Central European Journal of Economic Modelling and Econometrics

The Purchasing Power Parity Puzzle and Imperfect Knowledge: The Case of the Polish Zloty

Robert Kelm*

Submitted: 16.12.2016, Accepted: 11.01.2017

Abstract

A review of the contemporary mainstream literature on exchange rate modelling clearly indicates that the rational expectations hypothesis (RE) is almost invariably taken as a point of reference in empirical investigations. This paper tests the RE hypothesis for the Polish foreign exchange market within the Roman Frydman and Michael Goldberg model that builds on the hypothesis of imperfect knowledge economics (IKE). The employed modelling strategy consists in the formulation of assumptions about the persistence of nominal rate, prices and interest rates and of the verification of competing scenarios congruent with RE and IKE. As a result of the analysis, the RE hypothesis is rejected in favour of the IKE alternative.

Keywords: purchasing power parity, expectations, econometric modelling, cointegration, transition economies

JEL Classification: E31, F31, C51, C32

*University of Łódź; e-mail: emfrok@uni.lodz.pl

1 Introduction

The number of papers dealing with purchasing power parity (hereafter PPP) is substantial and keeps growing, but the hypothesis about international price arbitrage still gives rise to serious reservations. From the review of the literature it follows that empirical studies of PPP have not solved yet the puzzle formulated some 20 years ago by Kenneth Rogoff (1996) and that the nominal rigidities and market frictions are still unable to explain why real exchange rates (RER) deviate from the PPP level and high estimates of RERs' half-lives (3-5 years).

The empirical literature on the PPP hypothesis and the PPP model has several strands, but very broadly, it can be divided into studies that directly attempt to confirm the PPP hypothesis and those relaxing some of the overly restrictive assumptions of the law of one price. The historically earliest strand that now seems to be passing into oblivion utilizes the linear tests of the stationarity of real rates and employs vector error correction models (VEC) to investigate whether both nominal rates and domestic and foreign prices are driven by common stochastic trends. The conclusions from direct testing of the PPP hypothesis and of the PPP model are now deemed stylized facts that can be summed up by stating that the PPP hypothesis is difficult to confirm unless long-span time series or large panel data sets are available. Moreover, even when large data sets are used and RERs' difference-stationarity is rejected, the estimates of half-lives point to the persistence of real rates that is difficult to interpret; for extensive reviews of the early investigations see Froot and Rogoff (1995), Sarno and Taylor (2002), Taylor (2002), MacDonald (2007) and more recent monograph by James *et al.* (2012).

Recent literature provides evidence that studies relaxing some assumptions inherent to the PPP hypothesis and the PPP model can be more useful for explaining the PPP puzzle. This strand of the literature comprises (i) analyses that acknowledge the existence of non-zero transaction costs and allow RERs to adjust non-linearly to constant equilibrium level (following Dumas 1992, Sercu *et al.* 1995 and recently Pavlidis *et al.* 2011), and (ii) analyses that allow the possibility of non-linear smooth changes in the real rate equilibrium level. The number of studies that formally confirm the first type of non-linearity is increasing, but Stephen Norman's recent conclusion 'that N[onlinear] M[ean] R[eversion] is a resolution to the PPP puzzle' (Norman 2010, p. 936) seems premature, because the rejection of the hypothesis of real rates' difference-stationarity in favour of more or less 'capacious' non-linear alternatives is by no means a rule (Kapetanios *et al.* 2003, Bahmani-Oskooee *et al.* 2007, Kim and Moh 2010, Cuestas and Regis 2013). The second type of non-linearity appears to result from the impact of medium- and short-term factors on the real exchange rate, which are either approximated by non-linear deterministic trends (Sollis 2005, Cushman and Michael 2011) or by periodic functions (e.g. Fourier function in Su *et al.* 2011; Chang *et al.* 2012; Yilanci and Eris 2013). Serious doubts arise, again, because it is not clear how the unit root tests (hereafter URT) should be interpreted when the RER's difference-stationarity is not rejected in favour of any specific alternative hypothesis. Nor is it clear in what respect the non-linear analysis could be superior to analyses performed with the fully specified models.

Because the results of the univariate and panel unit root tests are ambiguous and the mainstream literature fails to provide a clear answer to the PPP puzzle, a much more general question must be asked about whether the existing theoretical framework for PPP analyses is adequate. This question is rather touchy, because the properties of most theoretical models are still determined by the rational expectations assumption, causing the empirical studies on purchasing power parity to seek 'any form' of RERs' stationarity. Doubts surrounding the rational expectations hypothesis are still there, though. Just before the global financial crisis that unfolded after the collapse of the Lehmann Brothers (hereafter the *subprime crisis*) . Frydman and Goldberg (2007) published a powerful critique of the RE hypothesis and proposed replacing it with the imperfect knowledge economics hypothesis (IKE). According to Frydman and Goldberg (2007), (2013a) and Frydman *et al.* (2015), the main weakness of the RE-based models lies in the presumption that the scale and timing of a non-repetitive and unforeseeable structural change can be characterized *ex ante* by means of some density function. In the real world, this assumption is violated by investors' imperfect knowledge and the RE-based scheme of expectations formation requires revisions. Frydman and Goldberg (2007) underline the role of the psychological determinants behind investors' conservatism and assume that heterogeneous individuals are rational in that they try to seize all opportunities for profits and use different forecasting strategies that vary in time and cannot be pre-specified in advance (Frydman and Goldberg 2007, 2013b, Frydman *et al.*; also Frydman *et al.* 2008; Juselius 2011, 2013, 2015). In line with Kahneman and Tversky's (1979) prospect theory, investors are loss-averse rather than risk-averse and make their entry to a foreign exchange market contingent on some minimum return on their capital (represented by an uncertainty premium). Increasing discrepancy between the current exchange rate and its long-term value causes that the uncertainty premium rises until investors 'invert' their forecasting strategies. Last but not least, the IKE-based model de-links prices adjustments in the commodity markets from the long-lasting swings in the exchange rates, thus making pointless the question about the persistence of the real exchange rate. In the IKE-world, the real exchange rate is generated by random walk with temporally unstable drifts, and the goods market equilibrium is defined by the real exchange rate's stationary relationship with the real interest rate differential.

This paper seeks to establish which of the two hypotheses - rational expectations or imperfect knowledge economics – is more efficient in explaining the Polish foreign exchange market in the free float period 1999:01-2011:06. The structure of the paper is the following. In section 2, the main properties and predictions of the Frydman-Goldberg model are outlined. Because the IKE hypothesis implies that some of the nominal variables are integrated of order 2 (hereafter $I(2)$ variables) and other exhibit near-$I(2)$-ness, some basic properties of the VEC models are revisited and the problems with structuring the cointegrating vectors are presented. Section 3 presents arguments against the use of overly simplified one-dimensional models in the exchange rate studies. The linear and non-linear unit root tests generally show that the real zloty/euro exchange rate is stationary, but the cointegration tests in the VEC models specified according to the RE hypothesis clearly point to the absence of cointegration and/or of evident symptoms of $I(2)$-ness. Because the PPP is finally rejected as an equilibrium relationship, section 4 thoroughly discusses the cointegration analysis of VEC models structured after the predictions of the Frydman-Goldberg model that also fails to identify equilibrium relationships characterizing the zloty/euro exchange rate unless two other specific gap effects – internal and external risk premiums – are considered. This result shows that in periods of deep risk adjustments equilibrium in the IKE-based models may be determined by a combination of equilibria in the goods and foreign exchange markets.

2 The RE and IKE: hypotheses and scenarios

The conclusions from the overview of empirical PPP analyses support the thesis that the non-linear models, various variants of the monetary model and the BEER-type (*behavioural equilibrium exchange rate*, see: Clark and MacDonald 1999) eclectic medium-term models allow overcoming some of the overly restrictive assumptions of the PPP model. However, the overview also shows quite clearly that the assumption about rational expectations of the agents remains 'unshakable'. The implications of the RE hypothesis are well known. In the Dornbusch-type monetary model, the domestic and foreign real interest rates (r and r^*, respectively) become equal in *steady state*$E(r_t - r_t^*) = 0$, which forces the RER throughout uncovered interest parity (UIP) go towards the PPP level, $E(q_t) = q^{PPP}$ (equivalently, $q_t \sim I(0)$, where $q = b - p + p^*$, b – log of nominal exchange rate, p, p^* – logs of domestic and foreign prices).
Juselius (2015) indicates that under very general conditions the rational expectations hypothesis still gives solid grounds to for expecting that domestic and foreign prices will be generated by processes with double-cumulated shocks $\sum_{j=1}^{t} \sum_{i=1}^{j} \varepsilon_i$, i.e. by the $I(2)$ trend that initially appears as $I(1)$ trend $\sum_{i=1}^{t} \varepsilon_i$ driving the nominal interest rate $i_t = i_{t-1} + \varepsilon_t$. In general, $I(2)$-ness of domestic and foreign prices does not violate rational expectations and purchasing power parity as long as domestic and foreign prices follow the same $I(2)$ trend that finally cancels in cointegrating relations.
An empirical investigation making use of the IKE assumptions is much more complex. Juselius (2015) outlines the consequences of introducing an uncertainty premium into the standard UIP relation, i.e. of replacing an RE-congruent process $i_t = i_{t-1} + \varepsilon_t$ with its IKE counterpart $i_t = i_{t-1} + \vartheta_t + \eta_t$, where ϑ denotes a change in the domestic uncertainty premium. The latter is characterised by substantial persistence and should be approximated by a near $I(1)$ process $\vartheta_t = \rho_t \vartheta_{t-1} + \varepsilon_t^{\vartheta}$,where the values of ρ_t are close to unity in some subperiods. The near difference-stationarity of ϑ_t has profound consequences, because the uncertainty premium cumulates to near-$I(2)$ stochastic trend $\sum_{i=1}^{t} \vartheta_i$ in the nominal exchange rate and does not disappear when

