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WEI LI*, HUAN ZHAO*#, MEILING LIU**, SIQI LI*, WENFENG SUN*, LEI WANG***

THREE-DIMENSIONAL SIMULATION OF THE FRACTURE SYSTEM DISTRIBUTION IN FORMATION ROCK BASED ON FRACTAL METHOD

TRÓJWYMIAROWE SYMULACJE ROZKŁADU SPĘKAŃ W SKAŁACH Z WYKORZYSTANIEM METODY FRAKTALI

During exploitation process of fractured reservoir, the complex distribution of natural fracture system may lead to a series of accidents, such as sand plug and multi fracture extension in hydraulic fracturing operation. Considering the difficulties of numerical analysis on formation rock mass fracture system distribution, three-dimensional geometry model of a single fracture formation is proposed in this paper, and fractal geometry method is introduced to build the three-dimensional fractal description model of formation fracture system distribution. On this basis, the effects of fractal parameters on natural fracture porosity, permeability and other properties are analyzed. The results show that: First, the number and propagation of natural fracture are controlled by the fractal dimension, the number of groups and the initial quantity. Second, the fractal dimension of natural fracture distribution has an obvious effect on natural fracture porosity and permeability. Third, porosity and permeability of natural fracture distribution both experience exponential growth as fractal dimension increases. Fourth, when the fractal dimension remains constant, the porosity and permeability of natural fractures both increase with the fracture scale.

Keywords: hydraulic fracturing, fractal dimension, natural fracture, fracturing fracture, numerical simulation

W trakcie eksploatacji złoża zalęgającego w spękanych warstwach i pokładach złożony system naturalnych spękań prowadzić może do licznych incydentów, np. powstawania zatorów piaskowych lub nadmiernego rozszerzenia spękań w trakcie szczelinowania hydraulicznego. Z uwagi na trudności związane z analizą numeryczną rozkładu spękań skał macierzystych, w pracy zaproponowano trójwymiarowy model geometryczny pojedynczego pęknięcia z wykorzystaniem metod geometrii fraktalnej do opracowania trójwymiarowego modelu opisującego powstawanie układu spękań i ich rozkład. Na tej podstawie przeanalizowano wpływ parametrów fraktalnych na naturalną porowatość pękniętych skał, ich przepuszczalność oraz pozostałe właściwości. Wyniki badań wskazują że, po pierwsze, liczba

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* NORTHEAST PETROLEUM UNIVERSITY

** DAQING OILFIELD PRODUCTION TECHNOLOGY INSTITUTE

*** NO. 2 OIL PRODUCTION PLANT, PETROCHINA DAQING OILFIELD COMPANY

Corresponding author: Zhaohuan7696@163.com

i tempo propagacji naturalnych spękań uzależnione są od wymiarów fraktalnych, liczby grup i wielkości początkowej. Po drugie, wymiary fraktalne naturalnego systemu spękań skał mają zdecydowany wpływ na porowatość i przepuszczalność. Po trzecie, porowatość i przepuszczalność naturalnych systemów pęknięć wykazują wzrost w miarę wzrastania wymiarów fraktalnych. Po czwarte, gdy wymiary fraktalne pozostają niezmienne, zarówno porowatość i przepuszczalność naturalnych spękań rosną wraz ze skalą fraktali.

Słowa kluczowe: szczelinowanie hydrauliczne, wymiary fraktali, naturalne spękania, spękania powstałe wskutek szczelinowania, symulacje numeryczne

1. Introduction

Exploitation of fractured reservoir is an important work for enhancing oil recovery in oil fields of China. However, fracture system distribution is always very complex, and it is affected by many factors, which greatly complicates the fracture system hydraulic fracturing operation (Sondergeld et al., 2010; King, 2010; Kassis, 2010; Rickman, 2008). In addition, accidents such as sand plug, multi fracture extension may happen occasionally (Reugel et al., 2000; Peacock et al., 2005; Chen et al., 2008; Yao et al., 2008), and cause unexpected disasters for human safety and production safety. Consequently, it is of great significance to investigate fractured reservoir hydraulic fracturing mechanism for both theoretical research and field accident prevention.

