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A Real-Business-Cycle Model with Efficiency Wages and a Government Sector: The Case of Bulgaria

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Abstract

In this paper we investigate the quantitative importance of efficiency wages of no-shirking type in explaining business cycle fluctuations in Bulgarian labor markets. This is done by augmenting a relatively standard real business cycle model with unobservable workers effort by employers and efficiency wage contracts, as well as through the inclusion of a detailed government sector. This imperfection in labor markets introduces a strong internal transmission mechanism that allows the model framework to capture the business cycles in Bulgarian data better than earlier models, and setups assuming perfectly-competitive labor markets in particular.

Keywords: general equilibrium, shirking, efficiency wages, unemployment, Bulgaria

JEL Classification: E24, E32, J41

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1 Introduction and Motivation

The standard real business cycle model, *e.g.* Kydland and Prescott (1982) and Long and Plosser (1983), was shown to be unable to capture the dynamics in the labor markets in the US. For Bulgaria, Vasilev (2009) documented a similar failure for wage and employment fluctuations. Most of those earlier studies have tried to explain the mismatch with the modeling choice assuming perfect information and market-clearing, and the absence of involuntary unemployment. Bulgaria, however, as many other Eastern European countries as well, exhibits a significant rate of involuntary unemployment, which was due to the process of structural transformation. In other words, being out of job is not an optimal choice, but rather represents an inefficient outcome, as it produces a waste of non-storable labor resources.

Modeling unemployment as inefficiency requires a departure from the Walrasian (market-clearing) models of labor markets. In other words, only when certain imperfections in labor market are present in the model, can involuntary unemployment appear. Since we are interested in studying the cyclical behavior of the labor market, we take those seriously. One aspect of labor market frictions are informational problems, connected to costly monitoring or imperfect monitoring of worker's effort by an employer. In the absence of perfect information, some workers may decide to shirk, and will be caught with some probability. To prevent shirking from happening, employers would design a "carrot-and-stick" strategy: given the limited information available, they would set a wage rate that would induce the worker to supply an optimal amount of effort. This wage rate would be generally higher than the competitive wage rate ("the carrot"), so if caught shirking, the worker will be penalized and be worse-off ("the stick"). Earlier examples of models with efficiency wages are Solow (1979), Shapiro and Stiglitz (1984), and Danthine and Donaldson (1990). In technical terms, the firms have to consider the incentive-compatibility constraint (ICC) of workers (conditions that affect workers in their decision whether to cheat or not) when decisions on wages and employment are made. More specifically, the equilibrium in the labor market is determined not by labor supply and demand, but the intersection of the labor demand and the worker's ICC, which produces an above-market wage. Therefore, the shirking model is one that leads to labor rationing, so the very introduction of efficiency wages creates a pool of involuntary unemployed. As shown in Paskaleva (2016), real wages in Bulgaria are indeed downward rigid. That is mostly due to collective agreements in place, which prohibit cuts in base wages. Such restrictions mean that adjusting labor costs needs to happen mostly through employment reductions. Lozev *et al.* (2011) also documents downward real wage rigidity in Bulgaria, even though it is lower than in the other EU member states. We use these empirical findings to motivate our modeling approach here. We follow Alexopoulos (2003), who incorporates Shapiro and Stiglitz (1984) mechanism into a general-equilibrium setup. The difference from the original study in Alexopoulos (2003) is that given the short time series available for Bulgaria after the introduction of the currency board arrangement (which corresponds to a period of stability), instead of estimating the model, in this paper we will calibrate it in order to produce simulated time series, which will be evaluated against their empirical counterparts. In this way, we are using the model as a laboratory that could provide some insights about the data generating process behind observed data. For better realism, we also add a detailed government sector, which is missing from Alexopoulos (2003).

In this paper we focus on the efficiency wages of "no-shirking" type. We are aware that there are alternative models of efficiency wages, which are not going to be explored further in this paper: For example, Akerlof (1982) regards labor contract as an exchange of gifts. On the one hand, a wage premium above the reference (reservation) wage rate can be viewed as a gift, which in turn inspires the worker to return a higher level of effort (relative to some minimally expected level). However, Danthine and Donaldson (1990) find that this gift exchange mechanism is not able to quantitatively account for the business cycle. For other more recent papers on efficiency wage theory, the reader is encouraged to consult Raurich and Sorolla (2014), Wu and Ho (2012), Brecher *et al.* (2010), Haruyama and Leith (2010), and the references therein.

