

MAŁGORZATA SKORUPA \*, TOMASZ MACHNIEWICZ \*\*

## APPLICATION OF THE STRIP YIELD MODEL TO CRACK GROWTH PREDICTIONS FOR STRUCTURAL STEEL

A strip yield model implementation by the present authors is applied to predict fatigue crack growth observed in structural steel specimens under various constant and variable amplitude loading conditions. Attention is paid to the model calibration using the constraint factors in view of the dependence of both the crack closure mechanism and the material stress-strain response on the load history. Prediction capabilities of the model are considered in the context of the incompatibility between the crack growth resistance for constant and variable amplitude loading.

### 1. Introduction

Among non-linear concepts proposed for fatigue crack growth predictions, the so-called strip yield (SY) model based on the Dugdale conception of crack tip plasticity, but modified to allow for the plasticity induced crack closure mechanism, remains a particularly versatile predictive tool convenient to use in the case of mode I fatigue crack growth under arbitrary variable amplitude (VA) loading histories.

According to the crack closure concept, the crack opening stress ( $S_{op}$ ) in a current load cycle depends on plastic deformations in the crack wake, which result from the loads experienced previously. With the SY model, all plastic deformation is confined within an infinitely thin strip located along the crack line and embedded in perfectly elastic material. The fatigue crack growth is simulated by severing the strip material over a distance corresponding to the fatigue crack growth increment. Consequently, layers of plastically elongated material are built up on the crack surfaces, as shown in Fig. 1.

\* Faculty of Mechanical Engineering and Robotics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland; E-mail: mskorupa@agh.edu.pl

\*\* Faculty of Mechanical Engineering and Robotics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland; E-mail: machniew@agh.edu.pl

The strip stresses and deformations are solved using numerical methods by considering compatibility conditions along the fictitious crack surface, Fig. 1. To this end, the plastic strip is divided into a number of bar elements. The elements in the plastic zone can carry both tensile and compressive stresses, whilst the broken elements in the crack wake can only undergo compressive stresses referred to as the contact stresses. The latter are generated if at the minimum level of a fatigue load cycle the element length ( $L_i$ ) exceeds the crack opening displacement ( $V_i$ ), see Fig. 1. The material's memory of the previous load history is accounted for by the distribution of the residual plastic stretches, which are related to the contact stresses through the aforementioned compatibility conditions. The  $S_{op}$  level for a given load cycle is determined from the contact stress distribution.

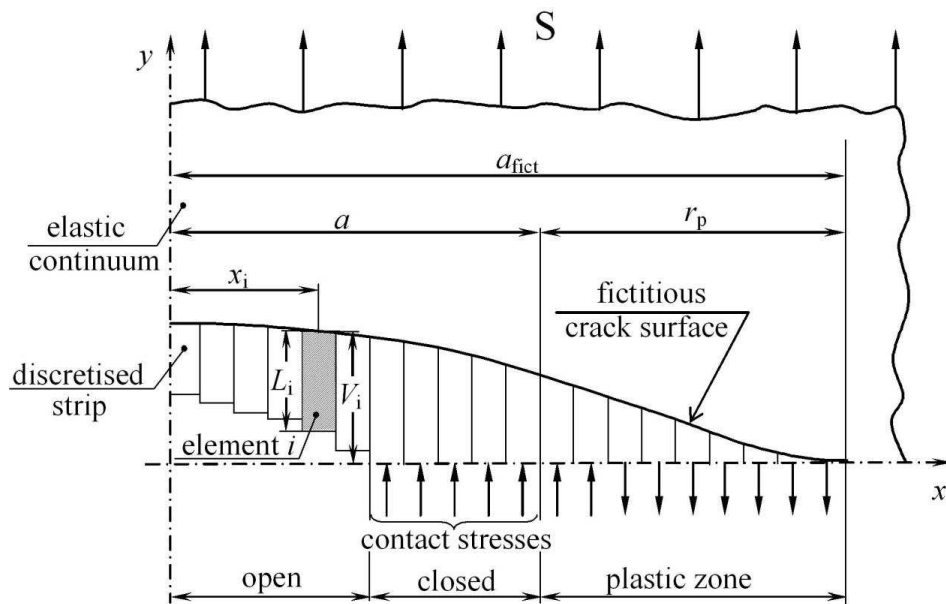


Fig. 1. Schematic illustrating of the discretized plastic strip

The SY model computation results can be largely affected by a number of choices concerning the plastic strip discretisation, various numerical aspects, the material constitutive response and the crack driving force parameter. The model calibration, which is usually done by imposing appropriate constraints on yielding the plastic strip elements, remains another crucial issue.

The present paper is focused on the SY model application to structural steel using the code by the present authors. Tuning the model by means of the constraint factors is considered first in the context of specific fea-

tures of the material constitutive response as well as of the fatigue crack growth behaviour and crack closure behaviour observed for this type of metal. Next, the constraint model proposed by the present authors is evaluated through comparisons between the SY model predictions and the experimental data from crack growth tests on structural steel for constant amplitude (CA) loading and a variety of VA load sequences. Based on the obtained results and the available literature evidence, the prospects of reliable predictions on crack growth under VA loading using the SY model are considered.

