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Seasonality Revisited - Statistical Testing for Almost Periodically Correlated Stochastic Processes

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Abstract

This article aims at constructing a new method for testing the statistical significance of seasonal fluctuations for non-stationary processes. The constructed test is based on a method of subsampling and on the spectral theory of Almost Periodically Correlated (APC) time series. In the article we consider an equation of a nonstationary process, containing a component which includes seasonal fluctuations and business cycle fluctuations, both described by an almost periodic function. We build subsampling test justifying the significance of frequencies obtained from the Fourier representation of the unconditional expectation of the process.
The empirical usefulness of the constructed test is examined for selected macroeconomic data. The article studies survey indicators of economic climate in industry, retail trade and consumption for European countries.

Keywords: seasonality, almost periodically correlated stochastic processes, subsampling, business cycle

JEL Classification: C14, C46, E32

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1 Introduction

In the majority of macroeconomic indices, apart from fluctuations due to a general trend or changing economic activity, one can also discern seasonal fluctuations as their inseparable element. Seasonality is related to fluctuations of a rather regular shape, constant length and similar amplitude and are primarily due to changing seasons. An in-depth characteristic of these fluctuations does not play a key role in empirical macroeconomics. However we deal very frequently with the interaction between the seasonal fluctuations with the business cycle fluctuations, while the latter being of extreme importance in empirical investigations. It takes place for the majority of macroeconomic time series on both quarterly and monthly basis.

In the empirical analysis, seasonal variations are removed by the application of some ad hoc procedures. The development of dynamic econometrics yields the vast literature concerning methods of seasonal adjustment. These methods assume the separation of seasonal fluctuations from the analysed phenomenon, without major interference with any long-term rising or declining trend or with business cycle. Known methods of clearing data of seasonal fluctuations include the X-11, X-11-ARIMA, X-12-ARIMA - developed by the United States Census Bureau, TRAMO/SEATS, initially proposed and applied by the Bank of Spain, and other methods. There are, however, numerous contesting opinions in the literature expressing reservations towards removing seasonal fluctuations from macroeconomic data. The topic of the effect of the seasonal adjustment has been taken up e.g. in the monograph by Franses (1996), where a separate chapter is devoted for this issue. Franses (1996) concentrates on the independence of seasonal variations and business cycle fluctuations and the possibility of sorting them out. The author emphasizes that in certain situations the separation of the seasonal fluctuations and the business cycle fluctuations is far from easy, and at times seemingly impossible due to the interaction between fluctuations of both nature. Suggestions have been formulated indicating when such a situation may occur.

At the end of the eighties and beginning of the nineties of XX century a series of publications addressed the topic of the importance of seasonal fluctuations in the analysis of macroeconomic time series. Ghysels (1988) argues that seasonal adjustment of macroeconomic data is not a harmless operation as many researchers had earlier assumed. What is more, the author indicates that an inadequately chosen seasonal adjustment method may lead to substantial loss of information. In subsequent years the importance of seasonal fluctuations, particularly in case of forecasting, was addressed by Barsky and Miron (1989), Braun and Evans (1990), Chattarjee and Ravikumar (1992), Canova and Hansen (1995), Miron (1996), Franses and Ooms (1997), Novales and Fruto (1997), Wells (1997) and Herwartz (1999). Canova and Hansen (1995) conclude that seasonality constitutes an integral part of the analysed phenomenon and cannot be disregarded in the construction of a model. Moreover, seasonally adjusted data may possess certain characteristics which do not constitute features of the analysed phenomenon but result from the properties of the method used for seasonal adjustment of the data. This issue was addressed by Franses and Ooms (1997), recommending analysis of data which have not been subject to seasonal adjustment. In turn, studies by Novales and Fruto (1997), Wells (1997) and Herwartz (1999) show the advantage of the so-called periodic models (i.e. models assuming the periodicity of the expected value function and autocovariance function) over non-periodic models in forecasting. Ghysels, Granger, Sikolos (1996) indicate that seasonal adjustment of data may lead to the creation of artefact nonlinear structures. Luginbuhl and de Vos (2003) reiterate that such components as growth, business cycle fluctuations and seasonal fluctuations should be modelled simultaneously.

Despite of the importance of seasonality effect in empirical modelling, the issue of its formal statistical identification is not explored in details. In the empirical literature, if some ad-hoc chosen procedure of seasonal adjustment is applied, usually the explanation of the existence of those fluctuations has intuitive and anecdotal origin. Hence, in the presence of such deficiency, development of statistical test indicating the significance of seasonal fluctuations is necessary. This paper tries to apply, in the problem stated above, the spectral theory of Almost Periodically Correlated (APC) stochastic processes in building formal statistical test of the existence of seasonality fluctuations in observed time series. Starting from the Fourier representation of the unconditional expectation of the process we build test statistics, to make formal inference about the significance of frequencies of length related with seasonal fluctuations. The distribution of the natural test statistics is approximated by the subsample distribution according to the method described in Politis, Romano, Wolf (1999). We discuss asymptotic consistency of both test statistics and subsampling approximations of quantiles, utilized in the procedure as critical values.