The main ingredients of the original argument from Shapiro and Stiglitz (1984), which was incorporated in Alexopoulos (2003), which are adopted in this paper as well, are as follows: (i) firms cannot produce positive levels of output unless workers supply a positive level of effort; (ii) a worker's effort is imperfectly observable by the employer; (iii) firms fire workers who are detected shirking (i.e. providing a level of effort below the one stipulated in the labor contract). One difficulty resulting from the utilization of such a mechanism, and when the punishment implemented is the dismissal of the worker, is that the model results come at odds with aggregate data. This necessitated changing the punishment to a monetary penalty, expressed as a proportion of a worker's wage income, as in Alexopoulos (2003), and Burnside *et al.* (2000), which was in turn motivated by empirical evidence (Weiss 1990); indeed, firms generally do not outright dismiss first-time shirkers. They would punish the worker by refusing promotions, or through other channels. Such behavior on the firm side could be explained with the tough restrictions on layoffs. Proving in court that the workers is shirking is not easy, so the efficiency wage mechanism in this paper could be regarded as trying to capture the fact that labor legislation in Bulgaria is heavily skewed in the interest of the worker. As we will demonstrate in this paper, the departure from perfect competition, and the use of efficiency wages in particular, could potentially rationalize the propagation mechanism of business cycle fluctuations, and point to an important phenomenon that could help us understand labor markets in Bulgaria. In addition, as shown in Alexopoulos (2003), the labor market frictions arising from the unobservability of effort, when incorporated in a RBC model, could mimic the effect of high labor supply elasticity. That is, one does not need to use aggregation lotteries as in Rogerson (1988) and Hansen (1985), or even need to stretch the value of labor supply elasticity parameter.

The paper proceeds to evaluate the quantitative importance of efficiency wages in the case of the Bulgarian business cycle after the introduction of the currency board arrangement, which, when complemented with other reforms, brought aggregate stability to the economy. This is one of the novelties relative to Alexopoulos (2003, 2004). The presence of efficiency wages in the labor market could be a quantitatively important propagation mechanism that can replicate data behavior, especially along the labor market dimension. Overall, as in the case with the search-and-matching model calibrated for Bulgaria in Vasilev (2016), the model with efficiency wages provides a tractable general-equilibrium setup which also generates persistence in output and both employment and unemployment, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that RBC models generally do not have a strong internal propagation mechanism (besides the strong persistence in the TFP process). The rest of the paper is organized as follows. Section 2 describes the model setup. Section 3 outlines the model parameterization and the calibration strategy employed. Section 4 presents the steady state results. Section 5 discusses the impulse responses, compares simulated to empirical moments and evaluates the model's overall goodness-of-fit. Section 6 concludes.

2 Model setup

The structure of the model economy is as follows: There is a unit mass of households, as well as a representative firm. The households own the physical capital and labor, that are supplied to the firm. The firm produces output using labor and capital, but cannot observe the effort exerted by workers. The firm sets a wage at par with the reservation wage to induce an optimal level of effort. The government uses tax revenues from labor and capital income to finance the wasteful government consumption and the lump-sum government transfers.

2.1 Households

There is a unit mass of household in this economy, who own all the capital, and decide how many hours to work. Each household derives utility out of consumption and leisure

$$\sum_{t=0}^{\infty} \beta^t \Big\{ \ln c_t + \eta \ln(1 - e_t h - \xi) \Big\}, \tag{1}$$

where $0 < \beta < 1$ is the discount factor, $\eta > 0$ is the weight attached to leisure, and as in Burnside *et al.* (1993, 1996), $\xi > 0$ is some fixed cost of working. Parameter ξ is to be interpreted as some kind of organizational or planning cost, e.g the time spent on planning how to spent the day productively. Note that if the household decides to supply zero hours of labor, then $\xi = 0$. Variable c_t denotes consumption in period t, h denotes hours worked in period t, and e_t is the amount of effort exerted. The time available to each worker is normalized to unity. In contrast to the standard

