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A METHODOLOGY OF DESIGNING THE TEETH CONJUGATION IN A PLANETARY ROLLER SCREW

The paper presents the methodology for designing the teeth conjunction of planetary gears in the planetary roller screw mechanism. A function of the planetary gears is to synchronize an operation of rollers in order to avoid axial displacements. A condition of the correct operation is no axial movement of rollers in relation to the nut. The planetary gears are integral parts of rollers and therefore an operation of the gear transmissions has a direct impact on cooperation of the screw, rollers and the nut. The proper design of gear engagements is essential for reducing slippage on surfaces of the cooperating threaded elements. For this purpose, in a designing method, both the limitations of operation and kinematic conditions of rollers' operation have to be taken into account.

1. Introduction

Planetary roller screw (PRS) is a high-efficiency mechanism, which allows for conversion of rotary motion into linear motion or in the other way round. It is a relatively new type of lead screw for demanding applications. The structure of this mechanism is shown in Fig. 1. The rollers (2) cooperate with the screw (1) and the nut (3). The planetary gear transmissions consist of planetary gears (7) and ring gears (5). The even distribution of rollers is provided by end plates (4) locked by the retaining ring (6). In comparison to the more popular ball screw, in general, PRS has better operating parameters. These are mainly: higher load carrying capacity, higher speed and acceleration, more accurate positioning, higher impact resistance and stiffness.

In the recent years, several authors of publications considered problems related to PRS design. In the paper [1] studied the capabilities and limitations of the mechanism. The authors analysed the axial displacement of the roller cooperating with

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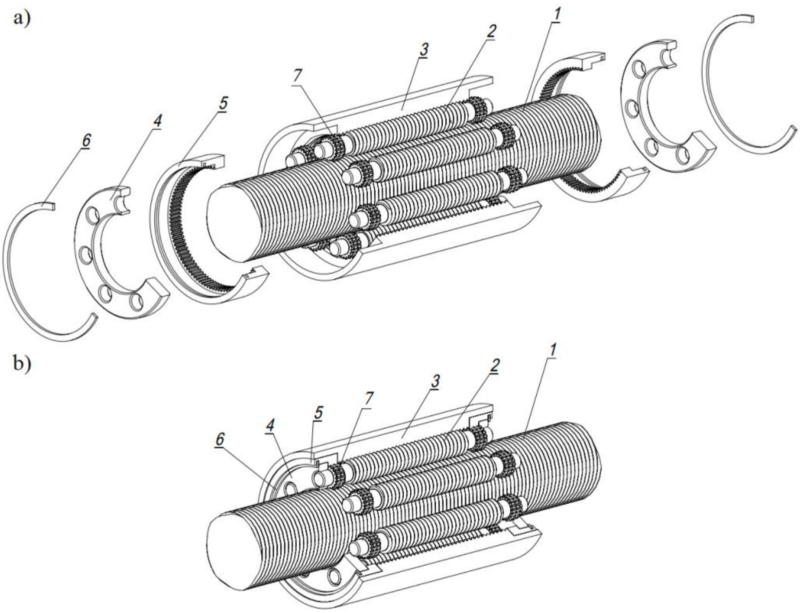

Fig. 1. Planetary roller screw; a) The main parts of roller screw, b) assembly view 1 – screw, 2 – roller, 3 – nut, 4 – end plate, 5 – ring gear, 6 – retaining ring, 7 – planetary gear

the screw according to the threads' parameters and also described the conditions of slip phenomenon. In the article [2], among other things, geometric parameters and kinematic relationships were analysed. Several authors studied also the load distribution between cooperating elements. The authors of paper [3] developed an analytical model for determining the load distribution intended for a preliminary design. In the article [4] an analysis of displacements and the load distribution based on the bar model including one roller was proposed. In the paper [5] an original approach to computation of the load distribution and axial stiffness in the planetary roller screw mechanism under the static load within the elastic range was presented. The authors have developed a hybrid model using one-dimensional finite elements and nonlinear spring elements. The authors of paper [6] presented a kinematic model to determine the axial displacement of the roller in relation to the nut, which results from manufacturing deviations. It was explained that this displacement is caused by a slippage between the surfaces of the rollers and the nut threads. This slippage is in turn caused by the pitch incompatibility of planetary and ring gears. An axial displacement of the roller leads to bending of the teeth, which may cause a damage. In paper [7] the nature of the contact between the screw and the roller and between the roller and the nut was examined. The authors presented a model to determine the accurate radii of cooperating components due to the limitation of the axial displacement of rollers.