2 Basic definitions and results

This chapter presents basic concepts and results related to Periodically Correlated (PC) time series, Almost Periodically Correlated (APC) time series and the subsampling method. Let us consider a real valued stochastic process $\{X_t : t \in \mathbb{Z}\}$ with the unconditional expectation $\mu(t) = E(X_t) < \infty$ and the autocovariance function $B(t,\tau) = \mathrm{cov}(X_t, X_{t+\tau}) < \infty$, where $\tau \in \mathbb{Z}$. According to Gladyshev (1961) and Gladyshev (1963), the process is periodically correlated (respectively almost periodically correlated) if its unconditional mean and autocovariance function is a periodic (respectively almost periodic) function of the variable t for any $\tau \in \mathbb{Z}$. The definition of an almost periodic function, relaxing periodic function case, may be found in the monograph Corduneanu (1989). The expected value function of the APC time series has the following Fourier representation:

$$\mu(t) \sim \sum_{\psi \in \Psi} m(\psi) e^{i\psi t}, \tag{1}$$

where coefficient $m(\psi) \in \mathbb{C}$ are given as follows:

$$m(\psi) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \mu(t) e^{-i\psi t}. \tag{2}$$

Corduneanu (1989) proved the most important result for decomposition (1) that the set $\Psi = \{\psi \in [0, 2\pi) : |m(\psi)| \neq 0\}$ is countable. In this article we do not require that the number of elements in the set Ψ is finite.

If the set Ψ includes at lest one element from among the frequencies defined as: $\Psi_T := \{2k\pi/T : \ k = 1, 2, \ldots, T-1\}$, then the time series observed T times a year is characterized by regular seasonal fluctuations. This property is crucial for the construction of the significance test for seasonal fluctuations. The problem of identification of the presence of frequencies from the Ψ_T set in an unknown set Ψ consist of the identification of non-zero values of the modulo of Fourier coefficients, namely of $|m(\psi)|$, for $\psi \in \Psi_T$, which is due to the following equivalence:

$$\psi \in \Psi \Leftrightarrow |m(\psi)| \neq 0.$$

We will therefore focus on the problem of estimation of the modulo of Fourier coefficients, namely of $|m(\psi)|$.

For a given frequency $\psi \in [0, 2\pi)$, based on a d_n-element sample $\{X_{c_n+1}, X_{c_n+2}, \ldots, X_{c_n+d_n}\}$ from the time series $\{X_t : t \in \mathbb{Z}\}$, with $\{c_n\}_{n\in\mathbb{N}}$ and $\{d_n\}_{n\in\mathbb{N}}$ being any non-negative sequences of integers where $d_n \to \infty$, the natural estimator $\hat{m}_n^{c_n,d_n}(\psi)$ (in short $\hat{m}_n^{c,d}(\psi)$) of the parameter $m(\psi)$ takes the form:

$$\hat{m}_n^{c,d}(\psi) = \frac{1}{d_n} \sum_{t=c_n+1}^{c_n+d_n} X_t e^{-it\psi};$$

see Lenart (2013). The non-parametric approach to testing the significance of the parameter $|m(\psi)|$, consisting in the use of asymptotic distribution of normalized estimator $|\hat{m}_n(\psi)| = |\hat{m}_n^{0,n}(\psi)|$, seems to be impossible due to the form of this distribution (see Lenart (2013)). This clearly motivates the use the subsampling method, where the exact form of the asymptotic distribution does not play such a crucial role as in the classical case. The idea of subsampling together with the general theoretical results was presented in Politis, Romano, Wolf (1999). Asymptotic properties of this method, such as its consistency when applied for estimation of the modulo of Fourier coefficients $|m(\psi)|$, were presented by Lenart (2013). An application of this method to formal statistical inference about the properties of the business cycle component for industrial production time series was discussed by Lenart and Pipień (2013). Therefore the presentation of the main assumptions and the concept of the subsampling method with respect to parameter $|m(\psi)|$ has been omitted.

Here we present the theoretical results that enable the construction of an asymptotically consistent significance test of seasonal fluctuations. The set of theorems discussed below are generalizations of theorems presented in Lenart (2013).

Theorem 2.1. Let all the assumptions of Theorem 2.1 from Lenart (2013) are met. Then for any set of frequencies $\tilde{\Psi} = \{\psi_1, \psi_2, \ldots, \psi_p\} \subset [0, 2\pi)$, where $p \in \mathbb{N}$ the following convergence is obtained:

$$\sqrt{d_n}\left(\begin{bmatrix} \mathrm{Re}[\hat{m}_n^{c,d}(\psi_1)] \\ \mathrm{Im}[\hat{m}_n^{c,d}(\psi_1)] \\ \mathrm{Re}[\hat{m}_n^{c,d}(\psi_2)] \\ \mathrm{Im}[\hat{m}_n^{c,d}(\psi_2)] \\ \vdots \\ \mathrm{Re}[\hat{m}_n^{c,d}(\psi_p)] \\ \mathrm{Im}[\hat{m}_n^{c,d}(\psi_p)] \end{bmatrix} - \begin{bmatrix} \mathrm{Re}[m(\psi_1)] \\ \mathrm{Im}[m(\psi_1)] \\ \mathrm{Re}[m(\psi_2)] \\ \mathrm{Im}[m(\psi_2)] \\ \vdots \\ \mathrm{Re}[m(\psi_p)] \\ \mathrm{Im}[m(\psi_p)] \end{bmatrix}\right) \xrightarrow{d} \mathcal{N}_{2p}(0, \Phi(\tilde{\Psi})), \tag{3}$$

where the exact formula of the variance-covariance matrix $\Phi(\tilde{\Psi})$ may be derived from Lemma A.3; see proof of Theorem 2.1 in Lenart (2013).