perfectly-competitive model with observable effort, here we follow Alexopoulos (2003) and assume that that worker's effort will be imperfectly observable by firms.
Each household invests in physical capital to collect capital income $r_t k_t$. The law of motion for capital accumulation is

$$k_{t+1} = i_t + (1 - \delta)k_t, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate. Aggregate after-tax capital income, together with government transfers, g_t^t, is first pooled together (within the "family" of households), and then distributed equally among all households. In this way, households can partially insure one another against unfavorable outcomes in the labor market, *e.g.* not being selected for work. The common consumption can be represented as

$$c_t^h = (1 - \tau^y)r_t k_t + (1 - \delta)k_t - k_{t+1} + g_t^t, \tag{3}$$

where $0 < \tau^y < 1$ is the proportional income tax rate. The other type of income is the labor income, and households would differ in each period depending on their employment status.
From the perspective of firms, all individuals are identical, so employment outcome could be viewed as random, *i.e.* the firm will choose a certain share of households for work, and leave the rest unemployed. Since the level of effort is not directly observable by firms, some of the employed workers will work and exert the required effort level, e_t, stipulated in the contract, while others may decide to shirk. If caught, which happens with probability d due to the imperfect technology of detection, the individual is fired and receives a fraction $0 < s < 1$ of the wage. As in Burnside *et al.* (2000), the household does not observe whether the others shirked, or were fired, only the initial employment status.
The labor contract that the firms then need to offer has to be one that induces workers not to cheat in equilibrium. The contract would specify a wage rate, an effort level, and an implementable rule that a worker caught cheating on the job will be fired and paid only a fraction s of the wage, $0 < s < 1$. All workers know this in advance, and take the terms of the contract and the labor demand as given. In general, the supply of labor will exceed labor demand, so in equilibrium there is going to be involuntary unemployment.
In addition, each employed transfers/contributes T_t units of income to the unemployment pool, where the proceeds are used to payout to the unemployed. The level of transfers is such that individuals who are not selected for work by the firm are at least as well off as employed workers who are caught shirking. Labor income is also taxed at the constant proportional income tax rate of τ^y. The consumption of an employed worker who does not decide to shirk then equals:

$$c_t = c_t^h + (1 - \tau^y)w_t h_t - T_t, \tag{4}$$

where w_t is the hourly wage rate. Note that an employed worker who decided to shirk, but is not caught, obtains the same consumption as the conscientious worker, but a higher utility of leisure due to the zero effort exerted and thus no fixed cost of work is incurred.

In contrast, a worker who is employed, decides to cheat, and is caught, receives

$$c_t^s = c_t^h + (1 - \tau^y)sw_t h_t - T_t. \tag{5}$$

Alternatively, as proposed in Alexopoulos (2004), this is identical to a case where the firm pays $sw_t h_t$ upfront, and $(1-s)w_t h_t$ at the end of the period, which is retained in case the worker is caught cheating.

Note that not everyone will be employed, thus the employment rate $n_t < 1$, and $0 < 1 - n_t < 1$ would denote the mass of unemployed. The consumption of unemployed individuals, c_t^u, is then

$$c_t^u = c_t^h + \frac{n_t}{1-n_t} T_t, \tag{6}$$

where the transfer received by each unemployed equals $\frac{n_t}{1-n_t} T_t$. Note that if a household is selected for work and rejects the job offer, there will be no unemployment insurance, or it would receive just the common consumption c_t^h. Therefore, no household selected for work would have an incentive to reject, so the participation constraint will be trivially satisfied.

Depending on whether a household is selected for work or not, the corresponding instantaneous utility levels are:

$$u(c^u, e^u = 0, h^u = 0) = \ln c^u + \eta \ln 1 = \ln c^u, \tag{7}$$

if unemployed,

$$u(c, e, h) = \ln c + \eta \ln(1 - eh - \xi), \tag{8}$$

if employed and the worker does not shirk,

$$u(c, e, h) = \ln c + \eta \ln(1) = \ln c, \tag{9}$$

if the person shirks, but is not caught, and

$$u(c^s, e^s = 0, h^s = 0) = \ln c^s + \eta \ln(1) = \ln c^s, \tag{10}$$

if the person shirks, and is caught.

