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SPECIFIC SOLUTION OF 3D DEFORMATION VECTOR IN MINE SUBSIDENCE: A CASE STUDY OF THE KOŠICE-BANKOV ABANDONED MAGNESITE MINE, SLOVAKIA

SZCZEGÓŁOWE ROZWIĄZANIE TRÓJWYMIAROWEGO WEKTORA ODKSZTAŁCENIA OPISUJĄCEGO OSIADANIE TERENU. STUDIUM PRZYPADKU: NIECZYNNA KOPALNIA MAGNEZYTU KOSICE-BANKOV, SŁOWACJA

Mining activity influence on the environment belongs to the most negative industrial influences. Land subsidence can be a consequence of many geotectonic processes as well as due to anthropogenic interference with rock massif in part or whole landscape. Mine subsidence on the surface can be a result of many deep underground mining activities. The presented study offers the theory to the specific case of the deformation vectors solution in a case of disruption of the data homogeneity of the geodetic network structure in the monitoring station during periodical measurements in mine subsidence.

The theory of the specific solution of the deformation vector was developed for the mine subsidence at the Košice-Bankov abandoned magnesite mine near the city of Košice in east Slovakia. The outputs from the deformation survey were implemented into Geographic Information System (GIS) applications to a process of gradual reclamation of whole mining landscape around the magnesite mine. After completion of the mining operations and liquidation of the mine company it was necessary to determine the exact edges of the Košice-Bankov mine subsidence with the zones of residual ground motion in order to implement a comprehensive reclamation of the devastated mining landscape. Requirement of knowledge about stability of the former mine subsidence was necessary for starting the reclamation works. Outputs from the presented specific solutions of the deformation vectors confirmed the multi-year stability of the mine subsidence in the area of interest. Some numerical and graphical results from the deformation vectors survey in the Košice-Bankov abandoned magnesite mine are presented. The obtained results were transformed into GIS for the needs of the self-government of the city of Košice to the implementation of the reclamation works in the Košice-Bankov mining area.

Keywords: mine subsidence, deformation vector, geodetic network, GIS, reclamation

Górnictwo pozostaje tą dziedziną działalności człowieka której wpływ na środowisko naturalne jest najbardziej niekorzystny. Osiadanie terenu może być skutkiem różnorakich procesów tektonicznych, a także działalności człowieka i jej ingerencji w integralność górotworu i krajobrazu w terenie. Osiadanie gruntu na powierzchni ponad kopalnią jest najczęściej skutkiem prowadzonego pod ziemią wydobycia.

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W pracy tej przedstawiono teorię mającą zastosowanie do szczególnego przypadku rozwiązania wektora odkształcenia w przypadku zaburzenia jednorodności sieci geodezyjnej i braku danych dla stacji monitoringu dokonującej okresowych pomiarów wielkości osiadania.

Teoria mająca zastosowanie do szczególnego rozwiązania wektora odkształcenia opracowana została w trakcie badania przypadku nieczynnej kopalni magnezytu Kosice-Bankov, znajdującej się w pobliżu miasta Koszyce, we wschodniej Słowacji. Dane wyjściowe z pomiarów odkształcenia zostały zaimplementowane w aplikacji GIS (Geographic Information System) w celu opracowania programu całościowej rekultywacji terenu wokół dawnej kopalni magnezytu. Po zakończeniu działalności górniczej i likwidacji kopalni konieczne było wyznaczenie dokładnych granic niecki osiadania w rejonie kopalni Kosice-Bankov, wraz ze strefami w których odnotowuje się rezydualne ruchy gruntów, w celu opracowania całościowego projektu rekultywacji terenu zniszczonego przez działalność górniczą. Przed rozpoczęciem prac nad rekultywacją terenu konieczne było uzyskanie wiedzy o stabilności procesu i zakresie osiadania terenu ponad kopalnią. Wyniki uzyskane dzięki zastosowaniu szczegółowego rozwiązania wektora odkształcenia potwierdziły wieloletnią stabilizację procesu osiadania terenu na badanym obszarze. Przedstawiono rezultaty procedur numerycznych i wyniki w postaci graficznej rozwiązania wektora odkształcenia dla nieczynnej kopalni magnezytu Kosice-Bankov. Otrzymane wyniki wprowadzono następnie do bazy GIS na użytek władz lokalnych w mieście Koszyce, by umożliwić rozpoczęcie prac nad skuteczną rekultywacją terenów pogórniczych w rejonie kopalni Koszyce-Bankov.