The next theorem is a natural extension of Theorem 2.2 from Lenart (2013). It provides a basis for the construction of a test related to the structure of the function that describes the unconditional mean of the observed time series.

Theorem 2.2. Let all the assumptions of Theorem 2.2 from Lenart (2013) be met. Then for any set of frequencies $\tilde{\Psi} = \{\psi_1, \psi_2, \ldots, \psi_p\} \subset [0, 2\pi)$, where $p \in \mathbb{N}$ the following convergence is obtained:

$$\sqrt{d_n}\left(\left(\sum_{\psi\in\tilde{\Psi}} |\hat{m}_n^{c,d}(\psi)|^2\right)^{\frac{1}{2}} - \left(\sum_{\psi\in\tilde{\Psi}} |m(\psi)|^2\right)^{\frac{1}{2}}\right) \xrightarrow{d} J^{\tilde{\Psi}}, \tag{4}$$

where

$$J^{\tilde{\Psi}} = \begin{cases} \mathcal{L}(\tilde{Z}), & \mathit{for} \sum\limits_{\psi\in\tilde{\Psi}} |m(\psi)|^2 = 0, \\ \mathcal{N}_1(0, A_0\Phi(\tilde{\Psi})A_0^T), & \mathit{for} \sum\limits_{\psi\in\tilde{\Psi}} |m(\psi)|^2 \neq 0, \end{cases} \tag{5}$$

$$A_0 = \frac{1}{\sqrt{\sum\limits_{\psi\in\tilde{\Psi}} |m(\psi)|^2}} \cdot$$
$$\cdot\big[\mathrm{Re}(m(\psi_1))\ \mathrm{Im}(m(\psi_1))\ \mathrm{Re}(m(\psi_2))\ \mathrm{Im}(m(\psi_2))\ \ldots\ \mathrm{Re}(m(\psi_p))\ \mathrm{Im}(m(\psi_p))\big],$$

$\tilde{Z} = \left(\sum_{j=1}^{2p} B_j^2\right)^{1/2}$, and the random vector $[B_1\, B_2\, \ldots\, B_{2p}]^T$ follows $2p$-dimensional Normal distribution with zero mean and covariance matrix $\Phi(\tilde{\Psi})$.

Let us take any $p \in \mathbb{N}$ and $\tilde{\Psi} = \{\psi_1, \psi_2, \ldots, \psi_p\} \subset [0, 2\pi)$. By

$$L^{\tilde{\Psi}}_{n,b}(x) = \frac{1}{n-b+1} \cdot$$

$$\cdot \sum_{t=1}^{n-b+1} \mathbf{1}\left\{\sqrt{b}\left(\left(\sum_{\psi\in\tilde{\Psi}} |\hat{m}_n^{t-1,b}(\psi)|^2\right)^{\frac{1}{2}} - \left(\sum_{\psi\in\tilde{\Psi}} |\hat{m}_n(\psi)|^2\right)^{\frac{1}{2}}\right) \leq x\right\} \tag{6}$$

let us define a subsample estimator of the cumulative distribution function of the following quantity:

$$\sqrt{n}\left(\left(\sum_{\psi\in\tilde{\Psi}} |\hat{m}_n(\psi)|^2\right)^{\frac{1}{2}} - \left(\sum_{\psi\in\tilde{\Psi}} |m(\psi)|^2\right)^{\frac{1}{2}}\right),$$

and by $J^{\tilde{\Psi}}(x)$ the value at point x of the cumulative distribution function of the asymptotic distribution of this quantity. Let $c^{\tilde{\Psi}}_{n,b}(1-\alpha)$ be a quantile of order $(1-\alpha)$ of the $L^{\tilde{\Psi}}_{n,b}(x)$ distribution, i.e.

$$c^{\tilde{\Psi}}_{n,b}(1-\alpha) = \inf\{x: \ L^{\tilde{\Psi}}_{n,b}(x) \geq 1-\alpha\}.$$

The following theorem states the consistency of the subsampling procedure for a more general setting than in the case considered previously in Theorem 2.3 by Lenart (2013).

Theorem 2.3. Let us take any $\tilde{\Psi} = \{\psi_1, \psi_2, \ldots, \psi_p\} \subset [0, 2\pi)$, where $p \in \mathbb{N}$. If the assumptions of Theorem 2.2 (including the additional conditions) are met and additionally the distribution $J^{\tilde{\Psi}}(\cdot)$ in not degenerate, then

1. $L^{\tilde{\Psi}}_{n,b}(x) \xrightarrow{p} J^{\tilde{\Psi}}(x)$, for any $x \in \mathbb{R}$,
2. $\sup_{x\in\mathbb{R}} |L^{\tilde{\Psi}}_{n,b}(x) - J^{\tilde{\Psi}}(x)| \xrightarrow{p} 0$,
3. subsample confidence intervals for parametr $\left(\sum_{\psi\in\tilde{\Psi}} |m(\psi)|^2\right)^{\frac{1}{2}}$ are consistent, which means that

$$P\left(\sqrt{n}\Big(\big(\sum_{\psi\in\tilde{\Psi}} |\hat{m}_n(\psi)|^2\big)^{\frac{1}{2}} - \big(\sum_{\psi\in\tilde{\Psi}} |m(\psi)|^2\big)^{\frac{1}{2}}\Big) \leq c^{\tilde{\Psi}}_{n,b}(1-\alpha)\right) \longrightarrow 1-\alpha, \tag{7}$$

where $b = b(n) \to \infty$ and $b/n \to 0$.