## 2. SY model

The major components of the present SY model implementation which involve solving for stresses and deformations of the strip elements at consecutive extrema of the fatigue loading history are the same as for other models of this category, e.g. [1]. However, aspects related to the plastic strip discretisation, like the rules for the plastic strip division into elements as well as updating the discretisation and setting the strip element lengths after the crack advance are distinct. Another original feature of the present model is accounting for the elastic deformation of the strip, which enables one to detect crack closure at high stress ratios  $R \geq 0.7$ . Besides  $S_{op}$  defined as the applied stress level at which the contact stresses vanish during the upward part of a load cycle, the model also computes stress levels corresponding to the onset of tensile stresses and to the onset of plastic straining ahead of the crack tip. All above-mentioned characteristic stress levels are calculated iteratively in order to improve the model accuracy.

To run the SY model computations, the material crack growth data must be defined in the input part of the program. With the present code, the crack growth description is in the form of the discrete fatigue crack growth rate ( $da/dN$ ) versus the effective stress intensity factor range ( $\Delta K_{eff}$ ) data. Here  $\Delta K_{eff} = K_{max} - K_{op}$ , where the stress intensity factor levels  $K_{max}$  and  $K_{op}$  values correspond to the maximum and the crack opening level of a fatigue cycle. The  $K_{op}$ -values are obtained from crack closure measurements.

The most favourable computation option has been selected based on a systematic study of the model sensitivity to various decisions. This involved over 700 crack growth simulations carried out under CA and VA loading for different combinations of the discretisation and computation-related assumptions, as detailed elsewhere [2].

### 3. SY model calibration for structural steel

As it is well known, the original concept of Dugdale assumes plane stress conditions at the crack tip. In order to accommodate in the SY model the more general case of triaxial stress state, constraints on yielding the strip elements are imposed. Another important role of the constraint factors is covering indirectly various processes which do influence crack growth but cannot be treated in a rigorous way. The description of the constraint behaviour makes a model in itself and is critical to the model predictive capabilities. For example, the present authors have concluded that altogether unsatisfactory prediction quality of SY models from the NASGRO software shown in applications to both aluminium alloys [3] and structural steel [4] under CA and VA loading stems from an inadequate conception of the constraint factors.

The fatigue crack growth behaviour of structural steel and of aluminium alloys differ in many respects. First, under CA loading the fatigue crack growth rates ( $da/dN$ ) for steel remain almost unaffected by the sheet thickness ( $t$ ) and are much less than for Al-alloys influenced by the stress ratio [5]. This is, however, not the case under VA loading on structural steel. For example, the amount of retardation observed after a single 100% overload (OL) during subsequent smaller amplitude cycles strongly depends on both  $t$  and  $R$ , as shown in Fig. 2 [5]. Elasto-plastic FE analyses reveal that the  $\Delta a_{OL} > r_{pOL}$  effect, where  $r_{pOL}$  is the plane stress plastic zone of the OL, seen in Fig. 2 stems from the strain hardening of the material due to the OL and cannot be simulated if the ideally plastic material is assumed [6].

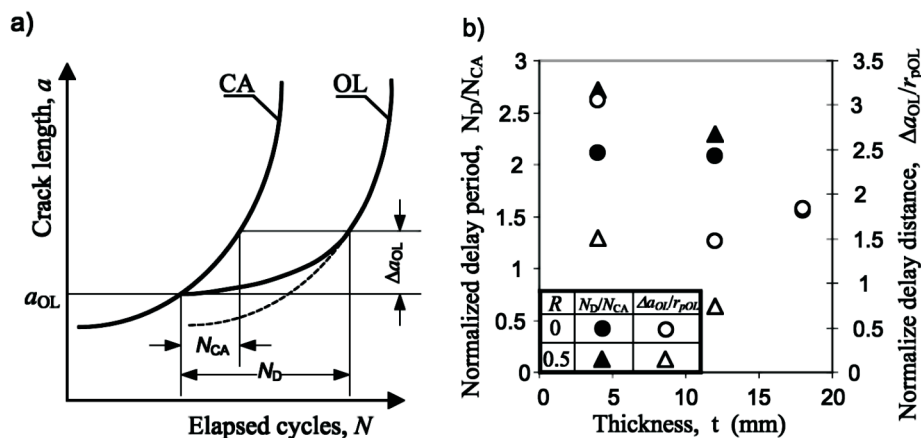


Fig. 2. Crack growth retardation in structural steel: (a) nomenclature; (b) effect of the specimen thickness and stress ratio