$\sum_{i=1}^{t} \vartheta_i$ is combined with stochastic trends driving the prices. Therefore, the real exchange rate is driven by the same near-$I(2)$ trend $q_t = q_{t-1} + \vartheta_t + \varsigma_t$. Even this cursory discussion of the main predictions of Frydman and Goldberg's model allows at least three conclusions to be drawn. First, the IKE world is free of the PPP puzzle. As long as the revisions of forecasts are conservative, the real exchange rate cannot be mean-reverting and the question about its persistence is pointless. Second, in the case of RERs' near I(2)-ness, the customary procedures for confronting real rates' difference-stationarity with some forms of mean-reversion are incomplete. Third, the second order of integration of the nominal exchange rates and of the domestic and foreign prices requires the appropriate econometric techniques to be applied. If the exchange rates swings continue for a longer time, a standard vector error correction model VEC-$I(1)$:

$$\Delta y_{(m)t} = \Pi y_{(m)t-1} + \sum_{s=1}^{S-1} \Gamma_s \Delta y_{(m)t-s} + \mu_{(m)} + \varepsilon_{(m)t} \tag{1}$$

needs to be replaced with a more flexible VEC-$I(2)$ model (e.g. Johansen 1995):

$$\Delta^2 y_{(m)t} = \Pi y_{(m)t-1} + \Gamma \Delta y_{(m)t-1} + \sum_{s=1}^{S-2} \Phi_s \Delta^2 y_{(m)t-s} + \mu_{(m)} + \varepsilon_{(m)t}, \tag{2}$$

where: $y_{(m)}$ – the vector of M variables, Π – total multipliers matrix, $\Gamma = -(I - \sum_{s=1}^{S-1} \Gamma_s)$ – the matrix of the medium-term multipliers, $\Gamma : [M \times M]$, $\Phi_s = -\sum_{j=s+1}^{S-1} \Gamma_j$ – short-term parameters, $\Phi_s : [M \times M]$, $\mu_{(m)}$ – constant term. Most empirical analyses focus exclusively on the difference-stationarity of $y_{(m)t}$. This amounts to assuming that the $y_{(m)t}$ components are determined by S pushing common stochastic $I(1)$ trends and by $V = M - S$ attracting cointegrating vectors. Consequently, the total multipliers matrix Π has reduced rank and can be decomposed into adjustments and cointegrating matrices (short term and deterministic components have been omitted):

$$\Delta y_{(m)t} = \alpha \left(\beta' y_{(m)t-1} \right) + \mu_{(m)} + \varepsilon_{(m)t} \tag{3}$$

with stationary $CI(1,1)$ cointegrating relations $\beta' y_{(m)t} \sim I(0)$, $\alpha, \beta : [M \times V]$. The analysis becomes more complicated, however, when $y_{(m)t}$ is driven by $I(2)$ stochastic trends and model (3) is replaced by its isomorphic $I(2)$ transformation:

$$\Delta^2 y_{(m)t} = \alpha \left(\beta' y_{(m)t-1} + \delta' \Delta y_{(m)t-1} \right) + \zeta \tau' \Delta y_{(m)t-1} + \varepsilon_{(m)t}, \tag{4}$$

where δ – the matrix of the dynamic equilibrium parameters, τ – the matrix of the medium-term equilibrium parameters, $\tau = (\beta, \beta_{\perp 1})$, $\beta_{\perp 1}$ – the submatrix of $\beta_\perp$, $\beta_\perp$ – the orthogonal complement of β, ζ – the matrix of the adjustment parameters, $\delta : [M \times V]$, $\tau : [M \times M - S_2]$, $\zeta : [M \times M - S_2]$, $\beta_{\perp 1} : [M \times S_1]$, S_2 – the number of the $I(2)$ stochastic trends, S_1 – the number of the autonomous stochastic $I(1)$ trends. Following from the assumption that the components of $y_{(m)t} \sim I(2)$ are cointegrated, it is possible to identify cointegrating vectors that in the general case define nonstationary $CI(2{,}1)$ relations, $\beta' y_{(m)t} \sim I(1)$, where $V = M - S_1 - S_2$. The identification of the stationary relations is possible when the linear combinations $\beta' y_{(m)t}$ cointegrate with the first differences of the $I(2)$ variables:

$$\left(\beta' y_{(m)t} + \delta' \Delta y_{(m)t}\right) \sim I(0). \tag{5}$$

Equation (5) defines the dynamic equilibrium of the VEC-$I(2)$ model (polynomial cointegration). Matrix $\tau = (\beta, \beta_{\perp 1})$ identifies V stationary linear combinations of first-differenced variables $\beta' \Delta y_{(m)t} \sim I(0)$ and S_1 medium-run equilibrium relations $\beta'_{\perp 1} \Delta y_{(m)t} \sim I(0)$.

The dynamic structure of the cointegrating component of the VEC-$I(2)$ model causes that the scale of the problems that need to be solved is much bigger than when dealing with the standard VEC- $I(1)$ model. A case in point is the situation when $V = 1$, $S_2 = 1$ and $S_1 = 1$ in a PPP model in which $p_t, p_t^* \sim I(2)$ and $b_t \sim I(1)$. Then, the nominal rate should be analysed using the following equation:

$$\begin{aligned} \Delta^2 b_t \quad = \quad & \alpha \left(\beta_1 b_{t-1} + \beta_2 p_{t-1} + \beta_3 p^*_{t-1} + \delta_1 \Delta b_{t-1} + \delta_2 \Delta p_{t-1} + \delta_3 \Delta p^*_{t-1}\right) + \\ & + \zeta_1 \left(\beta_1 \Delta b_{t-1} + \beta_2 \Delta p_{t-1} + \beta_3 \Delta p^*_{t-1}\right) + \\ & + \zeta_2 \left(\beta_{\perp 1,1} \Delta b_{t-1} + \beta_{\perp 1,2} \Delta p_{t-1} + \beta_{\perp 1,3} \Delta p^*_{t-1}\right) + \varepsilon_{Bt}. \end{aligned} \tag{6}$$

that is fundamentally different from the routinely estimated form:

$$\Delta b_t = \alpha \left(\beta_1 b_{t-1} + \beta_2 p_{t-1} + \beta_3 p^*_{t-1}\right) + \varepsilon_{Bt}. \tag{7}$$

It is easy to see that $I(2)$-ness introduces nonlinearity into the VEC model and that the imposition of *a priori* restrictions on the cointegrating vectors corresponding to the alternative variants of the monetary model may be somewhat problematic. This is why the focus of the solution proposed by Katarina Juselius (2006, 2015) – consisting in the construction of theory-consistent scenarios – shifts from structuring the cointegrating vectors to analysing the propagation of $I(2)$ and $I(1)$ shocks in the model. The idea behind the scenarios is the following: if the theoretical model is correct, it can be used to identify the direction of the diffusion of $I(2)$ and $I(1)$ shocks and to find out which variables absorb them. This means that the time series under consideration have testable regularities that allow discriminating between the variants of the monetary model and thereby between the IKE and RE hypotheses.

Tab. 1 provides a summary of the regularities that can be identified in different RE-based variants of the PPP model. The PPP1 scenario presents a case with nominal variables integrated of order one and a stationary real exchange rate. Scenarios 2–3 identify cointegrating vectors that are also appropriate for the RE hypothesis: (i) prices are driven by a common $I(2)$ trend ($S_2 = 1$) that cancels in relative prices $p - p^*$, (ii) the nominal exchange rate is affected by $I(1)$ trend that also drives

Table 1: Theory-consistent scenarios in the PPP model

PPP1	Assumptions:	$V=1,\ S_2=0,\ S_1=2$	$b_t,\ p_t,\ p_t^* \sim I(1)$
	Coint. vector 1:	$b-p+p^* \sim I(0)$	
PPP2	Assumptions:	$V=2,\ S_2=1,\ S_1=0$	$p_t,\ p_t^* \sim I(2),\ b_t \sim I(1)$
	Coint. vector 1:	$b-p+p^* \sim I(0)$	
	Coint. vector 2:	$b+\bar{c}_1\Delta p \sim I(0)$	
PPP3	Assumptions:	$V=1,\ S_2=1,\ S_1=1$	$p_t,\ p_t^* \sim I(2),\ b_t \sim I(1)$
	Coint. vector 1:	$(b-p+p^*)+\bar{c}_2\Delta p \sim I(0)$	
	Medium term:	$\beta_{\perp 1,1}\Delta b+\beta_{\perp 1,2}\Delta p+\beta_{\perp 1,3}\Delta p^* \sim I(0)$	

domestic (or foreign) prices and cancels in $(b+\bar{c}_1\Delta p) \sim I(0)$ or $(q+\bar{c}_2\Delta p) \sim I(0)$. These two polynomial cointegrating relations unveil the simplifications of the routine PPP-VEC-$I(1)$ models. For $y_{(m)t} \sim I(2)$, the omission of the dynamic components from the equilibrium relationships may lead to relations containing a moderate $I(1)$ component. In this case, the estimates of the error correction terms would be very precise and would 'testify' to strong persistence of the real rates that is difficult to explain within the RE hypothesis. The PPP2–PPP3 scenarios show non-linear adjustments in the polynomial cointegrating vectors.