Furthermore, the authors of publication [8] studied dynamics and efficiency of the planetary roller screw. In turn, the author of study [9] analysed, inter alia, dynamics of a typical drive system with the planetary roller screw under the impulse load. In the paper [10] a procedure for the preliminary design of the planetary roller screw was proposed.

The goal of this paper is to present the methodology for designing the teeth conjunction of planetary gears in the planetary roller screw mechanism. To consider the planetary gears' designing problem it is essential to pay attention to the principles of operation and limitations, from which arise the relations between the pitch diameters of the screw, roller and the nut. On the contrary, a kinematic analysis brings the dependencies among the pitch diameters of rollers and the nut as well as the pitch diameters of the planetary and the ring gears.

2. Principles of operation and limitations of PRS

The principle of the operation is no axial displacements of rollers in relation to the nut. This condition is fulfilled by assuming multi-start threads of the screw and the nut, single-start threads of rollers and the same helix angles of rollers and nut's threads [1]. In that case, the number of thread starts of the nut is obtained from Eq. 1.

$$n_n = \frac{d_s + 2d_r}{d_r} = \frac{d_n}{d_r}, \tag{1}$$

where: d_s, d_r, d_n – pitch diameters of the screw, rollers and the nut. Based on the assumption that the rollers cooperate with the screw without slippage, the starts of the screw and the nut's threads are equal $n_s = n_n$ [2]. An orbit, which includes the axes of the rollers, is concentric with respect to the screw and to the nut. The distances between axes of the rollers and the screw as well as between axes of rollers and the nut must be equal (Eq. 2)

$$a_{rn} = (d_n - d_r)/2, \quad a_{rs} = (d_s + d_r)/2, \quad a_{rn} = a_{rs}, \tag{2}$$

where: a_{rn}, a_{rs} – the distances between axes of the roller and the nut as well as the roller and the screw. Accepting the above conditions results in the limitation on the thread starts of the screw and the nut given by Eq. 3.

$$(d_s/d_r) = n_s - 2 = n_n - 2 \tag{3}$$

Accordingly to the above, the smallest thread start of the screw and the nut that can be used is $n_s = n_n = n = 3$. The relations between pitch diameters are defined as given by Eq. 4.

$$d_n = n_n d_r = n d_r, \quad d_s = (n_n - 2)\, d_r = (n - 2)\, d_r. \tag{4}$$

3. Kinematics of rollers movement

During the mechanism operation, rollers are in rolling contact with the screw and the nut for the purpose of transferring the load within threads cooperation. At the same time, planetary toothed wheels located at the ends of the rollers cooperate with the ring gears in the nut. Movement of the rollers is complex. A single roller performs rotary motion relative to its own axis as well as circular motion and linear motion relative to an axis of the screw simultaneously. The pure rolling occurs if the radii of pitch diameters of rollers and the nut are respectively equal to the radii of generating circles of the planetary toothed wheels and ring gears, as indicated by Eq. 5. The principle of slippage in PRS was developed in [6].

$$r_{w1} = r_r, \quad r_{w2} = r_n. \tag{5}$$

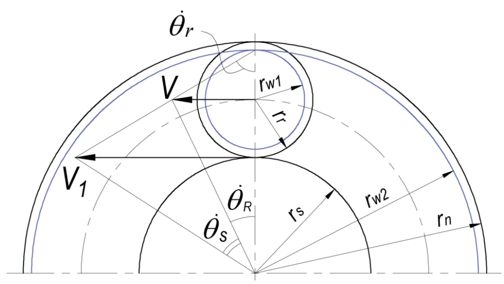

Fig. 2. Angular and linear velocities of PRS components

According to Fig. 2, the angular velocities of the roller in the rotary motion and circular motion as well as the angular velocity of the screw are respectively given by Eq. 6. The relations between angular velocities of the rollers and the screw depend on Eq. 7.

