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EVALUATION OF UNCERTAINTY OF PHASE DIFFERENCE DETERMINATION IN PRESENCE OF BIAS

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Abstract

Determination of the phase difference between two sinusoidal signals with noise components using samples of these signals is of interest in many measurement systems. The samples of signals are processed by one of many algorithms, such as 7PSF, UQDE and MSAL, to determine the phase difference. The phase difference result must be accompanied with estimation of the measurement uncertainty. The following issues are covered in this paper: the MSAL algorithm background, the ways of treating the bias influence on the phase difference result, comparison of results obtained by applying MSAL and the other mentioned algorithms to the same real signal samples, and evaluation of the uncertainty of the phase difference.

Keywords: sinusoidal electrical signals, phase difference, bias, uncertainty of measurement, MSAL algorithm.

1. Introduction

The phase difference between two sinusoidal signals is of interest in many measurement systems employed in such applications as: calibration of standards for electric power, determination of complex ratios between the output and input signals of transducers, voltage dividers and current shunts, flow measurement, source localization, *etc.* [1–9].

The phase difference between two continuous sinusoidal signals is usually determined by means of discrete samples of signals processed by one of many algorithms used for phase difference estimation. Some of the algorithms are: the *three-parameter sinus fitting algorithm* (3PSF), the *four-parameter sinus fitting algorithm* (4PSF), the seven-parameter sinus fitting algorithm (7PSF) [10–13], the *unbiased quadrature delay estimator* (UQDE) [9], and the *modified simple algorithm* (MSAL) [14]. According to the MSAL algorithm, two new signals' phases shifted to the existing signals are introduced, and a complex formula is applied to a large number of signal samples to determine the phase difference between two sinusoidal signals.

The result of phase difference measurement must be accompanied with estimation of the uncertainty according to the international recommendation GUM [15]. The measurement uncertainty is defined as a non-negative parameter characterizing the dispersion of quantity values being attributed to a measured value based on the information used [16]. The type A, type B, combined standard uncertainty and expanded uncertainty are defined in [15] and applied here. The measurement uncertainty can include components arising from systematic effects which are sometimes not corrected [15].

The rest of the paper is arranged as follows: Section 2 introduces the basic principles of MSAL algorithm. In Section 3 the bias of phase difference results determined from real signal samples is presented. Section 4 presents comparison of real signals' phase difference results obtained with the MSAL algorithm with those obtained with other algorithms. Section 5 introduces evaluation of the uncertainty of phase difference result. The uncertainty estimation

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results for samples of two voltages are presented and the uncertainty of phase difference result obtained by applying the MSAL algorithm to those samples is estimated. Section 6 presents the conclusions.

2. MSAL algorithm

Two real-valued sinusoidal signals without a dc offset can be expressed as:

$$u_1(t) = U_1 \sin(\omega_0 t + \varphi_1) + n_1(t),$$
$$u_2(t) = U_2 \sin(\omega_0 t + \varphi_2) + n_2(t), \tag{1}$$

where: U_1 and U_2 are the amplitudes, ω_0 is the angular frequency, φ_1 and φ_2 are the initial phases, $n_1(t)$ and $n_2(t)$ are the uncorrelated additive white Gaussian noise values [14]. The phase difference to be determined is $\varphi = \varphi_1 - \varphi_2$.

To evaluate the phase difference φ, the phase-delayed signals in respect to (1) are introduced:

$$u_1'(t) = U_1 \sin(\omega_0 t + \varphi_1 - \psi_1) + n_1'(t),$$
$$u_2'(t) = U_2 \sin(\omega_0 t + \varphi_2 - \psi_2) + n_2'(t), \tag{2}$$

where ψ_1 and ψ_2 are the known phase delays, and $n_1'(t)$ and $n_2'(t)$ are the phase-shifted noise values $n_1(t)$ and $n_2(t)$, respectively.

