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Solution of inverse heat conduction equation with the use of Chebyshev polynomials

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Abstract A direct problem and an inverse problem for the Laplace's equation was solved in this paper. Solution to the direct problem in a rectangle was sought in a form of finite linear combinations of Chebyshev polynomials. Calculations were made for a grid consisting of Chebyshev nodes, what allows us to use orthogonal properties of Chebyshev polynomials. Temperature distributions on the boundary for the inverse problem were determined using minimization of the functional being the measure of the difference between the measured and calculated values of temperature (boundary inverse problem). For the quasi-Cauchy problem, the distance between set values of temperature and heat flux on the boundary was minimized using the least square method. Influence of the value of random disturbance to the temperature measurement, of measurement points (distance from the boundary, where the temperature is not known) arrangement as well as of the thermocouple installation error on the stability of the inverse problem was analyzed.

Keywords: Laplace's equation; Boundary inverse problem; Quasi-Cauchy problem; Stability of the inverse problem

Nomenclature

a	–	multinomial coefficient of the function of distribution of temperature $\widetilde{T}(w)$
c	–	multinomial coefficient of the function of distribution of temperature $T(x,y)$

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G	–	element of a vector {G}
F	–	element of a vector {F}
J	–	functional, the sum of squares of the differences between the temperature calculated at the measurement point and the measured one
k	–	summing index, Chebyshev nodes
M	–	number of measuring points
m	–	number of Chebyshev nodes on the y-axis
$N_1 - 1$	–	degree of the polynomial describing unknown distribution of temperature on the Γ_1 boundary
n	–	number of Chebyshev nodes on the x-axis
p	–	summing index, pertains to the temperature measurement points
T	–	temperature, K
W_i	–	Chebyshev polynomial of the first kind of ith degree
w	–	Chebyshev node
$[x]_n$	–	integer part of the division of number x by n
$x \bmod n$	–	remainder of the division of number x by n
x, y	–	Cartesian coordinates
$\|\delta T\|$	–	Euclidean norm from the difference between the temperature assumed in a direct problem and that calculated with the inverse problem at points on edge Γ_1

Greek symbols

Γ	–	edge of the area
γ	–	multinomial coefficient, pertains to the sought temperature distribution on edge Γ_1
δ	–	absolute error
ε	–	distance of the temperature measurement points from edge Γ_1

Subscripts

c	–	calculated value
dp	–	assumed values in the direct problem
ip	–	values calculated with the use of the inverse problem
h, i, j, k	–	summing index
p	–	summing index
q	–	number of rows of the Chebyshev nodes on axis x, in which the temperature measurement is performed
$random$	–	values calculated with random disturbance to the temperature measurement

Superscripts

$*$	–	measured value
$\widetilde{T}$	–	temperature, function dependent on the Chebyshev node
$\widetilde{A}$	–	matrix A^{-1} element
$\widetilde{F}$	–	element of a vector $\{\widetilde{F}\}$
$\widetilde{G}$	–	element of a vector $\{\widetilde{G}\}$
$''$	–	second derivative
$\top$	–	transpose

1 Introduction

In many technological cases, it is impossible to measure temperature on the edge of considered domain or such measurements show significant uncertainties. It is due to the surface high temperature and heat flow by radiation. This problem occurs, for instance, in combustion chambers or in heat-turbine housings. Determination of temperature distribution on the region's boundary is possible through solving the inverse problem. In this paper the boundary-value inverse heat conduction problem with steady boundary was solved with the use of the Laplace and Fourier transforms [1]. A new approach to solving the inverse boundary problem with the use of the Laplace transform consisting in solving the first-order Volterra equation was proposed in [2]. On the other hand the Cauchy problem in the multilayer region for the one-dimensional heat conduction equation was analyzed in [3]. To solve the ill-posed problem, the Fourier transform and the modified Tikhonov regularization technique were used. The Cauchy problem for the Laplace equation was also solved by replacing it by the solution of the Poisson equation based on polyharmonic functions [4]. Paper [5] presents the solution of the mixed inverse problem consisting in the Cauchy problem, the backward heat conduction problem and the heat source recovery problem. To do so, a differential quadrature method and the Lie-group adaptive method were applied. Two new methods of regularization for ill-posed Cauchy problem were considered in [6]. In paper [7] Chebyshev polynomials and the least-squares method were applied to solve the inverse heat conduction problem. Sensitivity of solutions to inverse problems was analyzed in papers [5,8–11]. Coefficient inverse problem was a subject of study in [12], where the thermal conductivity function of a solid was approximated by a polynomial. Inverse problems are applied in technical problems, such as analysis of boilers operation [13], heat exchangers operation [14], processes related to changes of phase of solidifying metal [15].