$$\dot{\theta}_r = \frac{V}{r_{w2}}, \quad \dot{\theta}_R = \frac{V}{r_{w2} - r_{w1}}, \quad \dot{\theta}_s = \frac{V_1}{r_s}, \tag{6}$$

$$\dot{\theta}_r = \frac{r_{w2} - r_{w1}}{r_{w1}}\dot{\theta}_R, \quad \dot{\theta}_R = \frac{r_s r_{w1}}{r_r r_{w2} + r_s r_{w1}}\dot{\theta}_s, \tag{7}$$

where: $\dot{\theta}_s$, $\dot{\theta}_r$ – angular velocities of the screw and roller in rotational motion; $\dot{\theta}_R$ – angular velocity of roller in circular motion; $\dot{\theta}_d$ – angular velocity of driven shaft; V_1 – linear velocity of the contact point on the thread of the screw; V – linear velocity of the roller axis; r_s, r_r, r_n – radii of pitch circles of the screw, rollers and the nut; r_{w1}, r_{w2} – radii of generating (rolling) circles of the planetary gear and the ring gear.

When the radii of the pitch circles of rollers and the nut are respectively equal to the radii of the generating circles of planetary toothed gear and ring gear, Eq. 7 takes the form:

$$\dot{\theta}_r = \frac{r_n - r_r}{r_r}\dot{\theta}_R, \quad \dot{\theta}_R = \frac{r_s}{r_s + r_n}\dot{\theta}_S. \tag{8}$$

The linear velocity of the roller relative to the screw is the sum of the linear velocity of the contact point on the surface of the screw thread, linear velocity associated with the rotary motion and the circulating motion of the roller and linear velocity of the contact point on the surface of the roller thread [6], as given by Eq. 9.

$$V_{RS} = \frac{L_s}{2\pi}\dot{\theta}_s - \frac{L_s - L_r}{2\pi}\dot{\theta}_R + \frac{L_r}{2\pi}\dot{\theta}_r \tag{9}$$

where: V_{RS} – linear velocity of the roller relative to screw; L_s, L_r – number of thread starts of the screw and roller. Taking into account Eq. 6-8 and assuming the condition given by Eq. 5, the linear velocity of the roller relative to the screw takes the form as written by Eq. 10.

$$\begin{aligned} V_{RS} &= \frac{L_s}{2\pi}\dot{\theta}_s - \frac{L_s - L_r}{2\pi}\frac{r_s}{r_s + r_n}\dot{\theta}_S + \frac{L_r}{2\pi}\left(\frac{r_n}{r_r} - 1\right)\frac{r_s}{r_s + r_n}\dot{\theta}_s = \\ &= \frac{L_s}{2\pi}\frac{r_n}{r_s + r_n}\left[1 + \frac{L_r}{L_s}\frac{r_s}{r_r}\right]\dot{\theta}_s \end{aligned} \tag{10}$$

For a small thread pitch ($p \leq 1.5$ mm), it can be technically complicated to meet the condition (Eq. 5). In such a situation, the rollers' movement can be synchronized in an approximate manner, by allowing small slides on the threads. In that case, the contact zone of rollers moves towards the top of the nut thread. This is owing to positive gears' correction [11]. For that case, the linear velocity of the roller relative to the screw takes the form as given by Eq. 11.

$$\begin{aligned} V_{RS} &= \frac{L_s}{2\pi}\dot{\theta}_s - \frac{L_s - L_r}{2\pi}\frac{r_s r_{w1}}{r_r r_{w2} + r_s r_{w1}}\dot{\theta}_S + \frac{L_r}{2\pi}\left(\frac{r_{w2}}{r_{w1}} - 1\right)\frac{r_s r_{w1}}{r_s r_{w1} + r_r r_{w2}}\dot{\theta}_s = \\ &= \frac{L_s}{2\pi}\frac{r_r r_{w2}}{r_r r_{w2} + r_s r_{w1}}\left[1 + \frac{L_r}{L_s}\frac{r_s}{r_r}\right]\dot{\theta}_s \end{aligned} \tag{11}$$

4. Planetary gears

A function of the planetary gears is to synchronize the rollers' operations in order to prevent the axial displacements of rollers in the case of slippage on threads. This slippage can be caused by manufacturing errors and elastic deformations. For proper synchronization, planetary gears have to meet the following assumptions:

- The radii of the pitch circles of rollers and the nut have to be respectively equal to the radii of the generating (rolling) circles of planetary toothed gear and ring gear (Eq. 5.). External radii of rollers must equal to the radius of addendum circle of planetary gear as given by Eq. 12.