Using the signals defined by (1) and (2) the following combinations of signals are formed:

$$m_a(t) = u_1(t) \cdot u_2(t) + u_1'(t) \cdot u_2'(t),$$
$$m_r(t) = u_1'(t) \cdot u_2(t) - u_1(t) \cdot u_2'(t). \tag{3}$$

Substituting (1) and (2) in (3) gives the following results:

$$\begin{aligned} m_a(t) = {} & U_1 U_2 \cos\left(\varphi_1 - \varphi_2 + \frac{\psi_2 - \psi_1}{2}\right) \cos\left(\frac{\psi_2 - \psi_1}{2}\right) \\ & - U_1 U_2 \cos\left(2\omega_0 t + \varphi_1 + \varphi_2 + \frac{\psi_1 + \psi_2}{2}\right) \cos\left(\frac{\psi_1 + \psi_2}{2}\right), \\ & + n_a(t) \end{aligned} \tag{4}$$

$$\begin{aligned} m_r(t) = {} & U_1 U_2 \sin\left(\varphi_1 - \varphi_2 + \frac{\psi_2 - \psi_1}{2}\right) \sin\left(\frac{\psi_1 + \psi_2}{2}\right) \\ & - U_1 U_2 \sin\left(2\omega_0 t + \varphi_1 + \varphi_2 + \frac{\psi_1 + \psi_2}{2}\right) \sin\left(\frac{\psi_2 - \psi_1}{2}\right), \\ & + n_r(t) \end{aligned}$$

where $n_a(t)$ and $n_r(t)$ contain all components with multiplicative noise. For uncorrelated white Gaussian noise the mean values of these components are zero.

For the sampled signals (1) and (2), with the sampling period T_s, the same results as given by (4) can be obtained for $m_a(kT_s)$ and $m_r(kT_s)$, computed at the instant kT_s, where k is an integer. In a digital system, it is easy to achieve the same introduced phase delay for both

Metrol. Meas. Syst., Vol. 23 (2016), No. 4, pp. 603–614.

introduced signals, *i.e.* $\psi_1 = \psi_2 = \psi$. For a discrete signal $u(kT_s)$, the phase delay can be obtained as a delay of its samples, so that $u'(kT_s) = u(kT_s - rT_s)$, where r is an integer.

When introduced, the phase delay is chosen to satisfy the condition:

$$\psi_1 = \psi_2 = \psi = (4l+1)\pi/2, \tag{5}$$

where l is an integer. Next, the equations can be obtained:

$$\begin{aligned} m_a(kT_s) &= U_1 U_2 \cos(\varphi_1 - \varphi_2) + n_a(kT_s), \\ m_r(kT_s) &= U_1 U_2 \sin(\varphi_1 - \varphi_2) + n_r(kT_s). \end{aligned} \tag{6}$$

For a high signal to noise ratio (*SNR*), the phase difference φ is obtained from (6):

$$\varphi = \varphi_1 - \varphi_2 \approx \arctan \frac{m_r(kT_s)}{m_a(kT_s)}, \tag{7}$$

and this formula is called the SAL algorithm. It should be noticed that this formula for calculations needs only four samples of signals, one sample of each of four signals.

In the case of coherent sampling, the relation between signal and sampling angular frequency is:

$$\omega_0/\omega_s = q/p, \tag{8}$$

where q and p are positive relative prime integers and $\omega_s = 2\pi/T_s$. According to (5) and (8), for the SAL algorithm, r must be equal to:

$$r = (4l+1)p/(4q). \tag{9}$$

When the equation (9) is not possible to satisfy, and/or *SNR* is low, M_a and M_r are defined instead of $m_a(kT_s)$ and $m_r(kT_s)$:

$$\begin{aligned} M_a &= \frac{1}{n}\sum_{k=0}^{n-1} m_a(kT_s) \approx U_1 U_2 \cos(\varphi_1 - \varphi_2), \\ M_r &= \frac{1}{n}\sum_{k=0}^{n-1} m_r(kT_s) \approx \sin(\psi) U_1 U_2 \sin(\varphi_1 - \varphi_2), \end{aligned} \tag{10}$$

where n is the number of samples and should be an integer multiple of p, and $\psi \neq a\pi$, where a is an integer. The introduced phase delay $\psi = r(q/p)2\pi$ may or may not satisfy (5). The estimated phase difference $\hat{\varphi}_{MSAL}$ can be calculated from (10) as [14]:

$$\hat{\varphi}_{MSAL} = \arctan \frac{\sum_{k=0}^{n-1} m_r(kT_s)}{\sin\left(r\frac{q}{p}2\pi\right)\sum_{k=0}^{n-1} m_a(kT_s)}. \tag{11}$$

This formula is called the *modified simple algorithm* (MSAL).