3 The significance test of seasonal fluctuations in the APC time series

Let Ψ be a *true* set of frequencies in the Fourier representation for the unconditional mean of the process $\{X_t : t \in \mathbb{Z}\}$. If the set Ψ contains at least one element from the set $\Psi_T = \{2k\pi/T : \ k = 1, 2, \ldots, T-1\}$, where T is the number of observations during a year then time series $\{X_t : t \in \mathbb{Z}\}$ is subject to seasonal fluctuations. Let us then state the following problem of testing the significance of seasonal fluctuations in the following way:

$$\begin{aligned} H_0 &: \Psi \cap \Psi_T = \emptyset \\ H_1 &: \Psi \cap \Psi_T \neq \emptyset, \end{aligned} \tag{8}$$

together with the natural test statistic

$$\Pi_n(\Psi_T) = \sqrt{n}\left(\sum_{\psi \in \Psi_T} |\hat{m}_n(\psi)|^2\right)^{1/2}.$$

Hypothesis H_0 is interpreted as a case when the series is not subject to seasonal fluctuations, while alternative hypothesis H_1 indicates theirs existence.
The distribution of the test statistics will be approximated by the subsample distribution, with the application of Theorem 2.3 and the results presented in Politis, Romano, Wolf (1999). We will use identical notation as in Lenart (2013) and Lenart and Pipień (2013).
Additionally, to compare our results we make use of the quantile

$$g_{n,b}^{\Psi_T}(1-\alpha) = \inf\{x : G_{n,b}^{\Psi_T}(x) \geq 1-\alpha\},$$

where subsampling distribution $G_{n,b}^{\Psi_T}(x)$ has the following form

$$G_{n,b}^{\Psi_T}(x) = \frac{1}{n-b+1}\sum_{t=1}^{n-b+1} \mathbf{1}\left\{\sqrt{b}\left(\sum_{\psi \in \Psi_T} |\hat{m}_n^{t-1,b}(\psi)|^2\right)^{\frac{1}{2}} \leq x\right\}. \tag{9}$$

The test based on the $g_{n,b}^{\Psi_T}(1-\alpha)$ quantile and the test statistic $\Pi_n(\Psi_T)$ is also asymptotically consistent. In order to prove this, it is enough to apply the same arguments as in the proof of Theorems 2.6.1 and 4.2.1 in the monograph Politis, Romano, Wolf (1999).

4 Model equation

Here we present a one-dimensional model equation that describes the dynamics of a selected macroeconomic variable and constitutes the basis for further considerations.

Let us assume that the examined variable is the time series $\{Y_t : t \in \mathbb{Z}\}$ with a representation:

$$(1-B)^d Y_t = \eta_t, \tag{10}$$

for non-negative real number d, $B^k Y_t = Y_{t-k}$ for any natural number k, while $\{\eta_t : t \in \mathbb{Z}\}$ is an APC time series. Let us recall that depending on the values of parameter d and properties of the time series $\{\eta_t : t \in \mathbb{Z}\}$, the time series $\{Y_t : t \in \mathbb{Z}\}$ may constitute $I(0)$ case, a process with long memory or I(1), I(2) etc. cases.
The application of the test presented in the previous chapter, to identify the form of the expectation for the time series $\{Y_t : t \in \mathbb{Z}\}$, is not possible automatically. It is necessary to make certain assumptions about the bounding of the respective unconditional moment and convergence of the α mixing, but in the case when no information about parameter d is given those assumptions do not hold for the time series $\{Y_t : t \in \mathbb{Z}\}$. However, the periodic structure resulting from the seasonality may be characterized by the parameters of the time series $\{\eta_t : t \in \mathbb{Z}\}$ only. Therefore, subsequently we will be considering the problem of identifying the structure of the expected value function for the time series $\{\eta_t : t \in \mathbb{Z}\}$, making a rather general assumption that it is an APC class time series. Such an assumption allows to interpret respective parameters of the structure of the expected value function for the time series $\{\eta_t : t \in \mathbb{Z}\}$ as responsible for the seasonal fluctuations and the business cycle fluctuations contained in the observed time series; see the model equation in Lenart and Pipień (2013).
The problem of identification of the periodic structure of the unconditional expectation for the time series $\{\eta_t : t \in \mathbb{Z}\}$, in case when the value of the parameter d is unknown, is difficult, since the time series $\{\eta_t : t \in \mathbb{Z}\}$ is not observed in this case. In order to identify the structure of the expected value function for the time series $\{\eta_t : t \in \mathbb{Z}\}$ we shall use a well-known differencing operator, as it does not change (non zero) elements of the set Ψ, i.e. the set of frequencies in the representation of the expected value function for the time series. Below, we present a theoretical result formulating this issue in a precise manner. It will allow later on to obtain a method for the identification of significant frequencies in the Fourier representation for the expected value function of the time series $\{\eta_t : t \in \mathbb{Z}\}$. The proofs can be found in the Appendix.

