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FREE VIBRATION ANALYSIS OF MINDLIN PLATES RESTING ON PASTERNAK FOUNDATION USING COUPLED DISPLACEMENT METHOD

The authors developed a simple and efficient method, called the Coupled Displacement method, to study the linear free vibration behavior of the moderately thick rectangular plates in which a single-term trigonometric/algebraic admissible displacement, such as total rotations, are assumed for one of the variables (in both X,Y directions), and the other displacement field, such as transverse displacement, is derived by making use of the coupling equations. The coupled displacement method makes the energy formulation to contain half the number of unknown independent coefficients in the case of a moderately thick plate, contrary to the conventional Rayleigh-Ritz method. The smaller number of undetermined coefficients significantly simplifies the vibration problem. The closed form expression in the form of fundamental frequency parameter is derived for all edges of simply supported moderately thick rectangular plate resting on Pasternak foundation. The results obtained by the present coupled displacement method are compared with existing open literature values wherever possible for various plate boundary conditions such as all edges simply supported, clamped and two opposite edges simply supported and clamped and the agreement found is good.

Nomenclature

θ assumed total rotation,

w transverse displacement,

G shear modulus,

k shear correction factor,

T total kinetic energy,

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U total strain energy,
ρ density of the plate,
ω_L linear radian frequency,
$\overline{K}_f$ Winkler foundation parameter,
$\overline{K}_s$ Pasternak foundation parameter,
K_f Winkler stiffness,
K_s shear layer stiffness,
ν Poisson ratio,
a length of the plate in x direction,
b length of the plate in y direction,
D plate flexural rigidity,
α undetermined coefficient,
h thickness of plate.

1. Introduction

Plates are used in various engineering applications such as aerospace, marine, civil, mechanical, nuclear, automobile and industrial. These structural members vibrate when they are subjected to severe dynamic environment. So, the knowledge of vibration is very important and essential to study and design of structural members. In case of classical thin plate theory shear deformation is neglected and it is considered in first order shear deformation theory. The authors used a simple method called the Coupled Displacement method to study the vibration characteristics of structural members by considering shear deformation. Many authors contributed different methodologies to study the free vibration behavior of thin plates.

Karasin [1] extended analytical solutions of the discrete one-dimensional beam elements resting on elastic foundation for solution of plate vibration problems by the so-called discrete parameter approach where the physical domain is broken down into discrete sub-domains. Ferreira et al. [2] considered the static and free vibration analysis of rectangular plates resting on Pasternak foundations described by a two-parameter model based on collocation with radial basis functions and based on a first-order shear deformation theory, where displacements and stresses, as well as natural frequencies and modes are produced. Matsunaga [3] analysed the natural frequencies of thick isotropic plates on two parameter elastic foundation by considering the effect of shear deformation and rotary inertia using the method of power series expansion of the displacement components using a set of fundamental dynamic equations, higher order theory for thick plates with the help of Hamilton's principle. Zhou et al. [4] deliberated the free-vibration characteristics of rectangular thick plates resting on elastic foundations based on the three-dimensional, linear and small strain elasticity theory. The foundation is described by the Pasternak (two-parameter) model and the Ritz method is used to derive the eigenvalue equation of the rectangular plate by augmenting the strain energy of the plate with the potential energy of the elastic foundation. Bahmyari [5] utilized the the element free Galerkin method to analyse free vibration of thin plates resting on Pasternak elastic foundations with all possible types of classical boundary conditions. Rao et al. [6] considered large-amplitude free vibrations of uniform Timoshenko beams with different beam boundary conditions by using coupled displacement field method.

Liu et al. [7] premeditated the vibration characteristics of square thick plates on Pasternak foundation with arbitrary boundary conditions on the basis of the three dimensional elasticity theory and the exact solutions are obtained based on the Rayleigh-Ritz procedure by the energy functions of the thick plate. Reuzegar et al. [8] used two-variable refined plate theory for the analysis of thick plates resting on elastic foundation; the theory contains only two unknown parameters and predicts parabolic variation of transverse shear stresses without using shear correction factor and used the principle of minimum potential energy, the governing equations for simply supported rectangular plates resting on Winkler elastic foundation are obtained.

Krishnabhaskar et al. [9] used coupled displacement method for the evaluation of large-amplitude free vibrations of Timoshenko beams at higher modes. Rajesh et al. [10] presented the vibration behavior of rectangular thick plates by using the Coupled Displacement method for simply supported beam boundary condition for various thickness ratios. Rajesh and Saheb [11] used Coupled Displacement method and successfully applied it to study the free vibrations of uniform Timoshenko beams resting on Pasternak foundation for hinged-hinged and clamped-clamped beam boundary conditions. Ozgan et al. [12] did the free vibration analysis of thick plates on elastic foundations using modified Vlasov model with higher order modified elements using 4-noded (PBQ4) and 8-noded (PBQ8) Mindlin plate elements for the analysis using Winkler foundation model. Two different integration rules, namely the full integration (FI) and the selective reduced integration (SRI) techniques, are used to obtain stiffness matrix of plates.

Ozgan et al. [13, 14] implemented Vlasov model with higher-order finite elements for free vibration analysis of thick plates on elastic foundations. Dehghany and Farajpour [15] dealt with exact solution for free vibration analysis of simply supported rectangular plates on elastic foundation on the basis of three dimensional elasticity theory for the Pasternak foundation (two-parameter) model. The Navier equations of motion are replaced by three decoupled equations in terms of displacement components and these equations are solved using a semi-inverse method to get the solution in the form of a double Fourier sine series. Omurtag et al. [16] studied the free vibrations of thin plates resting on Pasternak foundation using finite element method. Matsunaga [3] deliberated the vibration and stability of simply supported rectangular thick plates on a Pasternak foundation using a special higher order plate theory. Mindlin [17] proposed the so-called first-order shear deformation theory by assuming constant shear strain throughout the cross section and introduced shear correction factor to compensate errors and got more accurate results than that of classical plate theory. Zaman et al. [18] focused on free vibration analysis of plates resting on a two-parameter elastic medium through a finite element procedure. The associated shape functions are derived from the solution of a plate resting on the two-parameter medium and a parametric study is conducted to identify the influence of various factors on the mode shapes and natural frequencies. Wen [19] investigated the method of fundamental solution (MFS) applied to a shear deformable plate (Reissner/Mindlin's theories) resting on the elastic foundation under either a static or a dynamic load through fundamental solutions by the boundary element method. Buczkowski et al. [20] studied the natural frequencies of thick plates resting on Pasternak foundation by taking 16-node Mindlin plate element and 32-node zero thickness interface element representing the response of the foundation by using finite element method. Saheb and Krishnabhaskar [21] premeditated large amplitude free vibration of simply supported and clamped moderately thick square plates using coupled displacement method. Saheb et al. [22] successfully applied coupled displacement method for evaluation of large amplitude free vibration behavior of uniform Timoshenko beams. Wang et al. [23] derived exact and analytical solutions for simply supported rectangular plate on Pasternak foundation.

Xiang and Kitipornchai [24] worked on exact vibration solution for initially stressed Mindlin plates on Pasternak foundations. Xiang et al. [25] worked on Winkler and Pasternak foundation to study the soil behavior for different types of foundations like shallow and deep. Zhong and Yin [26] explored eigenfrequencies and vibration modes of a rectangular thin plate on an elastic foundation using classical thin plate theory and integral transformation method. In this paper, the authors presented the Coupled Displacement Field (CDF) method for analysing the vibration behaviour of Mindlin rectangular plates, which is suitable for analysing the vibration characteristics of structural members. The CDF method was successfully applied for analysing the vibration behaviour of Timoshenko beam with the effect of foundation parameter [11].