In this paper, the direct problem and the inverse problem for the Laplace's equation were solved using the Chebyshev polynomials. Sensitivity of the obtained solution to errors in measurement and thermocouple installation were also considered.

2 Direct problem

Given is the Laplace's equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

with the boundary conditions (Fig. 1):

$$\Gamma_1 : T\left(x = 1, y\right) = T_{\Gamma_1}\left(y\right) \ , \tag{2}$$

$$\Gamma_2 : T\left(x, y = 1\right) = T_{\Gamma_2}\left(x\right) \ , \tag{3}$$

$$\Gamma_3 : T\left(x = -1, y\right) = T_{\Gamma_3}\left(y\right) \ , \tag{4}$$

$$\Gamma_4 : T\left(x, y = -1\right) = T_{\Gamma_4}\left(x\right) \ . \tag{5}$$

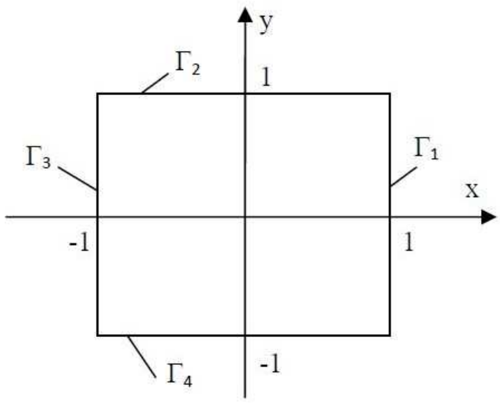


Figure 1: Considered domain.

Temperature assumed on the boundary $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ is a continuous function. Temperature distribution in the considered region can be written using Chebyshev polynomials [16]

$$T\left(x_i, y_j\right) = \sum_{p=0}^{n-1} \sum_{q=0}^{m-1} c_{pq} W_p\left(x_i\right) W_q\left(y_j\right) \ . \tag{6}$$

On the x and y axes there are n and m nodes, respectively. Inner nodes are the Chebyshev nodes [16]. Equation (1) is required to be satisfied

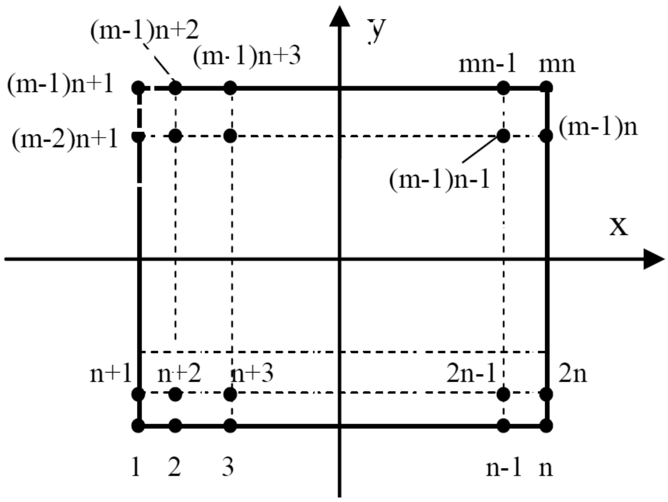


Figure 2: Node enumeration.

in these nodes. After renumerating nodes (Fig. 2), point (x_i, y_j) of the table is denoted as the node w_l, where $l = (j-1)\,n + i$. Dependences $i = (l-1)\,\mathrm{mod} n + 1$ and $j = [l-1]_n + 1$ occur there. After new numeration of nodes is introduced, temperature distribution can be written as

$$\widetilde{T}\,(w_l) = \sum_{k=1}^{mn} a_k W_{[k-1]_m}\left(x_{(l-1)\mathrm{mod} n+1}\right) W_{(k-1)\mathrm{mod} m}\left(y_{[l-1]_n+1}\right) \ . \tag{7}$$