Theorem 4.1. Let the Assumption 1.1 from Lenart (2013) be met for the APC time series $\{\eta_t : t \in \mathbb{Z}\}$. Then for any $\epsilon > 0$

$$\Psi_\eta \cap (0, 2\pi) = \Psi_{\tilde{\eta}} \cap (0, 2\pi),$$

where $\Psi_{\tilde{\eta}}$ (respectively Ψ_η) are frequencies in the Fourier representation of the expected value function for the time series $\{(1-B)^\epsilon \eta_t : t \in \mathbb{Z}\}$ (respectively the time series $\{\eta_t : t \in \mathbb{Z}\}$). Additionally for the time series $\{(1-B)^\epsilon \eta_t : t \in \mathbb{Z}\}$ the Assumption 1.1. from Lenart (2013) is met.

The above theorem leads to the conclusion that the identification of the structure of the expected value function of the time series $\{\eta_t : t \in \mathbb{Z}\}$ requires the identification of the structure of the expected value function for the time series of the following form:

$$(1-B)^{d_1} Y_t = (1-B)^{\epsilon} \eta_t, \tag{11}$$

where parameter d_1 has been selected so that $\epsilon = d_1 - d \geq 0$. The assumption that $\epsilon \geq 0$ ensures that the time series $\{(1-B)^{\epsilon}\eta_t : t \in \mathbb{Z}\}$ is also APC. Therefore, in the empirical analysis of data the procedure requires the appropriate choice of parameter d_1.
Below, one more theorem has been included, to be used later on in Chapter 5. It shows that an APC time series observed T times a year following the application of an annual aggregation filter does not include frequencies interpreted as the seasonal fluctuations.

Theorem 4.2. Let Assumption 1.1 from Lenart (2013) be met for the time series $\{X_t : t \in \mathbb{Z}\}$. Then, for the time series $\{Y_t = \tilde{L}(B)X_t : t \in \mathbb{Z}\}$ we have

$$\Psi_Y = \Psi_X \setminus \{2k\pi/T : \;\; k = 1, 2, \dots, T-1\},$$

where $\tilde{L}_T(B) = 1 + B + B^2 + \ldots + B^T$ denotes the filter of annual aggregation. Furthermore, the time series $\{Y_t = \tilde{L}_T(B)X_t : t \in \mathbb{Z}\}$ is also APC and meets the assumpion 1.1 from Lenart (2013).

5 Empirical analysis

We present the usefulness of the proposed method of estimation the seasonality effect on the basis of survey indices of economic climate for European countries. The dataset was taken from EUROSTAT and contains time series of survey indices of economic condition in industry, housing, retail trade and consumption. The data consist of at least 180 and not more than 310 observations taken for EU-27 and UE-18 regions, as well as, selected European countries. The length of the series attributed with the region or country is not the same and is determined by accessibility of survey indices. Tables 1, 2, 3 and 4 present the results of subsampling inference about frequencies interpreted as seasonal fluctuations, namely about elements of the set $\Psi_T = \{2k\pi/12 : k = 1, 2, \dots, 11\}$, in case of industry, housing, retail trade and consumption respectively. In tables we put test statistics $\Pi_n(\Psi_T)$ in both cases, where $d = 1$ and $d = 2$ in formula (10). Also in both cases of the order of integration of the observed proce4ss we put critical values $c_{n,b}$ and $g_{n,b}$ provided the level of significance $\alpha = 0.05$. The majority of inspected time series of survey indicators provide clear data evidence against hypothesis H_0 in (8) and consequently, not surprisingly, indicate the existence of seasonal fluctuations. The test statistics $\Pi_n(\Psi_T)$ is greater than critical values, making clear formal verification of stylised fact that survey indicators are mainly subject to seasonal fluctuations. In case of industry we see slightly weaker evidence about seasonality for Czech Republic, Denmark and Luxembourg, as the test statistics lies between $c_{n,b}$ and $g_{n,b}$ critical values; see Table 1. According to Table 2, the indicator of economic condition in housing is also subject to seasonal fluctuations in almost all series. The value of the test statistics $\Pi_n(\Psi_T)$ indicates the lack of seasonal fluctuations for Italy and Portugal only. There is also some uncertainty in case of Bulgaria. The survey indicator concerning retail trade seems to be the most irregular as compared to the other indices. The results of subsampling inference for this indicator was presented in Table 3. We see the strong data evidence in favour of seasonal fluctuations in case of EU-27, EU-16, Belgium, Denmark, Poland and Slovakia. There is no data evidence to reject hypothesis H_0 in (8) only in case of Bulgaria, but the results leaves great uncertainty about seasonal effect in case of German, Italy, Romania and UK.

The survey indicator of economic condition in consumption is generally subject to seasonal changes. The test statistics $\Pi_n(\Psi_T)$ calculated for this indicator is presented in Table 4. We report rejection of hypothesis H_0 in all cases, except Hungary, assuming $I(2)$ process ($d = 2$), for the series, the test statistics does not exceed both critical values $c_{n,b}$ and $g_{n,b}$.

In the next step observed series were subject to seasonal adjustment. We applied two seasonal filters of the following form:

$$(1 - B^{12})Y_t = (1 - B)(1 + B + B^2 + \ldots + B^{11})Y_t = \tilde{L}_{12}(B)(1 - B)Y_t \qquad (12)$$

and

$$(1-B^{12})(1-B)Y_t = (1-B)(1+B+B^2+\ldots+B^{11})(1-B)Y_t = \tilde{L}_{12}(B)(1-B)^2Y_t. \quad (13)$$

Seasonal differencig $1-B^{12}$ operator is commonly used in adjustment of the short term fluctuations; see Brockwell and Davis (2002), Makridakis, Wheelwright, Hyndman (1998). According to Theorem 4.1 and 4.2 unconditional means of filtered series (12) and (13), where the process Y_t follows (10), do not contain frequencies attributed with seasonal fluctuations.