Summing up, in the RE hypothesis the nominal exchange rate and nominal interest rates are $I(1)$ variables, but prices can be represented by $I(1)$ variables or $I(2)$ variables. Generally, the real exchange rate q_t, the real interest rates r_t and r_t^*, and the spread of the nominal interest rates $i_t - i_t^*$ need to be stationary. However, in relatively short samples the stationarity of these variables may not be evident. For example, the real exchange rate and the spread of nominal interest rates may behave like the $I(1)$ variables, but they should cointegrate:

$$(b_t - p_t + p_t^*) + \omega_1 (i_t - i_t^*) \sim I(0). \tag{8}$$

A comparison of the above RE-based scenarios with the predictions of Frydman and Goldberg's model enables the creation of clear-cut criteria for discriminating between the competing RE and IKE variants of the monetary model. To construct IKE-based scenarios, the nominal interest rates must be explicitly included into the model and the presence of $I(2)$ trend in prices, $p_t, p_t^* \sim I(2)$ and the near $I(2)$ trend in nominal exchange rate and interest rates, $b_t, i_t, i_t^* \sim$ near $I(2)$ must be taken into account. Then, depending on the number of autonomous $I(1)$ trends, two similar types of regularities can be identified in the time series. For $S_1 = 1$, two polynomial cointegrating vectors:

$$(b_t - p_t + p_t^*) + \omega_1 (i_t - i_t^*) + \omega_2 (\Delta p_t - \Delta p_t^*) \sim I(0) \tag{9}$$

$$i_t + \omega_3 \Delta p_t + \omega_4 p_t + \omega_5 b_t + \omega_6 t \sim I(0) \tag{10}$$

and one medium-term condition can be identified (Juselius 2015). For $S_1 = 0$, one can construct a VEC model 'spanned' by (9)–(10) and a third relation:

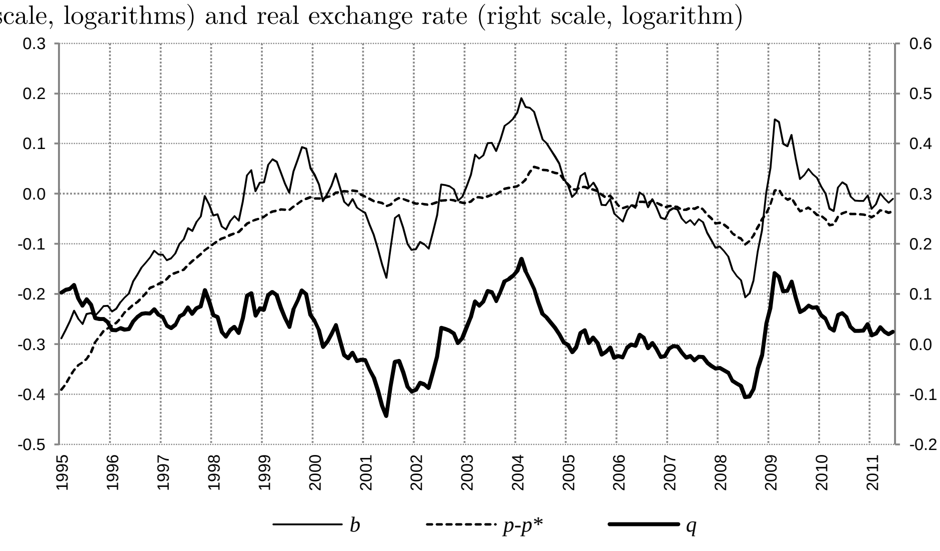
$$i_t^* + \omega_7 \Delta p_t^* + \omega_8 p_t^* + \omega_9 b_t + \omega_{10} t \sim I(0). \tag{11}$$

None of these three stationary relations (9)–(11) can be reduced to stationary parities implied by the RE variant of the model; relation (8) in the IKE model is non-stationary either.

3 An unsuccessful REH

The review of the literature shows that before the subprime crisis the studies on the exchange rates of the currencies of transition countries in Central and Eastern Europe mainly sought to explain the causes of appreciation trends in the rates deflated by general price indices (e.g. Égert *et al.* 2006 and references therein). To avoid a discussion about the medium-term effect productivity gains and changes in the structure of demand in Poland on the zloty/euro rate that is secondary at this point, in the next part of the paper the analysis will focus on the relationships between the nominal zloty/euro rate and price indices in tradable sectors (manufacturing) in Poland and the euro zone. The definitions of variables and data sources are covered at length in the Appendix.

Figure 1: Nominal exchange rate and relative prices in Poland and the euro area (left scale, logarithms) and real exchange rate (right scale, logarithm)



A visual inspection of the monthly real and nominal zloty/euro exchange rates in the period 1995:01-2011:06 (Fig. 1) confirms the heterogeneous nature of the exchange rate regime in Poland – until 1998, mean-reversion of the RER in the crawling peg and crawling band clearly contrasts with RER fluctuations between 1999 and 2011, i.e. in the period of effectively free and free float (the free float regime, officially announced in April 2000, became effective after the last major intervention NBP made in the foreign exchange market in February 1998). It is easy to see that between 1999 and 2011 both rates' fluctuations involved one long-lasting appreciation trend (after Poland's entry into the EU and before the *subprime* crisis), three shorter one-direction drifts and a deep depreciation of the zloty at the height of the *subprime* crisis. Two interpretations of this behaviour of the zloty/euro exchange rate are possible. *Firstly*, because the RER seems to be generally mean-reverting, one can simply adopt an 'orthodox' RE perspective and run different Dickey-Fuller-type unit root tests (URTs). Because the results of the linear URTs for the period 1999:01-2011:06 are borderline and the outcomes of the non-linear URTs proposed by Kapetanios et al. (2003) and Sollis (2009) explicitly point to RERs' global stationarity, a strong supporter of the REH-based interpretation of the PPP could terminate investigation exactly at this point and interpret the results as generally supporting the RE hypothesis.

There are several serious reasons why deriving preference for the RE hypothesis from the URTs might be premature, not in the least because of the tests' low power. It is particularly doubtful whether the standard URTs are useful at all for assessing the order of integration of the RERs. Both linear and non-linear tests of RERs' stationarity yield results that are conditional on arbitrary assumptions of long- and short-term homogeneity. A long-term homogeneity restriction means that the verification of the PPP hypothesis comes down to the simultaneous verification of (i) the hypothesis about common stochastic trends being present in the processes generating nominal exchange rate, domestic and foreign prices, and (ii) the hypothesis about the equality of equilibrium parameters β. At the same time, the short-term homogeneity restriction implies that the nominal variables respond identically to disequilibria (via α) and that the short-term dynamic patterns are identical. The effects of enforced short-term homogeneity restriction may be consequential. For example, the results of the linear DF-type tests presented in this paper are borderline, but the results of Johansen's stationarity test (Johansen and Juselius, 1992) for all variants of the VEC with real exchange rate q explicitly reject the null about stationary RER. The differences between these two approaches are considerable, because the URTs impose short-term homogeneity and 'strip' the tested variable of all economic 'context', unlike Johansen's test that does not assume short-term homogeneity and – performed with fully-specified VEC models – additionally 'enhances' the variable with a set of 'contextual' covariates. For these reasons, even when an exchange rate study is limited to the RE hypothesis, it is the VEC model with the nominal exchange rate and prices, $y = [b, p, p^*]'$ that must be analysed rather than one-dimensional models with $y = [q]'$.

The *second* interpretation of the fluctuations of the zloty/euro exchange rates can follow the IKE hypothesis according to which there is no mean-reversion in the real zloty/euro rate but only (i) a few one-direction, 'conservative' drifts reflecting the formation of the IKE-type expectations, and (ii) a few adjustments that individuals occasionally make to their forecasting strategies, which change the directions of these drifts. As a result, the real exchange rate temporarily moves towards the mean, but as parity is reached, the rate continues to increase or decrease showing no tendency to stabilize around the parity. Frydman *et al.* (2008) refer to a range of factors (psychological, political, etc.) that may cause the economic agents to revise their forecasts, placing at the top of the list attaching the greatest importance to the uncertainty premium that increases as the deviation of the real exchange rate from its long-run PPP value grows stronger. It is also likely that the turning points in the RERs occurred not only because of RERs' considerable deviations from parity, but also due to the impact of specific supplementary gap effects (e.g. *gap-plus* model in Frydman and Goldberg 2007. chapter 12). In the case of the zloty/euro real exchange rate, Kelm (2010) has noticed that as the expectations of 'inevitable' currency appreciation before the subprime crisis were fanned by the increasing misperception of global risks, the abrupt depreciation of the exchange rate in the second half of 2008 may present itself as an equilibrium-restoring process. The two other changes of the signs of the RER's drifts may be attributed to expanding fiscal deficits in the early 2000s and Poland's entry into the European Union in May 2004.

Paradoxically, the aforementioned results of non-linear URTs indicate that the zloty/euro exchange rate can be shaped by the IKE-driven drifts and forecast revisions. Because the KSS test (Kapetanios et al. 2003) rejects the real rate's difference-stationarity in favour of the ESTAR-type nonlinear adjustments to the parity, it is possible to perform a deeper analysis of the estimates of the second-order logistic exponential smooth transition autoregressive (STAR) model, i.e.:

$$\Delta q_t = \alpha_1 q_{t-1} + \tilde{\alpha}_1 q_{t-1} \cdot G\left(\theta, c_1, c_2\right) + \varepsilon_t, \tag{12}$$

where $G\left(\theta, c_1, c_2\right) = \left(1 + \exp\left(-\theta\left(q_{t-D} - c_1\right)\left(q_{t-D} - c_2\right)\right)\right)^{-1}$. The estimation results rise some doubts as to whether the non-linear URTs are conclusive (see Tab. 2). The error correction terms in the outer regimes confirm RER's global stationarity ($ECT = -0.27$, $HL = 2.2$ months), but mean-reversion is only supported by a small number of extreme RER's deviations from the parity observed in the short periods when the one-way drifts of the real rate were diverting. Further, the value of the optimal delay $D = 12$ is difficult to interpret, because by introducing substantial 'hidden persistence' into the model it prevents the STAR model from solving the PPP puzzle. The inner regime embraces about 85% of RER's observations resulting from the random walk and forming one-direction drifts. Lastly, the positive estimate of the ECT for the inner regime ($ECT = 0.03$) implies that a small explosive root may be present in the data generation process (DGP) and that the same root may be 'smoothly' driving the real exchange rate outside the non-arbitrage interval. Summing up, the estimation results of the STAR model of the zloty/euro real exchange rate can easily interpreted as directly following from the predictions of the Frydman-Goldberg model: (i) rare mean-reversions occur only when the baseline drifts revert, implying that the agents are 'non-rational' for most of the analysed period, (ii) the transient explosive root appears to confirm the occurrence of non-repetitive and unforeseeable structural changes in the zloty/euro exchange rate that cannot be characterized *ex ante* by means of some known density function.