Unlike for non-ferrous alloys, the crack growth response of structural steel is almost uniform along the whole crack front even for thick specimens, as evidenced by fractographic examinations [7] and by the influence of surface removal during the overload tests\* [5]. A specific feature of the structural steel constitutive behaviour is that the cyclic stress-strain response never gets stabilized throughout the fatigue life. At a given fraction of the fatigue life the cyclic properties are significantly affected by the loading history [5]. In terms of the SY model, the above characteristics imply that the stress-strain response may be different for different elements of the plastic strip depending on the stress history experienced at a given location. In view of the uncertainty about the actual constitutive behaviour of the material, accounting in the SY model for strain hardening, though feasible (e.g. [8]), could hardly be expected to bring improved predictions on crack growth for structural steel. For the same reason, there is no point in employing a constraint factor that varies along the plastic zone, as in the so-called variable constraint-loss model from the NASGRO software [1]. It should be emphasized that even advanced FE analyses cannot account for the load history dependent cyclic stress-strain response of structural steel.

A previous work by the authors [9] has revealed that in order that the observed  $R$ -ratio effect on crack growth for structural steel be covered by the SY model, three independent constraint factors are required, namely on tensile yielding ( $\alpha_t$ ) and on compressive yielding ahead and behind the crack tip ( $\alpha_c$  and  $\alpha_w$  respectively). Because, however, the required  $S_{op}$ -value can be obtained for many combinations of the three independent  $\alpha$ -values, an additional criterion for their selection is adopted, namely matching the experimentally observed and predicted by the SY model local cyclic stress-strain behaviour. The observed cyclic stress-strain response of the material is represented by the stress vs. offset strain ( $S - \varepsilon_{offset}$ ) diagrams derived from compliance measurements using a strain gauge positioned near to the crack tip [10]. The predicted  $S - \varepsilon_{offset}$  loop at the gauge location is obtained based on the SY model solution on stresses and its shape depends on the constraint factor values, as exemplified in Fig. 3 [5].

The above concept is applied in the SY model to extract variations of the three  $\alpha$ -values required to correlate by the model  $S - \varepsilon_{offset}$  data derived from the compliance measurements for CA loading at a range of the  $R$ -ratio values. The corresponding tests were performed on M(T) specimens in two low-carbon structural steels, 18G2A (PN-EN 10028) and Fe430D (UNI 7070). The measured mechanical properties of both steels are the following:

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\* For structural steel machining away the surface, plane stress regions of a specimen after an OL application does not affect crack growth retardation during the subsequent baseline loading, whilst for Al and Ti alloys the retardation becomes drastically reduced.

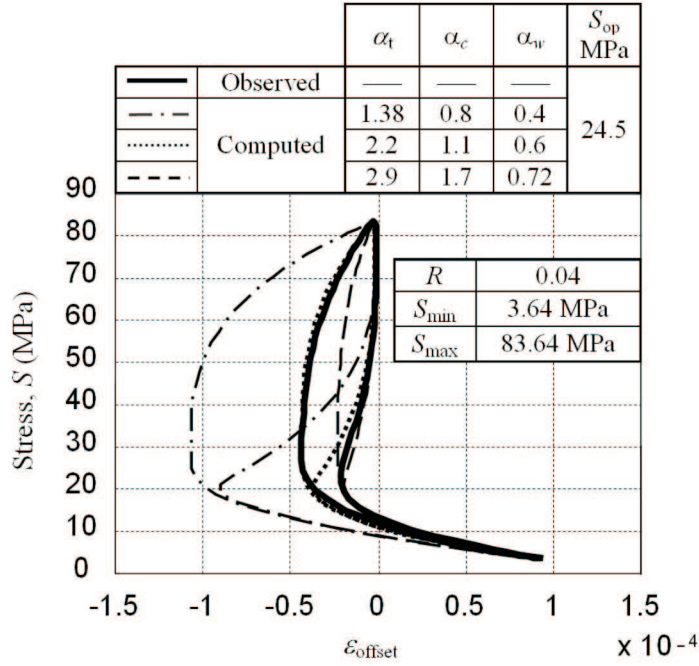


Fig. 3. Exemplary comparisons between the observed and simulated  $S - \varepsilon_{offset}$  loops for various combinations of the three constraint factors

yield stress  $S_y=398$  and  $320$  MPa, ultimate strength  $S_u=540$  and  $475$  MPa, elongation  $25$  and  $29\%$  for  $18G2A$  and  $Fe430D$  respectively. The chosen  $\alpha$ -factors can be approximated as:

$$\alpha_t = 2.0 \quad (1a)$$

$$\alpha_c = \begin{cases} 0.98 & \text{for } R \leq 0 \\ 0.57978 \cdot R + 0.98 & \text{for } R > 0 \end{cases} \quad (1b)$$

$$\alpha_w = \begin{cases} 0.2689 \cdot R + 0.523 & \text{for } R \leq 0 \\ 1.224 \cdot R + 0.523 & \text{for } R > 0 \end{cases} \quad (1c)$$

for  $18G2A$ , whilst for  $Fe430D$

$$\alpha_w = 0.183 \cdot R + 0.523 \quad \text{for } R \leq 0 \quad (1d)$$

Due to the limited sensitivity of crack closure measurements at high stress ratios, the  $\alpha$ -values for  $R > 0.5$  have been obtained by the extrapolation of the results for lower  $R$ -values. Because specimen thickness does not affect

CA crack growth [5], Eq. (1) is valid at least within the thickness range of 4 to 18 mm considered in the present tests.