Nowadays, there are mainly three kinds of model in reservoir modeling, namely equivalent continuous model, discrete model and comprehensive model, of which equivalent continuous model and discrete model are based on the appearance of fractures in the model (Wolfsberg, 1997). As the research on natural fracture distribution goes on, scholars begin to develop more suitable techniques to simulate the surface of rock fracture under the condition of three-dimensional rock mass fracture face, and a series of new simulation theories are put forward (Hart et al., 1988; Zhou et al., 1997). Further, Feng et al.(2005)conducted the research on three-dimensional distribution of rock mass fracture face and developed a fractal simulation software to simulate fracture quantity and distribution in rock mass plane (Feng et al., 2005; Zhao et al., 2005). Since then, mechanism of rock mass fracture distribution has gradually been revealed.

Considering the difficulties of numerical analysis on fractured reservoir hydraulic fracturing, fractal geometry theory is introduced to analyze the fractal dimension of fault and fracture system in this paper. Then, the regression relation is built between fractal dimensions of fractures with different scale. According to the discrete random method, a program is developed to study the two-dimensional trace distribution of natural fracture and the effects of fractal parameters on natural fracture character are discussed. Further, the influencing rule of fractal parameters on fracture geometrical feature is analyzed based on two-dimensional PKN and KGD models.

2. The mathematical model

2.1. Space geometry model of single fracture rock

At present, only two types of three-dimensional model can be applied in fracture face seepage calculation, namely disk structural plane network model and Dershowitz polygon structural plane network model. Considering that formation fractures in the neighbor of the borehole mainly consist of small and medium sized fractures, Baecherdisk fracture model is used, as shown in Figure 1.

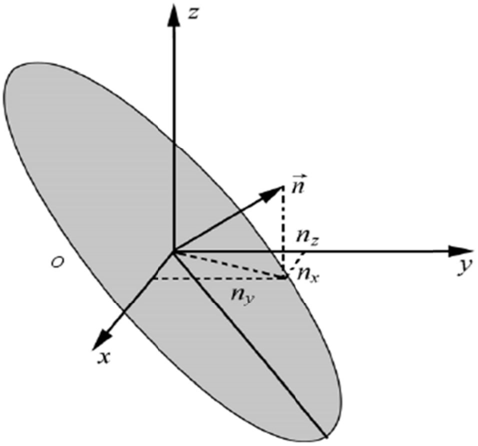

Fig. 1. Three-dimensional space layout of single fracture

Three-dimensional space direction vector of the surface of disk fracture can be expressed by three components (n_x, n_y, n_z) of the three axes (x, y, z). The spatial position of disk fracture is determined by the coordinates of the fracture midpoint, the radius and the fracture occurrence. The fracture occurrence can be described by dip angle θ_{ST} and trend angle θ_{SP}, and the relationship between fracture occurrence and normal vector can be seen in Figure 2.

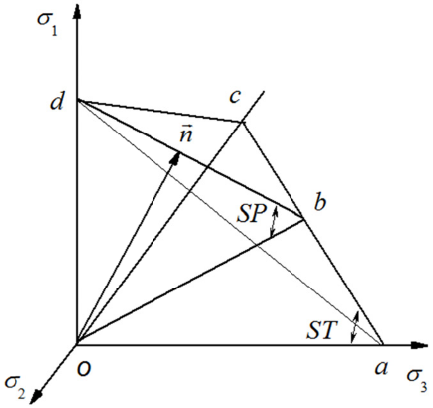

Fig. 2. The geometry relationship among dip angle, azimuth angle and normal vector

Assume that o (x_o, y_o, z_o) is the center point coordinate, r is the radius, and that $\vec{n}(n_x, n_y, n_z)$ are normal vectors of three coordinate axes. Then, two Eqs. can be obtained as follows:

$$n_x(x - x_o) + n_y(y - y_o) + n_z(z - z_o) = 0 \tag{1}$$

$$(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 \le r^2 \tag{2}$$

As Figure 2 shows, dip angle θ_{ST} and trend angle θ_{SP} of the fracture are both in the range of 0°~180°. If the direction of azimuth is the same as the negative half shaft of x axis, it is an acute angle, or it is an obtuse angle. If tilt direction of fracture is the same as the negative half shaft of x axis, the dip angle is an acute angle, or it is an obtuse angle.