Table 2: The STAR model (16), 1999:01-2011:06

Inner regime	Outer regime (change)	Transition parameters		
α_1	$\tilde{\alpha}_1$	θ_2	c_2^1	c_2^2
0.03 (1.0)	−0.30 (4.4)	11.0 (0.5)	−0.10 (10.9)	0.08 (26.2)
Diagnostics:	AR(1)= 0.835, AR(2)= 0.932, ARCH(1)= 0.652 JB= 0.553, Fnon= 0.633			

Notes: Tildes mark estimates' change in the outer regime and t-ratios are given in the parentheses. Dots stand for parameters with t-ratios smaller than 2. P-values are reported for AR, ARCH, JB and F(non) tests; AR(s) – the test of the errors autocorrelation of order s, ARCH(s) – the test of the ARCH effect of order s, JB – the Jarque-Bera normality test, Fnon – the test of non-remaining nonlinearity in the STAR model.

Table 3: Cointegration test in the VEC model $y_{(m)} = [b, p, p^*; t]'$, 1999:01-2011:06

v	s_2 3	2	1	0
0	213.8 (0.000)	138.6 (0.000)	81.18 (0.000)	38.76 (0.123)
1		73.99 (0.000)	19.80 (0.757)	12.73 (0.760)
2			10.40 (0.617)	2.95 (0.871)
Largest moduli of characteristic roots:				
1999:01-2008:06	1.04	0.94	0.94	
-2011:06	1.01	0.93	0.93	

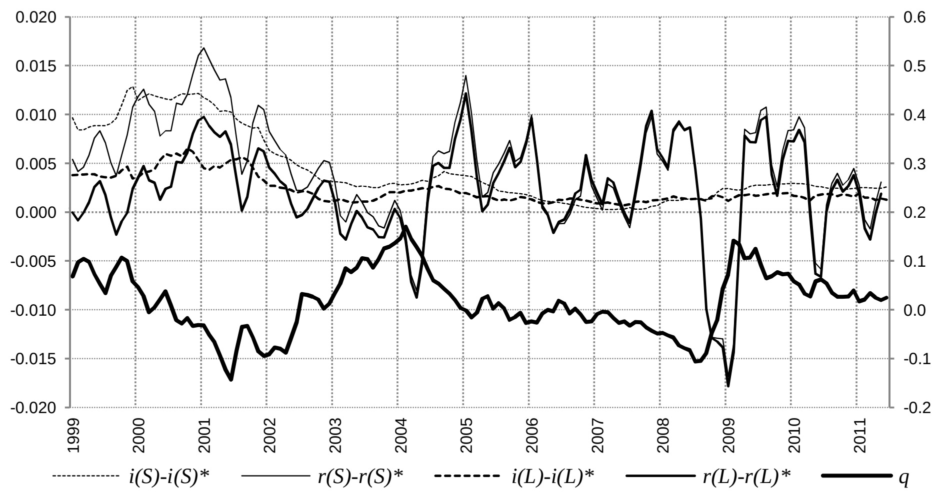
Notes: All VEC models in the paper were subjected to a standard estimation procedure. In the first step, outliers were neutralised with dummies, the optimal lag ($S = 3$) was defined and, subsequently, the normality, autocorrelation and heteroskedasticity tests were performed. Afterwards, the Greenslade *et al.* (2002) modelling strategy consisting of the recurring sequences of (i) cointegration tests, (ii) weak exogeneity tests, and (iii) over-identifying restrictions in succeeding restricted models was adopted.
The table provides the results of cointegration test presented by Johansen (1995) and Paruolo (1996) in VEC-$I(2)$ models. The consecutive null hypotheses imply that the number of polynomial cointegrating vectors V and the number of $I(2)$ stochastic trends S_2 are equal $\{v, s_2\}$ for decreasing values of v and s_2 (the values of s_2 are set for given v). The first non-rejected null $\{v, s_2\}$ implies that $V = v$ and $S_2 = s_2$. P-values are reported in parentheses.

Cointegration analysis of the VEC model for $y = [b, p, p^*]'$ offers more arguments against analysing monetary models of exchange rates limited to their REH-based variants. Tab. 3 contains the values of the three largest characteristic roots of the companion matrix in the sample that ends before the subprime crisis and in the sample accounting for the crisis years. In both models, the largest characteristic roots are outside the unit circle and the moduli of the next two roots are close to unity. The presence of the explosive roots is not easy to explain, but a closer look at the models estimated for shorter periods reveals an almost explicit connection between the building-up explosive tendencies and the deepening 'anomaly in appreciation' in the pre-crisis period 2007:04-2008:07. It might be therefore argued that the explosive roots resulted from the speculative strengthening of the long-lasting, one-direction appreciation drift after Poland's entry into the EU. The presence of this drift is the founding assumption of the Frydman-Goldberg model, but if the interpretation of the results in Tab. 3 were favourable to the strict PPP model, it would be possible, a good-sized significance level in the cointegration test (e.g. 0.15) having been accepted, to arrive at a VEC model with one polynomial cointegrating vector:

$$\Delta p_t - 0.063\left(b_{t-1} - p_{t-1} + p^*_{t-1}\right) + \Delta b_t + 0.01\Delta p^*_t + 0.00004t - 0.006 \sim I(0). \quad (13)$$

Indeed, the above relation is similar to that identified in the scenario PPP3 (Tab. 1) but it has little to do with the relation $b_t - p_t + p^*_t \sim I(0)$ presupposed in the unit root tests.

Figure 2: Spreads of nominal and real short-term interest rates (left scale) and real exchange rate (right scale, logarithm)



When the real zloty/euro exchange rate is compared with the differentials of the long- and short term *nominal* interest rates in Poland and euro zone suspicions arise that the fourth PPP-based scenario (8) is at odds with the DGP too (Fig. 2). The shapes of both nominal interest rates' differentials ($i^L - i^{L*}$ and $i^S - i^{S*}$) confirm that the Polish rates consistently converge to their counterparts in the euro-area and that they seem to have little in common with the drifts that occurred in the real and nominal

exchange rates in the 2000s. A different conclusion can be drawn when the oscillations of differentials of the long- and short-term *real* interest rates are analysed. It is easy to see that the one-direction drifts in $r^L - r^{L*}$ and $r^S - r^{S*}$ are symmetrical to those observed in the real exchange rate, and that the turning points in the real rates' differentials approximately correspond to the turning points in the RER.
In order to formally verify the RE hypothesis against the IKE, alternative VEC models comprising the nominal exchange rate, domestic and foreign prices and interest rates were estimated. The cointegration test in the VEC model:

$$y_{(m)} = [b, p, p^*, i^L, i^{L*}]', \tag{14}$$

where: $i^L = \ln(1 + I^L/1200)$, $i^{L*} = \ln(1 + I^{L*}/1200)$, I^L, I^{L*}– annual nominal interest on 10Y bonds in the zloty and the euro (%) is representative of this stage of investigation (Tab. 4). The conclusions are similar to those drawn with respect to the strict PPP model: (i) a large explosive root is still present in the pre-crisis period but it diminishes after the height of the *subprime* crisis, and (ii), the cointegration tests clearly indicate that when the entire sample is considered the variables included in the VEC model (14) are driven by two $I(2)$ stochastic trends at least. This result clearly shows that the IKE hypothesis adequately described the DGP in the Polish zloty market but before it can be ultimately accepted the equilibrium relations predicted in scenario (9)–(10) or (9)–(11) must be identified.