2. Coupled Displacement method (CDF)

Knowledge of fundamental frequency parameters of moderately thick rectangular plates is important and has to be considered in the initial design phase of structural members. The energy methods provide a convenient means for computing the fundamental frequency parameters of structural members and the solutions obtained using this approach are upper bound and the accuracy of the solution depends on the admissible functions chosen for the lateral displacement and total rotations.

In this paper, the authors used CDF method and that significantly simplifies the formulation of the vibration problem of moderately thick rectangular plates. In this paper, the solutions for the total rotations and the lateral displacement are coupled by using coupling equations, which are derived for obtaining the static solution of shear deformable moderately thick rectangular plates. This method reduces the magnitude of the problem by reducing the number of unknowns from $2n$ to n, for the multi-term admissible functions and from two to one for the single-term admissible functions and decreases the effort involved in the solution procedure considerably to obtain the fundamental frequency parameter.

2.1. First order shear deformation theory of plates

The simplest shear deformation plate theory is the first-order shear deformation plate theory (or FSDT), also referred to as the Mindlin plate theory (Mindlin, 1951) and it is based on the displacement.

$$u(x, y, z) = z\theta_x(x, y), \tag{1}$$

$$v(x, y, z) = z\theta_y(x, y), \tag{2}$$

$$w(x, y, z) = w(x, y), \tag{3}$$

where u and v are in plane displacements in x and y direction, w is transverse displacement along z direction, θ_x and θ_y denote rotations about the y and x axes respectively. In FSDT, shear correction factor is introduced to correct the discrepancy between the actual transverse shear stress distribution and that computed using the kinematic relations of FSDT. The shear correction factor (k) depends not only on the geometric parameters, but also on the loading and boundary conditions of the plate. However, a value of $k = 5/6$, the widely used value of the shear correction factor, is used in the present study.

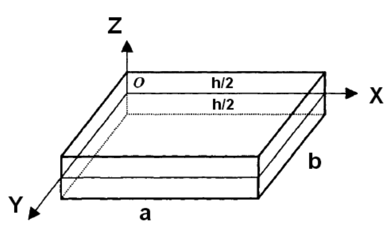

Fig. 1. Geometry of thick plate

2.2. Coupled Displacement (CDF) method for Mindlin rectangular plates

In this method, an admissible functions for θ_x and θ_y which satisfies all the geometric boundary conditions of plate domain is assumed. Note that these functions may satisfy some or all the natural boundary conditions also. The for lateral displacement (w) is evaluated using the coupling equations derived from the static equilibrium equation which is independent of the externally applied load term.

$$\frac{dw}{dx} = -\theta_x + \frac{h^2}{3.5}\left[\frac{\partial^2\theta_x}{\partial x^2} + \nu\frac{\partial^2\theta_y}{\partial y\partial x}\right] + \frac{h^2}{10}\left[\frac{\partial^2\theta_x}{\partial y^2} + \frac{\partial^2\theta_y}{\partial y\partial x}\right], \tag{4}$$

$$\frac{dw}{dy} = -\theta_y + \frac{h^2}{3.5}\left[\frac{\partial^2\theta_y}{\partial y^2} + \nu\frac{\partial^2\theta_x}{\partial y\partial x}\right] + \frac{h^2}{10}\left[\frac{\partial^2\theta_y}{\partial x^2} + \frac{\partial^2\theta_x}{\partial y\partial x}\right]. \tag{5}$$

It may be noted that two coupling equations are obtained for the moderately thick rectangular plate. Though the admissible functions θ_x and θ_y, in general can be written in a series form, here a single-term admissible functions for θ_x and θ_y is chosen again with same intention of simplicity and better understanding of the CDF method as

$$\theta_x = \alpha f_1(x, y), \tag{6}$$

$$\theta_y = \alpha f_2(x, y), \tag{7}$$

where α is the undetermined coefficient and $f_1(x, y)$ is the single-term admissible function. Note that the functions for θ_x and θ_y are the different, as the rectangular plate is considered in the present study.

Substituting the admissible functions for θ_x and θ_y as given in (6) and (7) in (4) and (5) coupled displacement for the lateral displacement w is obtained as

$$w = \alpha f_3(x, y). \tag{8}$$

Note that because of the use of the coupling equations, the transverse displacement distribution (w) contains the same undetermined coefficient α as existing in the θ_x and θ_y distribution. In general, a $2n$ undetermined coefficients problem in the

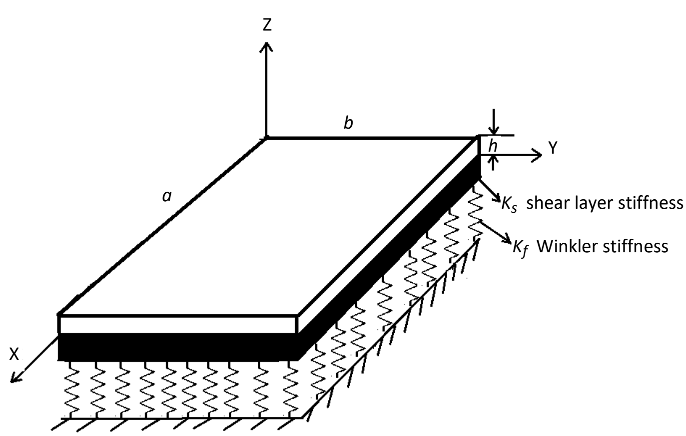

Fig. 2. Moderately thick rectangular plate resting on Pasternak foundation

R-R method reduces to an n undetermined coefficient problem in the CDF method. The linear fundamental frequency parameter is obtained from the following simple equation

$$\frac{d(U-T)}{d\alpha}=0\,. \tag{9}$$

3. Linear Free Vibrations of moderately thick rectangular Plates

The detailed procedure for the CDF method is discussed in this section for the case of evaluating the fundamental linear frequency parameter of a uniform all edges simply supported moderately thick rectangular plate for which the exact vibration mode shape (for shear flexible plate) for the transverse displacement w is well known. In the CDF method, the admissible functions for θ_x and θ_y are assumed in the functional form, noting the similarity between $\frac{dw}{dx}$, θ_x and, $\frac{dw}{dy}$, θ_y and satisfy the boundary conditions and symmetric conditions for m, n mode of vibration as

$$\theta_x=\alpha\frac{m\pi}{a}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \tag{10}$$

$$\theta_y=\alpha\frac{n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \tag{11}$$

where m is number of half sine waves in x direction and n is number of half sine waves in y direction.

For fundamental mode the above equations become ($m=1$, $n=1$)

$$\theta_x=\alpha\frac{\pi}{a}\cos\frac{\pi x}{a}\sin\frac{\pi y}{b}, \tag{12}$$

$$\theta_y=\alpha\frac{\pi}{b}\sin\frac{\pi x}{a}\cos\frac{\pi y}{b}, \tag{13}$$

$$\frac{dw}{dx}=-\theta_x+\frac{h^2}{3.5}\left[\frac{\partial^2\theta_x}{\partial x^2}+\nu\frac{\partial^2\theta_y}{\partial y\partial x}\right]+\frac{h^2}{10}\left[\frac{\partial^2\theta_x}{\partial y^2}+\frac{\partial^2\theta_y}{\partial y\partial x}\right], \tag{14}$$

$$\frac{dw}{dy}=-\theta_y+\frac{h^2}{3.5}\left[\frac{\partial^2\theta_y}{\partial y^2}+\nu\frac{\partial^2\theta_x}{\partial y\partial x}\right]+\frac{h^2}{10}\left[\frac{\partial^2\theta_y}{\partial x^2}+\frac{\partial^2\theta_x}{\partial y\partial x}\right]. \tag{15}$$