According to the relationship between fracture occurrence and normal vector in Figure 2, normal unit vector $\vec{n}(n_x, n_y, n_z)$ can be expressed by dip angle and strike angle of the fracture:

$$\begin{cases} n_x = \sin\theta_{ST}\sin\theta_{SP} \\ n_y = \cos\theta_{ST} \\ n_z = -\sin\theta_{ST}\cos\theta_{SP} \end{cases} \tag{3}$$

Based on Eqs. (2) and (3), the size and location of fracture face can be determined.

2.2. Three-dimensional fractal description model of formation fracture system distribution

A large number of studies show that (Jin et al., 2005; 2006; Li et al., 2000) the distribution of formation fault is similar to that of fracture. Development of reservoir fracture system can be quantitatively described according to fractal dimension values of large scale formation fracture and small scale core fracture (Fig. 3).

Fig. 3. The fracture distribution of rock surface

According to the fractal geometry theory, there are a group of fractal fracture media in the Euclidean space of D_T ($D_T \leq 3$) dimension. Thus, a power law relationship between fractal fracture medium measurement scale (r) and cumulative number (N) as well as cumulative length (L) can be obtained (Jin et al., 2006; Li et al., 2000):

$$N = N_0 r^{D_T - D} \tag{4}$$

$$L = L_0 r^{-D} \tag{5}$$

where N_0 is the initial number of fractal fracture medium; L_0 is the initial scale of fractal fracture medium; and D is the fractal dimension of fractal fracture medium distribution, $D < D_T$.

In the three-dimensional Euclidean space ($D_T = 3$), there are several discrete elements with a side length of $L_T (L_{T\min} \leq L_T \ll L_{T\max})$. Each discrete element includes m groups of fractal fracture media as shown in Figure 1. For the fractal fracture of group i:

$$N_i = N_{0i} r^{D_T - D} \qquad 1 \leq i \leq m \tag{6}$$

$$L_i = L_{0i} r^{-D_i} \qquad 1 \leq i \leq m \tag{7}$$

where $L_{T\min}$ is the lower limit of discrete element size; $L_{T\max}$ is the upper limit of discrete element size; N_{0i} is the initial number of the fractal fracture in group i; L_{0i} is the initial scale of the fractal fracture in group i; N_i is the accumulated quantity of the fractal fracture in group i; L_i is the accumulated length of the fractal fracture in group i; and D_i is the fractal dimension of fractal fracture distribution in group i.

3. Formation fractal simulation

3.1. Two-dimensional simulation of formation rock mass fracture system distribution

According to Eqs. (6) and (7), a program is developed to obtain the distribution map of two-dimensional trace of natural fracture system under horizontal section in the near-wellbore region. Then, the effects of fractal dimension, the number of groups and fracture initial value on the natural fracture distribution are analyzed. The fractal dimension of each following figure union is 1.6.

According to Figures 4-6, the number and development of natural fracture distribution are controlled by the fractal dimension, the number of groups and the initial quantity, and the number of natural fracture distribution increases with the fractal dimension.

