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Optimization of Short Probe Linear Frequency Modulated Signal Parameters

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In many physical experiments, linear frequency modulated (LFM) signals are widely used to probe objects in different environments, from outer-space to underwater. These signals allow a significant improvement in measurement resolution, even when the observation distance is great. For example, using LFM probe signals in underwater investigations enables discovery of even small objects covered by bottom sediments.

Recognition of LFM (chirp) signals depends on their compression based on matched filtering. This work presents two simple solutions to improve the resolution of the short chirp signals recognition. These methods are effective only if synchronization between the signal and matched filter (MF) is obtained. This work describes both the aforementioned methods and a method of minimizing the effects of the lack of synchronization.

The proposed matched filtering method, with the use of n parallel MFs and other techniques, allows only one sample to be obtained in the main lobe and to accurately locate its position in the appropriate sampling period T_s with accuracy T_s/n. These approaches are appropriate for use in probe signal processing.

Keywords: sonar, LFM signal, matched filtering.

1. Introduction

For many physical investigations, short chirp signals are especially expedient because they enable the probe rate to be increased (Grelowska, Kozaczka, 2010; Mahafza, Elsherbeni, 2004; Pogribny *et al.*, 2002; Schock, LeBlanc, 1990). Recognition of these signals is connected with their pulse compression ratio based on matched filtering whose convolutions form the main and side lobes. The peak to side lobe ratio (PSR) (that is the ratio of the main lobe to the maximal one of the side lobes), expressed in dB, defines the degree of chirp compression and strongly influences chirp recognition. Thus,

$$PSR = 20 \log y_c/y_i,$$

where y_c is the central sample of the main lobe (the result of a central convolution operation) and y_i is the maximal value of the convolution result appointed from other convolutions which form the side lobes. At the same time, the sharpness of the main lobe defines the resolution of the method. Therefore, it is necessary to try to decrease the width of the main lobe, ideally to one sample only, formed by the central convolution result. Next, it is very useful to localize the sample position at the boundaries of an appropriate sampling period T_s.

Chirp signals with large products of the signal bandwidth B and duration T (that is $BT \gg 100$) have already been sufficiently examined. In most cases, an increase in BT leads to compression improvement, but this work shows that significant compression can be obtained even if BT is low.

This article aims to work out methods that ensure both high compression of single LFM signals with low BTs even to one sample in the main lobe and maximal recognition resolution (one sample main lobe should be localized within the boundaries of the appropriate period T_s). It is accomplished by matching both chirp and impulse response (IR) spectra to a spectrum of rectangular window and by using nonlinear operations on convolution results in the time domain. The aforementioned resolution is obtained by matching of the beginnings of both the received signal and IR of the matched filter (MF) on the basis of a garland of n MFs with suitably shifted IRs. It allows to choose the maximal main lobe and identify its position among all MFs.

Usually, for chirp signal compression, "fast convolutions" in the frequency domain on the basis of FFT and inverse FFT (IFFT) are used (Tortoli *et al.*, 1997). However, such an approach requires about $N\log_2 N$ operations for N samples of the chirp, and this number limits the recognition speed, as well as the use of this method in fast-acting location systems. On the other hand, in this case the main lobe always consists of a few (not one) samples, which limits recognition resolution. This is connected with the fact that the FFT output terms are multiplied by the frequency response (FR) terms and afterwards squared, added up, square-rooted, and subjected to IFFT (Tortoli *et al.*, 1997). As a result, close to the central sample, squared negative values always come together, and the main lobe consists of a few samples which worsen the resolution.

Therefore, we consider that the optimal way to compress single short chip signals with small BT is to use digital matched filtering on the convolutions in the time domain on the basis of direct parallel algorithms. An N-channel parallel filter with one multiplier in each channel computes N convolution results within N periods T_s and this is $\log_2 N$ times faster than MF on "fast convolutions" if a sequential algorithm of FFT is used. In that parallel structure nonlinear operations on convolution results can easily be fulfilled.

