}) = \{\pi_3,\pi_4\}$.

# 3 Selling price of information for a risk averse decision maker

Building on the findings of previous value of information literature, we separate our discussion based on the initial decision made on the lottery. The degree of risk aversion at wealth level $w$ is measured by the absolute risk aversion function $r_u(w) = -u''(w)/u'(w)$. To simplify notation, $r_u \geq r_v$ is used to express $r_u(w) \geq r_v(w)$ holds for every $w$. We first consider the case in which the initial decision is to reject the lottery $\mathit{\Pi}$, and then discuss the relationship between risk aversion and the selling price when the initial decision is to accept $\mathit{\Pi}$.

## 3.1 When the lottery is rejected without information

The definition of the selling price indicates that the decision made without information may not coincide with the decision made after a compensation is given to the decision maker to sell information. Hence, whether or not a decision maker is decreasingly risk averse is important. The first proposition stated below indicates that monotonicity of risk aversion with the selling price of information is obtained when the decision maker is non-decreasingly risk averse and his decision is to reject the lottery without information.

**Proposition 1** *Consider two decision makers with non-decreasingly risk averse utility functions u and v such that* $r_u \geq r_v$*. Suppose that both decision makers reject the lottery* $\Pi$ *without information. Then* $S_v \geq S_u$*.*

**Proof:** See Appendix.

The requirement that the utility function $u$ exhibits non-decreasing risk aversion is imposed because such an attitude towards risk guarantees that the initial decision to reject at wealth level $w$ does not change at the wealth level $w + S_u$ as well. As such, it is also possible to extend Proposition 1 to hold for decreasingly risk averse utility functions if the decision without information is not sensitive to small changes in the wealth level $w$. We formalize this result in Proposition 2 and characterize the risk sensitive behavior of the selling price for decreasingly risk averse utility functions.

**Proposition 2** *Consider two decision makers with decreasingly risk averse utility functions u and v such that* $r_u \geq r_v$*. Suppose that both decision makers reject the lottery* $\Pi$ *without information at any wealth level in* $[w, w + \mathbb{E}\Pi)$*. Then* $S_v \geq S_u$*.*

**Proof:** See Appendix.

The following example demonstrates that if the decision is sensitive to small increases in wealth level, then a more risk averse decision maker may sell information at a higher price even when the decision is to reject the lottery by both decision makers.

**Example 2:** Consider two decision makers with utility functions $u(x) = x - 33.6e^{-0.05x}$ and $v(x) = x - 33.5e^{-0.05x}$. Both utility functions belong to the linear plus exponential family, satisfy $r_u \geq r_v$ and are decreasingly risk averse. Suppose a decision is made whether or not to accept the following lottery $\widetilde{\Pi}$:

Table 1: Lottery in Example 2

| Prob. | 0.1 | 0.3 | 0.3 | 0.3 |
|---|---|---|---|---|
| $\pi_i$, \$ | 9 | -4 | 10 | -7.2 |

At an initial wealth level of $w = 19$, both decision makers reject the lottery without information. The decision maker is presented with information generated by sets (or events) $\{9, -4\}$ and $\{10, -7.2\}$. In plain words, if a decision maker acquires this piece of information, then he learns whether the outcome of the lottery lies in the set $\{9, -4\}$ or the set $\{10, -7.2\}$. The selling price of this piece of information for the utility function $u$ is greater than the selling price for the utility function $v$ although $v$ is less risk averse. This follows because the decision to reject the lottery changes as a result of the change in the wealth level augmented by the selling price. In other words,

$$u(w + \widetilde{\Pi}) > \mathbb{E}[u(w + \widetilde{\Pi})], \text{whereas } u(w + S_u + \widetilde{\Pi}) < \mathbb{E}[u(w + S_u + \widetilde{\Pi})]$$

$$v(w + \widetilde{\Pi}) > \mathbb{E}[v(w + \widetilde{\Pi})], \text{whereas } v(w + S_v + \widetilde{\Pi}) < \mathbb{E}[v(w + S_v + \widetilde{\Pi})].$$

This is entirely possible in the decreasingly risk averse linear plus exponential utility family.

One final remark about decreasingly risk averse utility functions should be made here. Even for the decreasingly risk averse utility functions, such a change in the decision made against a risky lottery is relatively uncommon. In fact in Example 2, the magnitude of the difference in the selling price of two decision makers is extremely small when compared against lottery payoffs. Therefore, we could easily argue that monotonicity of selling price with the degree of risk aversion holds in overwhelming majority of cases for decreasingly risk averse utility functions when the decision without information is to reject.

## 3.2 When the lottery is accepted without information

In the case when the initial decision on the lottery is to accept, the benefit of information acquisition is compared against a lottery choice rather than a sure outcome. As such, the dynamics of information acquisition changes. The resulting implication is that we cannot prove a general monotonicity result to characterize the aforementioned relationship. However, we present a result on lotteries with potential losses bounded from below. For non-increasingly risk averse utility functions, we show that if possible losses that could result from choosing to accept the lottery are bounded by some maximum amount, then selling price of that information is higher for a more risk averse decision maker. To this end, we define $\pi_{min} = sup\{\pi : P(\Pi < \pi) = 0\}$ and formally state the result below.