- Due to the condition of symmetrical position of the planetary gear in relation to the ring gear (which means that in a place of the planetary gear's tooth is a notch at the ring gear), the ratio of the ring gear teeth's number to the number of rollers has to be a natural number $(Z_2/N_0) \in \mathrm{N}$.
- According to the condition of gear ratio, the number of teeth of planetary gear is $Z_1 = Z_2\,(d_r/d_n)$, where: Z_1, Z_2 – teeth number of planetary and ring gears; N_0 – number of rollers; d_r, d_n – diameters of pitch circles of the roller and the nut.
- Since the pressure angle of threads $\alpha_0 = 45^\circ$ is accepted, the radius of addendum circle of planetary gear is given by Eq. 12.

$$r_{a1} = r_r + \frac{p}{4}, \tag{12}$$

where: p – thread pitch, r_{a1} – radius of addendum circle of planetary gear.

The teeth number of planetary gear is also limited by manufacturing conditions. The lower limit of teeth number for which there is no undercutting of teeth is $Z_g = h_a^*\left(2/\sin^2\alpha\right)$, where: h_a^* – tooth's head height factor, α – pressure angle of manufacturing tool. If a small undercutting of teeth is allowed, which does not affect working conditions, the practical number of teeth of planetary gear is $Z_g' = (5/6)\,Z_g$ [12]. Accepting $\alpha = 20^\circ$ and $h_a^* = 1$, one obtains the smallest number of teeth $Z_g = 17$ and practical number of teeth $Z_g' = 14$.

Taking into account the above conditions, the solution is limited by acceptable modification coefficients of gears, which results in limitations of the module. The recommended range of modification coefficients for the planetary and the ring gears can be assumed according to [10]. Based on the limit values of the modification coefficients, the allowable range of the module can be determined. The pressure angle of teeth cooperation α_w can be obtained from Fölmer's equation:

$$inv\,(\alpha_w) = inv\,(\alpha) + \frac{2\,(x_1 + x_2)}{Z_2 - Z_1}\tan(\alpha)\,. \tag{13}$$

where: x_1, x_2 – modification coefficients.
For this purpose, a transcendental equation (Eq. 14) has to be solved, where C– the number given by Eq. 15. The pressure angle (Eq. 17) is obtained as a root of the transcendental equation (Eq. 16).

$$\tan(\alpha_w) - \alpha_w = C, \tag{14}$$

$$C = \tan(\alpha) - \alpha + \frac{2\,(x_1 + x_2)}{Z_2 - Z_1}\tan(\alpha)\,, \tag{15}$$

$$f(x) = \tan(x) - x - C, \tag{16}$$

$$\alpha_w = \mathrm{root}\,(f(x), x)\,. \tag{17}$$

Theoretical zero distance of gears' axes a_0 as well as module m are given by equations:

$$a_0 = \frac{a_{rn} \cos(\alpha_w)}{\cos(\alpha)}, \quad m = \frac{2a_0}{Z_2 - Z_1}. \tag{18}$$

For the specified range of permissible module, a standardized value m_n [12] should be accepted. Then, the theoretical zero distance of gears' axes and pressure angle for the actual distance of gears' axes can be calculated.

$$a_0 = \frac{1}{2}(Z_2 - Z_1)\, m_n, \quad \alpha_w = \arccos\left(\frac{a_0}{a}\cos(\alpha)\right), \quad a = a_{rn} = r_n - r_r, \tag{19}$$

where: a_0 – theoretical zero distance of gears' axes, a – actual distance of gears' axes.

For the accepted standardized module and number of teeth, modification coefficients (x_1, x_2), radii of pitch circles (r_1, r_2) and reduction of teeth height due to the top clearance(Δ_h) can be calculated using formulas given by Eq. 20-22.

$$r_{a1} = r_r + \frac{p}{4}, \quad r_1 = \frac{1}{2} Z_1 m_n, \quad h_{a1} = r_{a1} - r_1, \quad x_1 = -1 + \frac{h_{a1}}{m_n}, \tag{20}$$

$$r_2 = \frac{1}{2} Z_2 m_n, \quad C_2 = (\tan(\alpha_w) - \alpha_w - \tan(\alpha) - \alpha)\, \frac{(Z_2 - Z_1)}{2\tan(\alpha)}, \tag{21}$$

$$\Delta_h = C_2 m_n - (a - a_0), \quad x_2 = C_2 + x_1. \tag{22}$$

After calculating the module on the rolling diameter m_w (Eq. 23), one should check whether the radii of rolling circles of planetary and ring gears (r_{w1}, r_{w2}) are respectively equal to the radii of pitch circles of the roller and nut (r_r, r_n).