Słowa kluczowe: osiadanie terenu, wektor odkształcenia, siec geodezyjna, GIS, rekultywacja terenu

1. Introduction

On the present in accretive exigencies to people and its property protection, there is security one from priority needs and tasks of all countries or their groupings around the world. In the environment protection, which an unspoiled ecosystem is a condition of human living, it is needed to protect people and its property against the negative industrial influences. Mining activity influence on the environment belongs to the most negative industrial influences. As a result of underground mining of the mineral deposits in the surface creates the land subsidence (mine subsidence), i.e. caving zone (area) dangerous for the movement of people in this zone (Cui et al., 2000; Díaz-Fernández et al., 2010; Djamaluddin et al., 2011; Knothe, 1984; Kratzsch, 1983; Reddish & Whittaker, 1989). The underground mining of coal and other minerals creates voids which are subject to collapse. In rare and specific cases, the collapse of these voids may occur at any time ranging from immediate (i.e., while the mineral is being extracted) to 100 or more years after mining (Pinto et al., 2016). If the collapse causes sinking of the ground surface, the settlement is called mine subsidence (Knothe, 1984; Reddish & Whittaker, 1989). Very great danger and threat to people's lives and their property can be caused by sudden unexpected caving fall of the earth surface over the abandoned mining works (Bauer, 2006; Colorado Geological Survey, 2016; Kratzsch, 1983).

According to the report of the Illinois Department of natural Resources (Pinto et al., 2016) and also according to many theoretical and practical knowledge and scientific studies (Donnelly & Reddish, 1994; Knothe, 1984; Reddish & Whittaker, 1989) currently, it is not possible to predict precisely how long the mine subsidence events will be finished. From the present experience about 60 to 90 % of the total ground movement occurs within the first few weeks or months of an event. The remaining ground movement continues to develop at a continually decreasing rate and may take relatively 1 to 6 years and more, in average 3 to 5 years (Pinto et al., 2016).

In order to protect the environment, in particular the protection of human life and property, it is necessary to examine mine subsidence on the surface (Alehossein, 2009; Reddish & Whittaker, 1989). The most mine subsidence worldwide with their prediction by means of their modelling are examined through the coal fields (Cui et al., 2000; Djamaluddin et al., 2011; Jung et al., 2007; Sedlák, 1998).