The detailed derivation of estimating the phase difference between two sinusoidal signals, as well as margins of errors for the SAL and MSAL algorithms in different situations is presented in [14]. The main advantage of these algorithms is reducing the computational complexity while maintaining good accuracy.

The values of phase difference determined using the MSAL algorithm applied to samples of two signals may contain a bias that comes from the measurement equipment and the fact that it

is not possible to get a sampling frequency satisfying the requirement (8) of absolutely coherent sampling [17–18]. The consequences are: the requirement (5), *i.e.* the introduced phase delay of exactly $\pi/2$, is not satisfied; the exact introduced phase delay is not known in (10).

The bias of an estimator is the difference between this estimator's expected value and the true value being estimated [19]. The measurement results can be corrected regarding the bias, but – since it is not required – an uncorrected bias can be included in the uncertainty statement [15].

3. Bias of phase difference measurement results

In the experimental measurement, two signals are generated by a phase standard, type 5500-2 Clarke Hess, with 50 Hz frequency and 10º phase difference. For each of the two signals 200 samples are taken in one signal period, simultaneously using two type 3458A Agilent digital multimeters as *sampling voltmeters* (SV). These sampling voltmeters have a high-resolution integrating ADC (IADC) that operates according to the dual slope principle. The sampling process is such that the sampling instant is not a discrete point in time, but the measured signal is integrated by IADC over the aperture time of AD converter [20–27].

The results of phase difference between two real sinusoidal signals determined using the MSAL algorithm, are shown in Table 1. For $r = 0$ and $r = 100$ samples, or the introduced phase delay equal to $a\pi$, the algorithm is not defined. Theoretically, the introduced phase delay may be contained in an interval $(a\pi, a\pi + \pi)$, but in practice the introduced phase delay near $(2l+1)\pi/2$ is recommended because the error of phase difference result that comes from the function sin(x) in (11) is minimal [14]. In this example, the number of samples in a signal period is divisible by 4, so that the introduced phase delay of approximately $\pi/2$ can be obtained with 50 sample delays. The result for this case was adopted as the reference value.

In Table 1 the bias values for phase difference results determined using the MSAL algorithm are presented in dependence of the introduced phase delay, for r samples. It can be seen that the bias has a value of –64 m° for $r = 99$ samples, or near π. The bias is much smaller when the introduced phase delay is obtained for near 50 samples, *i.e.* $\pi/2$.

Table 1. The bias value for phase difference between two sinusoidal signals determined using the MSAL algorithm for the phase delay (r samples) of introduced signals; an example with 200 samples in a period of each signal.

***r* (samples)**	1	10	15	30	40	50
φ [°]	10.002	10.002	10.002	10.001	10.001	10.001
bias δ [m°]	1	1	1	0	0	REF

***r* (samples)**	65	75	85	97	98	99
φ [°]	10.000	9.999	9.998	9.980	9.969	9.937
bias δ [m°]	–1	–2	–3	–21	–32	–64

4. Comparison of results

Next, there is presented the case of experimental measurement when the generated phase difference between two real sinusoidal signals with 53 Hz frequency is 59.9°. The samples are taken in 19 periods of each signal with 18 samples in a signal period. In contrast to the measurement in Section 3, the number of samples in a signal period is not divisible by 4 and because of that it is not possible to get $\pi/2$ for the introduced phase delay when applying the UQDE and MSAL algorithms.

Metrol. Meas. Syst., Vol. 23 (2016), No. 4, pp. 603–614.

The results of phase difference obtained by applying 7PSF, UQDE and MSAL to the same samples of two signals are shown in Table 2. MSAL4 denotes the MSAL algorithm when the introduced signals have the phase delay related to the original signals equal to 4 samples; MSAL5 denotes the MSAL algorithm when the introduced signals have the phase delay related to the original signals equal to 5 samples; UQDE4 and UQDE5 are denoted in an analogous way.

Table 2. The results of phase difference between two real sinusoidal signals determined by the 7PSF, UQDE4, UQDE5, MSAL4 and MSAL5 algorithms for chosen numbers of signal samples; an example with 18 samples in a period of each signal.