Experiments by Skorupa et al. [7] have revealed that the observed  $da/dN$  values following a single OL cycle or a block of OLs systematically exceeded the rates inferred from the measured  $S_{op}$  levels when the master  $da/dN$  vs.  $\Delta K_{eff}$  relationship was based on crack closure measurements for CA loading, Fig. 4. Under such conditions, choosing the  $\alpha$ -factors based on the similarity between the predicted and observed  $S$ - $\varepsilon_{offset}$  data would yield overestimated post-OL  $S_{op}$  stresses and, consequently, overly low predicted  $da/dN$  values. Considering that, the concept of the constraint factors in the OL affected zone has been based on the aforementioned FE results [6]. These indicate that material hardening within the OL plastic zone leads to an intensification of the compressive residual stresses ahead of the crack tip. At the same time, the OL promotes a shift of the plastic zone behind the crack tip, which yields enhanced contact stresses. In an attempt to model both above trends, the post-OL  $\alpha_c$  and  $\alpha_w$  factors have been elevated compared to the values adopted for CA loading, as shown in Fig. 5. Here,  $\Delta a^*$  is the post-OL crack growth increment for which the current plastic zone reaches the boundary of the OL plastic zone ( $r_{pOL}$ ). In Fig. 5, the  $\alpha_c$  and  $\alpha_w$ -levels corresponding to CA loading (according to Eqs (1b-d)) have been provided with the additional subscript CA.

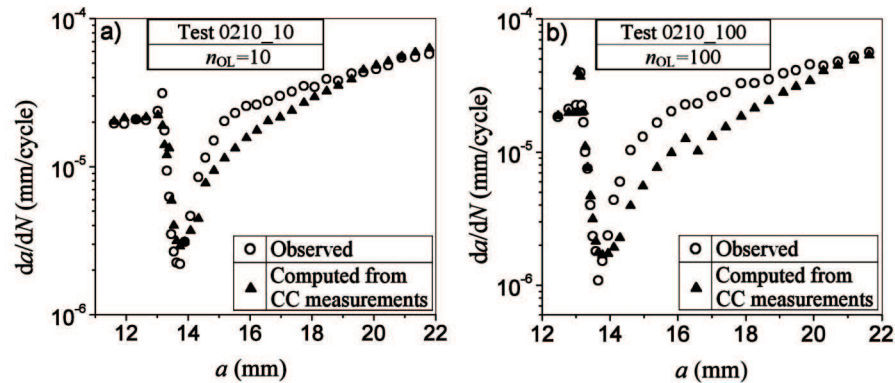


Fig. 4. Fatigue crack growth rates for structural steel measured and computed from the crack closure (CC) measurements after the block of: (a) 10 and (b) 100 OL cycles. Loading condition specified in Table 2

The concept enables to predict the retarded crack growth after an OL over the distance exceeding the OL plastic zone, see Fig. 2. It is worth noting that modelling the  $\Delta a_{OL} > r_{pOL}$  effect is unfeasible using the NASGRO code [3,4]. The post-OL  $\alpha$ -variations according to Fig. 5 are fully defined by two parameters, namely  $\alpha_{OL}$  given by:

$$\alpha_{OL} = \max \begin{cases} \alpha_{wCA} \\ \alpha_{wCA} \cdot (3.5 - 2.8 \cdot R - 0.0867 \cdot t), t \text{ in mm} \end{cases} \quad (2)$$

and  $d\alpha_w/da$ . The latter parameter defines the rate of the return to the pre-OL (CA) value by the constraint factor in the crack wake.  $d\alpha_w/da$  is considered to be a material constant equal to  $0.4 \text{ mm}^{-1}$  for 18G2A and  $1 \text{ mm}^{-1}$  for Fe430D.