Inserting temperature function in the form (8) into Eq. (1) in Chebyshev nodes, we have

$$\sum_{k=1}^{mn} a_k W''_{[k-1]_m}\left(x_{(l-1)\mathrm{mod} n+1}\right) W_{(k-1)\mathrm{mod} m}\left(y_{[l-1]_n+1}\right) + \\ + \sum_{k=1}^{mn} a_k W_{[k-1]_m}\left(x_{(l-1)\mathrm{mod} n+1}\right) W''_{(k-1)\mathrm{mod} m}\left(y_{[l-1]_n+1}\right) = 0\ . \tag{8}$$

Moreover, if boundary conditions (2) – (6) are taken into account, the algebraic system of linear equations is obtained

$$\mathbf{A}x = b\ , \tag{9}$$

where

$$x = \{a_1, a_2, \ldots, a_{mn}\}^\top\ , \tag{10}$$

where superscript ⊤ denotes transpose operation, and

$$b = \Big\{\widetilde{T}\left(w_1\right), \widetilde{T}\left(w_2\right), \ldots, \widetilde{T}\left(w_n\right), \widetilde{T}\left(w_{n+1}\right), 0, \ldots,$$
$$0, \widetilde{T}\left(w_{2n}\right), \widetilde{T}\left(w_{2n+1}\right), 0, \ldots, 0, \widetilde{T}\left(w_{3n}\right), \widetilde{T}\left(w_{3n+1}\right), 0 \ldots \tag{11}$$
$$\ldots, 0, \widetilde{T}\left(w_{(m-1)n}\right), \widetilde{T}\left(w_{(m-1)n+1}\right), \widetilde{T}\left(w_{(m-1)n+2}\right), \ldots, \widetilde{T}\left(w_{mn}\right)\Big\}^{\top},$$

$$[A_{ij}]_{\substack{i=1,\ldots,mn \\ j=1,\cdots,mn}} . \tag{12}$$

Elements of matrix **A** corresponding to nodes on the boundary assume the following form:

$$A_{lk} = W_{[k-1]_m}\left(x_{(l-1)\mathrm{mod}n+1}\right) W_{(k-1)\mathrm{mod}m}\left(y_{[l-1]_n+1}\right), \tag{13}$$

and for inner nodes we have

$$A_{lk} = W''_{[k-1]_m}\left(x_{(l-1)\mathrm{mod}n+1}\right) W_{(k-1)\mathrm{mod}m}\left(y_{[l-1]_n+1}\right)$$
$$+ W_{[k-1]_m}\left(x_{(l-1)\mathrm{mod}n+1}\right) W''_{(k-1)\mathrm{mod}m}\left(y_{[l-1]_n+1}\right). \tag{14}$$

If there is an inverse matrix to **A** then the solution is of the following form

$$x = \mathbf{A}^{-1} b . \tag{15}$$

3 Inverse problem

It is difficult to perform temperature measurements on the edges of many machines' components. Temperature may be therefore determined by solving boundary inverse problem based on temperature measurements at inner points of the body (Fig. 3) or by solving an inverse Cauchy-type problem based on the known value of the heat flux density and the temperature on the edge Γ_3 (Fig. 4).

The inverse problem was solved taking into account conditions (3)–(6) and temperature measurements in Chebyshev nodes. Measuring points were situated in one row (the first type of problem – the inverse boundary problem) or in two rows being close to each other (the second type of problem – the quasi-Cauchy problem). For the first-type problem, calculations included temperature measurements at points (x_q, y_i), where $i = 2, 3, \ldots, M+1$ (Fig. 5). In the second-type problem, points of temperature measurement

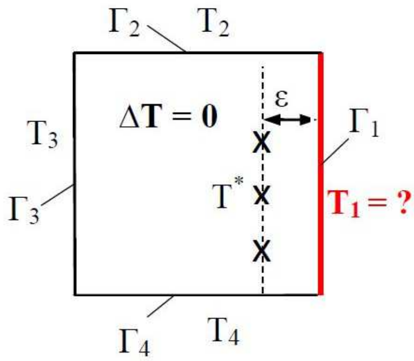


Figure 3: Boundary inverse problem.