**Proposition 3** *Consider two decision makers with non-increasingly risk averse utility functions $u$ and $v$ such that $r_u \geq r_v$. Suppose both are presented with a lottery $\Pi$ bounded from below. If both decision makers accept the lottery without information at the wealth level $w$ and consider acquisition of information $\mathcal{I}$ on $\Pi$, then $S_u \geq S_v$.*

**Proof:** See Appendix.

This result indicates that as far as the relationship between risk aversion and value of information is concerned, selling price exhibits a more predictable behavior than the buying price in two action decision problems. Existence of such a monotonic behavior of the selling price which is not necessarily observed with the buying price is another point of difference between these two approaches. It is difficult to explain the intuition behind this apparent divergence in behavior, but here is a rather technical explanation. First, let us recall the definition of the buying price, $B_u$, of the lottery $\Pi$ for the utility function $u$. It is defined as solution to the following equation

$$\max\{u(w), \mathbb{E}[u(w + \Pi)]\} = \sum_i P\{A_i\} \cdot \max\{\mathbb{E}[u(w + \Pi - B_u)|A_i], u(w - B_u)\}. \quad (2)$$

One major difference between the selling price and the buying price in assigning a monetary value to uncertain prospects including information acquisition is the compensation made for the benefit of information acquisition. In selling price, the decision maker is compensated for forgoing the opportunity of making an informed decision whereas in buying price, the decision maker makes the informed decision at a cost without receiving any compensation. Accordingly, adding the selling price as a compensation to the decision maker's wealth level has an influence on only one decision made, that is the decision made between the original lottery and the sure outcome (which appears on the left hand side of the value of information equation, see Equation 1). On the other hand, the buying price appears on the right hand side of the value of information equation (see Equation 2). This in fact causes the buying price to shift the risk preferences of the decision maker in $k$ conditional decisions made. To see this, recall that the events $A_1$ to $A_k$ provide a partition of entire outcome space E and the right hand side of Equation 2 involves $k$ conditional decisions on the lottery $\Pi$. Therefore, in comparison to the selling price, the buying price has a more risk preference shifting influence on the value of information equation, which reduces its tendency to move monotonically with respect to the degree of the decision maker's risk aversion.

Proposition 3 is important because in many practical applications lotteries under consideration are bounded. A question to ask is whether this result can naturally be extended to decisions involving unbounded lotteries. While it looks trivial to show that extension at the first glance, a comparative analysis of the selling price equation for two utility functions $u$ and $v$ such that $r_u \geq r_v$ indicates that monotonicity may not necessarily hold for unbounded lotteries. We currently have no proof or counterexample to support this conjecture. However, there is a technical note in the appendix that discusses why a trivial extension cannot be made here. The case for unbounded lotteries remains an open research question.

As in the case in which the initial decision is to reject the lottery, we present an analogous result for the increasingly risk averse utility functions that requires robustness of the initial decision to accept at any wealth level in $[w, w + \mathbb{E}\Pi)$.

**Proposition 4** *Consider two decision makers with increasingly risk averse utility functions $u$ and $v$ such that $r_u \geq r_v$. Suppose both are presented with a lottery $\Pi$ bounded from below. If both decision makers accept the lottery without information at any wealth level in $[w, w + \mathbb{E}\Pi)$ and consider acquisition of information $\mathcal{I}$ on $\Pi$, then $S_u \geq S_v$.*

**Proof:** See Appendix.

There is indeed one utility function family that is widely used in literature, increasingly risk averse and for which the selling price exhibits a monotonic behavior as a function of risk aversion even in the case of unbounded lotteries. Quadratic utility functions constitute a special increasingly risk averse family that possess the one-switch property (see Bell 1988 for the families of one-switch utility functions). In Abbas *et al.* (2013), quadratic utility functions were shown to be the only family for which a monotonicity relationship is observed for the buying price of information. We

have already shown that monotonicity between risk aversion and selling price is not elusive, however, quadratic utility function is the only widely used utility function in which a less risk averse decision maker never pays more for information when the initial decision is to accept and remains robust to small changes in wealth. The most general form of this utility function is $u(x) = ax^2+bx+c$. Since $u'' < 0$, we have $a < 0$ and for obvious reasons we eliminate parameter $c$. Thus, we use $u(x) = -ax^2 + bx$, $a > 0$ in our treatment of quadratic utility functions below.
In its general form, the selling price is the value of $S_u$ that satisfies the following equation

$$
\begin{aligned}
&-a\mathbb{E}[(w + \mathit{\Pi} + S_u)^2] + b\mathbb{E}[w + \mathit{\Pi} + S_u] = \\
&\quad = P\{\Gamma_u(w,\mathcal{I})\} \cdot \big( - a\mathbb{E}[(w + \mathit{\Pi})^2|\Gamma_u(w,\mathcal{I})] + b\mathbb{E}[w + \mathit{\Pi}|\Gamma_u(w,\mathcal{I})]\big)+ \\
&\quad +P\{\Gamma_u^c(w,\mathcal{I})\} \cdot \big( - aw^2 + bw\big).
\end{aligned} \tag{3}
$$