Character and size of the subsidence on the surface depends mainly on the geotectonic ratios of rock massif above the mined out area. Knowing the extent of the subsidence trough in mining areas is determining to prevent the entry of persons into these danger zones. Conditioning factors to establish the extent of the movement of the earth surface above the mined out area are a geodetic way surveyed deformation vectors which can be derived from the processing of measurements at monitoring stations based on these mining tangent territories. 3D (three-dimensional) deformation vectors most adequately characterize movements of ground, buildings and other engineering structures above the mined out area. Deformation modelling[1] is mostly based on periodic monitoring space changes of various engineering structures, buildings or terrain surfaces by means of using the surveying classic terrestrial methods, i.e. measuring 3D observation data elements by using classic optic theodolites and levelling instruments in the 40's up-to 80's of the last century or universal electronic measuring instruments – total stations since the 80's of the last century, or by up-to-date progressive surveying satellite navigation technologies and systems, i.e. Global Positioning System (GPS) and Global Navigation Satellite Systems (GNSS) or very seldom and specific surveying technologies such as the surveying technology – Interferometric synthetic aperture radar (InSAR) or using other advanced specific terrestrial and aerial and space technologies and techniques (Cai et al., 2008; Can et al., 2013; Hu, 2000; Jung et al., 2007; Lü et al., 2008; Marschalko et al., 2008; Ng et al., 2011; Sedlák, 2004; Wright & Stow, 1999). The deformation vectors are the result of such deformation investigations. The deformation vector with its value gives a global review about the deformation character of the monitored object of interest (earth surface, buildings, engineering structures, etc.) and it also can be used for modelling a future deformation development of such monitored object (Can et al., 2013; Donnelly & Reddish, 1994; Knothe, 1984; Reddish & Whittaker, 1989). Certain specific methods (especially geophysical) for monitoring ground motion must be carried out under controlled large-scale underground works, such as destress blasting or large-chamber mining in ore and industrial mineral deposits (Koníček et al., 2013; Strazalowski & Scigala, 2005).

Repeating geodetic measurements in some monitoring stations under deformation investigation of engineering structures, buildings or terrain surfaces can be often complicated in the individual time (periodic) epochs. Monitoring station is presented by a geodetic network with the given structure of the geodetic points on which various geodetic/surveying measurements are realised to determine earth movements or movements of other objects of interest (Djamaluddin et al., 2011). During the implementation of long-term periodic deformation measurements can occur in various unpredictable obstacles, for e.g. loss or damage or building-up some established geodetic network points due to construction of new engineering structures and buildings or other construction earth works on the monitoring station area. It means that the geodetic network with points at a monitoring station has the non-homogenous structure during all periodical geodetic measurements (during deformation survey).

All these or other unpredictable obstacles make it impossible for periodic execution of the original measurement sights realized at the geodetic network of the monitoring station in time

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[1] Deformation modelling is the process of creating 3D geometric models of earth surface subsidence over extracted mining space from periodically obtained geodetic data in expression of 3D deformation vectors. 3D models of the subsidence at certain time periods define the development and nature of the subsidence in time dependency. Deformations present the movements of the earth surface affected by the mining activity as well as the movements of the building objects located in the intervention area, in the periphery zones of the subsidence.

of the first (primary, zero) measuring epoch. It means that for any periodic measurements cannot be maintained equal conditions for realizing measurement sights. The data homogeneity of whole geodetic network in the monitoring station was disrupted. In these cases neither a renewal of the destroyed points (reference and object points) at other places and neither substitution of some values in the geodetic network of the monitoring station (which are not measurable in the successive monitoring epochs) by other variables do not make possible to use a standard method in calculation of the deformation vector (Donnelly & Reddish, 1994; Knothe, 1984; Li et al., 2011; Sedlák, 2004).

The analysis of time factor of the gradual subsidence development continuing with underground exploitation allows production of more exact model situations in each separate subsidence processes and especially, it provides an upper degree in a prevention of deformations in the surface. Possibility in improving polynomial modelling the subsidence is conditioned by the knowledge to detect position of so-called "break points", i.e. the points in the surface in which the subsidence border with a zone of breaches and bursts start to develop over the mineral deposit exploitation. It means that the break-points determine a place of the subsidence, where it occurs to the expressive fracture of the continuous surface consistence. 3D deformation vectors locate the places of the break points presenting the subsidence edges (Alehossein, 2009; Cai et al., 2008; Donnelly & Reddish, 1994; Lü et al., 2008). Also, GIS projects with 3D visualization of mine subsidence with their precise boundaries and relevance to the immediate vicinity of subsidence bring progressive elements into the decision making process of environmental protection (Blachowski, 2016; Yang et al., 2009, 2012).