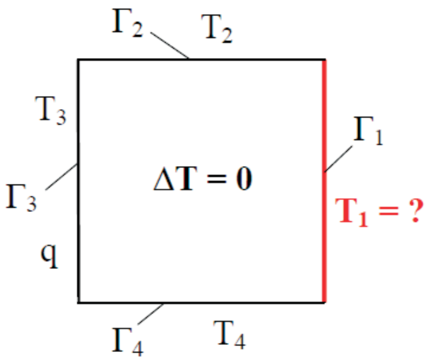


Figure 4: Cauchy-type inverse problem.

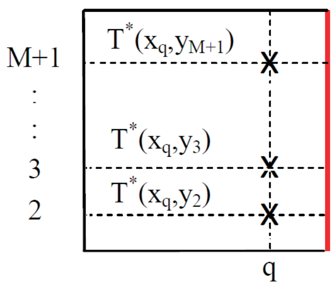


Figure 5: Points of temperature measurement for the inverse boundary problem.

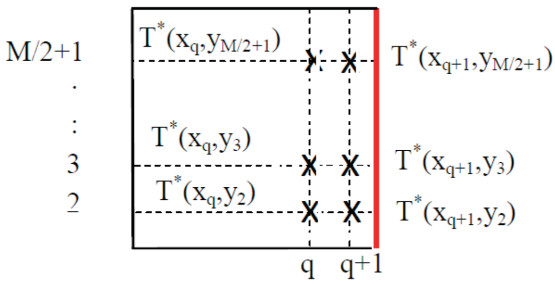


Figure 6: Points of temperature measurement for the quasi-Cauchy problem.

were situated in Chebyshev nodes of coordinates (x_q, y_i) and (x_{q+1}, y_i) for $i = 2, 3, \ldots, M/2 + 1$, where M is the even number (Fig. 6).

Based on temperature measurement inside the region, unknown temperature distribution on the Γ_1 boundary was determined. Unknown course of the boundary condition was approximated by Chebyshev polynomials

$$T_{\Gamma_1}(y) = \sum_{i=1}^{N_1} \gamma_i W_{i-1}(y) \ , \tag{16}$$

where the values of coefficients γ_i for $i = 1, 2, \ldots, N_1$ are unknown. On the basis of the direct problem, temperature distribution was described by the formula (8). Unknown values of coefficients a_k for $k = 1, 2, \ldots, mn$ were determined from the system of linear equations (16), including elements of matrix A and of vector b described by formulae (12)–(15). Unknown temperature distribution on the Γ_1 boundary corresponds to nodes $2n, 3n, \ldots, (m-1)\,n$. Based on (18), the following equalities occur at points on the Γ_1 boundary

$$\widetilde{T}\left(w_{jn}\right) = \sum_{i=1}^{N_1} \gamma_i W_{i-1}\left(y_j\right) \tag{17}$$

for $j = 2, 3, \ldots, (m-1)$. Therefore, Eq. (16) can be written in the form of

$$\{x\} = \mathbf{A}^{-1}\left\{\widetilde{F}\right\} + \mathbf{A}^{-1}\left\{\widetilde{G}\right\} \ , \tag{18}$$

where $\left\{\widetilde{G}\right\} = \left\{0, \cdots, 0, \widetilde{T}\left(w_{2n}\right), 0, \ldots, 0, \widetilde{T}\left(w_{3n}\right), 0, \ldots \ \ldots, 0, \widetilde{T}\left(w_{(m-1)n}\right),\right.$ $\left. 0, \ldots, 0\right\}^T$ and $\widetilde{F}_i = b_i - \widetilde{G}_i$ for each $i = 1, 2, \ldots, mn$. Assuming that $\{F\} = \mathbf{A}^{-1}\left\{\widetilde{F}\right\}$ and $\{G\} = \mathbf{A}^{-1}\left\{\widetilde{G}\right\}$, we obtain the solution of the form of

$$\{x\} = \{F\} + \{G\} \ , \tag{19}$$

where the values of the vector $\{F\}$ are known, while values of the vector $\{G\}$ are unknown and can be described by the following relation:

$$G_k = \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{i=1}^{N_1} \gamma_i W_{i-1}\left(y_j\right) \ . \tag{20}$$