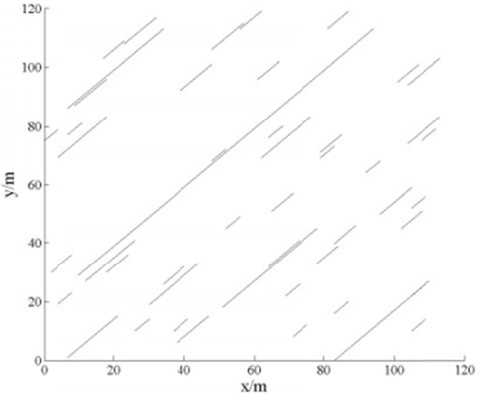

Fig. 4. Number 1, the initial number is 1, the initial length is 120, trend is 45°

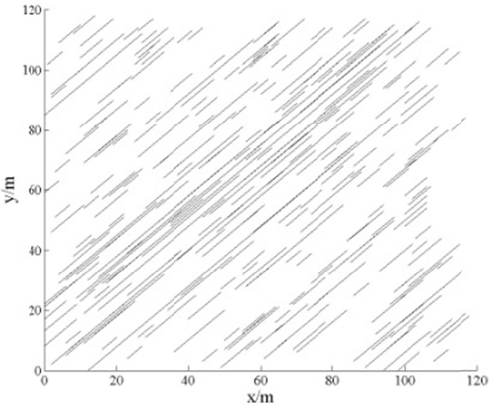

Fig. 5. Number 1, the initial number is 6, the initial length is 120, trend is 45°

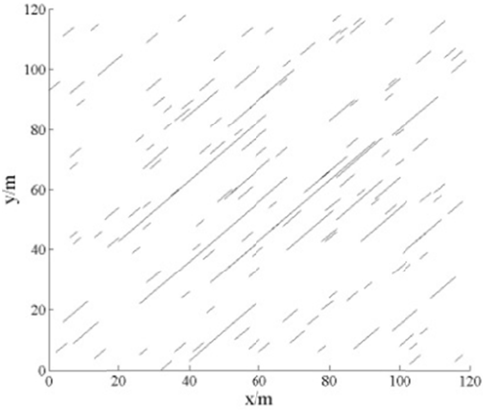

Fig. 6. Number 1,.the initial number is 3, the initial length is 60, trend is 45°

Figure 4, 5 and 7 show that the quantity of a single group of fracture distribution increases with the initial quantity. Figures 6 and 7 indicate that the length of a single group of the fracture distribution increases with the initial length. Figures 7-9 shows that for multiple groups of natural fractures, the distribution of natural fracture becomes more and more complex with the increase of the number of fracture groups. Fracture system that consists of multiple sets of natural fractures is very similar to the natural fracture system in the formation.

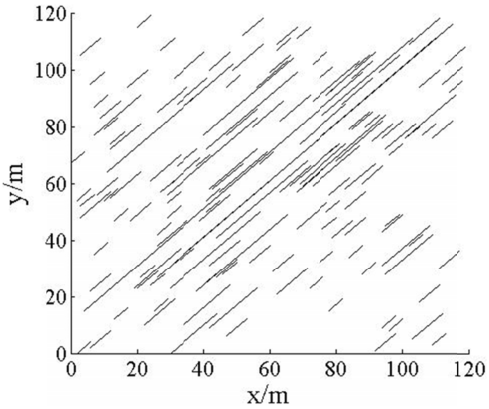

Fig. 7. Number 1, the initial number is 3, the initial length is 120, trend is 45°

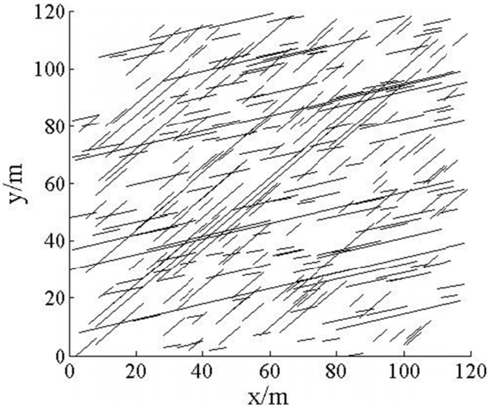

Fig. 8. Number 2, the initial number is 3, the initial length is 120, trend is 45°, trend is 15°