2. Theory for a specific deformation vector solution

The geodetic network structure of a monitoring station can be expressively changed between monitoring epochs (epochs with periodic measurements of the observed geodetic data in the geodetic network) by the above-mentioned changes in an original geodetic network and interference with the geodetic points of such network. The most common and efficient way of geodetic networks processing in geodesy and engineering surveying is the network structures estimate based on Gauss-Markov model. The statistics formulation of Gauss-Markov model is as follows (Christensen, 2011; Gene et al., 2013; Groß, 2004; Lindgren et al., 2011; Sedlák, 1998)

$$\mathbf{v} = \mathbf{A}\left(\hat{\mathbf{C}} - \mathbf{C}^{o}\right) - \left(\mathbf{L}_{(0)} - \mathbf{L}^{o}\right) = \mathbf{A}d\hat{\mathbf{C}} - d\mathbf{L} \tag{1}$$

$$\mathbf{\Sigma}_L = \sigma_0^2 \mathbf{Q}_L \tag{2}$$

where $\mathbf{v}$ is the vector of corrections of the measured (observed) values $\mathbf{L}$, $\mathbf{A}$ are the configuration (modelling) matrix of the geodetic network or also called Jacobian matrix, i.e. the matrix of partial derivatives of functions $\mathbf{L}^o = f(\mathbf{C}^o)$ by the vector $\mathbf{C}^o$, $\hat{\mathbf{C}}$ is the vector of the aligned 3D coordinate values, $\mathbf{C}^o$ is the vector of the approximate 3D coordinate values, $\mathbf{L}_{(0)}$ is the vector of the approximate observation magnitude values of the observed elements in of the first measuring epoch $t_{(0)}$, $\mathbf{L}^o$ is the vector of the approximate observation magnitude values of the observed elements, $d\hat{\mathbf{C}}$ is the deformation vector, $d\mathbf{L}$ is the vector of the measured values supplements, $\mathbf{\Sigma}_L$ is the covariance matrix of the measured values, σ_0^2 is a priori variance, $\mathbf{Q}_L$ is the cofactor matrix of the observations.

It will also be appeared in the changed structures, let us say in a size of the matrixes and vectors **A**, $\mathbf{Q}_L$, $\mathbf{C}^o$ and $\mathbf{L}^o$. These matrixes and vectors enter into the presupposed model of the network adjustment following out from Gauss-Markov model (Christensen 2011; Gene et al., 2013; Li et al., 2011, Sedlák, 1998).

2.1. Deformation vector

If between monitoring epochs there are no changes in the geometrical and observational structure of the geodetic network, then the matrixes and vectors **A**, $\mathbf{Q}_L$, $\mathbf{C}^o$ and $\mathbf{L}^o$ remain identical for each epoch. Only in such case the deformation vector $d\hat{\mathbf{C}}$ can be determined by a conventional procedure according to the following model (Reddish & Whittaker, 1989; Sedlák, 1998):

- in the basic (first) monitoring epoch $t_{(0)}$, we have the vector $\hat{\mathbf{C}}_{(0)}$ of the adjusted 3D coordinates of the observed points which are obtained according to Gauss-Markov model

$$\hat{\mathbf{C}}_{(0)} = \mathbf{C}^o + \left(\mathbf{A}^T\mathbf{Q}_L^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Q}_\mathbf{L}^{-1}\left(\mathbf{L}_{(0)} - \mathbf{L}^o\right) = \mathbf{C}^o + \mathbf{G}\left(\mathbf{L}_{(0)} - \mathbf{L}^o\right) \tag{3}$$