Substituting equations (12) and (13) in equation (14) and (15) and after simplification

$$\frac{dw}{dx}=-\alpha\frac{\pi}{a}\cos\frac{\pi x}{a}\sin\frac{\pi y}{b}\left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{a^2}+\frac{0.3\pi^2}{b^2}\right)+\frac{h^2}{5}\frac{\pi^2}{b^2}\right], \tag{16}$$

$$\frac{dw}{dy}=-\alpha\frac{\pi}{b}\sin\frac{\pi x}{a}\cos\frac{\pi y}{b}\left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{b^2}+\frac{0.3\pi^2}{a^2}\right)+\frac{h^2}{5}\frac{\pi^2}{a^2}\right]. \tag{17}$$

Integration of equation (16) and after evaluation of the constant of integration

$$w_x = -\alpha \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{a^2}+\frac{0.3\pi^2}{b^2}\right)+\frac{h^2}{5}\frac{\pi^2}{b^2}\right]. \tag{18}$$

The expression for strain energy due to bending moment and transverse shear force is given by

$$U_E = \frac{D}{2}\int_0^a\int_0^b\left\{\left(\frac{\partial\theta_x}{\partial x}\right)^2+\left(\frac{\partial\theta_y}{\partial y}\right)^2+2\nu\frac{\partial\theta_x}{\partial x}\frac{\partial\theta_y}{\partial y}+2(1-\nu)\frac{\partial\theta_x}{\partial y}\frac{\partial\theta_y}{\partial x}\right\}\mathrm{d}x\,\mathrm{d}y+$$
$$\frac{kGh}{2}\int_0^a\int_0^b\left\{\left(\frac{dw}{dx}+\theta_x\right)^2+\left(\frac{dw}{dy}+\theta_y\right)^2\right\}\mathrm{d}x\,\mathrm{d}y. \tag{19}$$

Substituting equations (12), (13), (16), (17) in the above equation and after simplification

$$U_E = \frac{D\alpha^2 ab}{2}\left[\left(\frac{\pi^2}{a^2}+\frac{\pi^2}{b^2}\right)^2+\frac{6k(1-\nu)}{h^2}\left\{\frac{\pi^2}{a^2}(A-1)^2+\frac{\pi^2}{b^2}(B-1)^2\right\}\right], \tag{20}$$

where

$$A = \left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{a^2}+\frac{0.3\pi^2}{b^2}\right)+\frac{h^2}{5}\frac{\pi^2}{b^2}\right],$$
$$B = \left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{b^2}+\frac{0.3\pi^2}{a^2}\right)+\frac{h^2}{5}\frac{\pi^2}{a^2}\right],$$

s is denoted as the aspect ratio ($b/a = s$) then

$$A = \left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{a^2}+\frac{0.3\pi^2}{a^2 s^2}\right)+\frac{h^2}{5}\frac{\pi^2}{a^2 s^2}\right],$$
$$B = \left[1+\frac{h^2}{3.5}\left(\frac{\pi^2}{a^2 s^2}+\frac{0.3\pi^2}{a^2}\right)+\frac{h^2}{5}\frac{\pi^2}{a^2}\right].$$

Rewriting the equation (20) in terms of aspect ratio s and thickness ratio (h/a)

$$U_E = \frac{D\alpha^2 a^2 s}{2}\left[\left(\frac{\pi^2}{a^2}+\frac{\pi^2}{a^2 s^2}\right)^2+\frac{6k(1-\nu)}{h^2}\left\{\frac{\pi^2}{a^2}(A-1)^2+\frac{\pi^2}{a^2 s^2}(B-1)^2\right\}\right]. \tag{21}$$

The expression for strain energy due to Pasternak foundation is given by

$$U_F = \frac{1}{2}\int_0^a\int_0^b\left[K_f w_x^2+K_s\left\{\left(\frac{dw}{dx}\right)^2+\left(\frac{dw}{dy}\right)^2\right\}\right]\mathrm{d}x\,\mathrm{d}y. \tag{22}$$

Winkler and Pasternak foundation parameters are given respectively as

$$\overline{K}_f = \frac{K_f a^4}{D}, \qquad \overline{K}_s = \frac{K_s a^2}{D}. \tag{23}$$

Substituting equation (14), (15), (18) and (23) in equation (22), and after simplification (for $b/a = s$)

$$U_F = \frac{\alpha^2 D s}{8a^2}\left[\left(\overline{K}_f + \overline{K}_s \pi^2\right)(A)^2 + \pi^2 \frac{\overline{K}_s}{s^2}(B)^2\right]. \tag{24}$$

The total strain energy $U = U_E + U_F$

$$U = \frac{D\alpha^2 a^2 s}{2}\left[\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{a^2 s^2}\right)^2 + \frac{6k(1-\nu)}{h^2}\frac{\pi^2}{a^2}\left\{(A-1)^2 + \frac{1}{s^2}(B-1)^2\right\}\right] +$$

$$\frac{\alpha^2 D s}{8a^2}\left[\left(\overline{K}_f + \overline{K}_s \pi^2\right)(A)^2 + \pi^2 \frac{\overline{K}_s}{s^2}(B)^2\right] \tag{25}$$

after further simplification the above equation becomes,

$$U = \frac{D\alpha^2 s \pi^4}{2a^2}\left[\begin{array}{l}\left[\frac{\left(s^2+1\right)^2}{s^4} + \frac{6k(1-\nu)}{h^2}\frac{a^2}{\pi^2}\left\{(A-1)^2 + \frac{1}{s^2}(B-1)^2\right\}\right] + \\ \frac{1}{4\pi^4}\left[\left(\overline{K}_f + \overline{K}_s \pi^2\right)(A)^2 + \pi^2 \frac{\overline{K}_s}{s^2}(B)^2\right]\end{array}\right]. \tag{26}$$

The expression for kinetic energy of the moderately thick rectangular plate is given as

$$T = \frac{\rho h \omega_L^2}{2}\int_0^a\int_0^b\left[w^2 + \frac{h^2}{12}\left(\theta_x^2 + \theta_y^2\right)\right]\mathrm{d}x\,\mathrm{d}y. \tag{27}$$

Substituting the equations (12), (13) and (18) in the above equation, and after simplification

$$T = \frac{\rho h \omega_L^2 \alpha^2 a^2 s}{8}\left[(A)^2 + \frac{h^2 \pi^2}{12a^2}\left(1 + \frac{1}{s^2}\right)\right]. \tag{28}$$

By minimizing the Lagranzian with respect to undetermined coefficient α

$$\frac{\partial(U-T)}{\partial\alpha} = 0, \tag{29}$$

$$\lambda = \frac{\rho h \omega_L^2 a^4}{\pi^4 D} = 4\left[\left[\frac{(s^2+1)^2}{s^4} + \frac{6k(1-\nu)}{h^2}\frac{a^2}{\pi^2}\left\{(A-1)^2 + \frac{1}{s^2}(B-1)^2\right\}\right] + \frac{1}{4\pi^4}\left[\left(\overline{K}_f + \overline{K}_s\pi^2\right)(A)^2 + \pi^2\frac{\overline{K}_s}{s^2}(B)^2\right]\right] \Bigg/ \left[(A)^2 + \frac{h^2\pi^2}{12a^2}\left(1+\frac{1}{s^2}\right)\right], \tag{30}$$

where

$$A = \left[1 + \frac{h^2}{3.5}\left(\frac{\pi^2}{a^2} + \frac{0.3\pi^2}{a^2 s^2}\right) + \frac{h^2}{5}\frac{\pi^2}{a^2 s^2}\right],$$

$$B = \left[1 + \frac{h^2}{3.5}\left(\frac{\pi^2}{a^2 s^2} + \frac{0.3\pi^2}{a^2}\right) + \frac{h^2}{5}\frac{\pi^2}{a^2}\right].$$

There are no exact trigonometric admissible functions available for plate boundary conditions such as all edges clamped and opposite edges clamped and simply supported. So, the author made an effort to derive algebraic admissible functions which satisfy all essential boundary conditions for the above mentioned plate boundary conditions instead of trigonometric admissible functions to show the efficacy of the coupled displacement method. The expressions for transverse displacement at different modes with different plate boundary conditions are shown in Table 1.