$$m_w = \frac{m_n \cos(\alpha)}{\cos(\alpha_w)}, \quad r_{w1} = \frac{1}{2} Z_1 m_w, \quad r_{w2} = \frac{1}{2} Z_2 m_w. \tag{23}$$

Using formulas given by Eq. 24, the following dimensions of gears can by calculated: r_{b1}, r_{b2} – radii of base circles; r_{a1}, r_{a2} – radii of addendum circles, r_{f1}, r_{f2} – radii of dedendum circles, h_z – tooth height; p_w – pitch on generating (rolling) circle, p_p – pitch on pitch circles, p_b – pitch on base circles.

$$\begin{gathered} r_{b1} = r_1 \cos(\alpha), \quad r_{b2} = r_2 \cos(\alpha), \\ r_{a1} = r_1 + (1 + x_1)\, m_n, \quad r_{a2} = r_2 - (1 - x_2)\, m_n, \\ h_z = 2m_n + 0.3m_n - \Delta_h, \\ r_{f1} = r_{a1} - h_z, \quad r_{f2} = r_{a2} + h_z, \\ p_w = \pi m_w, p_p = \pi m, \quad p_b = \pi m \cos(\alpha). \end{gathered} \tag{24}$$

The modification shift in cutting process of teeth, that is the distance between the pitch line of cutting tool and the gear, can be calculated for planetary and ring gears using Eq. 25.

$$X_1 = x_1 m_n, \quad X_2 = x_2 m_n. \tag{25}$$

The condition of meshing continuity is the position of gears for which two pairs of teeth work simultaneously. This will take place when the tooth contact ratio $\varepsilon > 1$. The above condition can be checked using Eq. 26-28.

$$\alpha_{a1} = \arccos\left(\frac{r_{b1}}{r_{a1}}\right), \quad e_1 = r_{b1}\left(\tan\left(\alpha_{a1}\right) - \tan\left(\alpha_w\right)\right), \tag{26}$$

$$\alpha_{a2} = \arccos\left(\frac{r_{b2}}{r_{a2}}\right), \quad e_2 = r_{b2}\left(\tan\left(\alpha_w\right) - \tan\left(\alpha_{a2}\right)\right), \tag{27}$$

$$e = e_1 + e_2, \quad \varepsilon = \frac{e}{p_b}, \tag{28}$$

where: ε – tooth contact ratio; α_{a1}, α_{a2} – angles defining the position of the tooth tips; e – path of contact; e_1, e_2 – parts of the path of contact measured from rolling point.

In the case when the radii of generating circles of planetary and ring gears are not equal to the radii of pitch circles of the roller and nut ($r_{w1} \neq r_r$ and $r_{w2} \neq r_n$), the slippage may occur on the surfaces of threads. In that situation, due to small diameter of roller, it is preferable to accept $Z_1 = 10 \div 14$ and apply correction P0 [13]. For the lower limit of teeth number $Z_g = 17$, the modification coefficient of planetary gear can be determined using Eq. 29.

$$x_1 \geq 1 - \frac{Z_1}{17} \quad \text{and} \quad x_1 \leq \frac{7}{17} + \frac{10\left(Z_1 - 10\right)}{289} \tag{29}$$

5. Example of the planetary gear calculations

Table 1.

Parameters of PRS

Pitch diameter of the screw	$r_s = 15$ mm
Pitch diameter of the roller	$r_r = 5$ mm
Pitch diameter of the nut	$r_n = 25$ mm
Lead	$p = 3$ mm
Pressure angle	$\alpha_0 = 45^\circ$

The outside radius of the roller must to be equal to the radius of the planetary addendum circle. The radius of the planetary addendum circle for the accepted pressure angle $\alpha_0 = 45^\circ$:

$$r_{a1} = r_r + \frac{p}{4}, \quad r_{a1} = 5.75 \text{ mm}. \tag{30}$$

The upper limit number of rollers:

$$N_0 < \pi / \arcsin\left(\frac{D_r}{d_r + d_s}\right) = \pi / \arcsin\left(\frac{2r_r + 0.5p}{2\left(r_s + r_r\right)}\right), \qquad N_0 < 10.773; \tag{31}$$

The accepted number of rollers: $N_0 = 10$.