		7PSF algorithm	(UQDE4 –7PSF)	(UQDE5 –7PSF)	**(MSAL4 –7PSF)**	**(MSAL5 –7PSF)**	MSALav –7PSF
Samples	Periods	**φ [°]**	[m°]	[m°]	[m°]	[m°]	[m°]
18	1	**59.90009**	–381.11	–380.99	**0.90**	**1.01**	0.95
36	2	**59.90055**	–382.01	–381.86	**–0.004**	**0.15**	0.07
54	3	**59.90069**	–382.09	–381.97	**–0.09**	**0.04**	–0.02
72	4	**59.90076**	–382.04	–381.98	**–0.03**	**0.03**	–0.002
90	5	**59.90077**	–382.01	–382.00	**0.0001**	**0.01**	0.003
108	6	**59.90069**	–381.99	–381.99	**0.02**	**0.02**	0.02
126	7	**59.90038**	–381.98	–381.95	**0.03**	**0.06**	0.05
144	8	**59.90010**	–382.00	–381.97	**0.01**	**0.04**	0.03
162	9	**59.89989**	–382.03	–382.00	**–0.01**	**0.02**	0.001
180	10	**59.89968**	–382.01	–381.98	**0.01**	**0.04**	0.02
198	11	**59.89960**	–381.98	–381.97	**0.03**	**0.04**	0.04
342	19	**59.90114**	–381.96	–381.92	**0.05**	**0.08**	0.07

The phase difference results obtained using the UQDE and MSAL algorithms are compared with those obtained using the 7PSF algorithm because it is shown in [13] that the 7PSF algorithm gives the most accurate results.

In Table 2: (UQDE4 – 7PSF) in millidegrees [m°] denotes the difference between the phase difference results obtained using the UQDE4 and 7PSF algorithms; analogous meanings have: (UQDE5 – 7PSF), (MSAL4 – 7PSF), (MSAL5 – 7PSF), and (MSALav – 7PSF), where MSALav = (MSAL4+MSAL5)/2. The bias results obtained using all algorithms are compared with those obtained using 7PSF for estimation of phase difference. It can be seen in Table 2 that the minimum values are obtained in (MSAL4 – 7PSF) and (MSALav – 7PSF) columns. That means that the MSAL4 algorithm gives the phase difference results with almost the same accuracy as the 7PSF algorithm in the measurement points given in Table 2.

5. Uncertainty of phase difference measurement

Estimation of the uncertainty presented in this section concerns the phase difference measurement results obtained by applying the MSAL algorithm. The experimental measurement results from Section 4 are shown in dependence of the introduced phase delay in intervals: $(0, \pi)$ and $(\pi, 2\pi)$.

5.1. Estimation of uncertainty for samples of voltages

The sinusoidal voltage, without a dc offset and noise, applied at the SV input is given by [28]:

$$u(t) = U\sin(\omega_0 t) = U\sin\left(\frac{2\pi}{T_0 + \varepsilon_{JTo}}t\right), \tag{12}$$

where T_0 is a signal period and ε_{JTo} is a time jitter of the period T_0. The model of internal IADC of SV is given by the equation [25, 28]:

$$U_k = \frac{1}{T_i + \varepsilon_{JTi}}\left(1 + \varepsilon_{REF} + \varepsilon_G + \frac{\varepsilon_{LIN} + \varepsilon_{RES}}{Abs[U_k]}U_{FS}\right)\cdot \int_{kT_s + \varepsilon_{JTs}}^{kT_s + \varepsilon_{JTs} + T_i + \varepsilon_{JTi}} u(t)dt, \tag{13}$$

where: U_k is a sampled value at the instant kT_s; ε_{REF} is the deviation of ideal internal DC reference of SV; ε_G is the gain error of SV dependent on the integration time T_i; ε_{RES} is an error due to SV resolution; ε_{LIN} is an error due to linearity of SV; U_{FS} is the voltage range of SV; ε_{JTs} is the time jitter of the sampling period T_s; ε_{JTi} is the time jitter of the integration time T_i.