Elements of the inverse of the matrix $\mathbf{A}$ are written as $\widetilde{A}_{k,jn}$. Hence,

$$a_k = F_k + G_k = F_k + \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{i=1}^{N_1} \gamma_i W_{i-1}\left(y_j\right) \ . \tag{21}$$

Values γ_i for $i = 1, 2, \ldots, N_1$ should be determined from the minimum of functional

$$J = \sum_{p=2}^{M+1} \left[T_c\left(x_{q+f(p)}, y_{g(p)}\right) - T^*\left(x_{q+f(p)}, y_{g(p)}\right)\right]^2, \tag{22}$$

where $f\left(p\right) = 0$ for the first-type problem (inverse boundary problem), and $f\left(p\right) = \left[p-1\right]_{M/2+1}$ for the second-type problem (quasi-Cauchy problem);

and $g(p) = p$ for the first-type problem and $g(p) = 1+(p-1) \bmod (M/2+1) + [p-1]_{M/2+1}$ for the second-type problem; the asterisk denotes the measured value. On the basis of the formula (8) we have

$$J = \sum_{p=2}^{M+1} \left[\sum_{k=1}^{mn} a_k W_{[k-1]_m} \left(x_{q+f(p)} \right) W_{(k-1) \mathrm{mod} m} \left(y_{g(p)} \right) - T^* \left(x_{q+f(p)}, y_{g(p)} \right) \right]^2 . \tag{23}$$

Substituting the dependence (23) into Eq. (25) we have obtained

$$J = \sum_{p=2}^{M+1} \left[\sum_{k=1}^{mn} \left(F_k + \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j) \right) \times W_{[k-1]_m} \left(x_{q+f(p)} \right) W_{(k-1) \mathrm{mod} m} \left(y_{g(p)} \right) - T^* \left(x_{q+f(p)}, y_{g(p)} \right) \right]^2 . \tag{24}$$

Functional has its minimum (necessary condition), if for each $i = 1, 2, \ldots, N_1$ the following equality occurs:

$$\frac{\partial J}{\partial \gamma_i} = 0 . \tag{25}$$

Hence,

$$\frac{\partial J}{\partial \gamma_i} = 2 \sum_{p=2}^{M+1} \left[\sum_{k=1}^{mn} \left(F_k + \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j) \right) \times W_{[k-1]_m} \left(x_{q+f(p)} \right) W_{(k-1) \mathrm{mod} m} \left(y_{g(p)} \right) - T^* \left(x_{q+f(p)}, y_{g(p)} \right) \right] \times \sum_{k=1}^{mn} W_{[k-1]_m} \left(x_{q+f(p)} \right) W_{(k-1) \mathrm{mod} m} \left(y_{g(p)} \right) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} W_{i-1}(y_j) \tag{26}$$

Applying substitutions $C_1(i,p) = \sum_{k=1}^{mn} W_{[k-1]_m} \left(x_{q+f(p)} \right) W_{(k-1) \mathrm{mod} m} \left(y_{g(p)} \right) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} W_{i-1}(y_j)$ and $C_2(k,p) = W_{[k-1]_m} \left(x_{q+f(p)} \right) W_{(k-1) \mathrm{mod} m} \left(y_{g(p)} \right)$ we have obtained

$$\frac{\partial J}{\partial \gamma_i} = 2 \sum_{p=2}^{M+1} C_1(i,p) \left[\sum_{k=1}^{mn} \left(F_k + \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j) \right) \times C_2(k,p) - T^* \left(x_{q+f(p)}, y_{g(p)} \right) \right] . \tag{27}$$

Therefore, taking into account the equality

$$C_3(i) = \sum_{p=2}^{M+1} T^*\left(x_{q+f(p)}, y_{g(p)}\right) C_1(i,p)$$

we have

$$\frac{1}{2}\frac{\partial J}{\partial \gamma_i} = -C_3(i) + \sum_{p=2}^{M+1} C_1(i,p) \sum_{k=1}^{mn} C_2(k,p) \left(F_k + \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j) \right). \tag{28}$$

Suppose that $C_4(p) = \sum_{k=1}^{mn} C_2(k,p) F_k$, then

$$\frac{1}{2}\frac{\partial J}{\partial \gamma_i} = -C_3(i) + \sum_{p=2}^{M+1} C_1(i,p) \left[C_4(p) + \sum_{k=1}^{mn} C_2(k,p) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j) \right]. \tag{29}$$