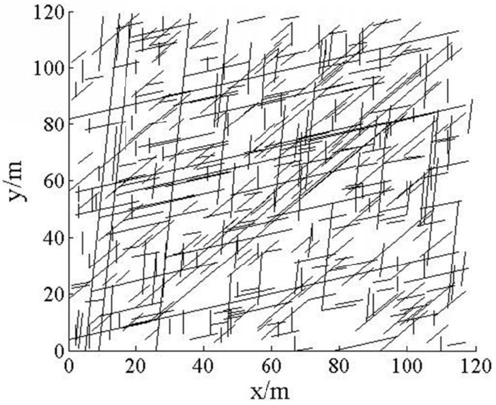

Fig. 9. Number 3, the initial number is 3, the initial length is 120, trend s 45°, trend is 15°, trend is 85°

3.2. Three-dimensional simulation of formation rock mass fracture system distribution

According to Eqs. (6) and (7), a program is developed to obtain the three-dimensional distribution map of natural fracture systems in the near-wellbore region (Figs 10 and 11). Then, the effects of fractal parameters, such as the fractal dimension, the number of groups, the fracture initial value, the direction and dip angle, on the three-dimensional distribution of natural fractures are analyzed.

For the convenience of analysis, the first group of fractures is colored in green and yellow, while the second group is in red and purple. In order to execute different sorts of research tasks, the fracture shape in Figures 10 and 11 is disc and that in Figure 12-13 is hexagonal.

Figures 10 and 13 show that, natural fracture group has similar direction and dip angle under three-dimensional space. However, their size is different, and their overall distribution is

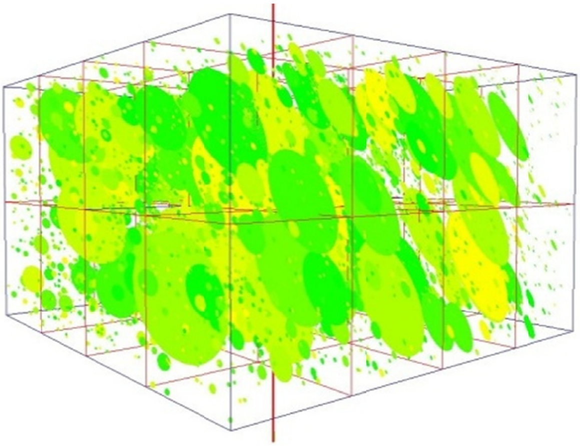

Fig. 10. Number 1, the initial number is 3, the initial length is 120, trend is 45°, tendency is 30°, disc

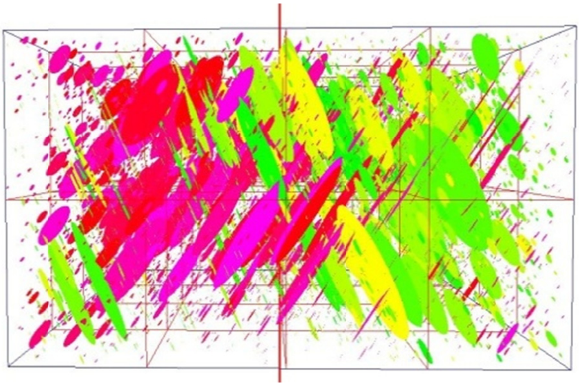

Fig. 11. Number 2, the initial number is 3, the initial length is 120, trend is 45º, tendency is 30º, disc

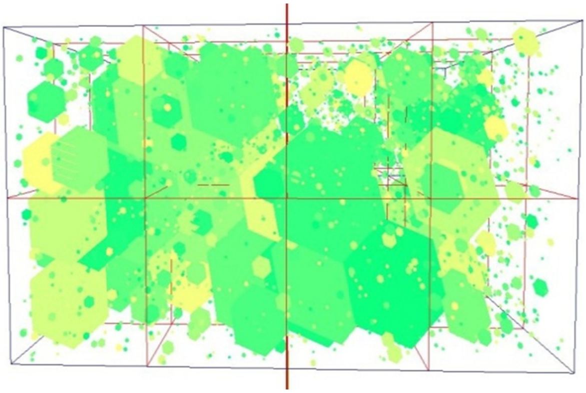

Fig. 12. Number 1, the initial number is 3,
the initial length is 120, trend is 45º, tendency is 30º, hexagon