Using the usual shorthand notation $\Gamma_u = \Gamma_u(w,\mathcal{I})$ and $\Gamma_u^c = \Gamma_u^c(w,\mathcal{I})$ and further simplifying (3), we obtain

$$
\begin{aligned}
&(b - 2aw) \cdot \big(\mathbb{E}[\mathit{\Pi}] + S_u\big) - a \cdot \big(\mathbb{E}[\mathit{\Pi}^2] + S_u^2\big) - 2aS_u\mathbb{E}[\mathit{\Pi}] = \\
&\quad = (b - 2aw) \cdot P\{\Gamma_u\}\mathbb{E}[\mathit{\Pi}|\Gamma_u] - aP\{\Gamma_u\}\mathbb{E}[\mathit{\Pi}^2|\Gamma_u].
\end{aligned} \tag{4}
$$

Using (4), we prove the next monotonicity result.

**Proposition 5** *Consider a decision maker with a quadratic utility function $u(x) = -ax^2 + bx$, $a > 0$ whose initial decision on lottery $\mathit{\Pi}$ is to accept at the wealth level $w$. If the initial decision to accept remains insensitive to wealth changes in the range $[w, w + \mathbb{E}\mathit{\Pi})$, then the selling price of information $\mathcal{I}$, $S_u$ is non-decreasing in $r_u$.*

**Proof:** See Appendix.

This nice result in the quadratic utility case cannot be extended to other one-switch utility functions. The below example is provided to illustrate the necessity of decision robustness in the wealth range $[w, w+\mathbb{E}\mathit{\Pi})$ when we consider increasingly risk averse utility functions.

**Example 3** Consider again two decision makers with quadratic utility functions $u(x) = -x + 43x^2$ and $v(x) = -x + 43.1x^2$. We observe that $r_u \geq r_v$. Suppose a decision is made whether or not to accept the lottery in Table 2.
The decision made at the wealth level $w = 0$ is to accept the lottery for both decision makers. However, this decision is quite sensitive to changes in the wealth level; both decision makers are almost equally well off rejecting the lottery. They have the

Table 2: Lottery in Example 3

| Prob. | 0.1 | 0.3 | 0.3 | 0.3 |
|---|---|---|---|---|
| $\pi_i$, \$ | 9 | -4 | 10 | -5 |

opportunity to gather information generated by sets (or events) $\{9, -4\}$ and $\{10, -5\}$. Because of the sensitivity of the decision to changes in the wealth level, selling price is higher for the less risk averse decision maker. Therefore, the wealth level condition imposed in propositions presented for the accept case on increasingly risk averse utility functions is necessary.

# 4 Conclusions

Selling price of information measures the compensation that the decision maker asks to forgo the opportunity to make an informed lottery decision. It is well documented in the value of information literature that information value is not monotonic as a function of the degree of risk aversion. It has been shown that in the context of the buying price of information, the relationship between the two depends on the initial lottery decision. We show in this paper that such a behavior is also valid for the selling price of information. When two non-decreasingly risk averse decision makers decide to reject the lottery before acquiring further information, the selling price is higher for a less risk averse decision maker. This result holds largely because the characterization of how a risky prospect relates to a sure outcome is well defined for all risk averse utility functions. When the initial decision is to reject the lottery, the selling price is largely a function of how the certainty equivalent of the lottery introduced by information acquisition compares to the sure outcome. For the decreasingly risk averse decision makers, though, we need the initial decision to remain robust to wealth level changes to obtain a parallel result.

When the initial decision is to accept the lottery, we obtain a similar monotonicity result for lotteries bounded from below. In particular, we find that more risk averse decision makers are willing to sell information on bounded lotteries at a higher price. This result is important for two reasons. First, lotteries with a finite maximum loss are observed in many practical decision problems. Second, as known from similar studies using the buying price approach, monotonicity between risk aversion and the value of information does not necessarily follow. However, we obtain such a strong monotonicity result with the selling price approach. Finally, when we restrict ourselves to the quadratic family of utility functions, we show that the boundedness condition can also be eliminated.