5.1. The number of gears' teeth

The number of gears' teeth due to the condition of symmetry position of the planetary gear in relation to the ring gear:

$$Z_2 = N_0 N; \qquad N = 3, 4, 5, 6, 7\ldots \tag{32}$$

$$Z_1 = Z_2 \frac{r_r}{r_n}; \qquad Z_1 = \frac{Z_2}{5}. \tag{33}$$

The fulfilment of the above condition gives the following statement for the number of teeth:

$$Z_1 = 10,\ Z_2 = 50;\ \ Z_1 = 12,\ Z_2 = 60;\ \ Z_1 = 14,\ Z_2 = 70;\ \ Z_1 = 16,\ Z_2 = 80.$$

Due to the radii of the generating circles of planetary and ring gears, only the last two cases have a solution limited by modification coefficients. The accepted number of gear teeth and the radii of generating circles, e.g.:

$$Z_1 = 14, \qquad Z_2 = 70, \qquad r_{w1} = r_r = 5 \text{ mm}, \qquad r_{w2} = r_n = 25 \text{ mm}. \tag{34}$$

For the assumed geometry of gears, the recommended modification coefficients were accepted according to [11]. The lower limit of modification coefficients is relevant to prevent undercutting the bottom of tooth, whereas the upper limit is due to the acceptable tightening of the tooth top.

$$x_1 \in \langle 0.1 \sim 0.8 \rangle \qquad x_2 \in \langle -0.1 \sim 1.0 \rangle \tag{35}$$

5.2. Calculations of the module limitation for the upper value of modifications coefficients

$$Z_2 = 70, \qquad Z_1 = 14, \qquad x_1 + x_2 = 1.8, \qquad \alpha = 20^\circ. \tag{36}$$

$$C = \tan(\alpha) - \alpha + \frac{2\left(x_1 + x_2\right)}{Z_2 - Z_1} \tan(\alpha), \qquad C = 0.038. \tag{37}$$

$$\alpha_w = \text{root}\left(f(x), x\right), \qquad \alpha_w = 27.145^\circ. \tag{38}$$

$$a = r_r - r_n, \qquad a = 20 \text{ mm}, \tag{39}$$

$$a_0 = \frac{a \cdot \cos(\alpha_w)}{\cos(\alpha)}, \qquad a_0 = 18.939 \text{ mm}, \tag{40}$$

$$m = \frac{2a_0}{Z_2 - Z_1}, \qquad m = 0.676 \text{ mm}. \tag{41}$$

5.3. Calculations of the module limitation for the lower value of modifications coefficients

$$Z_2 = 70, \quad Z_1 = 14, \quad x_1 + x_2 = -1.0, \quad \alpha = 20^\circ. \tag{42}$$

$$f(x) = \tan(x) - x - C. \tag{43}$$

$$C = \tan(\alpha) - \alpha + \frac{2(x_1 + x_2)}{Z_2 - Z_1}\tan(\alpha), \quad C = 0.0019. \tag{44}$$

$$\alpha_w = \mathrm{root}(f(x), x), \quad \alpha_w = 8.33^\circ. \tag{45}$$

The distance between axes of the roller and the nut:

$$a = r_r - r_n, \quad a = 20 \text{ mm}. \tag{46}$$

Theoretical zero distance between axes of gears:

$$a_0 = \frac{a \cos(\alpha_w)}{\cos(\alpha)}, \quad a_0 = 21.059 \text{ mm}. \tag{47}$$

$$m = \frac{2a_0}{Z_2 - Z_1}, \quad m = 0.752 \text{ mm}. \tag{48}$$

The obtained range of permissible module:

$$m \in \langle 0.675 \sim 0.752 \rangle \text{ mm}. \tag{49}$$

The accepted standard module: $m_n = 0.7$ mm.
The pressure angle:

$$\alpha_w = \arccos\left(\frac{a_0}{a}\cos(\alpha)\right), \quad \alpha_w = 22.942^\circ. \tag{50}$$

$$a_0 = \frac{a \cos(\alpha_w)}{\cos(\alpha)}, \quad a_0 = 19.6 \text{ mm}. \tag{51}$$