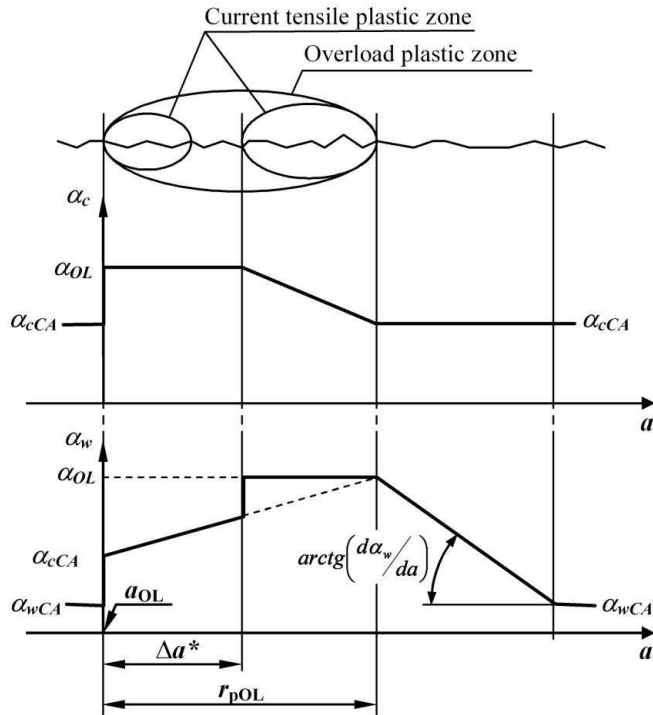


Fig. 5. Modification of the  $\alpha_c$  and  $\alpha_w$  factors after the OL

#### 4. Prediction results

Comparisons between the fatigue test results and predictions from the SY model incorporating the constraint factors according to Eqs (1) and (2) are presented in terms of the predicted-to-observed crack growth lives ( $N_{SY}/N_{EXP}$ ) in Table 1 for CA loading and in Table 2 for several types of VA loading sequences with OLs. All fatigue crack growth tests were done on M(T) specimens of various thicknesses. The loading conditions for all tests are specified in Table 1 and 2.



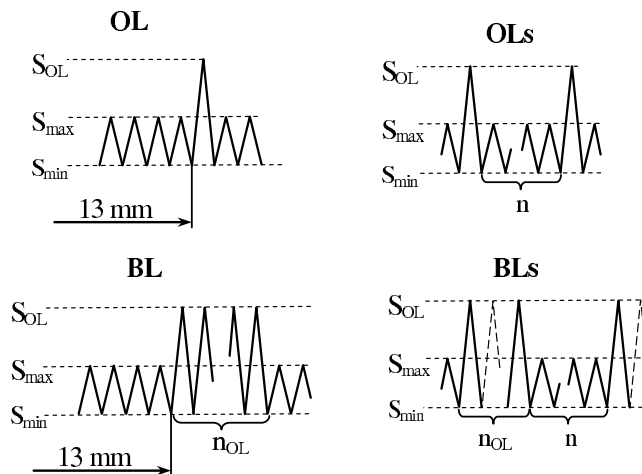
Table 1.  
 Comparisons between the predicted ( $N_{SY}$ ) and observed ( $N_{EXP}$ ) crack growth lives for CA loading

Material:	Test No.	Specimen thickness $t$ , mm	Stress ratio $R=S_{min}/S_{max}$	Stress levels, MPa		$N_{SY}/N_{EXP}$
				$S_{min}$	$S_{max}$	
18G2A	0225	4	-1	-55	55	<b>1.15</b>
	0227		-0.5	-25	50	<b>1.09</b>
	0230		-0.5	-42.15	84.3	<b>0.92</b>
	0205		0.05	4.3	84.3	<b>1.22</b>
	G1		0.15	9.12	59.52	<b>1.00</b>
	0220		0.15	14.1	94.1	<b>1.19</b>
	G3		0.5	52	102	<b>0.82</b>
	0221		0.5	80	160	<b>1.14</b>
	0211		0.7	116.67	166.67	<b>0.98</b>
	0223-1		0.15	9.125	59.525	<b>0.97</b>
	0223-2		0.5	52	102	<b>1.01</b>
	0228-1		0.15	11.5	76.5	<b>0.96</b>
	0228-2		0.5	65	130	<b>1.07</b>
	0204		8	0.05	4.3	84.3
	Fe430D	IT04	6	-1	-55	55
IT07		0.1		7.292	72.92	<b>1.14</b>
IT03		0.5		72.5	145	<b>1.09</b>
IT05		0.7	116.57	166.53	<b>0.97</b>	
IT08	9	-1	-43.5	43.5	<b>1.01</b>	
		IT01	0.5	60	120	<b>0.94</b>

As seen in Table 1, the  $N_{SY}/N_{EXP}$  ratios for 18 out of a complement of 20 CA tests fall between 0.82 and 1.2 and only in two cases are slightly above 1.2. It can be concluded that the model correctly describes in a quantitative way the effects of all test variables considered, namely of the stress ratio, sheet thickness, material and stress level.