Table 1.
Algebraic admissible functions for different plate boundary conditions at fundamental and higher modes

Boundary condition	Transverse displacement w	Mode sequence
C-C-C-C	$x^2(a-x)^2y^2(b-y)^2\alpha$	1, 1
	$x^2(a-x)^2y^2(b-y)^2(0.5-x)\alpha$	1, 2
	$x^2(a-x)^2y^2(b-y)^2(0.5-y)\alpha$	2, 1
	$x^2(a-x)^2y^2(b-y)^2(0.5-y)(0.5-x)\alpha$	2, 2
S-C-S-C	$x(x-a)y^2(b-y)^2\alpha$	1, 1
	$x(x-a)y^2(b-y)^2(y-0.5)\alpha$	1, 2
	$x(x-a)y^2(b-y)^2(x-0.5)\alpha$	2, 1
	$x(x-a)y^2(b-y)^2(x-0.5)(y-0.5)\alpha$	2, 2

By using equation (30), the fundamental frequency parameter values (λ), for S-S-S-S plate boundary condition are calculated in terms of plate thickness ratio (h/a), aspect ratio (s) and foundation parameters, and are shown in Table 2, Table 3.

Similar procedure is adopted for other plate boundary conditions such as all edges clamped (C-C-C-C), opposite edges clamped and simply supported (S-C-S-C) boundary conditions, and the corresponding frequency parameter values are calculated by developing code in Mathmatica (Mathmatical tool), and are shown in Table 4, Table 5 and Table 6, Table 7, and the agreement is good between the other researchers' results and the results generated by present CDF method.

4. Numerical results and discussion

Using the formulation described above, linear free vibration behavior of a uniform thin and moderately thick rectangular plates is obtained in terms of plate thickness ratio (h/a), aspect ratio (s) and foundation parameters. As a demonstration of the proposed formulation, the plate is considered with axially immovable edges.

The present results in terms of fundamental frequency parameter (λ) are presented in Table 2 for all edges of simply supported moderately thick square and rectangular plates with variation of Winkler foundation stiffness. For thin plates resting on elastic foundation, both classical plate theory and Mindlin plate theory give accurate results, but the Mindlin plate theory requires more efforts, similarly as other methods available in the literature, than the present CDF method. For the sake of comparison and validation of the proposed method, the same results obtained by the other researchers are included in Table 2, Table 3 for all edges S-S-S-S and Table 4, Table 5 for all edges C-C-C-C plate boundary conditions. Table 6, Table 7 shows the frequency parameter values for S-C-S-C boundary condition for different modes.

Table 2 shows frequency parameter values at fundamental and higher modes for thin and moderately thick square and rectangular plates with all edges S-S-S-S boundary condition resting on Winkler foundation. Different combinations of plate thickness ratios such as $h/a = 0.01, 0.1, 0.15, 0.2$; and Winkler foundation parameters such as $\overline{K}_f = 100, 200, 300, 400, 500, 1000$; Pasternak foundation parameters such as $\overline{K}_S = 0$, for various aspect ratios as $a/b = 1, 1.5, 2$ were considered. To show the accuracy and convergence rate of the present CDF method, the results are examined by varying the non-dimensional foundation parameters to find the influence of foundation parameter on non-dimensional frequency parameter and are compared with other researchers [4, 23] and [16]. It is observed from Table 2 that the frequency parameter value increases with the increase in Winkler foundation parameter for a given plate thickness ratio and aspect ratio. It is noticed from Table 2 that the fundamental frequency parameter value decreases with the

Table 2.

$\lambda^{1/2}$ values of thin and moderately thick S-S-S-S Mindlin square and rectangular plates resting on Winkler foundation

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1)		Mode (1,2)		Mode (2,1)		Mode (2,2)	
				CDF	Ref.	CDF	Ref.	CDF	Ref.	CDF	Ref.
1	0.01	100	0	2.2413	2.2413[a] 2.2413[b]	5.0971	5.0973[a] 5.0971[b]	5.0971	5.0973[a] 5.0971[b]	8.0523	8.0527[a] 8.0523[b]
		200	0	2.4596	–	5.1968	–	5.1968	–	8.1157	
		300	0	2.6601	–	5.2946	–	5.2946	–	8.1787	
		400	0	2.8465	–	5.3906	–	5.3906	–	8.2412	
		500	0	3.0214	3.0214[a] 3.0215[b]	5.4850	5.4850[a] 5.4850[b]	5.4850	5.4850[a] 5.4850[b]	8.3032	8.3035[a] 8.3032[b]
		1000	0	3.7764	–	5.9343	–	5.9343	–	8.6065	
	0.1	100	0	2.1778	–	4.7154	–	4.7154	–	7.1422	
		200	0	2.3989	2.3951[a] 2.3989[b]	4.8197	4.8262[a] 4.8194[b]	4.8197	4.8262[a] 4.8194[b]	7.2107	7.2338[a] 7.2093[b]
		300	0	2.6012	–	4.9219	–	4.9219	–	7.2786	
		400	0	2.7889	–	5.0220	–	5.0220	–	7.3459	
		500	0	2.9647	–	5.1201	–	5.1201	–	7.4125	
		1000	0	3.7213	3.7008[a] 3.7212[b]	5.5849	5.5661[a] 5.5844[b]	5.5849	5.5661[a] 5.5844[b]	7.7371	7.7335[a] 7.7353[b]
	0.15	100	0	2.1086	–	4.3580	–	4.3580	–	6.3890	–
		200	0	2.3332	–	4.4685	–	4.4685	–	6.4640	–
		300	0	2.5380	–	4.5762	–	4.5762	–	6.5381	–
		400	0	2.7274	–	4.6815	–	4.6815	–	6.6114	–
		500	0	2.9046	–	4.7844	–	4.7844	–	6.6839	–
		1000	0	3.6639	–	5.2692	–	5.2692	–	7.0352	–
	0.2	10	0	1.7958	1.8020[a] 1.7955[b] 1.8020[c]	3.8825	3.9103[a] 3.8780[b] 3.9103[c]	3.8825	3.9103[a] 3.8780[b] 3.9103[c]	5.6086	–
		100	0	2.0273	2.0216[a] 2.0268[b] 2.0216[c]	3.9924	4.0090[a] 3.9875[b] 4.0090[c]	3.9924	4.0090[a] 3.9875[b] 4.0090[c]	5.6849	–
		1000	0	3.6002	3.4793[a] 3.5972[b] 3.4793[c]	4.9595	4.8499[a] 4.9499[b] 4.8499[c]	4.9595	4.8499[a] 4.9499[b] 4.8499[c]	6.3977	–
1.5	0.01	100	0	1.5126	–	4.4636	–	4.4636	–	5.7892	–
		1000	0	2.0280	–	4.6635	–	4.6635	–	5.9447	–
	0.1	100	0	1.4777	–	4.1542	–	4.1542	–	5.2846	–
		1000	0	1.9972	–	4.3625	–	4.3625	–	5.4488	–
	0.15	100	0	1.4379	–	3.8557	–	3.8557	–	4.8266	–
		1000	0	1.9628	–	4.0749	–	4.0749	–	5.0023	–
	0.2	100	0	1.3889	–	3.5420	–	3.5420	–	4.3685	–
		1000	0	1.9215	–	3.7762	–	3.7762	–	4.5596	–

Table 2.
[cont.]