The standard uncertainty, type B [15], of each sample U_{1k} of the first voltage $u_1(t)$, taken by the first SV, denoted $u_{cB}(U_{1k})$ is:

$$\begin{aligned} u_{cB}(U_{1k}) = [&c_1^2u^2(\varepsilon_{REF1}) + c_2^2u^2(\varepsilon_{G1}) + c_3^2u^2(\varepsilon_{RES1}) + c_4^2u^2(\varepsilon_{LIN1}) + c_5^2u^2(\varepsilon_{JTo1}) \\ &+ c_6^2u^2(\varepsilon_{JTs1}) + c_7^2u^2(\varepsilon_{JTi1}) + u_{w1}^2]^{1/2} \end{aligned}, \tag{14}$$

where: c_1 to c_7 are sensitivity coefficients for each sample U_{1k}, $u(\varepsilon_{REF1})$, $u(\varepsilon_{G1})$, $u(\varepsilon_{RES1})$, $u(\varepsilon_{LIN1})$, $u(\varepsilon_{JTo1})$, $u(\varepsilon_{JTs1})$, $u(\varepsilon_{JTi1})$ are standard uncertainties due to ε_{REF1}, ε_{G1}, ε_{RES1}, ε_{LIN1}, ε_{JTs1} ε_{JTo1}, ε_{JTi1}, respectively, and u_{w1}^2 is the variance due to noise from the signal source and sampling the first signal. Some values of the uncertainties in (14) are derived from the manufacturer specifications of SV, whereas others – from the calibration certificate of SV.

The sensitivity coefficients [15] for each sample U_{1k} are calculated using (12) and (13) according to:

$$\begin{aligned} c_1 &= \frac{\partial U_{1k}}{\partial \varepsilon_{REF1}},\ c_2 = \frac{\partial U_{1k}}{\partial \varepsilon_{G1}},\ c_3 = \frac{\partial U_{1k}}{\partial \varepsilon_{RES1}},\ c_4 = \frac{\partial U_{1k}}{\partial \varepsilon_{LIN1}}, \\ c_5 &= \frac{\partial U_{1k}}{\partial \varepsilon_{JTo1}},\ c_6 = \frac{\partial U_{1k}}{\partial \varepsilon_{JTs1}},\ c_7 = \frac{\partial U_{1k}}{\partial \varepsilon_{JTi1}}. \end{aligned} \tag{15}$$

The standard uncertainty, type B, of each sample U_{2k} of the second voltage $u_2(t)$, taken by the second SV, denoted $u_{cB}(U_{2k})$, is given in an analogous way to (14):

$$\begin{aligned} u_{cB}(U_{2k}) = [&c_8^2u^2(\varepsilon_{REF2}) + c_9^2u^2(\varepsilon_{G2}) + c_{10}^2u^2(\varepsilon_{RES2}) + c_{11}^2u^2(\varepsilon_{LIN2}) + c_{12}^2u^2(\varepsilon_{JTo2}) \\ &+ c_{13}^2u^2(\varepsilon_{JTs2}) + c_{14}^2u^2(\varepsilon_{JTi2}) + u_{w2}^2]^{1/2}, \end{aligned} \tag{16}$$

where: c_8 to c_{14} are sensitivity coefficients for each sample U_{2k} and standard uncertainties have analogous explanations to those in (14). The sensitivity coefficients are calculated in an analogous way to (15).

The standard uncertainty type A of each sample is calculated from 19 periods of signal according to GUM [15]. The combined standard uncertainties $u_c(U_{1k})$ and $u_c(U_{2k})$ for each of 18 samples in a period of two signals, are calculated from the type A and type B uncertainties, and are shown in Table 3.

Metrol. Meas. Syst., Vol. 23 (2016), No. 4, pp. 603–614.

Table 3. The voltage samples U_{1k} and U_{2k} of two sinusoidal signals and the combined standard uncertainties $u_c(U_{1k})$ and $u_c(U_{2k})$ for an example with 18 samples in a period of each signal.