- in other following epochs $t_{(i)}$ we also obtain the vector $\hat{\mathbf{C}}_{(i)}$ of the adjusted 3D coordinates of the observed points according to the equation

$$\hat{\mathbf{C}}_{(i)} = \mathbf{C}^o + \left(\mathbf{A}^T\mathbf{Q}_L^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Q}_L^{-1}\left(\mathbf{L}_{(i)} - \mathbf{L}^o\right) = \mathbf{C}^o + \mathbf{G}\left(\mathbf{L}_{(i)} - \mathbf{L}^o\right) \tag{4}$$

thus, for the deformation vector $d\hat{\mathbf{C}}$ will be valid the following equation

$$d\hat{\mathbf{C}} = \hat{\mathbf{C}}_{(i)} - \hat{\mathbf{C}}_{(0)} = \mathbf{G}\left(\mathbf{L}_{(i)} - \mathbf{L}^o\right) \tag{5}$$

where $\mathbf{L}_{(0)}$ and $\mathbf{L}_{(i)}$ are the vectors of the observed magnitude values in the epochs $t_{(0)}$ and $t_{(i)}$.

Now we presuppose a case in which some changes in the established geodetic network structure of the monitoring station are occurred during the monitoring observation epochs, i.e. the geodetic network structure between the basic epoch $t_{(0)}$ and the epoch $t_{(i)}$ is changed. Then the origin matrixes and vectors **A**, $\mathbf{Q}_L$, $\mathbf{C}^o$ and $\mathbf{L}^o$ will be transformed into the following equations

$$\bar{\mathbf{A}} = \mathbf{A} + d\mathbf{A} \tag{6}$$

$$\bar{\mathbf{Q}}_L = \mathbf{Q}_L + d\mathbf{Q}_L \tag{7}$$

$$\bar{\mathbf{C}}^o = \mathbf{C}^o + d\mathbf{C}^o \tag{8}$$

$$\bar{\mathbf{L}}^o = \mathbf{L}^o + d\mathbf{L}^o \tag{9}$$

According to Equations from (6) to (9) the vectors $\hat{\bar{\mathbf{C}}}_{(0)}$ and $\hat{\bar{\mathbf{C}}}_{(i)}$ of the adjusted 3D coordinates of the observed points in the epochs $t_{(0)}$ and $t_{(i)}$ will be determined

$$\hat{\bar{\mathbf{C}}}_{(0)} = \bar{\mathbf{C}}^o + \left(\bar{\mathbf{A}}^T\bar{\mathbf{Q}}_L^{-1}\bar{\mathbf{A}}\right)^{-1}\bar{\mathbf{A}}^T\bar{\mathbf{Q}}_L^{-1}\left(\mathbf{L}_{(0)} - \bar{\mathbf{L}}^o\right) = \bar{\mathbf{C}}^o + \bar{\mathbf{G}}\left(\mathbf{L}_{(0)} - \bar{\mathbf{L}}^o\right) \tag{10}$$

$$\hat{\bar{\mathbf{C}}}_{(i)} = \bar{\mathbf{C}}^o + \left(\bar{\mathbf{A}}^T\bar{\mathbf{Q}}_L^{-1}\bar{\mathbf{A}}\right)^{-1}\bar{\mathbf{A}}^T\bar{\mathbf{Q}}_L^{-1}\left(\mathbf{L}_{(i)} - \bar{\mathbf{L}}^o\right) = \bar{\mathbf{C}}^o + \bar{\mathbf{G}}\left(\mathbf{L}_{(i)} - \bar{\mathbf{L}}^0\right) \tag{11}$$

and then the deformation vector $d\hat{\mathbf{C}}$ is expressed according to Equation (5) in the form

$$d\hat{\bar{\mathbf{C}}} = \hat{\bar{\mathbf{C}}}_{(i)} - \hat{\bar{\mathbf{C}}}_{(0)} \tag{12}$$

which will not express only 3D changes of the geodetic network points between the particular epochs but the deformation vector can be distorted (biased) by an influence of the geodetic network structural changes. Then deformation vector $d\hat{\bar{\mathbf{C}}}$ will not afford the reliable testing information about the concrete deformation consequences.