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1)		Mode (1,2)		Mode (2,1)		Mode (2,2)	
				CDF	Ref.	CDF	Ref.	CDF	Ref.	CDF	Ref.
2	0.01	100	0	1.2751	–	4.2542	–	4.2542	–	5.0018	–
		1000	0	1.4843	–	4.3215	–	4.3215	–	5.0592	–
	0.1	100	0	1.2484	–	3.9691	–	3.9691	–	4.6154	–
		1000	0	1.4596	–	4.0393	–	4.0393	–	4.6756	–
	0.15	100	0	1.2175	–	3.6913	–	3.6913	–	4.2519	–
		1000	0	1.4313	–	3.7651	–	3.7651	–	4.3159	–
	0.2	100	0	1.1788	–	3.3965	–	3.3965	–	3.8778	–
		1000	0	1.3962	–	3.4755		3.4755	–	3.9470	–
Note: a, b, c, values are taken from Ref.: [4], [23], [16]											

increase in plate aspect ratio for a given plate thickness ratio and Winkler foundation parameter. And also it can be observed that as mode sequence increases, the frequency parameter value increases for a particular aspect ratio, plate thickness ratio and Winkler foundation parameter.

Table 3 shows frequency parameter values at fundamental and higher modes both for thin and moderately thick square and rectangular plates with all edges S-S-S-S boundary condition resting on Winkler and Pasternak foundation. Different combinations of (thickness to length ratio) $h/a = 0.01, 0.1, 0.15, 0.2$ and Winkler foundation parameters $\overline{K}_f = 100, 200, 300, 400, 500, 1000$; Pasternak foundation parameter $\overline{K}_S = 10$ for different aspect ratios like $a/b = 1, 1.5, 2$ were considered. It is observed from Table 3 that frequency values are increasing with the increase of Winkler foundation parameter for a given plate aspect ratio, plate thickness ratio, Pasternak foundation parameter and mode sequence. The results obtained by present CDF method are very closely matching with those of the existing literature [4, 7, 16, 23].

Table 4 shows frequency parameter values for thin and moderately thick square and rectangular plates with all edges C-C-C-C boundary condition resting on Winkler foundation. Different combinations of thickness to length ratio $h/a = 0.01$, 0.015, 0.1 and foundation parameters $\overline{K}_f = 100, 200, 300, 400, 500, 1000$; $\overline{K}_S = 0$ for $a/b = 1, 1.5, 2$ were considered. It can be observed that the more frequency parameter values are found in case of all edges clamped (C-C-C-C) boundary condition compared to all edges simply supported (S-S-S-S) plate boundary condition. Table 5 shows the frequency parameter values for thin and moderately thick square and rectangular plates with all edges C-C-C-C boundary condition resting on Winkler and Pasternak foundation.

Different combinations of thickness to length ratio $h/a = 0.01, 0.015, 0.1$ and foundation parameters $\overline{K}_f = 100, 200, 300, 400, 500, 1000$; $\overline{K}_S = 10$ for $a/b = 1$,

Table 3.
$\lambda^{1/2}$ values of thin and moderately thick S-S-S-S square and rectangular plates resting on Pasternak foundation

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1)		Mode (1,2)		Mode (2,1)		Mode (2,2)	
				CDF	Ref.	CDF	Ref.	CDF	Ref.	CDF	Ref.
1	0.01	100	10	2.6551	2.6551[a] 2.6551[b] 2.6551[e]	5.5717	5.5717[a] 5.5718[b] 5.5717[e]	5.5717	5.5717[a] 5.5718[b] 5.5717[e]	8.5405	8.5406[a] 8.5405[b] 8.5404[e]
		200	10	2.8418	–	5.6631	–	5.6631	–	8.6003	–
		300	10	3.0170	–	5.7530	–	5.7530	–	8.6598	–
		400	10	3.1825	–	5.8415	–	5.8415	–	8.7188	–
		500	10	3.3399	3.3398[a] 3.3400[b] 3.3400[e]	5.9287	5.9285[a] 5.9287[b] 5.9287[e]	5.9287	5.9285[a] 5.9287[b] 5.9287[e]	8.7774	8.7775[a] 8.7775[b] 8.7774[e]
		1000	10	4.0357	–	6.3466	–	6.3466	–	9.0649	–
	0.1	100	10	2.5961	–	5.2101	–	5.2101	–	7.6666	–
		200	10	2.7842	2.7756[a] 2.7842[b] 2.7837[e]	5.3048	5.2954[a] 5.3043[b] 5.3013[e]	5.3048	5.2954[a] 5.3043[b]	7.7305	7.7279[a] 7.7287[b] 7.7215[e]
		300	10	2.9603	–	5.3978	–	5.3978	–	7.7938	–
		400	10	3.1265	–	5.4892	–	5.4892	–	7.8567	–
		500	10	3.2843	–	5.5791	–	5.5791	–	7.9190	–
		1000	10	3.9806	3.9566[a] 3.9805[b] 3.9802[e]	6.0085	5.9757[a] 6.0078[b] 6.0052[e]	6.0085	5.9757[a] 6.0078[b] 6.0052[e]	8.2236	8.1954[a] 8.2214[b] 8.2148[e]
	0.15	100	10	2.5329	–	4.8787	–	4.8787	–	6.9591	–
		200	10	2.7227	–	4.9776	–	4.9776	–	7.0280	–
		300	10	2.9001	–	5.0746	–	5.0746	–	7.0963	–
		400	10	3.0672	–	5.1697	–	5.1697	–	7.1638	–
		500	10	3.2257	–	5.2631	–	5.2631	–	7.2308	–
		1000	10	3.9234	–	5.7073	–	5.7073	–	7.5567	–
	0.2	10	10	2.2729	2.2539[a] 2.2722[b] 2.2539[c]	4.4520	4.4150[a] 4.4452[b] 4.4150[c]	4.4520	4.4150[a] 4.4452[b] 4.4150[c]	6.2460	–
		100	10	2.4600	2.4300[a] 2.4591[b] 2.4300[c]	4.5482	4.4986[a] 4.5409[b] 4.4986[c]	4.5482	4.4986[a] 4.5409[b] 4.4986[c]	6.3146	–
		1000	10	3.8604	3.7111[a] 3.8567[b] 3.7112[c]	5.4169	5.2285[a] 5.4043[b] 5.2285[c]	5.4169	5.2285[a] 5.4043[b] 5.2285[c]	6.9633	–

Table 3.
[cont.]

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1)		Mode (1,2)		Mode (2,1)		Mode (2,2)	
				CDF	Ref.	CDF	Ref.	CDF	Ref.	CDF	Ref.
1.5	0.01	100	10	1.7141	–	4.6823	–	4.6823	–	6.0096	–
		1000	10	2.1825	–	4.8732	–	4.8732	–	6.1595	–
	0.1	100	10	1.6813	–	4.3821	–	4.3821	–	5.5173	–
		1000	10	2.1522	–	4.5800	–	4.5800	–	5.6748	–
	0.15	100	10	1.6443	–	4.0954	–	4.0954	–	5.0752	–
		1000	10	2.1187	–	4.3024	–	4.3024	–	5.2425	–
	0.2	100	10	1.5992	–	3.7981	–	3.7981	–	4.6385	–
		1000	10	2.0786	–	4.0174	–	4.0174	–	4.8189	–
2	0.01	100	10	1.3937	–	4.3789	–	4.3789	–	5.1268	–
		1000	10	1.5874	–	4.4443	–	4.4443	–	5.1828	–
	0.1	100	10	1.3682	–	4.0990	–	4.0990	–	4.7465	–
		1000	10	1.5633	–	4.1669	–	4.1669	–	4.8051	–
	0.15	100	10	1.3389	–	3.8278	–	3.8278	–	4.3910	–
		1000	10	1.5359	–	3.8990	–	3.8990	–	4.4530	–
	0.2	100	10	1.3025	–	3.5423	–	3.5423	–	4.0279	–
		1000	10	1.5021	–	3.6181	–	3.6181	–	4.0945	–
Note: a, b, c, e values are taken from Ref.: [4], [23], [16], [7]											

1.5, 2 were considered. Similar trend is observed for C-C-C-C plate resting on Pasternak foundation. It is observed that the frequency parameter values are in excellent agreement with the other researchers [4, 16].