According to the results presented in Tables 5, 6, 7 and 8, filtering operators (12) and (13) definitely wipes out the seasonal effect in all survey indicators. Since the test statistics $\Pi_n(\Psi_T)$ does not cross both critical values $c_{n,b}$ and $g_{n,b}$, there is no data evidence to reject hypothesis H_0.

6 Conclusions

The issue of seasonal fluctuations in macroeconomic data is important from the point of view of making inference about fluctuations of economic activity and has been brought up many times in the literature of the subject. The present article focused on the issue of constructing the significance test for seasonality effect. The construct was carried out under general assumptions with respect to the modelled time series. It was assumed that observed series may incorporate a stochastic trend, seasonal fluctuations and business cycle fluctuations. No assumptions about the independence or additiveness among these components were made, which makes the argument more general. The approach was applied for survey indicators of economic situation in industry, construction, retail trade and consumption for selected European areas and countries. The results clearly indicate the existence of seasonal fluctuations in the majority of these indicators. However, one may identify the series, where there is no evidence in favour of seasonal effects.

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Appendix

Proof of Theorem 4.1

Let us notice that:

$$\begin{aligned} E((1-B)^{\epsilon}\eta_t) &= E\left(\sum_{k=0}^{\infty}(-1)^k\binom{\epsilon}{k}\eta_{t-k}\right) \\ &= \sum_{k=0}^{\infty}(-1)^k\binom{\epsilon}{k}\mu_\eta(t-k) = \sum_{\psi\in\Psi_\eta}\sum_{k=0}^{\infty}(-1)^k\binom{\epsilon}{k}m_\eta(\psi)e^{i\psi(t-k)} \\ &= \sum_{\psi\in\Psi_\eta} m_\eta(\psi)(1-e^{-i\psi})^{\epsilon}e^{i\psi t}, \end{aligned} \tag{14}$$

which means that $\Psi_\eta \cap (0,2\pi) = \Psi_{\tilde{\eta}} \cap (0,2\pi)$ (considering that $|(1-e^{-i\psi})^{\epsilon}| \neq 0$ for $\psi \in (0,2\pi)$). Condition 1.1 from Lenart (2013) for the time series $\{(1-B)^{\epsilon}\eta_t : t \in \mathbb{Z}\}$ is obtained immediately applying equality (14).

□

Proof of Theorem 4.2

Applying the same steps as in the proof of theorem 4.1 we obtain:

$$E(\tilde{L}_T(B)X_t) = \sum_{\psi\in\Psi_X} m_X(\psi)\tilde{L}_T(1-e^{-i\psi})e^{i\psi t} = \sum_{\psi\in\Psi_X} m_Y(\psi)e^{i\psi t}, \tag{15}$$

where $m_Y(\psi) = m_X(\psi)\tilde{L}_T(1-e^{-i\psi})$. Then, using the equation $|L_T(1-e^{-i\psi})| = 0$ which is true for any $\psi \in \Psi_T$ we obtain $m_Y(\psi) = 0$, for $\psi \in \Psi_T$. Hence

$$\Psi_Y = \Psi_X \setminus \{2k\pi/T : \; k = 1, 2, \ldots, T-1\}.$$

Condition 1.1. from Lenart (2013) for the time series $\{Y_t : t \in \mathbb{Z}\}$ is obtained immediately.

□

Table 1: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \ k = 1, 2, \ldots, 11\}$ together with subsampling approximations of the critical values. The case of survey indicator of economic climate in industry

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B)Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)^2Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	EU-27	18.59(1***)(2***)	11.17	4.13	20.34(1***)(2***)	13.21	5.5
2	EU-16	19.98(1***)(2***)	12.04	4.47	21.15(1***)(2***)	13.94	5.92
3	Belgium	23.03(1***)(2***)	15.57	6.85	30.53(1***)(2***)	22.07	10.5
4	Bulgaria	16.05(1***)(2***)	14.95	8.21	24.71(2***)	24.83	14.42
5	Czech-Republic	36.71(2***)	37.27	21.31	49.65(2***)	58.1	36.78
6	Denmark	21.39(2***)	23.48	15.39	30.4(2***)	30.97	19.44
7	Germany	21.46(1***)(2***)	12.55	4.43	22.66(1***)(2***)	14.57	5.98
8	Greece	27.86(1***)(2***)	18.99	8.44	34.68(1***)(2***)	28.86	15.71
9	Spain	25.64(1***)(2***)	18.5	8.57	29.69(1***)(2***)	24.07	12.54
10	France	27.4(1***)(2***)	17.34	6.97	30.46(1***)(2***)	22.36	10.81
11	Italy	24.22(1***)(2***)	15.93	6.76	29.67(1***)(2***)	23.53	12.28
12	Latvia	16.57(1***)(2***)	16.31	9.39	21.03(2***)	23.64	14.84
13	Luxembourg	20.18(2***)	24.38	16.74	21.92	35.19	26.88
14	Netherlands	29.57(1***)(2***)	16.15	4.96	35.06(1***)(2***)	19.02	5.72
15	Austria	30.55(1***)(2***)	18.6	7.04	39.1(1***)(2***)	25.62	10.8
16	Portugal	18.77(1***)(2***)	15.61	8.29	24.92(1***)(2***)	21.33	11.71
17	Slovakia	56.96(1***)(2***)	55.05	31.04	69.63(2***)	71.79	42.36
18	Finland	45.49(1***)(2***)	31.21	12.1	58.25(1***)(2***)	51.32	26.79
19	UK	33.68(1***)(2***)	26.59	13.84	55.26(1***)(2***)	43.99	23.04