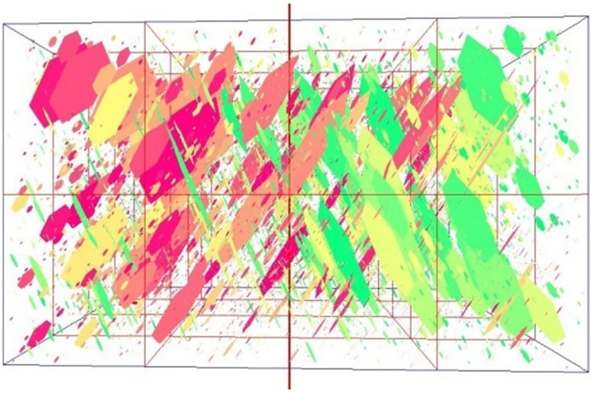

Fig. 13. Number 2, the initial number is 3, the initial length is 120, trend is 45º, trend is 15º,
tendency is 30º, tendency is 120º, hexagon

out of order, namely following a random distribution. It is noted that the simulation of fracture system agrees well with natural fracture system. Thus, the simulation is instructive to analysis on formation fracture distribution and can be used as an effective tool to investigate the effects of natural fractures on fracture initial and propagation. In addition, the influencing rule of fractal parameters, such as the initial quantity, the initial length and the fractal dimension, on fracture system distribution in three-dimension space agrees with that in two-dimension space.

4. Effects of fracture fractal parameters on porosity and permeability

Porosity and permeability of natural fractures are important parameters for analysis on the quality of fractured reservoir. The fracture development can be described by fractal parameters of natural fractures, which has close relationships with porosity and permeability.

Fracture density is an important parameter to represent fracture propagation. Conventional fracture density consists of three kinds of density, namely linear density, surface density and volume density. There into, volume density refers to the ratio of fracture total area to matrix total volume per unit volume:

$$D_{fV} = \frac{S}{V_b} \tag{8}$$

where D_{fV} is the volume density, S is the fracture total area, and V_b is the matrix total volume.

Fracture porosity is defined as the ratio of fracture volume in rock sample to the total volume of rock sample. Through observing the core, the average width and volume density of fractures can be obtained, and then

$$\phi_f = \frac{V_f}{V_b} \times 100\% = \frac{S\overline{b}}{V_b} \times 100\% = D_{fV}\overline{b} \times 100\% \tag{9}$$

where $\overline{b}$ is the average width of fracture.

Permeability of fractures reflects the fluid transmission ability of fracture. For a set of fractures with a certain dip angle, the permeability can be calculated by:

$$K_f = \frac{b^3}{12h}\cos^2\alpha_{ST} \tag{10}$$

For the rock with multiple groups of fractures, fracture permeability is actually the sum of permeability of each single fracture group. Thus,

$$K_f = \frac{1}{12h}\left[\cos^2\alpha_{ST1}\sum_{i=1}^{n} b_i^3 + \cos^2\alpha_{ST2}\sum_{j=1}^{m} b_j^3\right] \tag{11}$$

According to Eqs. (9) and (10), Figures 14 and 15 can be obtained. As can be seen, the porosity and permeability of natural fractures are proportional to the fractal parameters, and show a rising trend with the increase of fracture development degree. If the fractal dimension remains constant, the porosity and permeability of natural fractures increase with the scale.