In the hypothetical case of an N-channel parallel FFT/IFFT processor, which allows us to obtain a result close to the one obtained with the help of the direct parallel algorithm, it is also possible to realize nonlinear operations on the results of each channel. Furthermore, the FFT result must be multiplied by a filter FR which corresponds to IR and subjected to IFFT afterwards. In this way we obtain the result of matched filtering by $2\log_2 N+1$ simultaneous multiplication operations in each channel which are realized only after N sampling periods T_s that are necessary to enter all samples of a signal to a processor memory. Assuming that the duration of the multiplication and addition operations does not exceed the sampling period T_s (in this context it is a time or clock period), the result of processing on the basis of the parallel FFT/IFFT algorithm needs about $N+2\log_2 N+1$ clock periods, which is longer by $2\log_2 N+1$ than the aforementioned direct parallel algorithm. Apart from that, each channel of the FFT parallel processor contains $\log_2 N$ multipliers plus a complementary one to multiply an FFT result by the FR component. In contrast to this, the processor on direct convolutions has only one multiplier at each channel.

2. Matched filtering algorithm

To implement the digital matched filter, the input signal is represented in the form of a time series $\{x_n\}$ with the sampling rate $f_s = 1/T_s \geq 2f_2$. The number of samples is equal to N, where $N = ENT(\tau_i f_s)$ and ENT means the integer part of a number.

Each sample of the chirp-signal is given as follows:

$$x_r = x(rT_s) = A\cos\left[2\pi\left(\frac{\Delta f}{2N}r + f_1\right)rT_s + \varphi_0\right], \tag{1}$$

where $\Delta f = f_2 - f_1$ is the deviation of the frequency, f_1 is the initial frequency, f_2 is the final frequency, τ_i is the duration of the chirp signal, φ_0 is the initial phase, and $r = \overline{0, N-1}$.

The algorithm of the digital matched filter based on a convolution in the time domain can be shown as follows:

$$y_n = \sum_{m=0}^{N-1} x_{n-m} h_m w_m. \tag{2}$$

IR of the matched filter without a smoothing window is a mirror reflection of the input signal (1):

$$h_n = x_{N-n}, \tag{3}$$

where $n = \overline{1, N}$, y_n is the n-th convolution operation result, $\{x_n\}$ are the input signal samples, $\{h_n\}$ are the weight factors of IR, N is the number of weight factors and input signal samples, and $\{w_n\}$ are the smoothing window samples. In order to reduce the effect of Gibbs' oscillations, a smoothing window $\{w_n\}$ is used (Tortoli *et al.*, 1997). The total number of convolutions is $2N-1$. For a rectangular window, $\forall w_n = 1$ is assigned. As a result of the chirp signal matched filtering, both the main and side lobes are formed. Thus, a few central samples (results of the convolution operations) make the main lobe, whilst other samples form the side lobes. According to algorithm (2), the total number of multiplications needed to obtain one convolution is N. However, if MF consists of N parallel channels containing multipliers, one convolution is carried out in the time of one multiplication interval which does not exceed T_s.

3. Methods for increasing the short chirp signal compression

3.1. Matching of the chirp signal parameters

To improve the short chirp signal compression, the parameters of the signals and IRs are matched to the corresponding windows. Let us explain our approach. We choose the IR parameters to obtain a windowed IR $\{h_m w_m\}$ where the positive part of the envelope should be closest to the shape of the window. This results in the form of the windowed IR amplitude spectrum $\{|H_w(k)|\}$, that is FR of the filter, which would be closest to the window spectrum $\{|W(k)|\}$, whilst the chirp amplitude spectrum $\{|X(k)|\}$ should be rectangular. Then, in the frequency domain, the result of filtering has the shape of $\{|Y(k)|\}$, where $Y(k) = X(k)H_w(k)$, which should be as close as possible to $\{|H_w(k)|\}$. This means that in this case the degree of compression PSR of the signal $\{y_n\}$ is essentially defined by the smoothing window parameters. Therefore, the choice of the specific window depends on the compression and resolution requirements, as well as the parameters of the chirp.