Let n_t^s be the proportion of shirkers and given a detection probability d of a shirker being caught, this implies dn_t^s would be the proportion of shirkers being caught, and $(1-d)n_t^s$ are the shirkers not being caught. In turn, $n_t - n_t^s$ are the employed individuals who decide not to shirk.

Finally, note that the leisure (in efficiency units) of shirkers that are caught, and leisure enjoyed by unemployed individuals is the same. Thus, the lump-sum transfer should be chosen so that the consumption levels of the two groups is equalized. This assumption is imposed for tractability considerations: In order to perform the aggregation that follows, we need to assume that a shirker that is caught is punished so hard, that his/her consumption is as low as the consumption of an unemployed person. It follows that

$$\begin{aligned} c_t^s &= c_t^u & (11) \\ c_t^h + (1-\tau^y) s w_t h_t - T_t &= c_t^h + \frac{n_t}{1-n_t} T_t. & (12) \end{aligned}$$

or

$$T_t = (1-n_t)(1-\tau^y) s w_t h_t. \tag{13}$$

In this setup the aggregate household takes as given the effort level and wage rate $\{e_t, w_t\}_{t=0}^{\infty}$, which are specified in the contracts that the firm offers. This means that the household takes firm's labor demand as given, which would produce involuntary unemployment. Thus, by taking initial condition for capital, k_0 as given, the household chooses $\{c_t^h, k_{t+1}\}_{t=0}^{\infty}$ to maximize (where we have already used the fact that $c_t^u = c_t^s$)

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t \{ & (n_t - n_t^s) [\ln c_t + \eta \ln(1-\xi-e_t h)] + \\ & n_t^s [(1-d) \ln c_t + d \ln c_t^s] + (1-n_t) \ln c_t^s \} \end{aligned} \tag{14}$$

s.t

$$\begin{aligned} (n_t - dn_t^s) c_t + (dn_t^s + 1 - n_t) c_t^s = \ & (1-\tau^y) r_t k_t + (1-\delta) k_t - k_{t+1} + g_t^t + \\ & +(n_t - dn_t^s)(1-\tau^y) w_t h_t + \\ & +dn_t^s (1-\tau^y) s w_t h_t. \end{aligned} \tag{15}$$

The first-order conditions (FOCs) are as follows:

$$\begin{aligned} c_t^h &: \frac{(n_t - dn_t^s)}{c_t} + \frac{(dn_t^s + 1 - n_t)}{c_t^s} = \lambda_t & (16) \\ k_{t+1} &: \lambda_t = \beta E_t \lambda_{t+1} [(1-\tau^y) r_{t+1} + 1 - \delta] & (17) \\ TVC &: \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0, & (18) \end{aligned}$$

where the last equation is the transversality condition (TVC); This is a boundary condition that needs to be imposed to rule out explosive solutions. The other optimality conditions are standard: the first equates the marginal utility of consumption to marginal cost of wealth. The second equation describes how physical capital should be allocated across time (the so-called "Euler equation").

2.2 Firm

There is a perfectly competitive representative firm that produces output via the following Cobb-Douglas production function (in the case where workers exert positive effort; it follows trivially that without any effort exerted, no output can be produced)

$$y_t = A_t k_t^{\alpha} (n_t h e_t)^{1-\alpha}, \tag{19}$$

where A_t is the level of technology, determined by an AR(1) process, and $0 < \alpha, 1 - \alpha < 1$ are the capital and labor shares, respectively.
The firm chooses the employment rate, capital input, wage rate (and thus effort level) to maximize

$$A_t k_t^{\alpha} (n_t h e_t)^{1-\alpha} - w_t n_t h - r_t K_t \tag{20}$$

s.t. "no shirking condition" (the ICC):

$$\ln c_t + \eta \ln(1 - \xi - h e_t) \geq (1 - d) \ln c_t + d \ln c_t^s \tag{21}$$

or

$$d \ln c_t + \eta \ln(1 - \xi - h e_t) \geq d \ln c_t^s \tag{22}$$

In equilibrium, the firm chooses the optimal quantities of capital and employment. In addition the firm offers an efficiency wage rate w_t to induce a certain optimal effort level, i.e. $e_t = e(w_t)$. The firm's optimality conditions are as follows:

$$k_t : r_t = \alpha \frac{y_t}{k_t}, \tag{23}$$

$$n_t : w_t h = (1 - \alpha) \frac{y_t}{n_t}. \tag{24}$$

$$w_t : n_t h = (1 - \alpha) \frac{y_t}{e_t} e'(w_t) \tag{25}$$

Dividing the FOC for employment and wages, we obtain the standard Solow (1979) condition

$$\frac{w_t e'(w_t)}{e_t} = 1 \tag{26}$$

or

$$\frac{w_t}{e(w_t)} = (1 - \alpha) \frac{y_t}{e_t n_t h}. \tag{27}$$

In other words, this is an equation that characterizes firm's labor demand. Note that the firm minimizes cost per efficiency unit here: firms want to hire labor as cheaply as possible, and $w/e(w)$ is the cost per unit of effective labor.
Next, for a given wage rate, the "no-shirking" condition indicated a maximum effort level the firm can obtain from each worker. Rearranging further the constraint, we obtain

$$e_t \leq e(w_t) = \frac{1 - \xi}{h} - \frac{1}{h} (c_t^s / c_t)^{d/\eta}. \tag{28}$$

The firm takes T_t as given (the firm cannot influence T_t, as it is determined by the average compensation of a household), so the right-hand side is only a function of w_t, since

$$\frac{c_t}{c_t^s} = \frac{c_t^h + (1-\tau^y)w_t h_t - T_t}{c_t^h + (1-\tau^y)s w_t h_t - T_t} \tag{29}$$

Also

$$w_t = \frac{c_t - c_t^s}{(1-\tau^y)(1-s)h} \tag{30}$$

since the ratio of consumptions is a function of the wage rate. In addition, the ratio of consumptions is a constant (denoted by χ), and a function of model parameters, *i.e.*

$$\frac{c_t}{c_t^s} = \frac{c_t^h + (1-\tau)w_t h_t - T_t}{c_t^h + (1-\tau^y)s w_t h_t - T_t} = \chi > 1 \tag{31}$$

In general, the optimal level of employment, will not coincide with the proportion of workers wishing to accept the contract $(w_t, e(w_t))$. As long as firm's demand for labor is less than the labor supply, the "no-shirking" constraint will be binding (hold with equality), and there will be involuntary unemployment in equilibrium.

2.3 Government

The government will be assumed to be running a balanced budget in every period. The government collects revenue from levying taxes on capital and labor income, and then spends on government consumption and transfers, which are returned lump-sum to the households:

$$\tau^y\left[r_t k_t + w_t n_t h\right] = g_t^c + g_t^t, \tag{32}$$

where g_t^c are government purchases. Government spending share will be set equal to its long-run average, so the level will be varying with output. Government transfers will be residually determined and will always adjust to make sure the budget is balanced.

2.4 Decentralized Dynamic Equilibrium (DCE) with Efficiency Wages

Given the process followed by total factor productivity $\{A\}_{t=0}^{\infty}$, average effective income tax rate $\{\tau^y\}$, initial capital endowments stock k_0, hours worked per household h, the decentralized dynamic equilibrium with efficiency wages is a list of sequences $\{c_t, c_t^s, i_t, k_t, n_t, e_t\}_{t=0}^{\infty}$ for each household i, input levels $\{k_t, n_t\}$ chosen by the firm in each time period t, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) each household i maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit by setting an efficiency wage to satisfy the workers' incentive compatibility constraint and to induce an optimal effort level; (iii) government budget is balanced in each period; (iv) all markets clear.