Table 6 shows frequency parameter values for thin and moderately thick square and rectangular plates with all edges S-C-S-C plate boundary condition resting on Winkler foundation. Different combinations of thickness to length ratio $h/a = 0.01$, 0.015, 0.1, 0.2 and foundation parameters $\overline{K}_f = 100$, 200, 300, 400, 500, 1000; $\overline{K}_S = 0$ for $a/b = 1$, 1.5, 2 were considered. It is noticed that the frequency parameter values are increasing with the increase in plate thickness ratio and Winkler foundation parameter for a given particular aspect ratio, Winkler foundation parameter and plate thickness ratio. Table 7 shows frequency parameter values for thin and moderately thick square and rectangular plates with all edges S-C-S-C boundary condition resting on Winkler and Pasternak foundation. Different combinations of thickness to length ratio $h/a = 0.01$, 0.015, 0.1, 0.2; and foundation parameters $\overline{K}_f = 100$, 200, 300, 400, 500, 1000; $\overline{K}_S = 10$ for $a/b = 1$, 1.5, 2; were considered.

Table 4.

$\lambda^{1/2}$ values of thin and moderately thick C-C-C-C Mindlin square and rectangular plates resting on Winkler foundation

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1)		Mode (1,2)		Mode (2,1)		Mode (2,2)	
				CDF	Ref.	CDF	Ref.	CDF	Ref.	CDF	Ref.
1	0.01	100	0	3.7824	–	7.5993	–	7.5993	–	11.0548	–
		200	0	3.9157	–	7.6665	–	7.6665	–	11.1011	–
		300	0	4.0446	–	7.7332	–	7.7332	–	11.1472	–
		400	0	4.1696	–	7.7992	–	7.7992	–	11.1931	–
		500	0	4.2909	–	7.8647	–	7.8647	–	11.2389	–
		1000	0	4.8522	–	8.1844	–	8.1844	–	11.4648	–
	0.015	100	0	3.7783	–	7.6035	–	7.6035	–	11.0780	–
		200	0	3.9118	–	7.6707	–	7.6707	–	11.1241	–
		300	0	4.0408	–	7.7372	–	7.7372	–	11.1701	–
		400	0	4.1658	–	7.8032	–	7.8032	–	11.2159	–
		500	0	4.2872	–	7.8687	–	7.8687	–	11.2615	–
		1000	0	4.8488	–	8.1880	–	8.1880	–	11.4867	–
		1390.2	0 0	5.2455	5.2446[a] 5.2588[d]	8.4288	8.3156[a] 8.4322[d]	8.4288	8.3156[a] 8.4322[d]	11.6595	11.5410[a] 11.6740[d]
		2780.4	0 0	6.4638	6.4629[a] 6.4601[d]	9.2359	9.1324[a] 9.2482[d]	9.2359	9.1324[a] 9.2482[d]	12.2553	12.1420[a] 12.2630[d]
	0.1	100	0	3.4999	–	7.1966	–	7.1966	–	10.0721	–
		200	0	3.6417	–	7.2656	–	7.2656	–	10.1210	–
		300	0	3.7781	–	7.3339	–	7.3339	–	10.1696	–
		400	0	3.9098	–	7.4016	–	7.4016	–	10.2180	–
		500	0	4.0372	–	7.4687	–	7.4687	–	10.2662	–
		1000	0	4.6217	–	7.7954	–	7.7954	–	10.5038	–
1.5	0.01	100	0	4.3048	–	5.3190	–	5.3190	–	8.9517	–
		1000	0	4.5117	–	5.4878	–	5.4878	–	9.0530	–
	0.1	100	0	4.3617	–	5.7215	–	5.7215	–	9.3249	–
		1000	0	4.5580	–	5.8696	–	5.8696	–	9.4153	–
	0.15	100	0	4.3951	–	6.1888	–	6.1888	–	9.4881	–
		1000	0	4.5807	–	6.3130	–	6.3130	–	9.5707	–
	0.2	100	0	4.3788	–	6.6632	–	6.6632	–	9.3566	–
		1000	0	4.5540	–	6.7598	–	6.7598	–	9.4352	–
2	0.01	100	0	2.7444	–	3.0085	–	3.0085	–	7.0543	–
		1000	0	2.8476	–	3.1030	–	3.1030	–	7.0951	–
	0.1	100	0	2.8212	–	3.1006	–	3.1006	–	7.3791	–
		1000	0	2.9099	–	3.1903	–	3.1903	–	7.4165	–
	0.15	100	0	2.8949	–	3.1955	–	3.1955	–	7.4712	–
		1000	0	2.9891	–	3.2801	–	3.2801	–	7.5071	–
	0.2	100	0	2.9615	–	3.2908	–	3.2908	–	7.3321	–
		1000	0	3.0513	–	3.3700	–	3.3700	–	7.3679	–

Note: a, d values are taken from Ref.: [4], [16]

Table 5.

$\lambda^{1/2}$ values of thin and moderately thick C-C-C-C Mindlin square and rectangular plates resting on Pasternak foundation

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1)		Mode (1,2)		Mode (2,1)		Mode (2,2)	
				CDF	Ref.	CDF	Ref.	CDF	Ref.	CDF	Ref.
1	0.01	100	10	4.0949	–	7.9693	–	7.9693	–	11.4573	–
		200	10	4.2183	–	8.0334	–	8.0334	–	11.5020	–
		300	10	4.3383	–	8.0970	–	8.0970	–	11.5465	–
		400	10	4.4550	–	8.1601	–	8.1601	–	11.5908	–
		500	10	4.5688	–	8.2228	–	8.2228	–	11.6350	–
		1000	10	5.0996	–	8.5290	–	8.5290	–	11.8534	–
	0.015	100	10	4.0908	–	7.9742	–	7.9742	–	11.4853	–
		200	10	4.2144	–	8.0382	–	8.0382	–	11.5268	–
		300	10	4.3344	–	8.1018	–	8.1018	–	11.5712	–
		400	10	4.4512	–	8.1648	–	8.1648	–	11.6154	–
		500	10	4.5650	–	8.2274	–	8.2274	–	11.6594	
		1000	10	5.0961	–	8.5333	–	8.5333	–	11.8771	
		1390.2	166.83	8.2787	8.1675[a] 8.1375[d] 8.1397[e]	12.9371	12.823[a] 12.898[d] 12.899[e]	12.9371	12.823[a] 12.898[d] 12.899[e]	16.9739	16.833[a] 16.932[d] 16.934[e]
	0.1	100	10	3.8145	–	7.5987	–	7.5987	–	10.5308	–
		200	10	3.9450	–	7.6640	–	7.6640	–	10.5775	–
		300	10	4.0712	–	7.7288	–	7.7288	–	10.6241	–
		400	10	4.1937	–	7.7931	–	7.7931	–	10.6704	–
		500	10	4.3127	–	7.8568	–	7.8568	–	10.7166	–
		1000	10	4.8643	–	8.1680	–	8.1680	–	10.9444	–
1.5	0.01	100	10	4.5617	–	5.5867	–	5.5867		9.1931	–
		1000	10	4.7575	–	5.7476	–	5.7476		9.2918	–
	0.1	100	10	4.6182	–	5.9899	–	5.9899		9.5858	–
		1000	10	4.8041	–	6.1315	–	6.1315		9.6738	–
	0.15	100	10	4.6527	–	6.4583	–	6.4583		9.7850	–
		1000	10	4.8284	–	6.5774	–	6.5774		9.8651	–
	0.2	100	10	4.6403	–	6.9325	–	6.9325		9.7120	–
		1000	10	4.8060	–	7.0255	–	7.0255		9.7878	–
2	0.01	100	10	2.8545	–	3.1292	–	3.1292		7.1642	–
		1000	10	2.9538	–	3.2201	–	3.2201		7.2044	–
	0.1	100	10	2.9363	–	3.2257	–	3.2257		7.5015	–
		1000	10	3.0312	–	3.3119	–	3.3119		7.5384	–
	0.15	100	10	3.0173	–	3.3274	–	3.3274		7.6152	–
		1000	10	3.1078	–	3.4087	–	3.4087		7.6504	–
	0.2	100	10	3.0956	–	3.4338	–	3.4338		7.5086	–
		1000	10	3.1816	–	3.5098	–	3.5098		7.5436	–