On the other hand, it is a well-known fact that a rectangular window leads to a smaller PSR than a smoothing one, but at the same time it ensures a narrower main lobe of the signal $\{y_n\}$. So, the use of such a window is expedient to improve the recognition resolution. Therefore, we propose assigning the IR $\{h_n\}_{n=0}^{N-1}$ parameters in such a way that its amplitude spectrum $\{|\,H(k)|\}_{k=-(N/2-1)}^{N/2-1}$ should be rectangular. We estimate the degree of rectangularity on the basis of a variance value:

$$D_H = \frac{1}{N} \sum_{k=-(N/2-1)}^{N/2-1} \left(|H(k)| - \overline{|H|} \right)^2,$$

where

$$\overline{|H|} = \frac{1}{N} \sum_{k=-(N/2-1)}^{N/2-1} |H(k)|.$$

In the presented work, the matched filtering of short chirp signals was carried out using classical smoothing windows and a rectangular window. For each of these cases, the effect of the initial phase and sampling rate on the IR amplitude spectrum shape and the short chirp signal compression was examined. In the experiments, chirp signal durations ranging from 0.1 to 2.5 μs with linear changing of the instantaneous frequency from 0 to 15 MHz were used. In these cases, the BT product was rounded up and the condition $N = 2BT$ was met when one side band B was used. The sampling rate f_s was changed from 29.8 to 30.2 MHz and the initial phase of the signal was set in the range from 0° to 180°. The experiments were conducted with different windows.

Simulations were run using a specially created program. Figure 1 shows the dependence of PSR on the initial phase φ_0 for different windows and at the

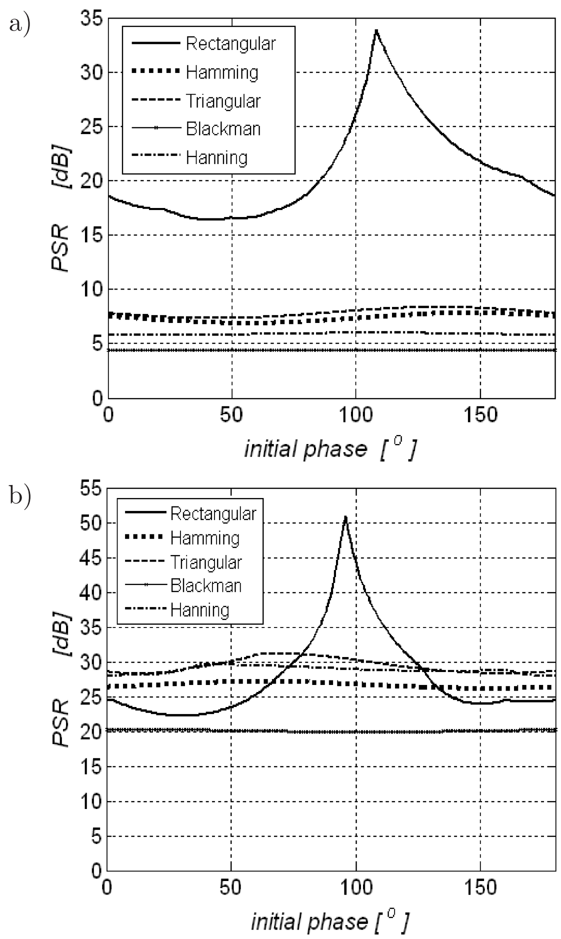

Fig. 1. PSR dependence on the initial phase for different windows when the chirp parameters are: $f_1 = 0$, $f_2 = 15$ MHz, $\tau_i = 2.5$ μs, $BT = 37.5$, optimal $f_s = 30.2$ MHz: a) without nonlinear operations; b) with nonlinear operations. In both cases the main lobe consists of one sample.

optimal f_s when the main lobe consists of one sample only. The results show that PSR depends on φ_0 practically only in the case of a rectangular window but strongly depends on the use of nonlinear operations.

3.2. Using nonlinear operations on matched filtering results

In such cases when a matched filtering result is obtained directly on the basis of algorithm (2), some convolution operation results $\{y_n\}$ have negative values. Traditionally, these values are converted to positive ones and afterwards combined with positive convolution operation results to form the set $\{|y_n|\}$ which is an envelope basis. Moreover, some former negative values of convolution operation are large enough, and are situated near the main lobe. Accordingly, some of them become a part of the main lobe and expand it, which worsens the resolution and PSR.