3 Data and model calibration

When modeling business cycle fluctuations in Bulgaria, we will focus on the period after the introduction of the currency board (1999-2014). Data on real output, consumption and investment was collected from National Statistical Institute (2015), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2015). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics, *e.g.* Kydland and Prescott (1982). First, the average income tax rate was set to its (average effective) rate $\tau^y = 0.100$. The depreciation rate of physical capital in Bulgaria, $\delta = 0.05$, was taken from Vasilev (2015a), where the rate was estimated as the average depreciation rate over the period 1999-2014. The discount factor, $\beta = 0.942$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 3.491$, in the steady-state Euler equation, describing oprimal capital allocation in two adjacent periods. The capital share parameter, $\alpha = 0.429$, was obtained as unity minus the average ratio labor income in aggregate output over the period 1999-2014. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital during the communist regime (1945-1989).
As in Burnside at al. (2000), we set $\xi = 0.012$. This is the fixed cost of the household supplying a positive level of effort, and the value chosen corresponds to 10 min per day. Next, we set χ to match the average c_t/c_t^u in data. The rate $s = 0.82$ was calibrated to match the average employment rate in Bulgaria over the period, which in Vasilev (2016) was obtained to equal $n = 0.533$. Note that by matching the average employment rate in the steady state, we have given up the possibility to match the average unemployment rate in the steady state. Thus by construction the unemployment rate in the model is higher than the rate in data, because it contains out of the labor force as well. In the model, there is no search/registration dimension, so as a result, there is no out-of-labor force component (it is zero in the model by construction): one cannot distinguish in the model between those individuals without a job and searching for one (and registering in job centers/unemployment bureaus), and those without a job but not searching/registering. In addition, in this class of infinitely-lived households, total population is of working age, and is either employed, or unemployed. In fact, there is one special case when the measurements of empirical and model unemployment rate are equivalent: if all out of the labor force consists of discouraged workers, who want to register as unemployment, but cannot. Unfortunately, in reality one can register in an unemployment bureau for a limited period of time (usually the period when one is eligible for unemployment benefits). Hence, we would not focus much on the steady-state level of the unemployment rate in the model, but instead put the emphasis on the business cycle fluctuations.

Next, the relative weight attached to the utility out of leisure in the household's utility function, η, and the shirking detection probability d, can only be determined as a ratio, which would be calibrated to match χ. Following Vasilev (2015b), the hours worked per person h is set to one-third. Finally, the moments of the total factor productivity process were obtained from running an AR(1) regression on the detrended Solow residuals.
Table 1 summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.942	Discount factor	Calibrated
α	0.429	Capital Share	Data average
$1-\alpha$	0.571	Labor Share	Calibrated
η	N/A	Relative weight attached to leisure	–
d	N/A	Shirking detection probability	–
d/η	0.062	Shirking detection probability to leisure weight ratio	Calibrated
s	0.72	Proportion of income retained if caught shirking	Calibrated
δ	0.050	Depreciation rate on physical capital	Data average
ξ	0.012	Fixed cost of working	Data average
χ	1.285	consumption ratio employed-to-unemployed	Set
h	0.333	Share of time spent working	Calibrated
n	0.533	Employment rate	Data average
τ^y	0.100	Average tax rate on income	Data average
ρ_a	0.701	AR(1) parameter, total factor productivity	Estimated
σ_a	0.044	st.dev, total factor productivity	Estimated

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.674	0.674
i/y	Investment-to-output ratio	0.201	0.175
g^c/y	Government cons-to-output ratio	0.159	0.151
k/y	Capital-to-output ratio	3.491	3.491
$wenh/y$	Labor income-to-output ratio	0.571	0.571
rk/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
n	Employment rate	0.533	0.533
u	Unemployment rate	0.467	0.467
e	Effort level	N/A	1.979
A	Scale parameter of the production function	N/A	1.062
$\tilde{r}$	After-tax net return on capital	0.056	0.061

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2. The steady-state level of output was normalized to unity (hence the level of technology A differs from unity), which greatly simplified the computations, and allows the steady-state to be solved by hand. Next, the model matches consumption-to-output ratio by construction; The investment and government purchases ratios are also closely approximated. The shares of income are also identical to those in data, which follows directly from the constant-returns to scale featured by the aggregate production function. The after-tax return, where $\tilde{r} = (1-\tau^y)r - \delta$ is also relatively well-captured by the model.