Table 2.  
 Comparisons between the predicted ( $N_{SY}$ ) and observed ( $N_{EXP}$ ) crack growth lives for VA loading

Material:	Test No.	Test type	$t$ , mm	$R$	Stress levels, MPa			Cycle No		$N_{SY}/N_{EXP}$
					$S_{min}$	$S_{max}$	$S_{OL}$	$n$	$n_{OL}$	
18G2A	0229	OL	4	-0.5	-25	50	125	—	1	<b>1.24</b>
	0210			0.05	4.3	84.3	164.3	—	1	<b>1.04</b>
	G4			0.07	3.6	53.6	102.6	—	1	<b>1.12</b>
	0209			0.5	80	160	240	—	1	<b>1.05</b>
	0208	OL	12	0.05	4.3	84.3	164.3	—	1	<b>1.12</b>
	0207			0.5	80	160	240	—	1	<b>1.11</b>
	0214	OL	18	0.05	4.1	79.6	155.2	—	1	<b>1.16</b>
	0217			0.5	75.6	151.1	226.7	—	1	<b>1.19</b>
	0104	OLs	4	0.05	7.5	137.5	175	$10^4$	1	<b>1.77</b>
	0107						200	$10^4$	1	<b>1.05</b>
	0110						250	$10^4$	1	<b>0.77</b>
	0111						200	$2 \cdot 10^3$	1	<b>0.77</b>
	0301			0.5	80	160	200	$10^4$	1	<b>1.25</b>
	0210_10	BL	4	0.05	4.3	84.3	164.3	—	10	<b>0.90</b>
	0210_100							—	100	<b>0.78</b>
	0209_10							0.5	80	160
0108	BLs	4	0.05	7.5	137.5	200	$10^4$	10	<b>1.06</b>	
0109								100	<b>0.92</b>	
Fe430D	IT06	OL	6	0.1	7.1	70.6	134.1	—	1	<b>1.23</b>



In Table 2, the life ratios for the single OL tests (type OL) ranging from 1.04 to 1.24 imply that the proposed conception of the model calibration ensures a good correlation of the observed influence of  $R$  and  $t$  on the

post-OL transient  $da/dN$  behaviour. The discrepancies between the predicted and observed lives for the thicker specimens stem from the crack growth acceleration which occurs when the crack grows outside the OL-affected zone, as seen in Figs 6a and b. This behaviour is difficult to understand in terms of the crack closure mechanism and, for that reason, cannot be predicted by the model.

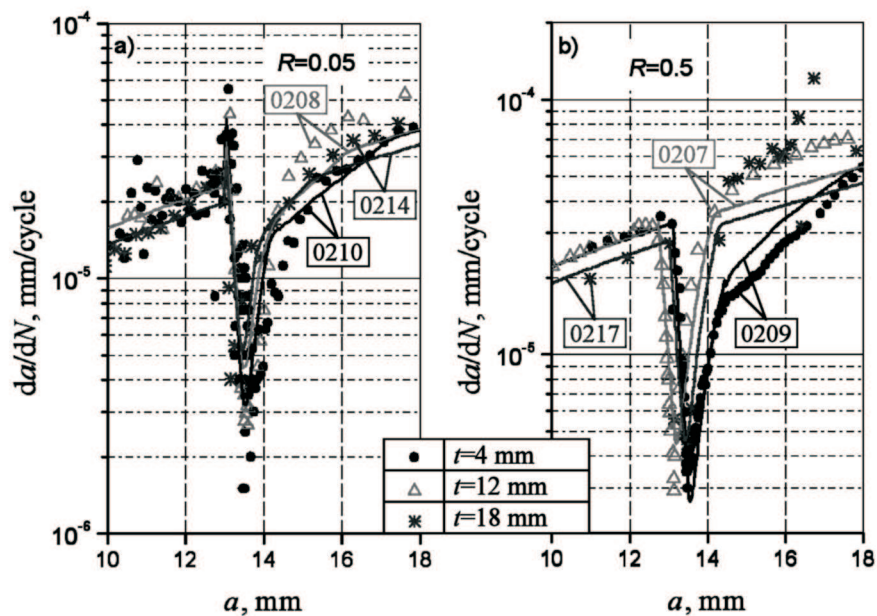


Fig. 6. Comparison between the observed and predicted crack growth rates for the OL tests:  
a)  $R=0.05$ ; b)  $R=0.5$

With the periodic single OL sequences (type OLs, Table 2), different reasons for the different tests are behind the discrepancies between the observed and predicted lives. A poor correlation by the model of the experimental results of Tests 0101 and 0104 stems primarily from the fact that the constraint factors for the OL-affected zone have been derived based on the experiments with 100% OLs, i.e. with the OL ratio ( $OLR = \frac{S_{OL} - S_{min}}{S_{max} - S_{min}}$ ) of 2, whilst  $OLR=1.29$  for Test 0101 and 0104. Overestimating the retardation effect for lower OLs currently revealed by the model can be easily avoided by relating the  $\alpha_{OL}$  factor (see Eq. 2) to the OL level.

An overly conservative prediction for Test 0110 is caused by the long range plasticity generated due to the very high OL stress which makes the linear-elastic fracture mechanics approach dubious. With Test 0111 characterized by very small cycle intervals ( $n$ ) between the OLs, the inaccurate

prediction stems from differences between the actual and computed  $da/dN$  values right after the OL application. Within that zone of a very large  $da/dN$  gradient (see Figs 6a and b) it would be extremely difficult to accurately reproduce by the SY model the actual crack growth rate variations. For frequently applied OLs, as in Test 0111, cumulating the minute discrepancies between the computed and observed crack rates may lead to a poor overall prediction result.