Table 4: Cointegration test in the VEC model $y_{(m)} = [b, p, p^{*L}, i, i^{*L};\ \iota'_{(k)}]'$, 1999:01-2011:06

v	s_2	5	4	3	2	1	0
0		336.2 (0.000)	260.8 (0.000)	197.2 (0.000)	138.0 (0.004)	105.6 (0.037)	84.53 (0.096)
1			195.4 (0.000)	133.4 (0.002)	84.13 (0.220)	62.28 (0.403)	52.11 (0.328)
2				80.38 (0.195)	56.49 (0.377)	39.97 (0.423)	31.13 (0.443)
3					37.08 (0.419)	25.70 (0.375)	14.83 (0.595)
4						12.79 (0.394)	4.68 (0.648)
Largest moduli of characteristic roots:							
1999:01-2008:06		1.04	0.95	0.95	0.88	0.88	
-2009:09		0.96	0.95	0.96	0.91	0.91	
-2011:06		0.95	0.95	0.93	0.92	0.86	

4 Does the IKE explain the Polish zloty's swings?

Some uncertainty involved in the economic interpretation of the 'cryptic' cointegrating vector (13) in the strict PPP model is only one of the problems that need to be tackled when more complex VEC-$I(2)$ systems are constructed. To overcome the problems, a

test of long-run homogeneity of the nominal exchange rate and domestic and foreign prices was run before passing to the next stages of investigation. The rationale for this approach lies in the fact that if the homogeneity restriction can be imposed on some components of β and δ in (5), the so-called 'nominal-to real' transformation can be performed (Juselius 2006) and instead of analysing the model $y_{(m)} = [b, p, p^*, i, i^*;\ t]'$ its 'real' counterpart $y_{(m)} = [q, \Delta p, \Delta p^*, i, i^*; t]'$ can be examined without the loss of information. Although the results of the long-term homogeneity test in the zloty/euro exchange rate model $y_{(m)} = [b, p, p^*, i^L, i^{*L}; t]'$ are borderline (p-values of 0.11 and 0.08 for $V = 2$ and $V = 3$, respectively), they provide sufficient arguments for the 'real' model $y_{(m)} = [q, \Delta p, \Delta p^*, i^L, i^{*L}; t]'$ to be analysed.
The cointegration tests and the estimates of the largest characteristic roots in $y_{(m)} = [q, \Delta p, \Delta p^*, i^L, i^{*L}; t]'$confirm that two cointegrating vectors are present in the model. The analysis of the adjustment matrix in the unrestricted model indicates that the first cointegrating vector can be normalized against price inflation in the domestic tradables sector and the second one with respect to the real exchange rate, the only domestic variable gravitating in this direction. Even so, the properties of the VEC system with over-identifying restrictions aligned with IKE scenario are unsatisfactory because of the near weak exogeneity of the real exchange rate (Tab. 5, upper panel). Qualitatively different results are obtained only when the long-term interest rates are replaced by their short-term counterparts, and only with the 1999:01-2009:09 sample (Tab. 5, lower panel). The major drawbacks of the $y_{(m)} = [q, \Delta p, \Delta p^*, i^S, i^{*S}; t]'$ model are the borderline results of the over-identifying restrictions test and, more importantly, the instability of the equilibrium parameters. Moreover, the model with short-term interest rates proves unacceptable when the full sample 1999:01-2011:06 is taken into consideration. The recursive tests of over-identifying restrictions give strong arguments for rejecting structural restrictions in the case of samples ending between the second half of 2009 and the end-point of the sample 2011:06.
Following a more specific analysis of the recursive estimation results, two complementary working hypotheses can be formulated. Firstly, even a cursory visual inspection of the recursive estimates of the equilibrium parameters reveals rapidly rising and then stabilising semi-elasticity on the real interest rate spread in the second cointegrating vector (Fig. 3). Therefore, an intuitive and fully justified working hypothesis is to link the instability of the estimated parameters with the eruption of the *subprime* crisis and a sudden change in the perception of uncertainty. Secondly, the 1999:01-2009:09 estimates of the parameters show that in the pre-crisis years semi-elasticity on the interest rate spread ranged significantly from 20 to 30. This implies that the exchange rate may have been affected by uncertainty in that period that, however, stemmed from different factors than at the height of the *subprime* crisis.

4.1 Working hypotheses and evidence

The estimates yielded by the VEC model $y_{(m)} = [q, \Delta p, \Delta p^*, i^S, i^{*S}; t]'$ indicate that the list of gap measures should have more items than deviations of the real zloty/euro

Table 5: The estimation of the IKE model $y_{(m)} = [q, \Delta p, \Delta p^*, i, i^*; t]'$

a. Long-term interest rates, 1999:01-2011:06

	q	Δp	i^L	Δp^*	i^{*L}	t
β_1'	-0.0329 (4.4)	**1**	0	0	0	0.0000 (0.2)
β_2'	**1**	-42.70 (9.5)	42.70 (9.5)	42.70 (9.5)	-42.70 (9.5)	0.0002 (0.4)
α_1'	-1.021 (2.5)	$\mathbf{-0.609}$ (4.8)	.	0.471 (5.9)	.	
α_2'	-0.023 (2.4)	.	-0.0003 (2.5)	-0.005 (2.7)	.	
LR= 0.403						
AR(1)= 0.523	AR(2)= 0.461			DH= 0.000		
AR(3)= 0.152	AR(4)= 0.306			ARCH(1)= 0.209	ARCH(2)= 0.046	

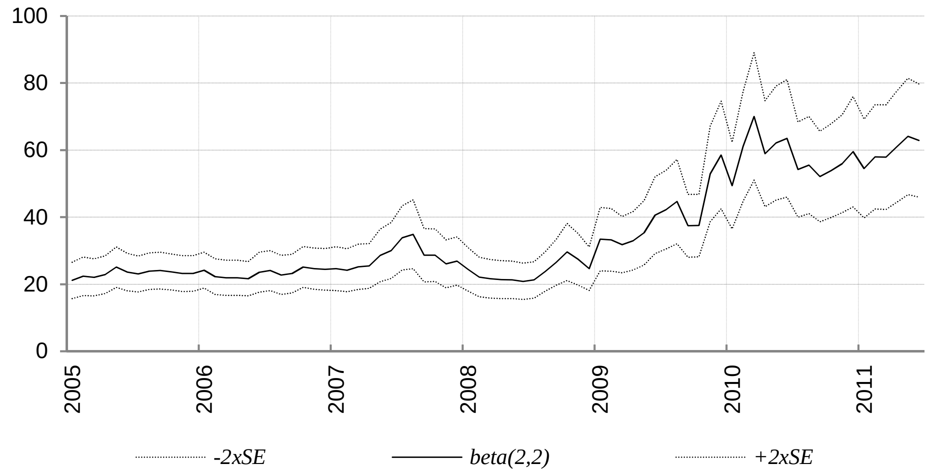
b. Short-term interest rates, 1999:01-2009:09

	q	Δp	i^S	Δp^*	i^{*S}	t
β_1'	-0.0102 (1.8)	**1**	0	0	0	0.0001 (4.0)
β_2'	**1**	-32.58 (9.5)	32.58 (9.5)	32.58 (9.5)	-32.58 (9.5)	0.0016 (2.7)
α_1'	-2.075 (3.2)	$\mathbf{-0.868}$ (5.4)	-0.018 (2.6)	-0.419 (3.7)	0.006 (2.0)	
α_2'	$\mathbf{-0.056}$ (3.3)	.	.	-0.010 (3.2)	.	
LR= 0.136						
AR(1)= 0.090	AR(2)= 0.190			DH= 0.130		
AR(3)= 0.104	AR(4)= 0.308			ARCH(1)= 0.128	ARCH(2)= 0.670	

Notes: t-ratios are reported in parentheses. Dots stand for the parameters with t-ratios smaller than 2. P-values are reported for LR, AR, DH and ARCH tests; LR – over-identifying restrictions test, AR(s) – test of the errors autocorrelation of order s, DH – Doornik-Hansen normality test, ARCH(s) – test of the ARCH effect of order s.

rate from the mean alone. Frydman and Goldberg (2007) accept the existence of supplementary gaps, the widening of which may induce investors into changing their forecasting strategies, and consider a gap-plus model with the uncertainty premium contingent on the PPP gap and the cumulated current account (Frydman and Goldberg 2007, chapter 12). Similar approaches can be found in earlier studies of uncovered interest rate parity. In most of them, the impacts of fiscal deficits, the significance of external or government debt, and the role of disequilibria in the external sector are hypothesized. The common feature of the studies is that they approximate fluctuations in the uncertainty premium (see Frydman and Golberg 2007) or the risk premium (e.g. Juselius 1995; Clark and MacDonald 1999; a review in Jongen et al. 2008) using different measures of disequilibria in the goods markets.

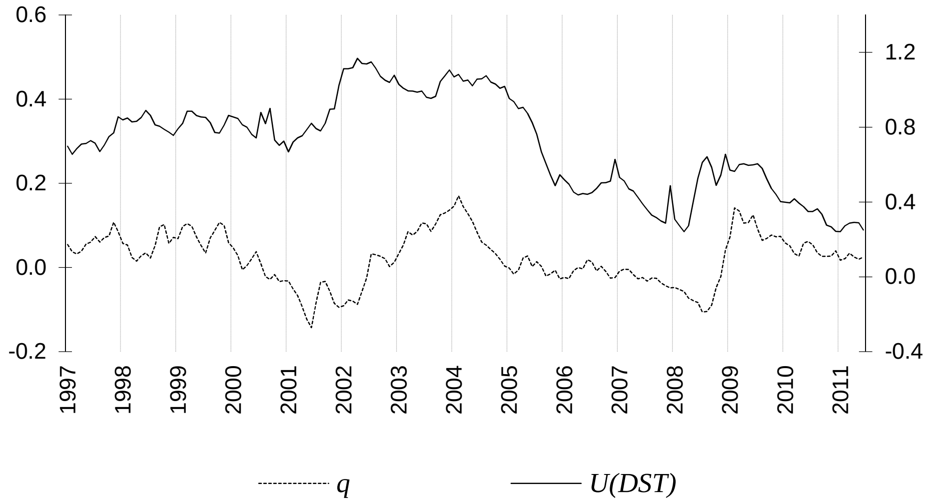
Figure 3: Recursive estimates of semi-elasticity on the real interest spread in the model $y_{(m)} = [q, \Delta p, \Delta p^{*}, i^{S}, i^{*S}; t]'$



A similar approach, although arising from different assumptions, was adopted in this investigation for the next steps of estimation of the zloty/euro exchange rate model. An overview of the history of the free float in Poland reveals that before the subprime crisis occurred the zloty exchange rate was strongly influenced by, chronologically, speculative short-term capital, expanding fiscal deficits, Poland's entry into the European Union in May 2004, and the expectations of the 'inevitable' appreciation of the currencies of catching-up economies in Central and Eastern Europe. As none of these factors was directly related to the disequilibria in the goods market (Poland's international investment position was steadily declining in all sample years), the first supplementary gap effect was defined as the short-term government debt-to-GDP ratio. Larger issues of T-bills increasing debt imply that either the government has more problem financing its current expenditures or that investors lose trust in securities of longer maturity. The global risk, too, may cause fluctuations in the short-term debt. Because a safer option for the government is to finance its expenditures through long-term securities, it may tend to make up for declining demand for bonds issue by issuing more T-bills.