k	U_{1k}	$u_c(U_{1k})$	U_{2k}	$u_c(U_{2k})$
	[V]	[mV]	[V]	[mV]
0	0.129951	0.1785	1.031419	0.0935
1	0.504598	0.1554	1.122462	0.0717
2	0.818463	0.1286	1.078107	0.0776
3	1.033572	0.0922	0.903626	0.1070
4	1.124001	0.0703	0.620167	0.1362
5	1.078850	0.0796	0.262030	0.1555
6	0.903546	0.1098	–0.127721	0.1601
7	0.619273	0.1399	–0.502052	0.1493
8	0.260405	0.1584	–0.815880	0.1245
9	–0.129899	0.1622	–1.031253	0.0938
10	–0.504526	0.1498	–1.122210	0.0716
11	–0.818356	0.1248	–1.077772	0.0793
12	–1.033465	0.0927	–0.903321	0.1076
13	–1.123912	0.0704	–0.619898	0.1364
14	–1.078824	0.0793	–0.261827	0.1549
15	–0.903589	0.1097	0.127784	0.1586
16	–0.619357	0.1398	0.502035	0.1479
17	–0.260512	0.1605	0.815872	0.1186

5.2. Uncertainty of phase difference result obtained by MSAL algorithm

The equation (11) can be written as:

$$\hat{\varphi}_{MSAL} = \arctan \frac{\sum_{k=0}^{n-1}\left(M_{1k} - M_{3k}\right)}{\sin\left(r\frac{q}{p}2\pi\right)\sum_{k=0}^{n-1}\left(M_{2k} + M_{4k}\right)} = \arctan \frac{M_1 - M_3}{\sin\left(r\frac{q}{p}2\pi\right)\left(M_2 + M_4\right)}, \tag{17}$$

where:

$$M_{1k} = U'_{1k}U_{2k},\ M_{3k} = U_{1k}U'_{2k},\ M_{3k} = U_{1k}U'_{2k},\ M_{4k} = U'_{1k}U'_{2k}, \tag{18}$$

U'_{1k} are phase delayed samples U_{1k}, U'_{2k} are phase delayed samples U_{2k} and

$$M_1 = \sum_{k=0}^{n-1} M_{1k},\ M_2 = \sum_{k=0}^{n-1} M_{2k},\ M_3 = \sum_{k=0}^{n-1} M_{3k},\ M_4 = \sum_{k=0}^{n-1} M_{4k}. \tag{19}$$

The combined standard uncertainty for the phase difference $u_c(\hat{\varphi}_{MSAL})$ obtained using the MSAL algorithm, according to GUM [15], is:

$$\begin{aligned} u_c(\hat{\varphi}_{MSAL}) = [&c_{15}^2 u^2(M_1) + c_{16}^2 u^2(M_2) + c_{17}^2 u^2(M_3) + c_{18}^2 u^2(M_4) \\ &+ u_{cA}^2(\varphi) + 2c_{15}c_{16}u(M_1)u(M_2)r_C(M_1,M_2) + 2c_{15}c_{17}u(M_1)u(M_3)r_C(M_1,M_3) \\ &+ 2c_{15}c_{18}u(M_1)u(M_4)r_C(M_1,M_4) + 2c_{16}c_{17}u(M_2)u(M_3)r_C(M_2,M_3) \\ &+ 2c_{16}c_{18}u(M_2)u(M_4)r_C(M_2,M_4) + 2c_{17}c_{18}u(M_3)u(M_4)r_C(M_3,M_4)]^{1/2} \end{aligned}, \tag{20}$$

where $u_{cA}(\varphi)$ is the standard uncertainty type A, the correlation coefficient is r_C and sensitivity coefficients are calculated from the formulas:

$$c_{15}=\frac{\partial\hat{\varphi}_{MSAL}}{\partial M_1},\ c_{16}=\frac{\partial\hat{\varphi}_{MSAL}}{\partial M_2},\ c_{17}=\frac{\partial\hat{\varphi}_{MSAL}}{\partial M_3},\ c_{18}=\frac{\partial\hat{\varphi}_{MSAL}}{\partial M_4}. \tag{21}$$

The standard uncertainty type A for the phase difference $u_{cA}(\varphi)$ is determined from the phase difference values calculated for each of 19 periods of signals using the MSAL algorithm. From those 19 values, the standard deviation and standard uncertainty type A are calculated according to GUM.

The square of standard uncertainty $u^2(M_1)$ in (20) can be written as:

$$u^2(M_1)=u^2\left(\sum_{k=0}^{n-1}M_{1k}\right)=\sum_{k=0}^{n-1}u^2(M_{1k}). \tag{22}$$

In an analogous way the squares of standard uncertainty components $u^2(M_2)$, $u^2(M_3)$ and $u^2(M_4)$ in (20) can be written.