For the type BL tests, Figs 7a and b demonstrate that though the observed delay distance does not change with increasing the OL block size ( $n_{OL}$ ), the minimum post-OL  $da/dN$  level becomes systematically lower. This results in a significant increase in life with increasing  $n_{OL}$ , as revealed in Figs 8a and b. The corresponding  $N_{SY}/N_{EXP}$  values in Table 2 and the predicted  $a$  vs.  $N$  curves in Figs 8a and b indicate that the SY model is fully incapable of covering this trend quantitatively.

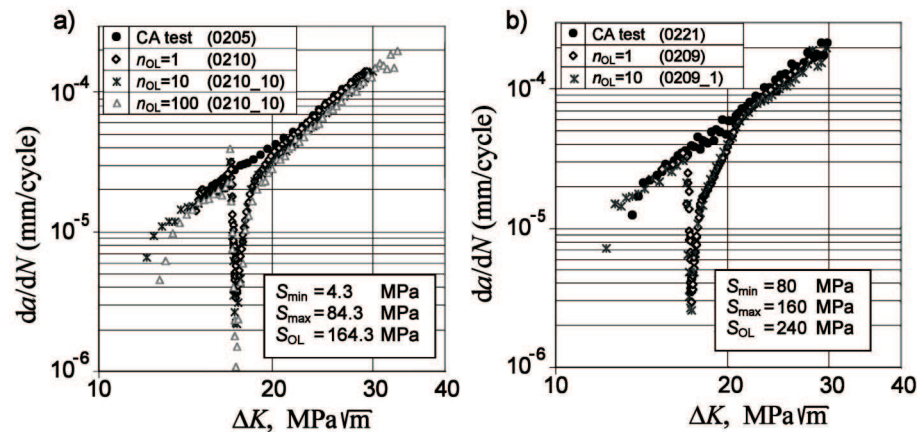


Fig. 7. Influence of the number of cycles in the OL block on the fatigue crack growth rates at: (a)  $R=0.05$ ; (b)  $R=0.5$

In view of the inadequate predictions for the type BL tests, the excellent computed results for the periodic OL blocks (test type BLs, Table 2) can be considered purely coincidental and following from the short, compared with the delay period after an OL block, intervals between the OL blocks ( $n=10^4$  cycles). As seen in Figs 8a and b, the delay periods are in the order of  $10^5$  cycles. Under the conditions of frequently applied OL blocks, the dramatic underestimate by the model of the beneficial effect of increasing the OL block size is not revealed in the predictions.

The above observations emphasize the need for checking the performance of a prediction model first of all for the most simple VA load sequences with well defined and easy to identify load interaction effects, like a single OL

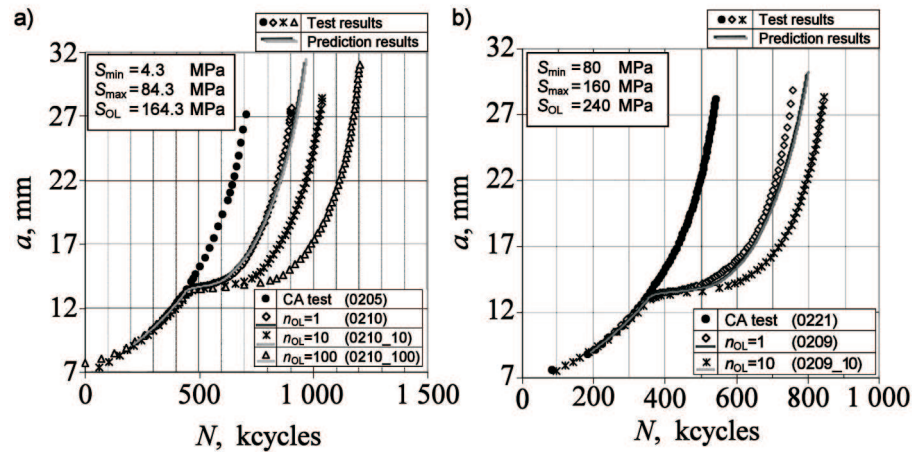


Fig. 8. Influence of the number of cycles in the OL block on the crack growth life at (a)  $R=0.05$ ; (b)  $R=0.5$

and a single block of OLs. Only then the model performance in the case of more complex load histories can be understood and properly evaluated.

### 5. Discussion and Conclusions

The prediction results presented in the previous section indicate that the constraint model according to Eqs (1) and (2) incorporated in the SY model provides an excellent correlation of crack growth observed for two different steels under CA loading conditions. For CA loading, the prediction quality of the model is at least as satisfactory as that observed for the best performing SY model implementation in the NASGRO software when applied to aircraft aluminium alloys [3]. However, it was shown in ref. [3] that none of the constraint factor conceptions in the NASGRO SY model enabled to account for the single OL-induced retardation if the observed  $R$ -ratio influence on crack growth for CA loading was adequately covered because for either loading type opposite actions on constraint factor values were required to improve the predictions. In this respect, the performance of the present SY model is far superior because adequate results are also obtained on the transient crack growth behaviour after a single OL. It is also worth noting that, in contrast to the present model, no computation option available in the NASGRO SY model was capable of giving at least qualitatively correct results on the effect of the OL block size.