Therefore, we propose to reject the negative values of the convolution operation in order to increase PSR and decrease the main lobe width to one sample, which is an extreme value for compressing. Nonlinear operations for the elimination of the negative values of convolution operations are realized on the basis of the following algorithm:

$$\forall y,\ \operatorname{sgn} y,\ \exists\, y^{+} \left((\operatorname{sgn} y_n = 1) \vdash (y_n^{+} = y_n) \vee (\operatorname{sgn} y_n = -1) \vdash (y_n^{+} = 0)\right), \tag{4}$$

where $y_n = \operatorname{sgn} y_n \cdot |y_n|$, $\operatorname{sgn} y_n \in \{1, -1\}$, $\vdash$ is the sequence sign, and y_n^{+} is the n-th processing result.

The simulations that have been run show that the use of an appropriately matched sampling rate, initial phase, rectangular window, and nonlinear operations in the digital matched filtering in the time domain results in nonlinear effects which give a significant improvement in short single chirp signal ($BT \ll 100$) compression. This enables us to obtain a main lobe consisting of one sample. In Fig. 2, some examples of chirp signal processing with the parameters listed below and the matched and unmatched spectra are shown (Pogribny *et al.*, 2007). The chirp with $BT = 37.5$ consists of $N = 76$ samples, the number of convolutions is $2N-1 = 151$, and the main lobe contains one sample only.

It is expedient to note that matching IR to a window allows a significant increase in PSR.

In digital systems, a receiver samples an input signal which comes into the receiver at unforeseen moments. This leads to sampling moments which differ from thoses used for determination of the IR weight factors; this is shown as an example in Fig. 3.

The weight factors of IR of MF are determined at moments $\{t_i\}$, i.e. in points $t_0, t_1, \ldots, t_5$. Sampling moments $\{t_{xi}\}$ of an input signal are identified by the dashed line and can fall on any moment within a period T_s. Then, for those moments, the following inequality takes place:

$$t_i \le t_{xi} \le t_i + T_s, \tag{5}$$

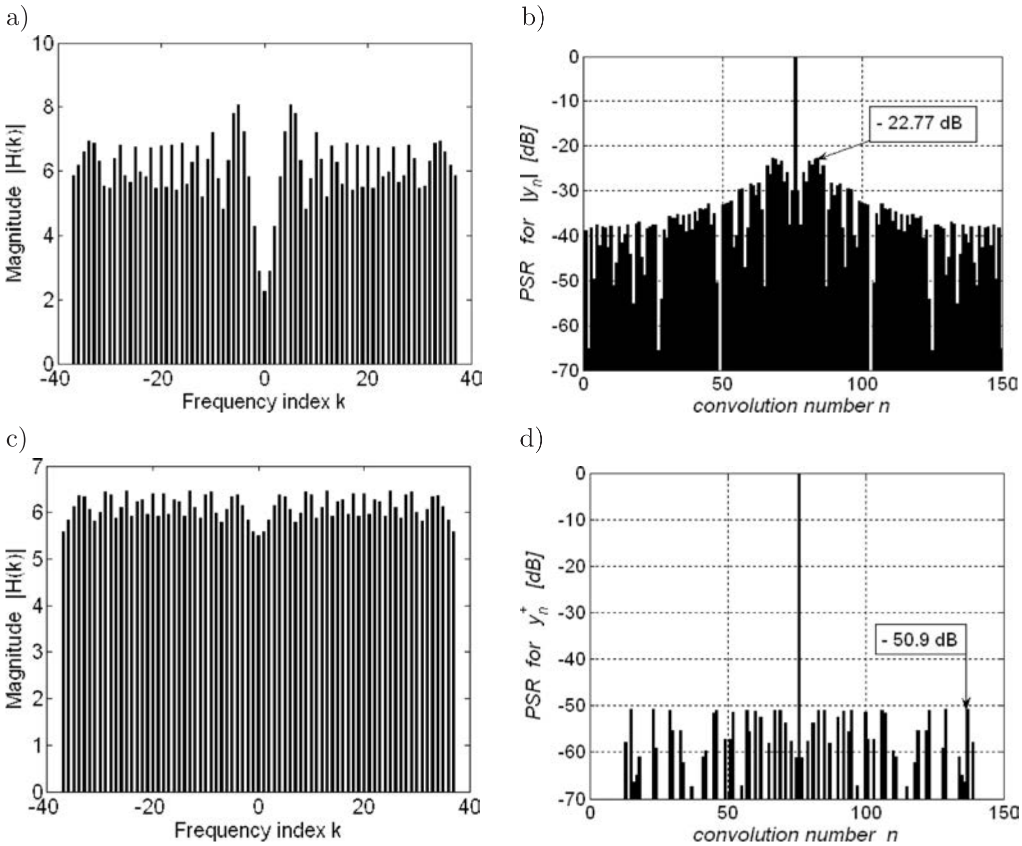

Fig. 2. a) Amplitude spectrum of unmatched IR to the rectangular window when $\varphi_0 = 30°$, $f_s = 30$ MHz, $D_H = 1.0765$; b) result of the chirp compression with the use of this IR and without any nonlinear operations; c) amplitude spectrum of the matched IR to the rectangular window when $\varphi_0 = 96°$, $f_s = 30.2$ MHz and minimized $D_H = 0.0711$; d) result of the chirp signal matched filtering with the use of this IR and nonlinear operations. The chirp parameters are: $f_1 = 0$, $f_2 = 15$ MHz, $\tau_i = 2.5$ µs, $BT = 37.5$.

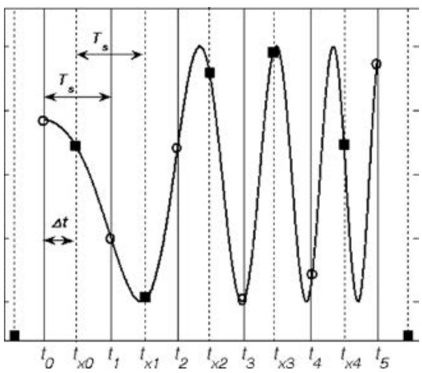

Fig. 3. Example of divergence between a signal sample and moments of MF IR weight factor determination. Here: ◯ – moments of the weight factors determining; ■ – signal samples.

where $i = 0, 1, 2, \ldots$, and all differences $\{\Delta t = t_i - t_{xi}\}$ between suitable moments are in the limits:

$$0 \leq \Delta t \leq T_s. \tag{6}$$

Adding $-T_s/2$ to both parts of (6) we obtain

$$-T_s/2 \leq \Delta t \leq T_s/2. \tag{7}$$

To study the influence of signal delay Δt on its compression, the period T_s is divided into n parts. Then the delay Δt is as follows:

$$\Delta t = \pm kT_s/n, \tag{8}$$

where $k = 0, 1, 2, \ldots, n/2$, whence $\Delta t_{\max} = T_s/2$. Figure 4 shows examples of a relation between *PSR* and Δt when the main lobe consists of one sample. Occurrences of shifted IRs according to the signal beginning and the optimal IR without any shifts were examined.

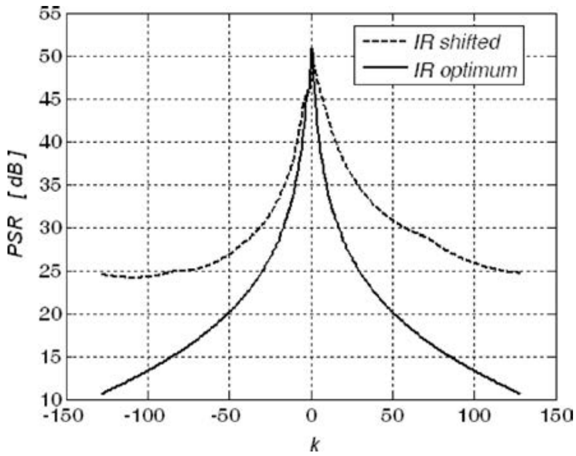

Fig. 4. Dependence of *PSR* on delay $\Delta t = \pm kT_s/128$ for the chirp parameters: $f_1 = 0$, $f_2 = 15$ MHz, $\tau_i = 2.5$ μs, $\varphi_0 = 96°$, $f_s = 30.2$ MHz. Maximal divergence: $\Delta t = T_s/256 = 0.12935$ ns.