The square of standard uncertainty of M_{1k} is given by [15]:

$$\begin{aligned}u^2(M_{1k})&=\left(\frac{\partial M_{1k}}{\partial U'_{1k}}\right)^2u^2(U'_{1k})+\left(\frac{\partial M_{1k}}{\partial U_{2k}}\right)^2u^2(U_{2k})\\&+2\left(\frac{\partial M_{1k}}{\partial U'_{1k}}\right)\left(\frac{\partial M_{1k}}{\partial U_{2k}}\right)u(U'_{1k})\,u(U_{2k})\,r_C(U'_{1k},U_{2k})\\&=U_{2k}^2\,u^2(U'_{1k})+U_{1k}'^2\,u^2(U_{2k})+2U_{2k}\,U'_{1k}\,u(U'_{1k})\,u(U_{2k})\,r_C(U'_{1k},U_{2k})\end{aligned}, \tag{23}$$

where $r_C(U'_{1k},U_{2k})$ is the correlation coefficient between samples of two voltages: U'_{1k} and U_{2k}.

The squares of standard uncertainties: $u^2(M_{2k})$, $u^2(M_{3k})$, $u^2(M_{4k})$ can be obtained in an analogous way.

In the measurement system two SVs are used, whose bandwidths are not exactly known and – according to the manufacturer's specification, for a chosen measurement range - the instrument-to-instrument difference of the upper limit of bandwidth is estimated to be in the range between 140 kHz and 160 kHz. Simultaneous measurement of two voltages whose frequency is 53 Hz produces the phase difference error of 0.00271°, due to the influence of bandwidth [1]. Assuming a uniform distribution of possible values [15], the type B component of uncertainty in this case increases to 0.00157°.

The differences in delay time between EXT TRIG input channels of two SVs according to the manufacturer's specification may be up to 75 ns, and that can produce the phase angle error whose value is 0.00143° for signals with frequency 53 Hz [1]. Assuming a uniform distribution of possible values [15], the type B component of uncertainty in this case increases to 0.00083°.

The bias of results is a systematic error that can be applied as a correction of the measured values, but that is not essential and the bias can be expressed as a part of expanded uncertainty (GUM, F.2.4.5). Three methods are proposed for treating the uncorrected bias δ as a part of expanded uncertainty [29]:

1. „RSS u_c method" treats the uncorrected bias δ as another uncertainty source and sums it in an RSS (*root-sum-of-squares*) way with the combined standard uncertainty u_c:

$$U_{\mathrm{RSS}u_c}=k\sqrt{u_c^2+\delta^2} \tag{24}$$

2. „RSSU method" sums the bias δ in an RSS way with the expanded uncertainty ku_c:

$$U_{\mathrm{RSS}U}=\sqrt{k^2u_c^2+\delta^2} \tag{25}$$

Metrol. Meas. Syst., Vol. 23 (2016), No. 4, pp. 603–614.

3. „SUM*U* method“ algebraically sums the bias δ with the expanded uncertainty ku_c:

$$U_+ = \begin{cases} ku_c - \delta, & \text{if} \quad ku_c - \delta > 0 \\ 0, & \text{if} \quad ku_c - \delta \le 0 \end{cases} \quad \text{and} \quad U_- = \begin{cases} ku_c + \delta, & \text{if} \quad ku_c + \delta > 0 \\ 0, & \text{if} \quad ku_c + \delta \le 0. \end{cases} \tag{26}$$

In (24) to (26) u_c is the combined standard uncertainty, δ denotes the bias, k is a coverage factor, and ku_c is the expanded uncertainty.

The results for a phase difference of 59.9° between two sinusoidal signals determined using the MSAL algorithm applied to 19 periods of signals, together with the expanded uncertainties, are shown in Figs. 1–3.

It can be seen:

a) for „RSS u_c method“ the expanded uncertainty is:
from ± 4.2 m° (for r=4) to ± 14.9 m° (for r=17);

b) for „RSS*U* method“ the expanded uncertainty is:
from ± 3.9 m° (for r=1) to ± 8.2 m° (for r=17);

c) for „SUM*U* method“ the expanded uncertainties are:
for r =4, U_+ = 4.4 m° and U_- = 4.0 m°;
for r = 8, U_+ = 0.8 m° and U_- = 6.8 m°;
for r =14, U_+ = 3.8 m° and U_- = 4.6 m°;
for r =17, U_+ = 0.0 m° and U_- = 11.0 m°.