At the same time, the present study implies that the SY model tuned on the basis of only CA test results cannot adequately describe the transient behaviour after a single OL. Moreover, the constraint model which adequately

accounts for the crack growth retardation due to a single OL or periodic single OLs is not capable of predicting quantitatively the effect of a block of OLs. The above observations can be understood based on results of the FE analyses [11] which reveal that due to material hardening the plastic region behind the crack tip is significantly larger and the plastic zone range ahead of the crack tip becomes significantly smaller under CA cycling between  $K_{\min}$  and  $K_{\max}$  than after a single OL characterized by the same  $K$ -levels. Such a displacement of the plastic zone to behind the crack tip does not occur if the ideally plastic material is assumed. Consequently, for the strain hardening material, an OL block yields a much enhanced closure effect and, hence, a more severe retardation compared with the single OL. In other words, the FE results [6,11] indicate that the closure mechanism is different for each of the three loading types considered here, namely CA loading, a single OL and a block of OLs. Specifically, at the relatively low stress levels of CA loading applied in the present tests, the effect of strain hardening was insignificant which provided a good similarity between the experimental and computed  $S$ - $\varepsilon_{\text{offset}}$  loops. For the OL and BL sequences such a similarity could not be achieved because, due to the higher stress levels, significant strain hardening must have occurred. The incompatibility between the crack closure mechanism for CA and VA loading found in the FE analyses [6,11] may be behind the inconsistency of the measured closure behaviour and the observed crack growth behaviour illustrated in Fig. 4.

The basic axiom of prediction models is that if the same fatigue resistance of the material and the same crack driving force occur for CA and VA loading then the crack extension for both type load histories is also the same. The literature evidence referred to earlier in this paper suggests that for structural steel the similarity between the fatigue resistance under CA and VA loading is violated due to the dependence of both the closure mechanism and the material constitutive behaviour on the load history. An additional contribution may come from incompatibilities between the fracture surfaces occurring for CA and VA loading, as observed by Schijve for Al alloys on a macro level and a micro level [12]. At the same time, the legitimacy of utilizing  $\Delta K_{\text{eff}}$  as the crack driving force parameter for non-stationary crack growth is sometimes questioned, e.g. [13,14]. For example, a numerical experiment by Bos [14] reveals that under some VA loading conditions the relation between the cyclic crack tip opening displacement  $\Delta\text{CTOD}$  and  $\Delta K_{\text{eff}}$  may not be unique. He postulates that the crack growth rate should be correlated in the SY model in terms of  $\Delta\text{CTOD}$  rather than  $\Delta K_{\text{eff}}$ .

It can be concluded that CA test data based predictions on crack growth for VA load histories are the far extrapolations associated with many variables involved. The constraint factors in the SY model actually serve as an

extrapolation tool. The load history dependence of the fatigue crack growth implies the need of introducing into the constraint model a variety of the material and load sequence related parameters. The associated calibration procedure for a new material would then require an extremely large number of carefully planned experiments in order to cover the spectrum of events which may occur in VA load histories. This type approach is applied in the model of Lang [13] who, however, neglects the dominant role of crack closure and employs a crack driving force parameter alternative to Elber's  $\Delta K_{\text{eff}}$ .

Rather than to try to develop a "universal" SY model suitable for arbitrary load histories, which is an attempt hardly rewarded with success so far, Schijve [12] suggests a more pragmatic approach, namely to tailor the model to a particular load spectrum type. Reliable predictions could then be obtained for practical purposes, as for example comparing the spectrum severity or studying the influence of certain load variables and material properties in order to evaluate the damage tolerance of the structure.

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#### **Zastosowanie modelu pasmowego płynięcia do prognozowanie wzrostu pęknięć zmęczeniowych z stali konstrukcyjnej**

##### **Streszczenie**

Opracowany przez Autorów model pasmowego płynięcia został zastosowany do prognozowania rozwoju pęknięć zmęczeniowych obserwowanych w badaniach zmęczeniowych próbek ze stali konstrukcyjnych w warunkach obciążeń stało- i zmiennoodprężeniowych. Skoncentrowano się głównie na kalibracji modelu przy użyciu odpowiednio dobranych współczynników skrzepowania uwzględniających zarówno mechanizm zamykania się pęknięcia jak i naprężeniowo-odkształceniową charakterystykę materiału właściwą dla danej historii obciążenia. Wyniki prognoz przy użyciu tak skalibrowanego modelu zostały poddane gruntownej ocenie z uwzględnieniem różnic w rozwoju pęknięć obserwowanych w przypadku obciążeń stało- i zmiennoodprężeniowych.