This observation should be used for the maximal increase of the recognition resolution of short chirp signals on the basis of a garland of n MFs. For this, we should use the fact that the MF number is directly connected with the time shift of its IR. Therefore, when a chirp signal appears in the moment kT_s/n of a current period T_s, a maximal main lobe will take place on an output of k-th MF. This lobe will be defined by the extreme analyzer *EAk*. Main lobes of other MFs are smaller than those of the k-th MF. Its position in time can be found by the number k.

4. MF structures calculated with minimisation of influence of a lack of synchronisation between short LFM signal and filter

Each of the proposed solutions needs a garland of n MFs together with one common or multiple ADC and a device which selects the output signal with the best compression.

The first structure (Fig. 5) designed for the recognition of short single chirps with $BT < 100$ consists of n channels with matched filters $F1-Fn$, a common ADC, and an analyzer of the highest main lobe (MAX) (Pogribny *et al.*, 2008; Pogribny, Leszczyński, 2010). Each MF works in the time domain and processes PCM samples $\{x_i\}$. It contains N parallel channels, a memory M of IR weight factors, a nonlinear operations block D, and a fuzzy extreme analyzer EA with an assigned threshold of fuzziness which chooses a maximal result of convolution operations (Pogribny, Drzycimski, 2008). Afterwards, a MAX unit picks out the highest maximum value.

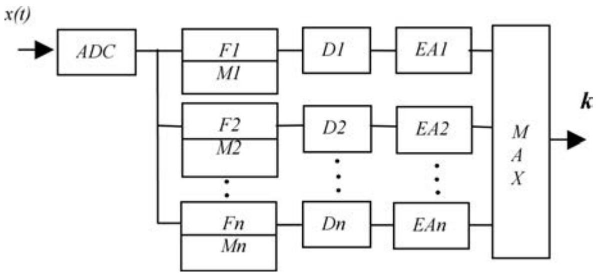

Fig. 5. Garland of n MFs with one ADC.

The ADC sampling rate f_s is assigned as above, and appropriate coefficients for IRs of each filter are shifted to neighboring ones in T_s/n to best match them to a chirp. To achieve this, the weight factors of each IR are defined by n-time decimation of the optimal IR, which was beforehand sampled with frequency nf_s. This allows us to obtain the weight factors general number nN. Then, IR of k-th MF is formed in the course of N periods T_s and consists only of k-th coefficient among all n ones in each period T_s. The output signal of each filter is subjected to nonlinear operations and afterwards to extreme analysis in order to find the maximal PSR among all MFs connected with the central sample of the main lobe. The extreme analysis is carried out within each current T_s, and the results are compared with the assigned threshold afterwards.

When, during a period T_s, the chirp appears, and PSR of the k-th MF exceeds the threshold and at the same time is maximal among other MFs, this means that the chirp appeared in kT_s/n moment of the current T_s. Apart from this, the MAX unit indicates the number of MF which corresponds to the time position in boundaries T_s. Then, the output signal k carries information about the moment kT_s/n of the appearance of the chirp signal.

Another worked out structure contains n ADCs together with corresponding MFs. The ADCs are controlled by shifted in time periodic clock impulses. In this garland, optimal IRs are used in all MFs, and the whole structure works with the same sampling rate f_s.

5. Conclusion

We have found that, in order to to improve the compression and recognition resolution of short single chirp signals with small BT, it is optimal to use digital matched filtration on convolutions in the time domain, not "fast convolutions" in the frequency domain. N-channel matched filters working in the time domain are faster (about $\log_2 N$ times) than filters operating on 'fast convolutions' and have a better resolution. The best results were obtained with a rectangular window, well-matched parameters of chirp and IR, and only positive results of convolutions. It has been shown that when the sampling rate of a chirp signal is slightly different from f_N we can obtain the optimum initial phase to achieve significant PSR, and the main lobe width equals one sampling period only.

Besides this, we have developed a method of minimizing the effects of a lack of synchronization between a single short chirp and filter on its compression. This is based on the use of n parallel MFs with suitable shifted IRs and other techniques to allow only one sample to be obtained in the main lobe located at the boundaries of the appropriate sampling period T_s with the accuracy T_s/n.

It should be noted that these simple filters have a regular structure consisting of uniform blocks and are convenient for implementation in different multiprocessor systems for solving echo-location tasks.

These approaches are expedient for use in different kinds of probe signal processing.

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