The fluctuations in the real exchange rate q and in the second proxy of uncertainty $U^{DST} = D^{ST}/F^{ST}$ (D^{ST}, F^{ST} – short-term government debt-to-GDP ratios in Poland and the euro zone) deserve three comments (Fig. 4). First, the shapes of the RER and of the proxy of uncertainty are similar enough to allow a tentative working hypothesis that both these variables are driven by common stochastic trend. Second, the turning points in U^{DST} precede the turning points in the real rate. Third, the real depreciation that abruptly occurred between September 2008 and March 2009 exceeded the relative increase in U^{DST}. Therefore, the model can be extended by introducing U^{DST} and a shift dummy representing a structural break caused by a sudden rise in global uncertainty during the subprime crisis.

Figure 4: The real exchange rate q and the relative short-term government debt U^{DST}



The model $y_{(m)} = [q, \Delta p, \Delta p^{*}, i^{S}, i^{*S}, U^{DST}, t]'$, based on samples 1999:01-2009:09 and 1999:01-2011:06 (with a shift dummy for the period 2009:04-2011:06), was analysed in the same way as its previous versions were. The three-lag models proved to be the optimal system again. Regardless of the dimension of the cointegration space, the weak exogeneity tests consistently provided arguments for conditioning the model on U^{DST} at standard significance levels. In the samples ending in the successive months of 2005, 2006 and 2007 the recursive estimates of the cointegrating vectors show only minor drifts and then oscillate around levels corresponding to the final estimates (Tab. 6). The differences between models based on the pre-crisis sample and the full sample are moderate at best. The estimates of the second equilibrium condition indicate that the internal uncertainty proxy U^{DST} is significant and that the influence of real interest rates on the real exchange rate is much weaker in the models without the supplementary gap effect U^{DST}.

The interpretation of the cointegrating vectors and of the whole IKE model is straightforward. The rate of producer price inflation $\Delta^2 p$ adjusts to the first polynomial cointegrating vector:

$$\Delta p_t = \underset{(3.5)}{0.0213}(b_t - p_t + p_t^{*}) - \underset{(2.8)}{0.00002}\, t, \tag{15}$$

and the equilibrium relation for the real exchange rate is:

$$q_t = -\underset{(4.3)}{4.233}\left(\left(i_t^{S} - \Delta p_t\right) - \left(i_t^{*S} - \Delta p_t^{*}\right)\right) + \underset{(8.3)}{0.166}\, U_t^{DST} + \underset{(5.7)}{0.082}\ C(09.04)_t. \tag{16}$$

Table 6: Estimation of the model $y_{(m)} = [q, \Delta p, \Delta p^*, i^S, i^{*S}, U^{DST}, t]'$

a. 1999:01-2009:09

	q	Δp	i^S	Δp^*	i^{*S}	U^{DST}		t
β_1'	−0.022 (3.6)	**1**	0	0	0	0		0.0003 (3.1)
β_2'	**1**	−5.949 (4.5)	5.949 (4.5)	5.949 (4.5)	−5.949 (4.5)	−0.152 (5.7)		0
α_1'	−0.950 (2.1)	**−0.741** (6.6)	0.028 (5.6)	−0.194 (2.5)	0.004 (2.1)	0		
α_2'	**−0.148** (4.4)	.	0.002 (5.8)	0.013 (2.2)	.	0		
LR= 0.550								
	AR(1)= 0.198	AR(2)= 0.127				DH= 0.074		
	AR(3)= 0.124	AR(4)= 0.308			ARCH(1)= 0.730		ARCH(2)= 0.966	

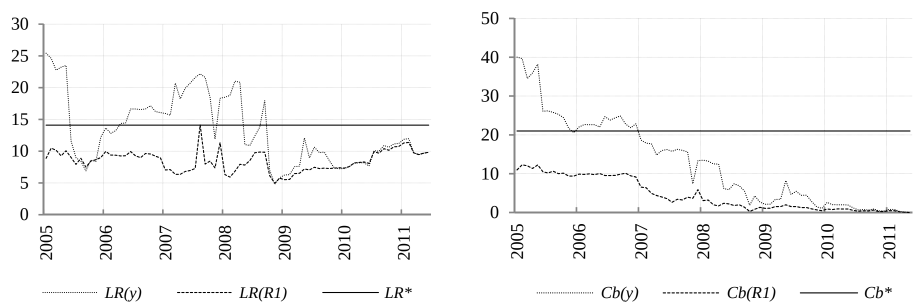
b. 1999:01-2011:06

	q	Δp	i^S	Δp^*	i^{*S}	U^{DST}	$C(09.04)$	t
β_1'	−0.021 (3.5)	**1**	0	0	0	0	0	0.0002 (2.8)
β_2'	**1**	−4.233 (4.3)	4.233 (4.3)	4.233 (4.3)	−4.233 (4.3)	−0.166 (8.3)	−0.082 (5.7)	0
α_1'	−0.590 (1.8)	**−0.725** (8.5)	0.019 (5.2)	−0.330 (5.3)	0.004 (2.2)	0		
α_2'	**−0.150** (4.6)	.	0.003 (7.2)	0.019 (3.3)	0.0004 (2.1)	0		
LR= 0.199								
	AR(1)= 0.155	AR(2)= 0.030				DH= 0.000		
	AR(3)= 0.490	AR(4)= 0.240			ARCH(1)= 0.084		ARCH(2)= 0.079	

The structure of the first equilibrium relation supports the hypothesis that prices in the tradables sector of a small and open economy are primarily determined by foreign prices and the nominal exchange rate. However, equation (15) is different from the standard PPP model in that the non-linear price adjustments within a so-called internal equilibrium correction mechanism link price inflation Δp with real exchange rate q. According to the estimate of the internal error correction term (0.0213), an increase in the nominal exchange rate (depreciation) or in the prices of the foreign tradables sector has a positive effect on domestic prices and increases inflation. At the same time, domestic prices rising above their PPP level bring inflation down, meaning that they converge to a level determined by price arbitrage in the tradables sector. Another indication of the non-linearity of prices is the presence of a deterministic trend. It can be very broadly interpreted as an 'autonomous' disinflationary process or, in a somewhat more complex manner, as relative productivity gains and falling relative unit labour costs in Poland (e.g. Kelm 2013, pp. 414-420). The interpretation

of the second equilibrium relationship in the context of the IKE model is obvious. Namely, in this model, the depreciation of the zloty against the euro initiates much stronger adjustments than those observed in the standard linear PPP models or in the IKE models without the short-term debt-to-GDP ratio. (see Tab. 5).
The IKE model with equilibrium relations (15)-(16) has satisfying properties. The recursive tests of over-identifying restrictions and the parameter constancy tests (for details see Juselius 2006, ch. 9) confirm the proposed structure of the cointegrating vectors (Fig. 5). The properties of the residuals are satisfying as well, however not without some reservations. In particular, the normality of errors is clearly rejected by the joint Doornik-Hansen test. Even so, the univariate tests show that the equations of the real rate (p-value = 0.13) and of domestic and foreign price inflation (p-values = 0.26 and 0.33) meet the normality assumption. The negative outcome of the joint DH test is caused by excessive kurtosis in the equations of the nominal interest rates.

Figure 5: Recursive tests of over-identifying restrictions (LR) and parameter constancy (Cb) in the model $y_{(m)} = [q, \Delta p, \Delta p^*, i^S, i^{*S}, U^{DST}, t]'$



4.2 The transmission of the global shock

The IKE model with cointegrating vectors (15)-(16) attracts criticism for one reason at least – the shift-dummy approximating changes in global uncertainty is included on an *ad-hoc* basis. This solution provokes questions about whether uncertainty pricing can change abruptly and why the short-term debt ratio U^{DST} fails to approximate variations in uncertainty over the 1999:01-2011:06 sample. To answer these questions, in the final stage of the investigation the observable 'external' uncertainty proxy for the subprime crisis period was identified. This part of analysis started with the identity (superscripts INT and EXT denote the domestic and foreign causes of premium changes, respectively):

$$\lambda^A = \lambda - \lambda^* = \left(\lambda^{INT} + \lambda^{EXT}\right) - \left(\lambda^{*INT} + \lambda^{*EXT}\right) \tag{17}$$

that defines fluctuations in the 'aggregate' uncertainty premium λ^A as a net effect of the changed perception of the uncertainty premium related to assets denominated in the Polish zloty λ and the euro λ^*. Because Poland's economy not only differs in size from the Eurozone economy but also strongly depends on it, a simplifying assumption $\lambda^{EXT} = \lambda^{*INT} + \lambda^{*EXT}$ can be made, which reduces equation (17) to:

$$\lambda^A = \lambda^{INT} \cong D^{ST}. \tag{18}$$

However, to account for global developments that significantly change uncertainty, equation (17) needs to be rearranged into:

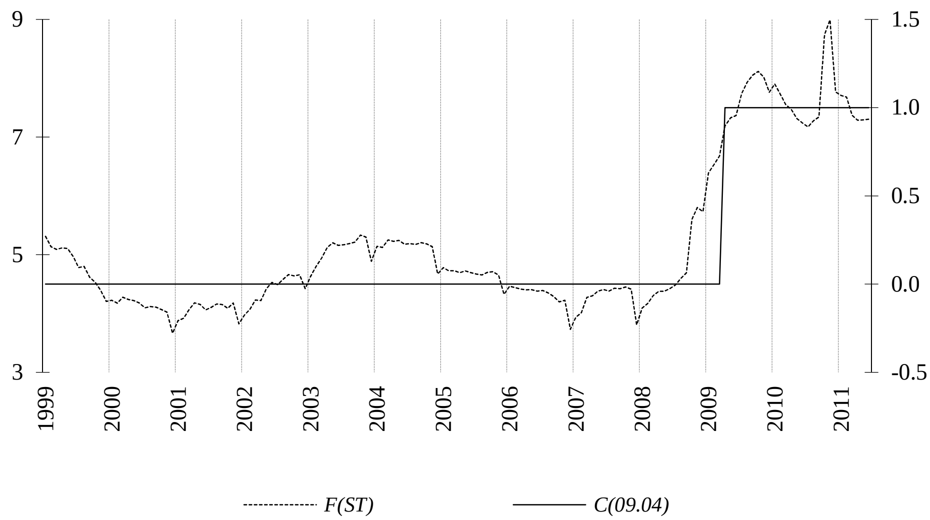
$$\lambda^A = \lambda^{INT} + \left(1 + m^U + v^U\right)\lambda^{EXT} - \left(1 + m^U\right)\left(\lambda^{*INT} + \lambda^{*EXT}\right), \tag{19}$$

where m^U denotes a crisis-induced mark-up in global uncertainty pricing (see eq. (22)) and v^U is an additional mark-up in uncertainty pricing on assets denominated in peripheral currencies. For $\lambda^{EXT} = \lambda^*$, global uncertainty:

$$\lambda^A = \lambda^{INT} + v^U\lambda^* \cong D^{ST} + v^U F^{ST}, \tag{20}$$

justifies subsequent extensions of the IKE model.

Figure 6: The global uncertainty proxy F^{ST} and the shift dummy $C(09.04)$



A visual inspection of trends in the global uncertainty proxy confirms that the shift dummy $C(09.04)$ 'started working' in the period when the global uncertainty was around its maximum level (Fig. 6). Because of that, in the final stage of the analysis of the zloty/euro exchange rate, the alternative IKE models with uncertainty premiums approximated by various combinations of U^{DST}, D^{ST}, F^{ST} and $C(09.04)$ were subjected to cointegration analysis. Tab. 7 summarizes the estimates obtained for one of the competing VEC models: $y_{(m)} = [q, \Delta p, \Delta p^*, i^S, i^{*S}, U^{DST}, F^{ST}]'$. Conclusions are obvious again. The probability value in the test of over-identifying restrictions is distinctly higher from the probability value in the model with a shift dummy. The downside of the model including F^{ST} lies in the mediocre stochastic properties of the residuals – error normality is rejected for excessive kurtosis and the results of the autocorrelation tests are borderline because of a rapid swing in F^{ST} at the end of 2010. Finally, the replacement of the shift dummy with F^{ST} does not have a major effect on the estimates of the equilibrium parameters and the adjustment coefficients, so earlier conclusions concerning the determinants of the zloty/euro exchange rate before and during the subprime crisis remain unchanged.

Table 7: Estimation of the IKE model $y_{(m)} = [q, \Delta p, \Delta p^*, i^S, i^{*S}, U^{DST}, F^{ST}; t]'$, 1999:01-2011:06

	q	Δp	i^S	Δp^*	i^{*S}	U^{DST}	F^{ST}	t
β_1'	-0.019 (3.0)	**1**	0	0	0	0	0	0.0003 (3.0)
β_2'	**1**	-3.101 (3.4)	3.101 (3.4)	3.101 (3.4)	-3.101 (3.4)	-0.144 (8.4)	-0.025 (6.1)	0
α_1'	-0.308 (1.0)	**-0.679** (7.9)	0.019 (5.2)	-0.292 (4.8)	.	0	0	–
α_2'	**-0.149** (4.3)	.	0.003 (7.2)	0.024 (3.8)	.	0	0	–
LR= 0.301								
AR(1)= 0.012	AR(2)= 0.077				DH= 0.000			
AR(3)= 0.338	AR(4)= 0.050				ARCH(1)= 0.029		ARCH(2)= 0.088	

4.3 Other studies

In closing the discussion it needs to be stressed that the above investigations are not the only ones to have confirmed the strong effect of short-term factors on the exchange rate of the Polish zloty at the height of the *subprime* crisis. Other studies make use of the uncovered interest parity model (UIP) that approximates the risk premium increase before the 2008-2009 crisis by means of increase in prices of the Credit Default Swaps (CDS; e.g. Kębłowski 2011, Kębłowski and Welfe 2012) or CDSs' prices and aggregated measures of disequilibria in the commodity and financial markets (Grabowski and Welfe 2016). Because the studies implicitly hypothesize that the influence of uncertainty premium on the exchange rate, they lead to the construction of models that are differently structured and interpreted than proposed in this paper. Kębłowski and Welfe 2012 have found equilibrium relation (t-ratios in parentheses):

$$q + \underset{(2.5)}{25.540}\,(i - i^{*}) - \underset{(3.8)}{0.129}\left(\lambda^{CDS} - \lambda^{*CDS}\right) + \mu \sim I(0), \tag{21}$$

which is consistent with the predictions of the Dornbusch-type monetary models; λ^{CDS} andλ^{*CDS} denote in (21) the prices of the CDS contracts hedging Polish and German governmental long-term bonds. As trajectory (21) attracts real exchange rate q and risk prices λ^{CDS} and λ^{*CDS}, the model describes 'tight' *risk premium–exchange rate* feedback loop. Although Grabowski and Welfe (2016) have confirmed the impact of risk prices on the real zloty exchange rate, in this case too the structure of the model becomes similar to the structure of the REH-based monetary models:

$$b - \underset{(4.8)}{0.974}\,(p - p^{*}) + \underset{(8.5)}{4.1}\,(i - i^{*}) - \underset{(5.0)}{0.1}\left(\lambda^{CDS} - \lambda^{*CDS}\right) - \underset{(3.7)}{0.021}\,N^{*} + \mu \sim I(0). \tag{22}$$

The measure of the state of the currency market N^{*} includes information on the deviations of b and $i - i^{*}$ from their equilibrium paths; the latter are predetermined outside the model, which makes the final interpretation of the equilibrium relation (22) somewhat problematic.

5 Concluding remarks

The paper represents an attempt at establishing which of the two hypotheses – rational expectations or imperfect knowledge economics – is more relevant, or sufficiently accurate, in describing processes observed in the Polish foreign exchange market in the free float period 1999-2011. Following the adoption of the strict RE perspective, the routine unit root tests were performed and a standard PPP model with nominal exchange rate and domestic and foreign prices was estimated. This introductory part of the paper can be summed up by noting that researchers who strongly support the RE-based interpretation of the PPP can terminate their investigations after URT tests and interpret the results as providing sufficient arguments in favour of the RE hypothesis. The next part shows, however, that there are several serious reasons why preferring to the RE hypothesis may be premature, at least as far as the modelling of the Polish zloty exchange rate is concerned. For instance, the results of the linear DF-type tests presented in this paper are borderline, but the results of Johansen's stationarity test (Johansen and Juselius, 1992) applied to all variants of the VEC models under consideration explicitly reject the null hypothesis about stationary RER. Different doubts arise in relation to the results of the non-linear unit root tests that show, like the linear UTRs, the RER to be stationary. The estimation results of the logistic STAR model of the zloty/euro real rate can be easily interpreted following the predictions of the Frydman-Goldberg model – rare and transient mean-reversions occur only when the baseline drifts revert, so agents would have to be 'non-rational' for most of the analysed period. The large delay parameter implies a substantial

'hidden' persistence of the RER, meaning that the STAR model is not capable of solving the PPP puzzle.
The evidence from the cointegration analysis of the RE-based VEC model with the nominal exchange rate and domestic and foreign prices indicates that it is not possible to build an empirical model capable of precisely replicating DGP's properties unless (i) the problem of non-stationary $CI(2,1)$ cointegration vectors is resolved, and (ii) the specification of the strict PPP model is extended. When both these conditions are met, the estimation results of the different variants of the Polish zloty/euro exchange rate model for the free float period 1999:01-2011:06 unequivocally justify the rejection of the RE hypothesis in favour of its IKE generalization. An analysis of the extended IKE model revealed cointegrating vectors that were basically similar in structure to those predicted by the Frydman-Goldberg model. The zloty/euro exchange rate model is different from other models with the IKE specifications (Frydman and Goldberg 2007, also: Juselius and MacDonald 2004, 2006) in that it accounts for the presence of two supplementary gap effects that strongly influence the formation of expectations in the Polish zloty currency market. Empirical evidence confirms that in the pre-crisis period the zloty/euro exchange rate was quite smoothly driven by short-term fiscal tensions, in contrast with the height of the subprime crisis when the rate abruptly overshot its equilibrium level responding to suddenly rising global risk. Both these supplementary gap effects are directly connected with foreign exchange market equilibrium, whereas in the Frydman-Goldberg model the PPP gap effect is determined by disequilibrium in the goods markets. This result implies that the core specification of the Frydman-Goldberg model may be incomplete in periods when exchange rates adjust more strongly to parity. This said, it needs to be noted that the main predictions of the Frydman-Goldberg model allow constructing an empirical model of the zloty/euro exchange rate where the half-life of RER's adjustments to the equilibrium path is only 41 months.

References

[1] Bahmani-Oskooee M., Kutan A.M., Zhou S., (2007), Testing PPP in the non-linear STAR framework, *Economics Letters* 94, 104–110.

[2] Chang H.-L., Liu D.-C., Su C.-W., (2012), Purchasing power parity with flexible Fourier stationarity test for Central and Eastern European countries, *Applied Economics* 44, 4249–4256.