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# Appendix

*Proof of Proposition 1*: The conclusion follows trivially when $S_u = 0$, so we restrict our attention to the case $S_u > 0$. Since the decision maker is non-increasingly risk averse, if the decision without information is to reject the lottery $\Pi$, then this decision remains the same at any level $w^* > w$ as well. Then the selling price equations are

$$u(w + S_u) = \int_{\Gamma_u(w,\mathcal{I})} u(w + \pi) \cdot f(\pi)\ d\pi + P\{\Gamma_u^c(w,\mathcal{I})\ \} \cdot u(w), \tag{5}$$

$$v(w + S_v) = \int_{\Gamma_v(w,\mathcal{I})} v(w + \pi) \cdot f(\pi)\ d\pi + P\{\Gamma_v^c(w,\mathcal{I})\ \} \cdot v(w), \tag{6}$$

where $f$ is the density function for $\Pi$. Since $r_u \geq r_v$ holds, we should have $\Gamma_u \subseteq \Gamma_v$. Then using the shorthand notation $\Gamma_u = \Gamma_u(w,\mathcal{I})$ and $\Gamma_v = \Gamma_v(w,\mathcal{I})$, we could rewrite (5) as

$$u(w + S_u) = \int_{\Gamma_u} u(w + \pi) \cdot f(\pi)\ d\pi + (P\{\Gamma_v\} - P\{\Gamma_u\}) \cdot u(w) + P\{\Gamma_v^c\ \} \cdot u(w). \tag{7}$$

Similarly, (6) could be rearranged as follows:

$$\begin{aligned} v(w + S_v) &= \int_{\Gamma_u} v(w + \pi) \cdot f(\pi)\ d\pi + \int_{\Gamma_v - \Gamma_u} v(w + \pi) \cdot f(\pi)\ d\pi + P\{\Gamma_v^c\} \cdot v(w) \\ &\geq \int_{\Gamma_u} v(w + \pi) \cdot f(\pi)\ d\pi + (P\{\Gamma_v\} - P\{\Gamma_u\}) \cdot v(w) + P\{\Gamma_v^c\} \cdot v(w). \end{aligned} \tag{8}$$

In expressions (7) and (8), lottery $\widetilde{\Pi}$ evaluated on the right hand side offers $\Pi$ on $\Gamma_u$ and $w$ on $\Gamma_u^c$. The certainty equivalent of $\widetilde{\Pi}$ for the utility function $u$, $CE_u(\widetilde{\Pi}) = w + S_u$. For $v$ though, $CE_v(\widetilde{\Pi}) \leq w + S_v$. If we combine these results with the fact that $CE_v(\widetilde{\Pi}) \geq CE_u(\widetilde{\Pi})$ (since $r_u \geq r_v$), the relationship $S_v \geq S_u$ is obtained. This completes the proof.

*Proof of Proposition 2*: Again, the conclusion follows trivially when $S_u = 0$, so we consider the case $S_u > 0$. First, note that $S_u \leq \mathbb{E}\Pi$ and $S_v \leq \mathbb{E}\Pi$ because any information alternative is less valuable than perfect information and the certainty equivalent of perfect information is strictly less than $\mathbb{E}\Pi$ for a decreasingly risk averse decision maker. This implies that since the decision maker does not switch from the reject decision to the accept decision in the wealth range $[w, w+\mathbb{E}\Pi)$, the selling price equations are

$$u(w+S_u) = \int_{\Gamma_u(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)\ d\pi + P\{\Gamma_u^c(w,\mathcal{I})\ \}\cdot u(w),$$

$$v(w+S_v) = \int_{\Gamma_v(w,\mathcal{I})} v(w+\pi)\cdot f(\pi)\ d\pi + P\{\Gamma_v^c(w,\mathcal{I})\ \}\cdot v(w).$$

The remaining line of arguments to conclude the proof are in fact identical to the proof of Proposition 1. Therefore, we argue at this point that $S_v \geq S_u$ in this case as well.

Before proving Proposition 3, we state a useful lemma. First, recall that an information alternative $\mathcal{I}$ is generated by disjoint events $A_1$ to $A_k$. These events may include both positive and negative elements. Note that the lottery $\Pi$ is now bounded below by $\pi_{min}$ defined in the paper. We define the lottery $\Pi^\Gamma$ as follows:

$$\Pi^\Gamma = \begin{cases} \Pi & if\ \ \Pi \in A_i \ \text{ s.t. }\ d_u(w, A_i) = +1, \\ 0 & o.w. \end{cases}$$

In words, $\Pi^\Gamma$ is the lottery that is evaluated on the right hand side of the selling price equation (1). Its value is identical to the value of the original lottery when the lottery outcome lies in an element $A_i$ of the information partition such that the decision is to play the lottery upon the occurrence of $A_i$. Otherwise, $\Pi^\Gamma$ takes on the value zero which indicates the decision to choose the sure outcome instead of the lottery. After defining $\Pi^\Gamma$, we now state the following useful lemma.