Substituting $C_5(i) = \sum_{p=2}^{M+1} C_1(i,p) C_4(p)$ we have obtained

$$\frac{1}{2}\frac{\partial J}{\partial \gamma_i} = -C_3(i) + C_5(i) + \sum_{p=2}^{M+1} C_1(i,p) \sum_{k=1}^{mn} C_2(k,p) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j). \tag{30}$$

For $C_6(i) = -C_3(i) + C_5(i)$ we have

$$\frac{1}{2}\frac{\partial J}{\partial \gamma_i} = C_6(i) + \sum_{p=2}^{M+1} C_1(i,p) \sum_{k=1}^{mn} C_2(k,p) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} \sum_{h=1}^{N_1} \gamma_h W_{h-1}(y_j). \tag{31}$$

After permuting summation, we have obtained

$$\frac{1}{2}\frac{\partial J}{\partial \gamma_i} = C_6(i) + \sum_{h=1}^{N_1} \gamma_h \sum_{p=2}^{M+1} C_1(i,p) \sum_{k=1}^{mn} C_2(k,p) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} W_{h-1}(y_j). \tag{32}$$

We have applied the substitution $C_7(i,h) = \sum_{p=2}^{M+1} C_1(i,p) \sum_{k=1}^{mn} C_2(k,p) \sum_{j=2}^{m-1} \widetilde{A}_{k,jn} W_{h-1}(y_j)$. Therefore,

$$0 = \frac{1}{2}\frac{\partial J}{\partial \gamma_i} = C_6(i) + \sum_{h=1}^{N_1} \gamma_h C_7(i,h) \ . \tag{33}$$

Hence, for each $i = 1, 2, \ldots, N_1$

$$-C_6(i) = \sum_{h=1}^{N_1} \gamma_h C_7(i,h) \ . \tag{34}$$

Matrix equation with unknown vector $\{\gamma\}$ can be written in the form:

$$\{-C_6\} = [C_7]\{\gamma\} \ . \tag{35}$$

If there is an inverse matrix to the matrix $[C_7]$, then

$$\{\gamma\} = [C_7]^{-1}\{-C_6\} \ . \tag{36}$$

4 Numerical example

It was assumed that temperature distribution in the whole region can be described by the following function

$$T(x, y) = \sinh x \sin y \ . \tag{37}$$

Calculations were made for $n = m = 10$ nodes along x and y axes. By means of solving the direct problem, values of temperature for selected inner nodes were determined; these values were assumed as measured values to test the program. Measured values were disturbed by values δ_{random} ranging from 0 to 0.05 of the temperature maximum value. The mean square deviation of set values in the direct problem from the values calculated with the use of inverse problem method in nodes on the Γ_1 boundary was described by the relation

$$\|\delta T\| = \sqrt{\sum_{i=1}^{m} \left[T_{dp}(x = 1, y_i) - T_{ip}(x = 1, y_i)\right]^2} \ . \tag{38}$$

Two types of solution were considered. The first one was the boundary inverse problem, where one row of measuring points was included (Fig. 7a). For the quasi-Cauchy problem, temperature measurement was performed in two rows (Fig. 7b). Measuring points for both problems coincided with Chebyshev nodes.

An influence of changes in measuring points arrangement on the accuracy of temperature distribution on the Γ_1 boundary was studied. The measure of accuracy are values $\|\delta T\|$ described by the formula (38).

For the first type of inverse problem, it was assumed that $M = 8$ and $N_1 = 8$. Values $\|\delta T\|$ were calculated; they grow with relocating temperature measuring points from the Γ_1 boundary to the Γ_3 boundary (Fig. 8). They reach their maximum for the second row of Chebyshev nodes. Decreasing value $\|\delta T\|$ for q amounting to 3, 5 or 7 results from nonuniform