The module check:

$$m_n = \frac{2a_0}{Z_2 - Z_1}, \quad m_n = 0.7 \text{ mm}. \tag{52}$$

5.4. Calculation of modification coefficients

$$r_{a1} = r_r + \frac{p}{4}, \quad r_{a1} = 5.75 \text{ mm}. \tag{53}$$

$$r_1 = \frac{1}{2} Z_1 m_n, \quad r_1 = 4.9 \text{ mm}. \tag{54}$$

$$h_{a1} = r_{a1} + r_1, \quad h_{a1} = 0.85 \text{ mm}. \tag{55}$$

$$x_1 = -1 + \frac{h_{a1}}{m_n}, \quad x_1 = 0.214. \tag{56}$$

$$r_2 = \frac{1}{2} Z_2 m_n, \quad r_2 = 24.5 \text{ mm}. \tag{57}$$

$$C_2 = (\tan(\alpha_w) - \alpha_w - \tan(\alpha) - \alpha) \frac{(Z_2 - Z_1)}{2\tan(\alpha)}, \quad C_2 = 0.613. \tag{58}$$

$$\Delta_h = C_2 m_n - (a - a_0), \quad \Delta_h = 0.029 \text{ mm}. \tag{59}$$

$$x_2 = C_2 + x_1, \quad x_2 = 0.827. \tag{60}$$

5.5. Checking the radii of generating circles

$$m_w = \frac{m_n \cos(\alpha_w)}{\cos(\alpha)}, \quad m_w = 0.714 \text{ mm}. \tag{61}$$

$$r_{w1} = \frac{1}{2} Z_1 m_w, \quad r_{w1} = 5 \text{ mm}. \tag{62}$$

$$r_{w2} = \frac{1}{2} Z_2 m_w, \quad r_{w2} = 25 \text{ mm}. \tag{63}$$

The radii of generating circles are consistent with the assumptions.

5.6. Dimensions of gears

Radii of base circle:

$$r_{b1} = r_1 \cos(\alpha), \quad r_{b1} = 4.604 \text{ mm}. \tag{64}$$

$$r_{b2} = r_2 \cos(\alpha), \quad r_{b2} = 23.022 \text{ mm}. \tag{65}$$

Radii of addendum circles:

$$r_{a1} = r_1 + (1 + x_1) m_n, \quad r_{a1} = 5.75 \text{ mm}. \tag{66}$$

$$r_{a2} = r_2 - (1 - x_2) m_n, \quad r_{a2} = 24.379 \text{ mm}. \tag{67}$$

Tooth height:

$$h_z = 2m_n + 0,3m_n - \Delta_h, \quad h_z = 1.58 \text{ mm}. \tag{68}$$

The radii of dedendum circles:

$$r_{f1} = r_{a1} - h_z, \quad r_{f1} = 4.17 \text{ mm}. \tag{69}$$

$$r_{f2} = r_{a2} + h_z, \quad r_{f2} = 25.959 \text{ mm}. \tag{70}$$

The pitch on generating circle :

$$p_t = \pi m_w, \quad p_t = 2.244 \text{ mm}. \tag{71}$$

The pitch on the pitch circle:

$$p_p = \pi m_n, \quad p_p = 2.199 \text{ mm}. \tag{72}$$

The pitch on the base circle

$$p_b = \pi m_n \cos(\alpha), \quad p_b = 2.066 \text{ mm}. \tag{73}$$

The modification shifts in manufacturing process (the distance between pitch lines of tool and gear):

$$X_1 = x_1 m_n, \quad X_1 = 0.15 \text{ mm}. \tag{74}$$

$$X_2 = x_2 m_n, \quad X_2 = 0.579 \text{ mm}. \tag{75}$$

5.7. Checking the condition of meshing continuity

$$\alpha_{a1} = \arccos\left(\frac{r_{b1}}{r_{a1}}\right), \quad \alpha_{a1} = 0.642 \text{ rad}. \tag{76}$$

$$e_1 = r_{b1}(\tan(\alpha_{a1}) - \tan(\alpha_w)), \quad e_1 = 1.495 \text{ mm}. \tag{77}$$

$$\alpha_{a2} = \arccos\left(\frac{r_{b2}}{r_{a2}}\right), \quad \alpha_{a2} = 0.335 \text{ rad}. \tag{78}$$

$$e_2 = r_{b2}(\tan(\alpha_w) - \tan(\alpha_{a2})), \quad e_2 = 1.723 \text{ mm}. \tag{79}$$

$$e = e_1 + e_2, \quad e = 3.218 \text{ mm}. \tag{80}$$

$$\varepsilon = \frac{e}{p_b}, \quad \varepsilon = 1.577. \tag{81}$$

Since $\varepsilon > 1$, the condition of meshing continuity is met.