Table 2: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \ k = 1, 2, \dots, 11\}$ together with subsampling approximations of the critical values. The case of survey indicator of economic climate in housing

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B)Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)^2 Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	EU-27	40.35(1***)(2***)	20.65	5.38	29.75(1***)(2***)	21.74	10.46
2	EU-16	42.97(1***)(2***)	22.45	6.18	31.36(1***)(2***)	23.01	11.12
3	Belgium	27.72(1***)(2***)	18.41	7.92	26.63(1***)(2***)	23.24	13.14
4	Bulgaria	30.34(2***)	34.12	21.37	37.03(2***)	51.57	35.98
5	Denmark	103.45(1***)(2***)	73.	33.83	119.92(1***)(2***)	102.59	57.12
6	Germany	80.25(1***)(2***)	39.33	8.95	57.11(1***)(2***)	34.43	12.78
7	Greece	42.87(1***)(2***)	42.73	26.5	51.97(2***)	60.52	40.81
8	Spain	57.97(1***)(2***)	56.72	33.62	83.31(2***)	91.88	58.62
9	France	36.91(1***)(2***)	22.72	8.75	47.04(1***)(2***)	33.25	15.41
10	Italy	33.	55.54	43.05	55.72	93.69	72.55
11	Latvia	68.13(1***)(2***)	49.07	20.43	74.49(1***)(2***)	69.22	37.82
12	Lithuania	58.83(1***)(2***)	52.48	27.13	58.27(2***)	74.36	49.17
13	Luxembourg	65.29(1***)(2***)	50.41	25.69	64.59(2***)	73.74	49.25
14	Netherlands	43.87(1***)(2***)	30.03	13.42	35.59(1***)(2***)	25.65	12.16
15	Portugal	16.54	28.33	21.75	19.24	48.01	40.33
16	Finland	88.79(1***)(2***)	65.57	31.95	108.84(1***)(2***)	94.62	53.34
17	Sweden	53.28(1***)(2***)	46.04	24.56	36.62	57.63	42.84
18	UK	24.13(1***)(2***)	23.97	14.84	22.4	38.	29.51

Table 3: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \ k = 1, 2, \dots, 11\}$ together with subsampling approximations of the critical values. The case of survey indicator of economic climate in retail trade

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B)Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)^2 Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	EU-27	18.07(1***)(2***)	15.58	8.68	22.55(1***)(2***)	22.42	13.79
2	EU-16	20.04(1***)(2***)	15.29	7.55	25.23(1***)(2***)	21.1	11.34
3	Belgium	60.39(1***)(2***)	43.57	20.71	69.53(1***)(2***)	58.08	31.72
4	Bulgaria	10.47	23.34	18.95	13.84	38.2	32.38
5	Denmark	31.51(1***)(2***)	27.15	14.34	41.28(2***)	44.66	27.85
6	Germany	18.86(2***)	23.62	16.48	26.76	41.72	31.57
7	Italy	37.3(2***)	43.5	29.16	58.02(2***)	71.11	48.77
8	Poland	28.13(1***)(2***)	21.3	9.38	28.06(2***)	30.64	18.72
9	Romania	21.56(2***)	23.22	14.02	31.62(2***)	37.57	24.04
10	Slovakia	40.51(1***)(2***)	38.76	21.64	51.45(2***)	59.56	37.76
11	UK	25.57(2***)	30.54	20.86	33.83	49.6	36.77

Table 4: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \; k = 1, 2, \ldots, 11\}$ together with subsampling approximations of the critical values. The case of survey indicator of economic climate in consumption

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B)Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)^2Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	EU-27	15.43(1***)(2***)	9.9	4.06	20.37(1***)(2***)	12.09	4.36
2	EU-16	15.52(1***)(2***)	9.99	4.12	19.85(1***)(2***)	11.72	4.19
3	Belgium	23.61(1***)(2***)	17.82	8.88	34.54(1***)(2***)	26.18	13.08
4	Denmark	18.67(1***)(2***)	16.02	8.95	30.35(1***)(2***)	24.82	13.31
5	Germany	15.22(1***)(2***)	12.81	7.04	19.65(1***)(2***)	19.04	11.59
6	Estonia	17.75(2***)	18.56	11.15	30.5(2***)	33.73	20.97
7	Greece	18.7(1***)(2***)	15.93	8.85	29.62(1***)(2***)	25.26	14.02
8	France	20.74(1***)(2***)	19.28	11.43	26.31(1***)(2***)	23.52	13.54
9	Italy	27.27(1***)(2***)	20.88	10.56	38.08(1***)(2***)	28.03	13.59
10	Hungary	18.67(2***)	22.7	14.84	25.99	40.4	29.43
11	Netherlands	24.79(1***)(2***)	19.01	9.63	34.43(1***)(2***)	27.71	14.65
12	UK	18.64(2***)	18.68	11.62	26.01(2***)	29.24	19.38

Table 5: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \; k = 1, 2, \ldots, 11\}$ together with subsampling approximations of the critical values. The case of seasonality adjusted survey indicator of economic climate in industry