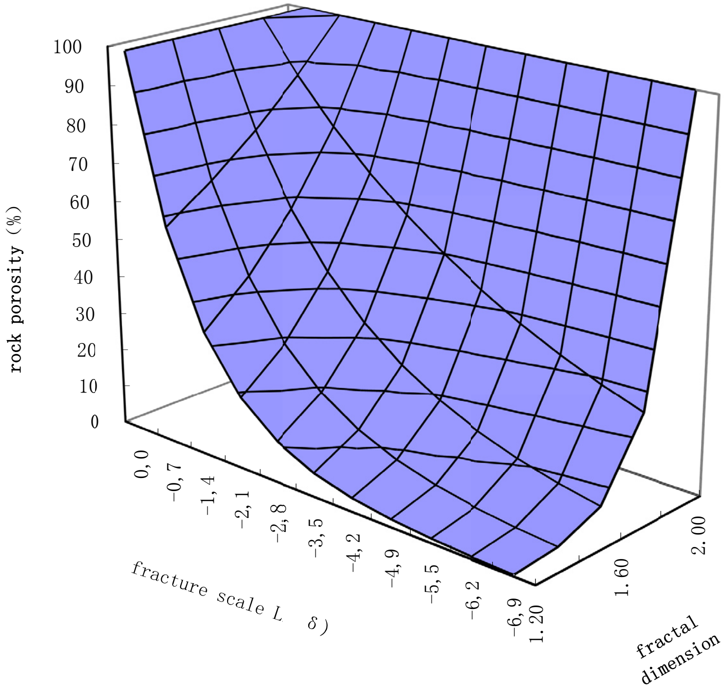

Fig. 14. Fracture porosity along with the change of scale under different fractal dimension

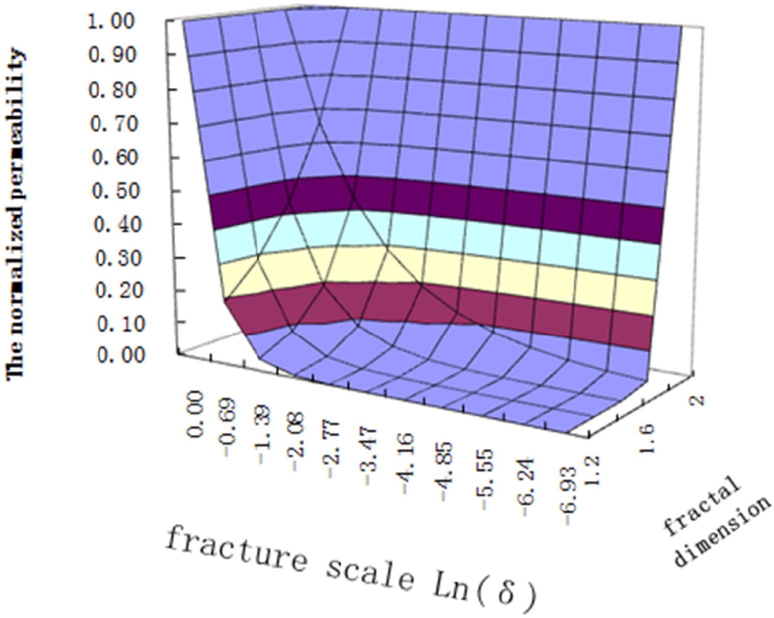

Fig. 15. Fracture permeability along with the change of scale under different fractal dimension

5. Conclusions

(1) The geometry model of a single rock fracture is put forward, and the relationships between the fracture parameters such as the fracture trend and tendency, the fracture location and the fracture size are described. On this basis, the fractal model that indicates the number, size and scale of rock mass fracture distribution is proposed, which lays foundation for two-dimensional and three-dimensional simulation of rock mass fractures.

(2) The two-dimensional trace diagrams of different rock fracture system distributions are proposed, and the effect of each parameter on the distribution of rock mass fracture is analyzed. The results show that the number and development of natural fractures are controlled by the fractal dimension, the number of groups and the initial quantity, and the number of natural fractures increases with the fractal dimension.

(3) The three-dimensional simulation of rock mass fracture system distribution is carried out, and the effect of each parameter on the distribution of rock mass fracture is analyzed. It is found that the simulation results of fracture system agrees well with natural fracture system, and that the influencing rule of fractal parameters on fracture system distribution is consistent with that in two-dimension space.

(4) The fractal dimension of natural fracture distribution has an obvious effect on porosity and permeability of natural fractures. Porosity and permeability of natural fracture distribution show an exponential growth with increasing fractal dimension. If the fractal dimension remains constant, the porosity and permeability of natural fractures increase with the scale.

Acknowledgments

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