The presented theory in the cases of some structural changes in the geodetic network can be likely to demonstrate by analytically way if we compare the deformation vector structures $d\hat{\mathbf{C}}$ and $d\hat{\bar{\mathbf{C}}}$ expressed according to Equations (5) and (12). Then the structure of the deformation vector $d\hat{\bar{\mathbf{C}}}$ is expressed according to Equation (12) and the further equation will be valid

$$\begin{aligned} d\hat{\bar{\mathbf{C}}} &= \left[\bar{\mathbf{C}}^{o} + \bar{\mathbf{G}}\left(\mathbf{L}_{(i)} - \bar{\mathbf{L}}^{o}\right)\right] - \left[\mathbf{C}^{o} + \mathbf{G}\left(\mathbf{L}_{(0)} - \mathbf{L}^{o}\right)\right] \\ &= \bar{\mathbf{G}}\left(\mathbf{L}_{(i)} - \mathbf{L}^{o}\right) - \mathbf{G}\left(\mathbf{L}_{(0)} - \mathbf{L}^{o}\right) + \bar{\mathbf{C}}^{o} - \mathbf{C}^{o} \end{aligned} \tag{13}$$

and on the base of Equations from (6) to (9) and also on the base of the linearization of $\bar{\mathbf{G}}$ into $\bar{\mathbf{G}} = \mathbf{G} + d\mathbf{G}$ the following derivation will be valid for the deformation vector $d\hat{\bar{\mathbf{C}}}$

$$\begin{aligned} d\hat{\bar{\mathbf{C}}} &= (\mathbf{G} + d\mathbf{G})\left(\mathbf{L}_{(i)} - \bar{\mathbf{L}}^{o}\right) - \mathbf{G}\left(\mathbf{L}_{(0)} - \mathbf{L}^{o}\right) + d\mathbf{C}^{o} = \\ &= \bar{\mathbf{G}}\left[\mathbf{L}_{(i)} - \left(\mathbf{L}^{o} + d\mathbf{L}^{o}\right)\right] + d\mathbf{G}\left(\mathbf{L}_{(i)} - \bar{\mathbf{L}}^{o}\right) - \mathbf{G}\left(\mathbf{L}_{(0)} - \mathbf{L}^{o}\right) + d\mathbf{C}^{o} = \\ &= \mathbf{G}\left(\mathbf{L}_{(i)} - \mathbf{L}^{o}\right) + \mathbf{G}d\mathbf{L}^{o} + d\mathbf{G}\left(\mathbf{L}_{(i)} - \mathbf{L}^{o}\right) - \mathbf{G}\left(\mathbf{L}_{(0)} - \mathbf{L}^{o}\right) + d\mathbf{C}^{o} = \\ &= \mathbf{G}\left(\mathbf{L}_{(i)} - \mathbf{L}_{(0)}\right) + \mathbf{G}d\mathbf{L}^{o} + d\mathbf{G}\left(\mathbf{L}_{(i)} - \bar{\mathbf{L}}^{o}\right) + d\mathbf{C}^{o} \end{aligned} \tag{14}$$

and finally the deformation vector $d\hat{\bar{\mathbf{C}}}$ will be calculated according to the following equation

$$d\hat{\bar{\mathbf{C}}} = d\hat{\mathbf{C}} + \delta\, d\hat{\mathbf{C}} \tag{15}$$

Equation (15) declares that the deformation vector $d\hat{\bar{\mathbf{C}}}$ (calculated with the changed geodetic network structure) is different from its vector of the correct values $d\hat{\mathbf{C}}$ only by the term $\delta d\hat{\mathbf{C}}$ (i.e. the correction component of the deformation vector corrections). In this case the term $\delta d\hat{\mathbf{C}}$ is not generated by spatial movements of the geodetic network points between the individual epochs of measurements, but it is currently generated by changes in the geometric and observational network structure between the particular epochs due to implementation of changes in its point field and also due to changes in measurements in the epochs.