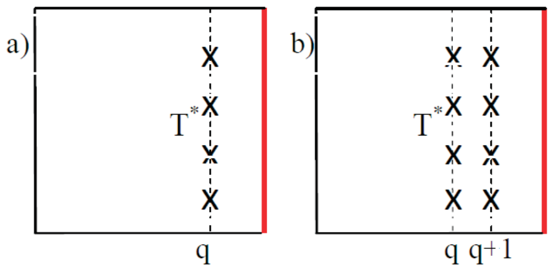
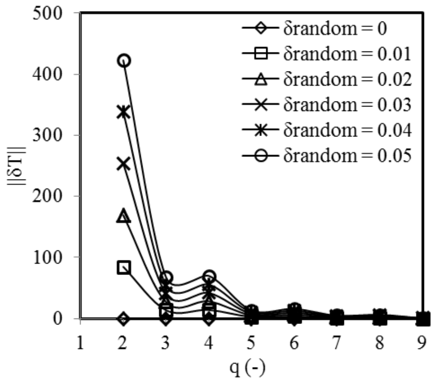
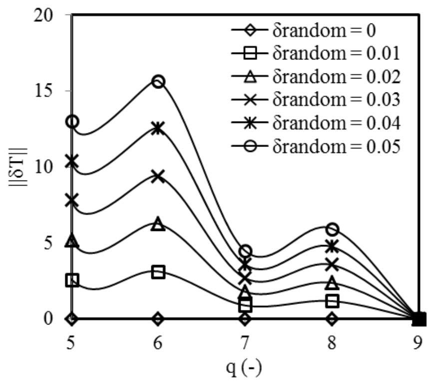


Figure 7: a) Boundary inverse problem, b) Quasi-Cauchy problem.



a)



b)

Figure 8: Values $\|\delta T\|$ for the boundary problem with temperature measurements in the Chebyshev nodes x_q for q: a) from 2 to 9; b) from 5 to 9, including a random disturbance to the temperature measurement δ_{random}.

arrangement of Chebyshev nodes enabling for fine meshing of collocation points close to the boundary of the region. Increase of the random disturbance to the temperature measurement δ_{random} from the value of 0 to 0.05 results in linear increase of the value $\|\delta T\|$ (Fig. 9). Values $\|\delta T\|$ amounted from 3.45×10^{-16} for temperature measurement in nodes closest to the Γ_1 boundary and $\delta_{random} = 0$ to 422.76 for measuring points situated in the second row of nodes and $\delta_{random} = 0.05$ (Tab. 1).

In calculations for the inverse problem of the second type, it was assumed that $M = 16$ and $N_1 = 8$. Calculations were made for two rows of measuring points of coordinates x coinciding with Chebyshev x_q and x_{q+1} nodes. For $q = 3$, the value $\|\delta T\|$ is maximal, what is presented in Fig. 10 and in Tab. 2. Increase of the random disturbance to the temperature mea-

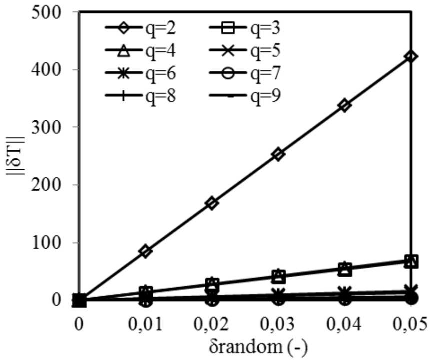
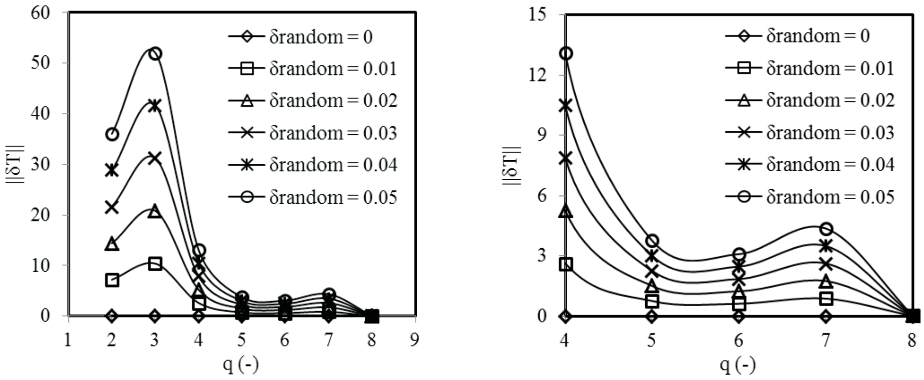


Figure 9: Values $\|\delta T\|$ for the boundary inverse problem depending on the random disturbance to the temperature measurement δ_{random}.