**Lemma 6** *Suppose $H$ is concave and increasing and $u$ is a utility function. Lottery $\Pi$ is bounded below by $\pi_{min}$. Then for any $y > 0$*

$$\begin{aligned}&\mathbb{E}[H(u(w+\Pi+y))] - \mathbb{E}[H(u(w+\Pi^\Gamma))] \\ &\qquad \leq H'(u(w+\pi_{min}))\cdot\big\{\mathbb{E}[u(w+\Pi+y)] - \mathbb{E}[u(w+\Pi^\Gamma)]\big\}.\end{aligned}$$

*Proof*: For arbitrary $\pi$ and $y > 0$

$$H(u(w+\pi+y)) - H(u(w+\pi)) \leq H'(u(w+\pi))\cdot\big\{u(w+\pi+y) - u(w+\pi)\big\}.$$

Since $u(w+\pi_{min}) \leq u(w+\pi)$, $H'(u(w+\pi_{min})) \geq H'(u(w+\pi))$ and

$$H(u(w+\pi+y)) - H(u(w+\pi)) \leq H'(u(w+\pi_{min}))\cdot\big\{u(w+\pi+y) - u(w+\pi)\big\}.$$

If we take expectations over the set $\{\Pi \in \Gamma\}$ and observe that $\Pi = \Pi^{\Gamma}$ when $\Pi \in \Gamma$,

$$\begin{aligned}&\mathbb{E}[H(u(w+\Pi+y))\cdot 1_{\{\Pi\in\Gamma\}}] - \mathbb{E}[H(u(w+\Pi^{\Gamma}))\cdot 1_{\{\Pi\in\Gamma\}}]\\ &\quad\leq H'(u(w+\pi_{min}))\cdot\Big\{\mathbb{E}[u(w+\Pi+y)\cdot 1_{\{\Pi\in\Gamma\}}] - E[u(w+\Pi^{\Gamma})\cdot 1_{\{\Pi\in\Gamma\}}]\Big\}.\end{aligned} \tag{9}$$

The outcomes in $\Gamma^c$ are also allowed to be mixed in sign. Again, for arbitrary $\pi$ and $y > 0$

$$H(u(w+\pi+y)) - H(u(w)) \leq H'(u(w))\cdot\big\{u(w+\pi+y) - u(w)\big\}.$$

Note that, there should be at least one negative outcome in $\Pi$ to make sure that information has positive value (which is the only interesting case to analyze). This implies $\pi_{min} < 0$ and $u(w+\pi_{min}) \leq u(w)$, $H'(u(w+\pi_{min})) \geq H'(u(w))$. Then

$$H(u(w+\pi+y)) - H(u(w)) \leq H'(u(w+\pi_{min}))\cdot\big\{u(w+\pi+y) - u(w)\big\}.$$

Taking expectations over the set $\{\Pi \in \Gamma^c\}$

$$\begin{aligned}&\mathbb{E}[H(u(w+\Pi+y))\cdot 1_{\{\Pi\in\Gamma^c\}}] - \mathbb{E}[H(u(w))\cdot 1_{\{\Pi\in\Gamma^c\}}]\\ &\quad\leq H'(u(w+\pi_{min}))\cdot\Big\{\mathbb{E}[u(w+\Pi+y)\cdot 1_{\{\Pi\in\Gamma^c\}}] - E[u(w)\cdot 1_{\{\Pi\in\Gamma^c\}}]\Big\}.\end{aligned}$$

Recall that $\Pi^{\Gamma} = 0$ on the set $\Gamma^c$. Then the last inequality can be rewritten as

$$\begin{aligned}&\mathbb{E}[H(u(w+\Pi+y))\cdot 1_{\{\Pi\in\Gamma^c\}}] - \mathbb{E}[H(u(w+\Pi^{\Gamma}))\cdot 1_{\{\Pi\in\Gamma^c\}}]\\ &\quad\leq H'(u(w+\pi_{min}))\cdot\Big\{\mathbb{E}[u(w+\Pi+y)\cdot 1_{\{\Pi\in\Gamma^c\}}] - E[u(w+\Pi^{\Gamma})\cdot 1_{\{\Pi\in\Gamma^c\}}]\Big\}.\end{aligned} \tag{10}$$

Finally, we combine expressions (9) and (10) and obtain the desired inequality. This completes the proof.
We use the above lemma to prove Proposition 3.

*Proof of Proposition 3*: Our first consideration will be the simpler case in which $\Gamma_u = \Gamma_v$. Since $u$ is more risk averse than $v$, there exists an increasing and concave function $H$ such that $u = H(v)$. We prove the proposition by considering the below difference

$$\begin{aligned}&\mathbb{E}[u(w+\Pi+S_v)] - \mathbb{E}[u(w+\Pi+S_u)] =\\ &= \mathbb{E}[H(v(w+\Pi+S_v))] - \mathbb{E}[u(w+\Pi^{\Gamma})] =\\ &= \mathbb{E}[H(v(w+\Pi+S_v))] - \mathbb{E}[H(v(w+\Pi^{\Gamma}))],\end{aligned} \tag{11}$$