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	UE27	5.49	16.42	14.32	1.85	5.27	4.56
2	UE16	4.87	18.25	16.4	1.4	5.68	5.14
3	Belgium	4.	20.35	18.83	1.94	8.36	7.62
4	Bulgaria	4.93	18.03	15.93	3.91	14.18	12.51
5	Czech-Republic	12.32	32.36	26.99	7.38	23.02	19.8
6	Denmark	8.25	24.62	21.48	4.36	13.49	11.82
7	Germany	5.78	21.26	19.05	1.66	7.25	6.61
8	Greece	3.65	17.82	16.43	2.89	11.14	10.04
9	Spain	3.91	18.55	17.02	2.26	9.54	8.66
10	France	4.72	23.4	21.6	2.32	9.25	8.36
11	Italy	5.09	18.19	16.25	2.53	9.25	8.29
12	Latvia	11.03	20.18	15.51	6.51	18.63	15.86
13	Luxembourg	8.92	36.33	32.92	5.32	19.87	17.84
14	Netherlands	5.79	13.32	11.12	2.97	5.79	4.66
15	Austria	3.25	17.06	15.82	2.34	8.21	7.32
16	Portugal	6.37	15.88	13.37	5.09	11.27	9.27
17	Slovakia	14.54	43.6	37.38	11.14	43.53	38.75
18	Finland	7.68	28.28	25.02	6.23	20.19	17.53
19	UK	8.85	22.49	19.12	4.53	15.34	13.61

Table 6: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \ k = 1, 2, \ldots, 11\}$ together with subsampling approximations of the critical values. The case of seasonality adjusted survey indicator of economic climate in housing

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	UE27	3.38	14.05	12.77	2.14	8.33	7.51
2	UE16	3.81	15.61	14.16	2.5	8.9	7.94
3	Belgium	4.61	12.48	10.72	3.25	8.93	7.69
4	Bulgaria	5.27	28.25	26.01	4.96	21.52	19.4
5	Denmark	10.48	27.35	23.35	8.44	27.67	24.45
6	Germany	6.74	18.11	15.54	2.73	9.74	8.7
7	Greece	7.24	39.38	36.62	6.67	29.75	27.2
8	Spain	8.97	39.88	36.27	13.85	46.52	40.93
9	France	6.54	21.68	19.19	4.21	10.47	8.87
10	Italy	4.	36.03	34.51	6.14	40.95	38.61
11	Latvia	7.88	33.18	29.82	3.62	20.35	18.8
12	Lithuania	8.47	42.98	39.27	4.61	33.65	31.63
13	Luxembourg	7.95	38.99	35.96	3.13	27.56	26.37
14	Netherlands	2.99	19.81	18.67	4.21	11.57	9.96
15	Portugal	10.99	22.97	18.55	7.62	23.06	19.98
16	Finland	9.35	49.57	46.	6.09	35.9	33.58
17	Sweden	7.24	52.73	49.78	10.41	31.51	27.26
18	UK	4.1	36.74	35.17	4.92	19.34	17.46

Table 7: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \ k = 1, 2, \ldots, 11\}$ together with subsampling approximations of the critical values. The case of seasonality adjusted survey indicator of economic climate in retail trade

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	EU-27	3.62	15.06	13.65	3.51	10.18	8.81
2	EU-16	3.31	13.13	11.84	3.88	11.31	9.79
3	Belgium	5.38	17.19	15.14	5.32	19.83	17.79
4	Bulgaria	7.91	24.4	21.03	6.52	20.83	18.05
5	Denmark	10.12	23.88	19.72	6.4	20.89	18.25
6	Germany	3.32	16.11	14.85	2.71	16.05	15.02
7	Italy	9.06	37.15	33.64	8.01	43.38	40.28
8	Poland	4.71	10.61	8.58	6.13	10.93	8.29
9	Romania	9.43	29.01	24.93	10.6	22.06	17.45
10	Slovakia	7.27	33.06	29.95	7.46	29.7	26.49
11	UK	10.17	33.37	29.49	8.	19.14	16.08

Table 8: Test statistics $\Pi_n(\Psi_T)$, where $\Psi_T = \{2k\pi/12 : \ k = 1, 2, \ldots, 11\}$ together with subsampling approximations of the critical values. The case of seasonality adjusted survey indicator of economic climate in consumption

Lp.	Country or region	$\Pi_n(\Psi_T)$ for $(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$	$\Pi_n(\Psi_T)$ for $(1-B)(1-B^{12})Y_t$	$g_{n,b}(0.95)$	$c_{n,b}(0.95)$
1	EU-27	3.36	11.61	10.33	1.26	6.1	5.62
2	EU-16	3.41	12.52	11.22	1.64	6.22	5.59
3	Belgium	4.27	17.23	15.6	3.78	11.69	10.25
4	Denmark	3.64	11.29	9.91	3.15	11.17	9.97
5	Germany	3.42	15.59	14.29	1.97	8.04	7.29
6	Estonia	10.65	19.89	15.38	5.91	15.77	13.27
7	Greece	9.38	17.59	14.02	8.13	13.88	10.77
8	France	3.8	17.04	15.59	3.22	11.78	10.55
9	Italy	3.04	15.02	13.86	2.4	10.27	9.36
10	Hungary	4.86	37.25	35.18	4.18	18.06	16.27
11	Netherlands	5.2	18.2	16.22	4.39	11.98	10.3
12	UK	3.78	14.99	13.55	3.19	12.87	11.65