Note: a, d values are taken from Ref.: [4], [16], [7]

Table 6.
$\lambda^{1/2}$ values of thin and moderately thick S-C-S-C Mindlin square and rectangular plates resting on Winkler foundation

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1) CDF Method	Mode (1,2) CDF Method	Mode (2,1) CDF Method	Mode (2,2) CDF Method
1	0.01	100	0	3.1467	7.2128	6.5051	10.2609
		200	0	3.3058	7.2836	6.5835	10.3108
		300	0	3.4576	7.3537	6.6610	10.3604
		400	0	3.6029	7.4232	6.7376	10.4098
		500	0	3.7427	7.4920	6.8133	10.4590
		1000	0	4.3749	7.8269	7.1800	10.7014
	0.015	100	0	3.1479	7.2197	6.4954	10.2483
		200	0	3.3069	7.2903	6.5738	10.2982
		300	0	3.4585	7.3603	6.6514	10.3478
		400	0	3.6038	7.4297	6.7280	10.3972
		500	0	3.7435	7.4984	6.8038	10.4464
		1000	0	4.3755	7.8328	7.1708	10.6889
	0.1	100	0	3.2245	7.5324	5.7184	9.1961
		200	0	3.3773	7.5979	5.8047	9.2494
		300	0	3.5236	7.6629	5.8898	9.3025
		400	0	3.6640	7.7274	5.9736	9.3552
		500	0	3.7993	7.7913	6.0563	9.4076
		1000	0	4.4137	8.1033	6.4538	9.6654
	0.2	100	0	3.3366	7.3525	4.0152	6.8726
		200	0	3.4794	7.4174	4.1354	6.9431
		300	0	3.6166	7.4817	4.2522	7.0129
		400	0	3.7487	7.5455	4.3659	7.0820
		500	0	3.8764	7.6087	4.4766	7.1504
		1000	0	4.4602	7.9173	4.9938	7.4831
1.5	0.01	100	0	2.8684	7.0638	3.2386	7.3416
		1000	0	3.1705	7.1918	3.5090	7.4648
	0.1	100	0	2.9224	7.3034	3.3708	7.7208
		1000	0	3.2140	7.4229	3.6245	7.8329
	0.15	100	0	2.9639	7.2866	3.5009	7.8428
		1000	0	3.2464	7.4035	3.7376	7.9494
	0.2	100	0	2.9863	7.0440	3.6148	7.7038
		1000	0	3.2611	7.1634	3.8346	7.8099
2	0.01	100	0	2.4457	6.5797	2.5235	6.6531
		1000	0	2.5610	6.6235	2.6355	6.6963
	0.1	100	0	2.5201	6.8968	2.6196	7.0225
		1000	0	2.6308	6.9373	2.7261	7.0622
	0.15	100	0	2.5842	6.9410	2.7125	7.1233
		1000	0	2.6908	6.9804	2.8137	7.1615
	0.2	100	0	2.6329	6.7496	2.7992	6.9808
		1000	0	2.7362	6.7897	2.8955	7.0193

Table 7.
$\lambda^{1/2}$ values of thin and moderately thick S-C-S-C Mindlin square and rectangular plates resting on Pasternak foundation

a/b	h/a	$\overline{K}_f$	$\overline{K}_s$	Mode (1,1) CDF Method	Mode (1,2) CDF Method	Mode (2,1) CDF Method	Mode (2,2) CDF Method
1	0.01	100	10	3.4875	7.5882	6.9173	10.6823
		200	10	3.6317	7.6555	6.9911	10.7302
		300	10	3.7704	7.7222	7.0641	10.7779
		400	10	3.9041	7.7884	7.1364	10.8254
		500	10	4.0334	7.8540	7.2079	10.8726
		1000	10	4.6261	8.1741	7.5555	11.1060
	0.015	100	10	3.4890	7.5957	6.9072	10.6699
		200	10	3.6331	7.6629	6.9810	10.7178
		300	10	3.7717	7.7295	7.0541	10.7655
		400	10	3.9053	7.7956	7.1264	10.8130
		500	10	4.0346	7.8611	7.1980	10.8603
		1000	10	4.6269	8.1807	7.5458	11.0937
	0.1	100	10	3.5921	7.9714	6.1016	9.6441
		200	10	3.7299	8.0334	6.1826	9.6949
		300	10	3.8629	8.0949	6.2625	9.7455
		400	10	3.9914	8.1559	6.3414	9.7959
		500	10	4.1159	8.2165	6.4194	9.8459
		1000	10	4.6890	8.5129	6.7957	10.0926
	0.2	100	10	3.7960	8.0142	4.3546	7.4288
		200	10	3.9222	8.0738	4.4657	7.4941
		300	10	4.0443	8.1329	4.5741	7.5588
		400	10	4.1629	8.1916	4.6799	7.6229
		500	10	4.2782	8.2499	4.7834	7.6865
		1000	10	4.8135	8.5353	5.2706	7.9969
1.5	0.01	100	10	3.0633	7.2469	3.4673	7.5430
		1000	10	3.3479	7.3717	3.7211	7.6629
	0.1	100	10	3.1254	7.5097	3.6119	7.9475
		1000	10	3.3997	7.6260	3.8497	8.0565
	0.15	100	10	3.1791	7.5306	3.7590	8.1107
		1000	10	3.4440	7.6438	3.9804	8.2139
	0.2	100	10	3.2207	7.3424	3.8974	8.0321
		1000	10	3.4770	7.4571	4.1021	8.1339
2	0.01	100	10	2.5302	6.6736	2.6180	6.7507
		1000	10	2.6419	6.7167	2.7260	6.7933
	0.1	100	10	2.6109	7.0046	2.7207	7.1344
		1000	10	2.7179	7.0444	2.8233	7.1735
	0.15	100	10	2.6837	7.0707	2.8227	7.2579
		1000	10	2.7866	7.1093	2.9201	7.2954
	0.2	100	10	2.7452	6.9105	2.9234	7.1482
		1000	10	2.8445	6.9497	3.0157	7.1857

5. Conclusions

In the present study, another attempt is made by the author to evaluate vibration characteristics by considering the foundation effect of uniform plates (shear flexible plates with edges immovable), i.e., resting on Winkler and Pasternak foundation. Elegant and accurate closed form expression for λ for all edges simply supported boundary condition is obtained in terms of plate thickness ratio, aspect ratio and Winkler and Pasternak foundation parameters. The frequency parameter values for C-C-C-C and S-C-S-C boundary conditions are also calculated by the above formulation for different various modes and are compared with the results given by other researchers. Higher order plate theories give more accurate results than classical plate theories without using shear correction factor, but more computational efforts are to be required to get even small amount of accuracy. This problem can be overcame by using CDF method, and is applied successfully by the authors to study the vibration response of thin and moderately thick rectangle plates resting on Pasternak foundation.The results obtained using the present (CDF method) formulation are found to be in good agreement with those results obtained by the other researchers for a wide range of plate thickness ratios, Winkler, Pasternak foundation parameters and for different aspect ratios including the thin plates and moderately thick plates.The authors derived two coupling equations for Mindlin rectangular plates for two dimensional shear deformable structural members and the effectiveness of the Coupled Displacement method has been successfully applied and demonstrated for most practically used different edge boundary conditions of the plate such as all edges simply supported, clamped and two opposite edges simply supported and clamped.The numbers of undetermined coefficients are reduced to the half in the proposed Coupled Displacement method when compared to the famous conventional Rayleigh-Ritz method. The proposed Coupled Displacement method also involves less computational efforts when compared to the famous conventional Rayleigh-Ritz method.