All three expressions are equal because $u = H(v)$ and $\mathbb{E}[u(w+\Pi+S_u)] = \mathbb{E}[u(w+\Pi^{\Gamma})]$

by the definition of $\Pi^\Gamma$ and the selling price equation (1). We now use Lemma 6

$$\begin{aligned}&\mathbb{E}[H(v(w+\Pi+S_v))] - \mathbb{E}[H(v(w+\Pi^\Gamma))]\\ &\quad \le H'(v(w+\pi_{min}))\cdot\Big\{\mathbb{E}[v(w+\Pi+S_v)] - E[v(w+\Pi^\Gamma)]\Big\} = 0,\end{aligned} \tag{12}$$

where the inequality is the implication of Lemma 6 and the equality is the direct result of the selling price equation (1). Hence, $S_u \ge S_v$ follows in this case.
However, $\Gamma_u$ and $\Gamma_v$ might be different. We define $\Gamma_v - \Gamma_u = \Gamma_d$. Let's recall the selling price equations in this case:

$$\begin{aligned}\mathbb{E}[u(w+\Pi+S_u)] &= \int\limits_{\Gamma_u(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)d\pi + P\{\Gamma_d(w,\mathcal{I})\}\cdot u(w)+\\ &+ P\{\Gamma_v^c(w,\mathcal{I})\}\cdot u(w),\\ \mathbb{E}[v(w+\Pi+S_v)] &= \int\limits_{\Gamma_u(w,\mathcal{I})} v(w+\pi)\cdot f(\pi)d\pi+\\ &+ \int\limits_{\Gamma_d(w,\mathcal{I})} v(w+\pi)\cdot f(\pi)d\pi + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot v(w).\end{aligned}$$

On set $\Gamma_d$, the decision maker with the utility function $u$ prefers to reject the lottery. Therefore, the following inequality should hold:

$$\mathbb{E}[u(w+\Pi+S_u)] \ge \int\limits_{\Gamma_u(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)d\pi + \int\limits_{\Gamma_d(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)d\pi + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot u(w)$$

In fact, there exists $0 < \widetilde{S} \le S_u$ such that the following equality holds:

$$\mathbb{E}[u(w+\Pi+\widetilde{S})] = \int\limits_{\Gamma_u(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)d\pi + \int\limits_{\Gamma_d(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)d\pi + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot u(w) \tag{13}$$

Now, $\widetilde{S}$ in equation (13) and $S_v$ in the selling price equation for $v$ are comparable because both equations resemble the first case analyzed above where we assumed $\Gamma_u = \Gamma_v$. As such, we have $\widetilde{S} \ge S_v$. Therefore, $S_u \ge S_v$ and this completes the proof.
Next, we present a technical note that argues that an extension of Proposition 3 does not necessarily follow for unbounded lotteries. Assume $\Pi$ is unbounded and $u$ is a more risk averse utility function than $v$. If $S_v > S_u$, then there exists $\bar{S}$, $S_u < \bar{S} < S_v$,

such that

$$\begin{aligned} \mathbb{E}[v(w+\mathit{\Pi}+\bar{S})] &< \int\limits_{\Gamma_v(w,\mathcal{I})} v(w+\pi)\cdot f(\pi)d\pi + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot v(w), \\ \mathbb{E}[u(w+\mathit{\Pi}+\bar{S})] &> \int\limits_{\Gamma_v(w,\mathcal{I})} u(w+\pi)\cdot f(\pi)d\pi + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot u(w). \end{aligned} \tag{14}$$

We focus on the first of the above inequalities. It can be rewritten as follows:

$$\begin{aligned} &P\{\Gamma_v(w,\mathcal{I})\}\cdot\mathbb{E}[v(w+\mathit{\Pi}+\bar{S})|\Gamma_v(w,\mathcal{I})] + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot\mathbb{E}[v(w+\mathit{\Pi}+\bar{S})|\Gamma_v^c(w,\mathcal{I})] \\ &< P\{\Gamma_v(w,\mathcal{I})\}\cdot\mathbb{E}[v(w+\mathit{\Pi})|\Gamma_v(w,\mathcal{I})] + P\{\Gamma_v^c(w,\mathcal{I})\}\cdot v(w). \end{aligned} \tag{15}$$

In this form, inequality (15) involves three separate lotteries: Lottery $\mathit{\Pi}+\bar{S}$ truncated to sets $\Gamma_v(w,\mathcal{I})$ and $\Gamma_v^c(w,\mathcal{I})$, respectively and lottery $\mathit{\Pi}$ truncated to $\Gamma_v(w,\mathcal{I})$. In the same order, let $CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v(w,\mathcal{I}))$, $CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v^c(w,\mathcal{I}))$, and $CE_v(w,\mathit{\Pi},\Gamma_v(w,\mathcal{I}))$ be the certainty equivalents of the respective lotteries for the utility function $v$ at wealth level $w$. Then inequality (15) can be rearranged to obtain

$$\frac{v(CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v(w,\mathcal{I}))) - v(CE_v(w,\mathit{\Pi},\Gamma_v(w,\mathcal{I})))}{v(w) - v(CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v^c(w,\mathcal{I})))} < \frac{\Gamma_v^c(w,\mathcal{I})}{\Gamma_v(w,\mathcal{I})}. \tag{16}$$

Note that

$$CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v(w,\mathcal{I})) \geq CE_v(w,\mathit{\Pi},\Gamma_v(w,\mathcal{I})) \geq w \geq CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v^c(w,\mathcal{I}))$$