5 Out of steady-state model dynamics

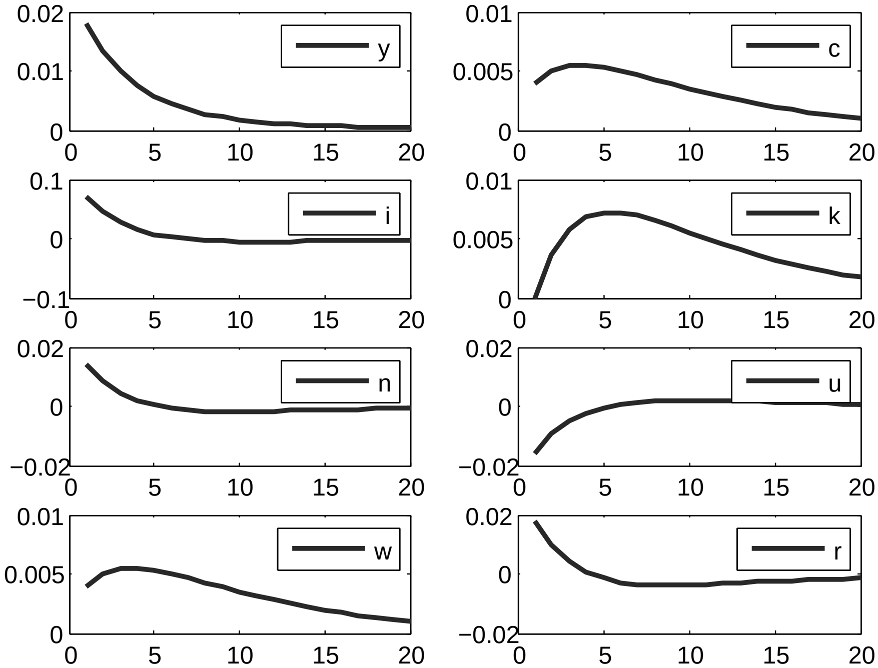
Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations (described in the definition of DCE in Section 2.4) around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts. Special focus is put on the cyclical behavior of labor market variables.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response function (IRFs) are presented in Fig. 1. As a result of the one-time unexpected positive shock to total factor productivity, output increases. This expands the availability of resources in the economy, so consumption, investment and government consumption also increase upon impact.

At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. The aggregate households responds to the incentives contained in prices and start accumulating capital, and supplying more hours worked. In turn, the increase in capital input feeds back in output through the production function and further adds to the positive effect of the technology shock. In the labor market, which is characterized by the presence of efficiency wages, to prevent workers from shirking, firms increase the wage rate (to make the cost of punishment higher), and in turn households increase their work effort. On the firm side, the increase in the marginal product of labor also makes the value of

Figure 1: Impulse Responses to a 1% surprise innovation in technology



marginal product of labor higher, so firms increase employment. In turn, the increase in employment further increases output. Note that in the shirking model, after a surprise innovation in technology, there is a large response to employment with a relatively smaller increase in the real wage. In other words, the wage rate exhibits real rigidity.

Over time, as capital is being accumulated, its marginal product starts to decrease, which lowers the aggregate household's incentives to save. As a result, capital eventually returns to its steady-state, and exhibits a hump-shaped dynamics over the transition path. Consumption also exhibits the same shape in its dynamic pattern. With efficiency wages, the variation in the wage rate follows exactly the variations in consumption. The rest of the variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

5.2 Simulation and moment-matching

We will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency. To minimize the sample error, the simulated moments are averaged out over the computer-generated draws. The model matches quite well the absolute volatility of output. However, the model overestimates the variability in consumption, and investment. This shortcoming of the model could be explained by structural factors in Bulgaria, such as privatization of state assets. In addition, public investment in infrastructure has been also substantial in the last few years due to the EU accession funds. Still, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output. By construction, government spending in the model varies as much as in data.

Table 3: Business Cycle Moments

	Data	Model
σ_y	0.05	0.05
σ_c/σ_y	0.55	0.81
σ_i/σ_y	1.77	2.43
σ_g/σ_y	1.21	1.00
σ_n/σ_y	0.63	0.44
σ_w/σ_y	0.83	0.81
$\sigma_{y/n}/\sigma_y$	0.86	0.81
σ_u/σ_y	3.22	0.50
σ_w/σ_n	1.32	1.85
$corr(c,y)$	0.85	0.89
$corr(i,y)$	0.61	0.83
$corr(g,y)$	0.31	1.00
$corr(n,y)$	0.49	0.61
$corr(w,y)$	-0.01	0.89
$corr(u,y)$	-0.47	-0.61
$corr(n,y/n)$	-0.14	0.22

With respect to the labor market variables, the variability of employment predicted by the model is less than in data, but the variability of wages is well-matched: In the efficiency wage model, it varies as much as consumption. Thus, the efficiency wage could be serving as a good approximation for the behavior of workers whose income is mostly labor earnings.