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References

[1] A.Karasin. Vibration of rectangular plates on elastic foundations by finite grid solution. *International Journal of Mathematical and Computational Methods*, 1:140–145, 2016.

[2] A.J.M. Ferreira, C.M.C. Roque, A.M.A. Neves, R.M.N. Jorge, and C.M.M. Soares. Analysis of plates on Pasternak foundations by radial basis functions. *Journal of Computational Mechanics*, 46(6):791–803, 2010. doi: 10.1007/s00466-010-0518-9.

[3] H. Matsunaga. Vibration and stability of thick plates on elastic foundations. *Journal of Engineering Mechanics*, 126(1):27–34, 2000. doi: 10.1061/(ASCE)0733-9399(2000)126:1(27).

[4] D. Zhou, Y.K. Cheung, S.H. Lo, and F.T.K. Au. Three dimensional vibration analysis of rectangular thick plates on Pasternak foundation. *International Journal for Numerical Methods in Engineering*, 59(10):1313–1334, 2004. doi: 10.1002/nme.915.

[5] E. Bahmyari, M.M. Banatehrani, M. Ahmadi, and M. Bahmyari. The Vibration analysis of thin plates resting on Pasternak foundations by element free Galerkin method. *Shock and Vibration*, 20(2):309–326, 2013. doi: 10.3233/SAV-2012-00746.

[6] G.V. Rao. M.K. Saheb, and R.G. Janardhan. Large-amplitude free vibrations of uniform Timoshenko beams: A novel formulation. *AIAA Journal*, 45(11):2810–2812, 2007. doi: 10.2514/1.27718.

[7] H. Liu, F. Liu, X. Jing, Z. Wang, and L. Xia. Three Dimensional vibration analysis of rectangular thick plates on Pasternak foundation with arbitrary boundary conditions. *Shock and Vibration*, 2017:1–10,2017. doi: 10.1155/2017/3425298.

[8] J. Rouzegar and R.A. Sharifpoor. Flexure of thick plates resting on elastic foundation using two-variable refined plate theory. *Archive of Mechanical Engineering*, 62(2):181–203, 2015. doi: 10.1515/meceng-2015-0011.

[9] K.K. Bhaskar and K.M. Saheb. Large amplitude free vibrations of Timoshenko beams at higher modes using coupled displacement method. *Journal of Physics: Conference Series*, 662(1), 2015. Article number 012019. doi: 10.1088/1742-6596/662/1/012019.

[10] K. Rajesh and K.M. Saheb. Linear free vibration analysis of rectangular Mindlin plates using coupled displacement method. *Mathematical Models in Engineering*, 2(1):41–47, 2016.

[11] K. Rajesh and K.M. Saheb. Free vibrations of uniform Timoshenko beams on Pasternak foundation using coupled displacement method. *Archive of Mechanical Engineering*, 64(3):359–373, 2017. doi: 10.1515/meceng-2017-0022.

[12] K. Özgan and A.T. Daloglu. Free vibration analysis of thick plates resting on winkler elastic foundation. *Challenge Journal of Structural Mechanics*, 1(2):78–83, 2015. doi: 10.20528/cjsmec.2015.06.015.

[13] K. Özgan and A.T. Daloglu. Free vibration analysis of thick plates on elastic foundations using modified Vlasov model with higher order finite elements. *Indian Journal of Engineering & Materials Sciences*, 19:279–291, 2012.

[14] K. Özgan and A.T. Daloglu. Application of modified Vlasov model to free vibration analysis of thick plates resting on elastic foundations. *Shock and Vibration*, 16(5):439–454, 2009. doi: 10.3233/SAV-2009-0479.

[15] M. Dehghany and A. Farajpour. Free vibration of simply supported rectangular plates on Pasternak foundation: An exact and three-dimensional solution. *Engineering Solid Mechanics*, 2(1):29–42, 2014.

[16] M.H. Omurtag, A. Özütok, A.Y. Aköz, and Y. Özçelikörs. Free vibration analysis of Kirchhoff plates resting on elastic foundation by mixed finite element formulation based on Gateaux differential. *International Journal for Numerical Methods in Engineering*, 40(2): 295–317, 1997. doi: 10.1002/(SICI)1097-0207(19970130)40:2<295::AID-NME66>3.0.CO;2-2.

[17] R.D. Mindlin. Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates. *Journal of Applied Mechanics*, 18:1031–1036, 1951.

[18] M. Zaman, M.O. Faruque, A. Agrawal, and J. Laguros. Free vibration analysis of rectangular plates resting on two-parameter elastic medium. *Transactions on the Built Environment*, 20:583–592, 1996.

[19] P.H. Wen. The fundamental solution of Mindlin plates resting on an elastic foundation in the Laplace domain and its applications. *International Journal of Solids and Structures*, 45(3–4):1032–1050, 2008. doi: 10.1016/j.ijsolstr.2007.09.020.

[20] R. Buczkowski, M. Taczała, and M. Kleiber. A 16-node locking-free Mindlin plate resting on two parameter elastic foundation – static and eigenvalue analysis. *Computers Assisted Methods in Engineering and Science*, 22(2):99–114, 2015.

[21] K.M. Saheb and K. Krishnabhaskar. Large amplitude free vibration of simply supported and clamped moderately thick square plates using coupled displacement method. *Journal of Vibration Engineering and Technologies*, 4(2):153–159, 2016.

[22] K.M. Saheb, G.V. Rao, and G.R. Janardhana. Evaluation of large amplitude free vibration behavior of uniform Timoshenko beams using coupled displacement method. *Advances in Vibration Engineering*, 9(1):15–23, 2010.

[23] Y.H. Wang, L.G. Tham, Y.K. Cheung. Beams and plates on elastic foundation: a review. *Progress in Structural Engineering and Materials*, 7(4):174–182, 2005. doi: 10.1002/pse.202.

[24] Y. Xiang, S. Kitipornchai, and K.M. Liew. Buckling and vibration of thick laminates on Pasternak foundations. *Journal of Engineering Mechanics*, 122(1):54–63, 1996. doi: 10.1061/(ASCE)0733-9399(1996)122:1(54).

[25] Y. Xiang, C.M. Wang, and S. Kitipornchai. Exact vibration solution for initially stressed Mindlin plates on Pasternak foundations. *International Journal of Mechanical Sciences*, 36(4):311–316,1994. doi: 10.1016/0020-7403(94)90037-X.

[26] Y. Zhong and J.H. Yin. Free vibration analysis of plate on foundation with completely free boundary by finite integral transformation method. *Mechanics Research Communications*, 35(4):268–275, 2008. doi: 10.1016/j.mechrescom.2008.01.004.