To prevent this problem (so that any depreciation of the deformation vector $d\hat{\mathbf{C}}$ is not occurred), which is frequently occurred at the deformation investigation, the following procedures are to using:

The geodetic network must be carefully projected from the point of view of a maximum and permanent providing its reference points and the line sights between the reference and object points during whole monitoring period, especially.

If some reference points were lost or destroyed, new points should be established in enough proximity of these lost or destroyed reference points as possible. Same principle is held for the object points.

If matrixes **A** and $\mathbf{Q}_L$ are expressively changed between the monitoring epochs $t_{(0)}$ and $t_{(i)}$ (for example, in $t_{(0)}$ the geodetic network was measured by a trilateration measurement way, and in $t_{(i)}$ by traverse measurement way, it is necessary to observe more new magnitudes, etc.), then the deformation vector $d\hat{\mathbf{C}}$ is determined according the following equations

$$d\hat{\mathbf{C}} = \mathbf{C}^o + \left(\mathbf{A}^T\mathbf{Q}_L^{-1}\mathbf{A}\right)_{(i)}^{-1}\mathbf{A}_{(i)}^T\mathbf{Q}_{L(i)}^{-1}\left(\mathbf{L}_{(i)} - \mathbf{L}^o\right) - \\ -\left[\mathbf{C}^o + \left(\mathbf{A}^\mathbf{T}\mathbf{Q}_L^{-1}\mathbf{A}\right)_{(0)}^{-1}\mathbf{A}_{(0)}^\mathbf{T}\mathbf{Q}_{L(0)}^{-1}\left(\mathbf{L}_{(0)} - \mathbf{L}^o\right)\right] \tag{16}$$

and

$$d\hat{\mathbf{C}} = \mathbf{G}_{(i)}\mathbf{L}_{(i)} - \mathbf{G}_{(0)}\mathbf{L}_{(0)} - \mathbf{L}^o\left(\mathbf{G}_{(i)} - \mathbf{G}_{(0)}\right) \tag{17}$$

because using the identical $\mathbf{C}^o$ and $\mathbf{L}^o$ is not problem to adhere in the individual epochs. Or the deformation vector corrections $\delta d\hat{\mathbf{C}}$ are calculated according to Equations (10), (11) and (13), so that the deformation vector $d\hat{\mathbf{C}}$ is then corrected according to the introduced Equation (15).

3. Study case example

3.1. Study territory description

The Košice-Bankov monitoring deformation station covers an area around the mine field of the magnesite mine in the Košice-Bankov territory. The Košice-Bankov city territory is in the northern part of the city of Košice where a popular city recreational and tourist centre of the city of Košice is situated. This popular urban recreational area is located in close proximity to the mine field of the Košice-Bankov magnesite mine (Fig. 1).

The magnesite deposit Košice-Bankov belongs to one of the largest magnesite deposits not only in Slovakia but also in Europe. Magnesite was initially mined by the openwork mining (stage working, 1908-1929) and later it went to an underground mining. Magnesite deposit is located in the main deposit horizon in the western part of Gemerikum. The immediate footwall and hanging wall of the deposit body are sericite-graphetic phyllites with unequal representation of the psamit component and the fluctuating content of the carbonate admixture and the sites of the dark dolomitic limestone to the dolomite with a gradual transition into the sediments. Higher in the hanging wall of deposit body are found black and chlorite phyllites and the massive position of greenstones (redepared diabases). A strong layer of the classical and volcanoclastic rocks of the Neogen covers the rocks of the upper Carbon (Grecula et al., 1995). The basic tectonic structure of the carbonate bearing bodies is part of the tectonic structure of the wider environment, which is characterized as a nappe-scaly. An important fault system on the bearing is the system of failures in the north-east and south-west direction, with a slope of approximately 60° to the south-west direction.