On the basis of the standard rack, various cutting tools can be designed. In this case, these can be i.e. hobbing cutter, Fellow's cutter or gear-forming cutter. The process of generating the shape of teeth is presented in Fig. 3 and Fig. 4. The geometry of planetary and ring gears, as well as the geometry of teeth conjunction, generated on the basis of the above calculations, is shown in Fig. 5.

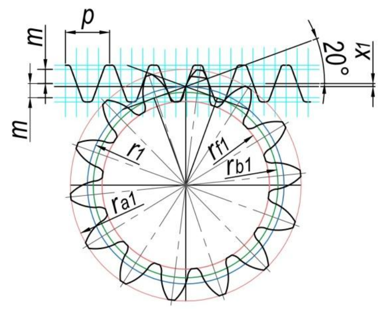

Fig. 3. Generating the shape of planetary gear's teeth on the basis of the standard rack

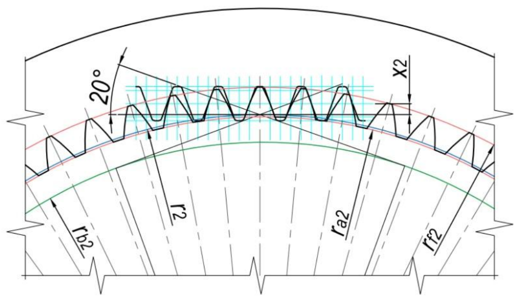

Fig. 4. Generating the shape of ring gear's teeth on the basis of the standard rack

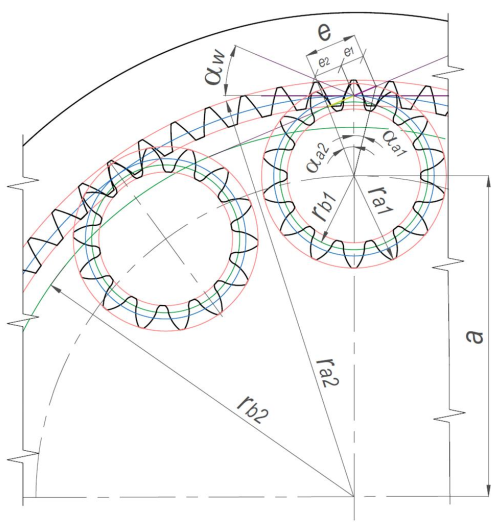

Fig. 5. The geometry of teeth conjunction of planetary gear in the planetary roller screw

6. Summary

The methodology for designing the geometry of teeth conjunction of planetary gears in the planetary roller screw mechanism was presented. The calculations include the condition of pure rolling of the nut and rollers that affect the values of radii of the generating circles of gears. Based on the condition of gear ratio as well as on the condition of symmetrical position of the planetary gear in relation to the ring gear, the possible combinations of gears' teeth numbers are obtained. The solution is limited by the permissible range of modification coefficients. The above considerations include the computational example. The presented methodology can be useful in the designing procedure of planetary roller screw mechanism.

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Metodologia projektowania zazębienia w planetarnej przekładni śrubowej rolkowej

S t r e s z c z e n i e

W artykule przedstawiono metodologię projektowania zazębienia planetarnych przekładni zębatych do zastosowania w planetarnej przekładni śrubowej rolkowej. Funkcją przekładni zębatych jest synchronizacja pracy rolek w celu uniknięcia ich przemieszczenia osiowego. Warunkiem poprawnej współpracy mechanizmu jest brak przemieszczenia osiowego rolek względem nakrętki. Koła satelitarne są integralnymi częściami rolek, a w związku z tym praca planetarnych przekładni zębatych ma bezpośredni wpływ na współpracę gwintów śruby, rolek i nakrętki. Prawidłowe projektowanie planetarnych przekładni zębatych jest szczególnie istotne ze względu na uniknięcie poślizgów na powierzchniach gwintów. W związku z powyższym, w przedstawionej metodzie projektowania planetarnych przekładni zębatych, zostały uwzględnione ograniczenia parametrów geometrycznych elementów przekładni śrubowej rolkowej oraz warunki kinematyczne.