The first inequality is trivial. The second inequality holds because the decision on lottery $\mathit{\Pi}$ given $\Gamma_v(w,\mathcal{I})$ is to accept at wealth level $w$. The last inequality is obtained because otherwise inequality (15) does not hold. Then since $u$ is more risk averse, it is a concave transformation of $v$ and thus the following inequality should hold:

$$\begin{aligned} &\frac{u(CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v(w,\mathcal{I}))) - u(CE_v(w,\mathit{\Pi},\Gamma_v(w,\mathcal{I})))}{u(w) - u(CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v^c(w,\mathcal{I})))} < \\ &\frac{v(CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v(w,\mathcal{I}))) - v(CE_v(w,\mathit{\Pi},\Gamma_v(w,\mathcal{I})))}{v(w) - v(CE_v(w,\mathit{\Pi}+\bar{S},\Gamma_v^c(w,\mathcal{I})))}. \end{aligned} \tag{17}$$

Since $CE_v(w, \Pi + \bar{S}, \Gamma_v^c(w, \mathcal{I})) > CE_u(w, \Pi + \bar{S}, \Gamma_v^c(w, \mathcal{I}))$, we have

$$\frac{u(CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_v(w, \Pi, \Gamma_v(w, \mathcal{I})))}{u(w) - u(CE_u(w, \Pi + \bar{S}, \Gamma_v^c(w, \mathcal{I})))} < \frac{u(CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_v(w, \Pi, \Gamma_v(w, \mathcal{I})))}{u(w) - u(CE_v(w, \Pi + \bar{S}, \Gamma_v^c(w, \mathcal{I})))}. \tag{18}$$

Now, if we could argue that

$$u(CE_u(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_u(w, \Pi, \Gamma_v(w, \mathcal{I}))) \leq \leq u(CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_v(w, \Pi, \Gamma_v(w, \mathcal{I})))$$

then the following relationship would be obtained:

$$\frac{u(CE_u(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_u(w, \Pi, \Gamma_v(w, \mathcal{I})))}{u(w) - u(CE_u(w, \Pi + \bar{S}, \Gamma_v^c(w, \mathcal{I})))} < \frac{u(CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_v(w, \Pi, \Gamma_v(w, \mathcal{I})))}{u(w) - u(CE_u(w, \Pi + \bar{S}, \Gamma_v^c(w, \mathcal{I})))} \leq \frac{\Gamma_v^c(w, \mathcal{I})}{\Gamma_v(w, \mathcal{I})}. \tag{19}$$

If (19) were to hold, that would contradict the second inequality in (14), which would eventually contradict the fact that $\bar{S}$ exists. Consequently, we would show that $S_u \geq S_v$ should hold for any unbounded lottery as well. However, we are short of this result because

$$u(CE_u(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_u(w, \Pi, \Gamma_v(w, \mathcal{I}))) \leq \leq u(CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_v(w, \Pi, \Gamma_v(w, \mathcal{I})))$$

does not necessarily follow for all concave utility functions. A good example comes from the exponential utility family. In this family of utility functions, since there is the property of constant absolute risk aversion, we observe

$$CE_u(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I})) - CE_u(w, \Pi, \Gamma_v(w, \mathcal{I})) = CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I})) - CE_v(w, \Pi, \Gamma_v(w, \mathcal{I})) = \bar{S}.$$

In addition

$$CE_u(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I})) \leq CE_v(w, \Pi + \bar{S}, \Gamma_v(w, \mathcal{I}))$$

and

$$CE_u(w, \Pi, \Gamma_v(w, \mathcal{I})) \leq CE_v(w, \Pi, \Gamma_v(w, \mathcal{I}))$$

Again, because of the concavity of $u$, these observations immediately lead to

$$u(CE_u(w, \mathit{\Pi} + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_u(w, \mathit{\Pi}, \Gamma_v(w, \mathcal{I}))) >$$
$$> u(CE_v(w, \mathit{\Pi} + \bar{S}, \Gamma_v(w, \mathcal{I}))) - u(CE_v(w, \mathit{\Pi}, \Gamma_v(w, \mathcal{I})))$$

Consequently, the monotonicity result cannot be stated for unbounded lotteries.
*Proof of Proposition 4*: Much like the similarity between the proofs of Propositions 1 and 2, the proofs of Propositions 3 and 4 are identical as long as the decision maker's initial decision to accept the lottery remains unchanged at a higher wealth level $w$ plus the selling price.
*Proof of Proposition 5*: The requirement that the decision to accept the lottery is robust to increases in the wealth level is the standard requirement that we proved in earlier propositions on increasingly risk averse utility functions. Recall that the risk aversion function for the quadratic utility is $r_u = 2a/(b - 2aw)$. If we divide both sides of the equation (4) by $(b - 2aw)$ and rearrange, we obtain

$$\mathbb{E}[\mathit{\Pi}] + S_u - \frac{r_u}{2} \cdot \left(\mathbb{E}[\mathit{\Pi}^2] + S_u^2\right) - r_u S_u \mathbb{E}[\mathit{\Pi}] = P\{\Gamma_u\}\mathbb{E}[\mathit{\Pi}|\Gamma_u] - \frac{r_u}{2} P\{\Gamma_u\}\mathbb{E}[\mathit{\Pi}^2|\Gamma_u]. \quad (20)$$