Table 1: Values $\|\delta T\|$ for the inverse boundary problem.

q	$\delta_{random} = 0$	$\delta_{random} = 0.01$	$\delta_{random} = 0.02$
2	1.72×10^{-9}	84.55	169.10
3	7.89×10^{-12}	13.48	26.96
4	1.04×10^{-11}	13.96	27.92
5	2.92×10^{-12}	2.60	5.21
6	7.21×10^{-11}	3.14	6.27
7	3.63×10^{-13}	0.90	1.80
8	6.97×10^{-13}	1.19	2.38
9	3.45×10^{-16}	0.90×10^{-2}	1.80×10^{-2}
q	$\delta_{random} = 0.03$	$\delta_{random} = 0.04$	$\delta_{random} = 0.05$
2	253.66	338.21	422.76
3	40.44	53.92	67.40
4	41.88	55.84	69.79
5	7.81	10.41	13.02
6	9.41	12.54	15.68
7	2.70	3.60	4.49
8	3.57	4.76	5.96
9	2.70×10^{-2}	3.60×10^{-2}	4.50×10^{-2}

surement δ_{random} causes linear increase of mean square deviation of the set values from the values calculated on the Γ_1 boundary (Fig. 11).



a) b)

Figure 10: Values $\|\delta T\|$ for quasi-Cauchy problem with temperature measurement in Chebyshev x_q and x_{q+1} nodes for q: a) from 2 and 3 to 8 and 9; b) from 4 and 5 to 8 and 9, including the random disturbance to the temperature measurement δ_{random}.

Table 2: Values $\|\delta T\|$ for quasi-Cauchy problem.

q	$\delta_{random} = 0$	$\delta_{random} = 0.01$	$\delta_{random} = 0.02$
2	8.14×10^{-12}	7.21	14.41
3	8.93×10^{-12}	10.42	20.84
4	3.28×10^{-12}	2.62	5.24
5	1.22×10^{-12}	0.75	1.50
6	7.73×10^{-13}	0.62	1.23
7	3.29×10^{-13}	0.87	1.75
8	3.42×10^{-16}	6.85×10^{-3}	1.37×10^{-2}
q	$\delta_{random} = 0.03$	$\delta_{random} = 0.04$	$\delta_{random} = 0.05$
2	21.62	28.82	36.03
3	31.25	41.67	52.09
4	7.87	10.49	13.11
5	2.25	3.01	3.76
6	1.85	2.47	3.09
7	2.62	3.49	4.37
8	2.06×10^{-2}	2.74×10^{-2}	3.4×10^{-2}

5 Conclusions

In this paper two types of inverse problem for the Laplace's equation in the rectangular domain were solved. The form of the solution was written as

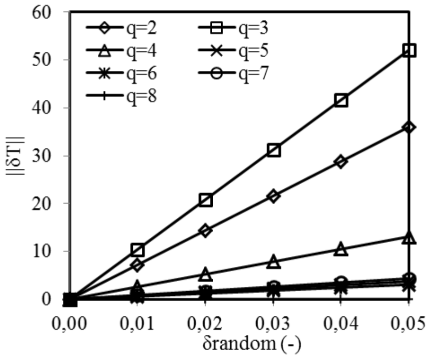


Figure 11: Values $\|\delta T\|$ for quasi-Cauchy problem depending on the random disturbance to the temperature measurement δ_{random}.

the linear combination of Chebyshev polynomials. Influence of changes in measuring points arrangement and random disturbance to the temperature measurement on the mean square deviation of the set values from the values calculated on the Γ_1 boundary was considered. Furthermore, the temperature distribution was sought on the Γ_1 boundary. Values $\|\delta T\|$ reach their maximum for $q = 2$ and $q = 3$, what results from the arrangement of collocation nodes. Calculation model brings positive results for measuring points situated in Chebyshev nodes closest to the $\Gamma_1$boundary, even for the disturbance to the temperature measurement $\delta_{random} = 0.05$. Relocation of measuring points into the Γ_3 boundary worsens significantly results of calculations. For calculations without disturbance to the measurement data ($\delta_{random} = 0$), the model returns positive results, irrespective of measuring points arrangement.

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