The model fails in matching unemployment volatility. In the model it varies as much as the employment rate. The reason behind this mismatch could be driven by several possible explanatory factors: the fact that the model misses the "out-of the-labor-force" segment, as well as the significant emigration to EU member states. Furthermore, as a model of employment, the real-business-cycle framework with efficiency wages matches employment volatility well, and not so well the unemployment volatility that well. Even though the model generates involuntary unemployment, it is determined residually as the number of total individuals minus the number of employed.

Next, in terms of contemporaneous correlations, the model slightly over-predicts the pro-cyclicality of the main aggregate variables - consumption, investment, and government consumption. This, however, is a common limitation of this class of models. However, along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, is relatively well-matched. With wages, the model predicts strong cyclicality, while wages in data are acyclical. This is an artifact of the efficiency wage which establishes a bi-directional link with labor productivity. In addition, wages in such setups are as variable as consumption.

In the next subsection, we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) on all model variables except for capital and effort, are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients of empirical ACFs and CCFs at different leads and lags are presented in Table 4 against the simulated AFCs and CCFs. Following Canova (2007), this comparison is used as a goodness-of-fit measure. As seen from Table 4, the model compares well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model.

Despite being outside the confidence bounds, the persistence of labor market variables are also somewhat captured by the model dynamics. Nevertheless, the model with efficiency wages generates persistence in output and both employment and unemployment, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. Furthermore, the efficiency wage mechanism dominates the setup with indivisible labor, developed by Rogerson (1988), and incorporated in the RBC setup by Hansen (1985). In those models, labor market is modeled in the Walrasian market-clearing spirit, and output and unemployment persistence is low.

Next, as seen from Table 5, over the business cycle, in data labor productivity leads employment. The efficiency wage model, however, cannot account for this fact. In the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$ (s.e.)	1.000 (0.000)	0.951 (0.040)	0.890 (0.057)	0.810 (0.083)
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$ (s.e.)	1.000 (0.000)	0.951 (0.040)	0.890 (0.057)	0.810 (0.083)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$ (s.e.)	1.000 (0.000)	0.957 (0.028)	0.904 (0.055)	0.844 (0.080)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$ (s.e.)	1.000 (0.000)	0.955 (0.029)	0.902 (0.055)	0.839 (0.080)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$ (s.e.)	1.000 (0.000)	0.959 (0.025)	0.910 (0.048)	0.854 (0.069)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$ (s.e.)	1.000 (0.000)	0.952 (0.030)	0.894 (0.057)	0.824 (0.082)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$ (s.e.)	1.000 (0.000)	0.959 (0.025)	0.910 (0.048)	0.854 (0.069)

between employment and labor productivity is only a contemporaneous one. Still, the model with efficiency wage is a clear improvement over the perfectly-competitive labor market paradigm used in Vasilev (2009).

Table 5: Dynamic correlations for Bulgarian data and the model economy

Method	Statistic	k						
		−3	−2	−1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	−0.342	−0.363	−0.187	−0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$ (s.e.)	0.080 (0.340)	0.080 (0.298)	0.080 (0.247)	0.212 (0.296)	−0.037 (0.197)	−0.050 (0.236)	−0.055 (0.236)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	−0.040	-0.390	−0.57
Model	$corr(n_t, w_{t-k})$ (s.e.)	0.080 (0.340)	0.080 (0.298)	0.080 (0.247)	0.212 (0.296)	−0.037 (0.197)	−0.050 (0.236)	−0.055 (0.236)

6 Conclusions

In this paper we investigate the quantitative importance of efficiency wages in explaining fluctuations in Bulgarian labor markets. This is done by augmenting an otherwise standard real business cycle model a la Long and Plosser (1983) with unobservable workers effort by employers and wage contracts as in Shapiro and Stiglitz (1984). This imperfection in labor markets introduces a strong propagation mechanism that allows the model to capture the business cycles in Bulgaria better than earlier models. The model performs well vis-a-vis data, especially along the labor market dimension, and in addition dominates both the market-clearing labor market framework featured in the standard RBC model, *e.g* Vasilev (2009).

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