[3] Clark P., MacDonald R., (1999), Exchange rates and economic fundamentals: A methodological comparison of BEER's and FEER's, Chapter 10, [in:] *Equilibrium exchange rates,* [eds.:] R. MacDonald, J. Stein, Kluwer Academic Publishers, Norwell, 285–322.

[4] Cuestas J.C., Regis P.J., (2013), Purchasing power parity in OECD countries: Nonlinear unit root tests revisited, *Economic Modelling* 32, 343–346.

[5] Cushman D.O., Michael N., (2011), Nonlinear trends in real exchange rates: A panel unit root test approach, *Journal of International Money and Finance* 30, 1619–1637.

[6] Dumas B., (1992), Dynamic equilibrium and the real exchange rate in a spatially separated world, *The Review of Financial Studies* 5, 153–180.

[7] Égert B., Halpern L., MacDonald R., (2006) Equilibrium exchange rates in transition economies: Taking stock of the issues, *Journal of Economic Surveys* 20, 257–324.

[8] Froot K., Rogoff K., (1995), Perspectives on PPP and long-run real exchange rates. Chapter 32, [in:] *Handbook of international economics*, vol. 3, [eds.:] G. Grossman, K. Rogoff, Elsevier, Amsterdam, 1647–1688.

[9] Frydman R., Goldberg M., (2007), *Imperfect knowledge economics. Exchange rates and risk*, Princeton University Press, Princeton.

[10] Frydman R, Goldberg M. 2013a. The imperfect knowledge imperative in modern macroeconomics and finance theory, Chapter 4, [in:] *Rethinking expectations: The way forward for macroeconomics*, [eds.:] R. Frydman, E. Phelps, Princeton University Press, Princeton, 130–165.

[11] Frydman R, Goldberg M. 2013b. Opening models of asset prices and risk to non-routine change, Chapter 6, [in:] *Rethinking expectations: The way forward for macroeconomics*, [eds.:] R. Frydman, E. Phelps, Princeton University Press, Princeton, 207–247.

[12] Frydman R., Goldberg M., Mangee N., (2015), Knightian uncertainty and stock-price movements: Why the REH present-value model failed empirically?, *Economics.: The Open Access, Open-Assessment E-Journal* 2015-24, 1–50.

[13] Frydman R., Goldberg M., Johansen S., Juselius K., (2008), A resolution of the purchasing power parity puzzle: Imperfect knowledge and long swings, *Unpublished Manuscript*, Discussion Paper no. 08-13, Department of Economics, University of Copenhagen, Copenhagen.

[14] Grabowski W., Welfe A., (2016), An exchange rate model with market pressures and a contagion effect, *Emerging Markets Finance and Trade* 52, 2706–2720.

[15] Greenslade J.V., Hall S.G., Henry S.G.B., (2002), On the identification of cointegrated systems in small samples: A modelling strategy with an application to UK wages and prices, *Journal of Economic Dynamics and Control* 26, 1517–1537.

[16] James J., Marsh I., Sarno L. (eds), (2012), *Handbook of exchange rates*, Wiley, Hoboken.

[17] Johansen S., (1995), A statistical analysis for $I(2)$ variables, *Econometric Theory* 11, 25–29.

[18] Johansen S., Juselius K., (1992), Testing structural hypotheses in a multivariate cointegration analysis of the PPP and the UIP for UK, *Journal of Econometrics* 53, 211–244.

[19] Jongen R., Verschoor W., Wolff C., (2008), Foreign exchange rate expectations: Survey and synthesis, *Journal of Economic Surveys* 22, 140–165.

[20] Juselius K., (1995), Do purchasing power parity and uncovered interest parity hold in the long run? An example of likelihood inference in a multivariate time-series model, *Journal of Econometrics* 69, 211–240.

[21] Juselius K., (2006), *The cointegrated VAR model: Methodology and applications*, Oxford University Press, Oxford.

[22] Juselius K., (2011), On the role of theory and evidence in macroeconomics, Chapter 17, [in:] *The Elgar companion to recent economic methodology*, [eds.:] W. Hands, J. Davis, Edward Elgar Publishing, Cheltenham, 404–436.

[23] Juselius K., (2013), Imperfect knowledge, asset price swings and structural slumps: A cointegrated VAR analysis of their interdependence, Chapter 10, [in:] *Rethinking expectations: The way forward for macroeconomics*, [eds.:] R. Frydman, E. Phelps, Princeton University Press, Princeton, 328-350.

[24] Juselius K., (2014), Testing for near $I(2)$ trends when the signal-to-noise ratio is small, *Economics.: The Open Access, Open-Assessment E-Journal* 2014-8, 1–22.

[25] Juselius K., (2015), Testing exchange rate models based on rational expectations versus imperfect knowlegde economics: A scenario analysis, *Unpublished Manuscript*, Department of Economics, University of Copenhagen, Copenhagen.

[26] Juselius K., MacDonald R., (2004), Interest rate and price linkages between USA and Japan, *Japan and the World Economy* 16, 17–34.

[27] Juselius K., MacDonald R., (2006), International parity relationships and a nonstationary real exchange rate. Germany versus the US in the Post Bretton Woods period, Chapter 5, [in:] *International macroeconomics: Recent developments*, [ed.:] A. Morales-Zumaquero, Nova Science Publishers, New York, 79–103.

[28] Kahneman D., Tversky A., (1979), Prospect Theory: An analysis of decision under risk, *Econometrica* 47, 263–291.

[29] Kapetanios G., Shin Y., Snell A., (2003), Testing for a unit root in the nonlinear STAR framework, *Journal of Econometrics* 112, 359–379.

[30] Kelm R., (2010), The exchange rate and two price inflations in Poland in the period 1999-2009. Do globalization and Balassa-Samuelson effect matter?, *Central European Journal of Economic Modelling and Econometrics* 4, 315–349.

[31] Kelm R., (2013), *Kurs złoty/euro: Teoria i empiria*, Wydawnictwo Uniwersytetu Łódzkiego, Łódź.

[32] Kębłowski P., (2011), The behaviour of exchange rates in the Central European countries and Credit Default Risk premiums, *Central European Journal of Economic Modelling and Econometrics* 3, 221–237.

[33] Kębłowski P., Welfe A., (2012), The risk-driven approach to exchange rate modelling, *Economic Modelling* 29, 1473–1482.

[34] Kim H., Moh Y.-K., (2010), A century of purchasing power parity confirmed: The role of nonlinearity, *Journal of International Money and Finance* 29, 1398–1405.

[35] MacDonald R., (2007), *Exchange rate economics. Theories and evidence*, Routledge, New York.

[36] Norman S., (2010), How well does nonlinear mean reversion solve the PPP puzzle, *Journal of International Money and Finance* 29, 919–937.

[37] Paruolo P., (1996), On the determination of integration indices in $I(2)$ systems, *Journal of Econometrics* 72: 313–356.

[38] Pavlidis E.G., Paya I., Peel D.A., (2011), Real exchange rates and time-varying trade costs, *Journal of International Money and Finance* 30, 1157–1179.

[39] Rogoff K., (1996), The purchasing power parity puzzle, *Journal of Economic Literature* 34, 647–668.

[40] Sarno L., Taylor M.P., (2002), *The economics of exchange rates*, Cambridge University Press, Cambridge.

[41] Sercu P., Uppal R., van Hulle C., (1995), The exchange rate in the presence of transaction costs: Implications for tests of purchasing power parity, *The Journal of Finance* 50, 1309–1319.

[42] Sollis R., (2005), Evidence on purchasing power parity from univariate models: The case of smooth transition trend-stationarity, *Journal of Applied Econometrics* 20, 79–98.

[43] Sollis R., (2009), A simple unit root test against asymmetric STAR nonlinearity with an application to real exchange rates in Nordic countries, *Economic Modelling* 26, 118–125.

[44] Su C.-W., Chang T., Chang H.-L., (2011), Purchasing power parity for fifteen Latin America countries: Stationarity test with a Fourier function, *International Review of Economics and Finance* 20, 839–845.

[45] Taylor A.M., (2002), A century of purchasing power parity, *Review of Economics and Statistics* 84, 139–150.

[46] Yilanci V., Eris Z.A., (2013), Purchasing power parity in African countries: Further evidence from Fourier unit root tests based on linear and nonlinear models, *South African Journal of Economics* 81, 20–34.

Appendix

Data sources and time series definitions:
p producer price index in manufacturing in Poland (log, 2000=0), MSO, Poland
p^* producer price index in manufacturing in euro area (log, 2000=0), OECD
b nominal exchange rate (price of 1 EUR in PLN, logarithm, 2000=0), NBP
$q = q(PT)$ PT-based real exchange rate zloty/euro, $q = b - p + p^*$
I^L, I^{*L} nominal interest rates on 10Y bonds denominated in zlotys and euros (%),OECD
I^S, I^{*S} three-month interbank rate denominated in zlotys and euros (%), OECD

$$i^J = \ln\left(1 + I^J/1200\right), i^{*J} = \ln\left(1 + I^{*J}/1200\right), J = \{L,\ S\}$$

D^{ST} short-term government debt to GDP ratio, Poland, $D^{ST} = ST^D/Y$
ST^D nominal short-term government debt in Poland, millions PLN, source: Narodowy Bank Polski
Y nominal gross domestic product in Poland, millions PLN, own monthly estimates on the basis of official quarterly data (source: Main Statistical Office)
F^{ST} short-term government debt to GDP ratio, euro area, $F^{ST} = ST^F/Y^*$
ST^F nominal short-term government debt in euro area, millions EUR, source: Bundesbank
Y^* nominal gross domestic product in euro area, millions EUR, own monthly estimates on the basis of official quarterly data (source: Eurostat)