To measure the sensitivity of $S_u$ as a function of $r_u$, we cannot proceed under differentiability assumptions because $\Gamma_u$ might be subject to a change as the risk tolerance shifts rendering $\mathbb{E}[\mathit{\Pi}|\Gamma_u]$ and $\mathbb{E}[\mathit{\Pi}^2|\Gamma_u]$ non-differentiable. Based on the impact on $\Gamma_u$, we consider two cases. Under case 1, $\Gamma_u$ is robust to small perturbations around $r_u$; $\exists\ \epsilon > 0$ s.t. $\Gamma_u(w, \mathcal{I})$ remains the same for all values of $r_u$ in $(r_u - \epsilon, r_u + \epsilon)$. We may handle case 1 under differentiability assumptions as long as the risk aversion function lies in $(r_u - \epsilon, r_u + \epsilon)$. Case 2 is the opposite of case 1, where such $\epsilon > 0$ does not exist and $\Gamma_u(w, \mathcal{I})$ is sensitive to any change in $r_u$.

*Case* 1*:* First, we make a slight arrangement of (20) to obtain:

$$S_u - \frac{r_u}{2} S_u^2 - r_u S_u \mathbb{E}[\mathit{\Pi}] = \frac{r_u}{2} P\{\Gamma_u^c\}\mathbb{E}[\mathit{\Pi}^2|\Gamma_u^c] - P\{\Gamma_u^c\}\mathbb{E}[\mathit{\Pi}|\Gamma_u^c]. \quad (21)$$

Since in this case, $\Gamma_u^c$ is insensitive to small changes in $r_u$, we may observe that an increase in $r_u$ results in an increase in the right hand side ($RHS$) of (21) whereas the left hand side ($LHS$) decreases. To reestablish the equality, $S_u$ on the $LHS$ should be perturbed and we should have $\partial LHS/\partial S_u > 0$. This follows if

$$\partial LHS/\partial S_u = 1 - r_u S_u - r_u \mathbb{E}[\mathit{\Pi}] = 1 - \frac{2a}{b - 2aw} S_u - \frac{2a}{b - 2aw} \mathbb{E}[\mathit{\Pi}] > 0,$$

or

$$b - 2a(w + S_u + \mathbb{E}[\mathit{\Pi}]) > 0. \quad (22)$$

Inequality (22) should hold because $u' > 0$ implies that $b > 2a\tilde{w}$ at any terminal wealth level $\tilde{w} = w + \pi + S_u$ of the decision maker at any lottery outcome $\pi$. Therefore, the quadratic utility result follows in case 1.

*Case 2:* In this case, $\Gamma_u$ is sensitive to even the slightest perturbation in $r_u$, so we should have at least one $A_c \in \{A_1, \ldots, A_k\}$ such that the decision maker is indifferent between accepting and rejecting the lottery $\Pi$. Without loss of generality, we assume that there exists only one such $A_c$. This implies that the decision maker moves from the point of indifference to an accept decision on $A_c$ whenever $r_u$ decreases and to a reject decision whenever $r_u$ increases. In such a case, the selling price can be calculated solving either one of the two equations below

$$S_u - \frac{r_u}{2} S_u^2 - r_u S_u \mathbb{E}[\Pi] = \frac{r_u}{2} P\{\widetilde{\Gamma}_u^c\} \mathbb{E}[\Pi^2 | \widetilde{\Gamma}_u^c] - P\{\widetilde{\Gamma}_u^c\} \mathbb{E}[\Pi | \widetilde{\Gamma}_u^c], \tag{23}$$

$$S_u - \frac{r_u}{2} S_u^2 - r_u S_u \mathbb{E}[\Pi] = \frac{r_u}{2} P\{\widehat{\Gamma}_u^c\} \mathbb{E}[\Pi^2 | \widehat{\Gamma}_u^c] - P\{\widehat{\Gamma}_u^c\} \mathbb{E}[\Pi | \widehat{\Gamma}_u^c]. \tag{24}$$

where $\widehat{\Gamma}_u - \widetilde{\Gamma}_u = A_c$ (i.e., $\widehat{\Gamma}_u$ includes $A_c$, but $\widetilde{\Gamma}_u$ does not). If $r_u$ is increased slightly, then the selling price equation becomes (23). Therefore, in that case we could analyze the selling price as in case 1 using equation (23) and argue that selling price increases. Conversely, when $r_u$ is decreased slightly, equation (24) becomes the relevant selling price equation. Again, from case 1, we know that the solution to $S_u$ decreases if $r_u$ is perturbed in the downward direction. Hence, we can conclude that the selling price is increasing in the degree of risk aversion in case 2 as well. This completes the proof.