The most realistic case, shown in Fig. 3, is obtained with application of „SUM*U* method“. The same conclusion is presented in [29].

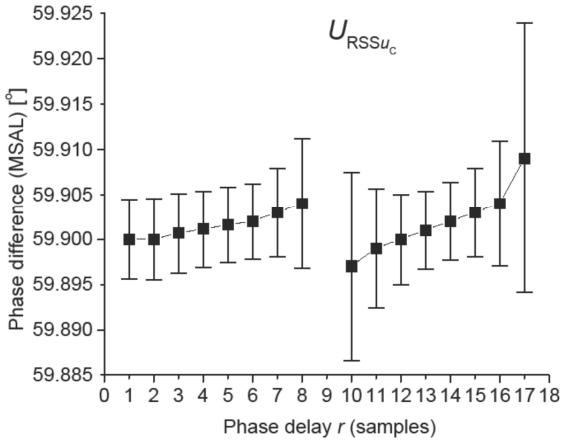

Fig. 1. The RSS u_c method for treating the uncorrected bias in evaluation of the uncertainty of the phase difference 59.9° between two real sinusoidal signals determined using the MSAL algorithm.

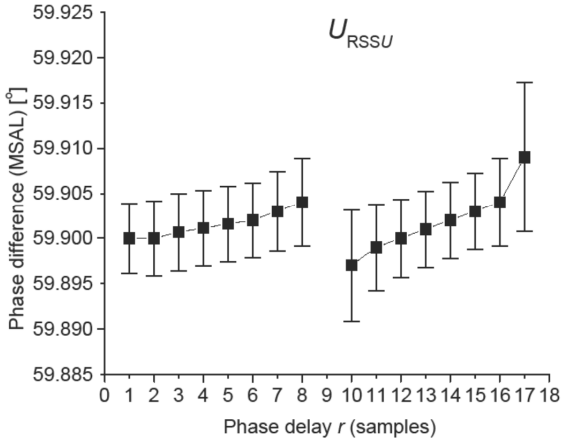

Fig. 2. The RSS*U* method for treating the uncorrected bias in evaluation of the uncertainty of the phase difference 59.9° between two real sinusoidal signals determined using the MSAL algorithm.

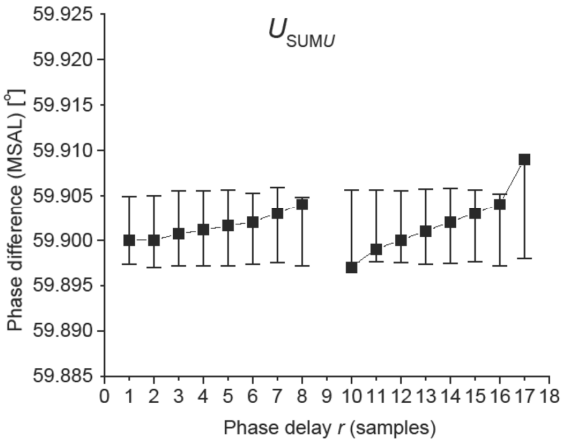

Fig. 3. The SUM*U* method for treating the uncorrected bias in evaluation of the uncertainty of the phase difference 59.9° between two real sinusoidal signals determined using the MSAL algorithm.

6. Conclusions

The phase difference between two sinusoidal signals is usually determined by using samples of signals processed by an algorithm estimating the phase difference. This paper presents application of the MSAL algorithm for determination of the phase difference and comparison of results with the ones obtained by using the 7PSF and UQDE algorithms. The differences between the phase difference results obtained by the MSAL algorithm, for which frequency is not known and those obtained by the 7PSF algorithm, for which frequency is known, are equal to parts of millidegrees.

The uncertainty of phase difference between two sinusoidal signals determined using the MSAL algorithm is estimated according to GUM. The bias of results is not applied as

Metrol. Meas. Syst., Vol. 23 (2016), No. 4, pp. 603–614.

a correction of the measured values, but it is expressed – using three methods – as a part of the expanded uncertainty. The conclusion is that the most realistic in this case is the „SUMU method“ which algebraically sums the bias with the expanded uncertainty.

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