The exploitation of the Košice-Bankov magnesite deposit after the completion of the surface mining (1929) was realized in an underground mining. From the beginning of the underground

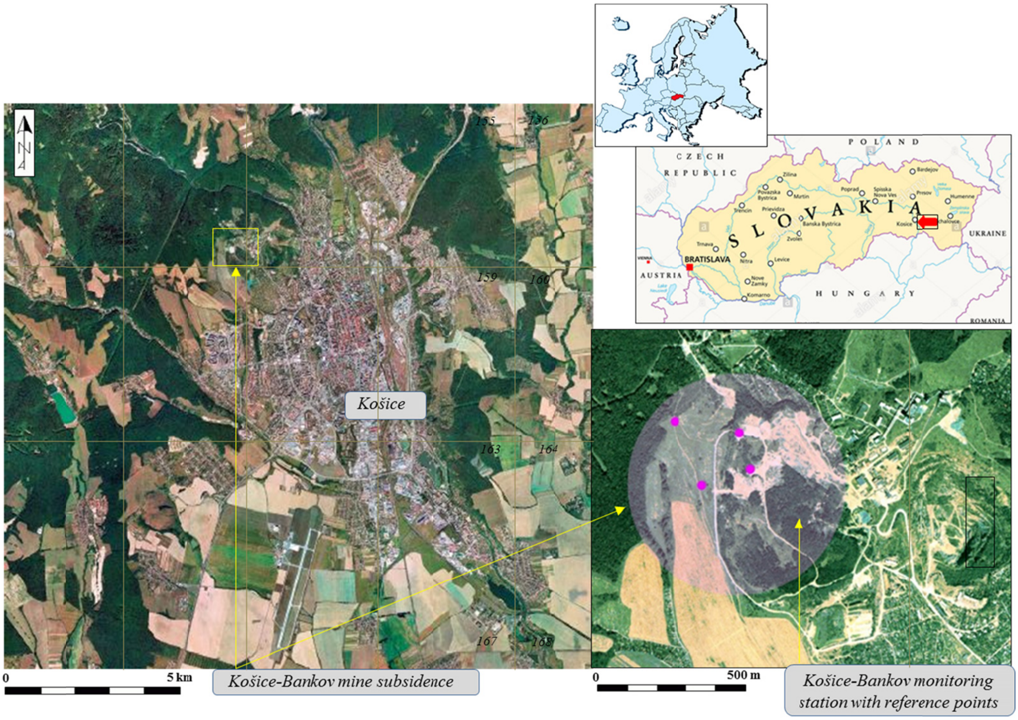

Fig. 1. Ortophoto map of the city of Košice with a detailed view of the Košice-Bankov mining field

mining, the technology of pillar and chamber mining system was applied. Later, in the second half of the 1990s, shortly before the ending of the mining operation on the Košice-Bankov magnesite deposit, in order to protect deformations of the earth surface the filled stope mining technology was transferred. On the earth surface, since the beginning of the magnesite mining process, a discontinuous subsidence has been created and, in order to protect the nearby urban recreation zone (in particular for the purpose of protection against human movement at the site of the basin and in its immediate vicinity), there was a need for permanent monitoring of this subsidence. Due to the economic unprofitability and technological demands, the mining of magnesite on the Košice-Bankov deposit was stopped and terminated at the end of the 90's. On the profitability of the magnesite deposit mining, besides the price of the raw material on the market, the used technologically demanding treatment, the degree of the deposit examination, the quality and the quantity of the extracted raw material were affected.

Problems of mine damages on the surface, dependent on the underground mine activities at the magnesite deposit, did not receive a systematic research attention in Slovakia till 1976. After that, the requirements for a scientific motivation in the subsidence development following out from rising exploitations and from introducing progressive mine technologies were taken in consideration. The gradual subsidence development at the Košice-Bankov mine region in the east region of Slovakia is monitored by geodetic measurements from the beginning (in the end of sixties of the 20th century) of the mine underground activities in the magnesite mineral deposit.

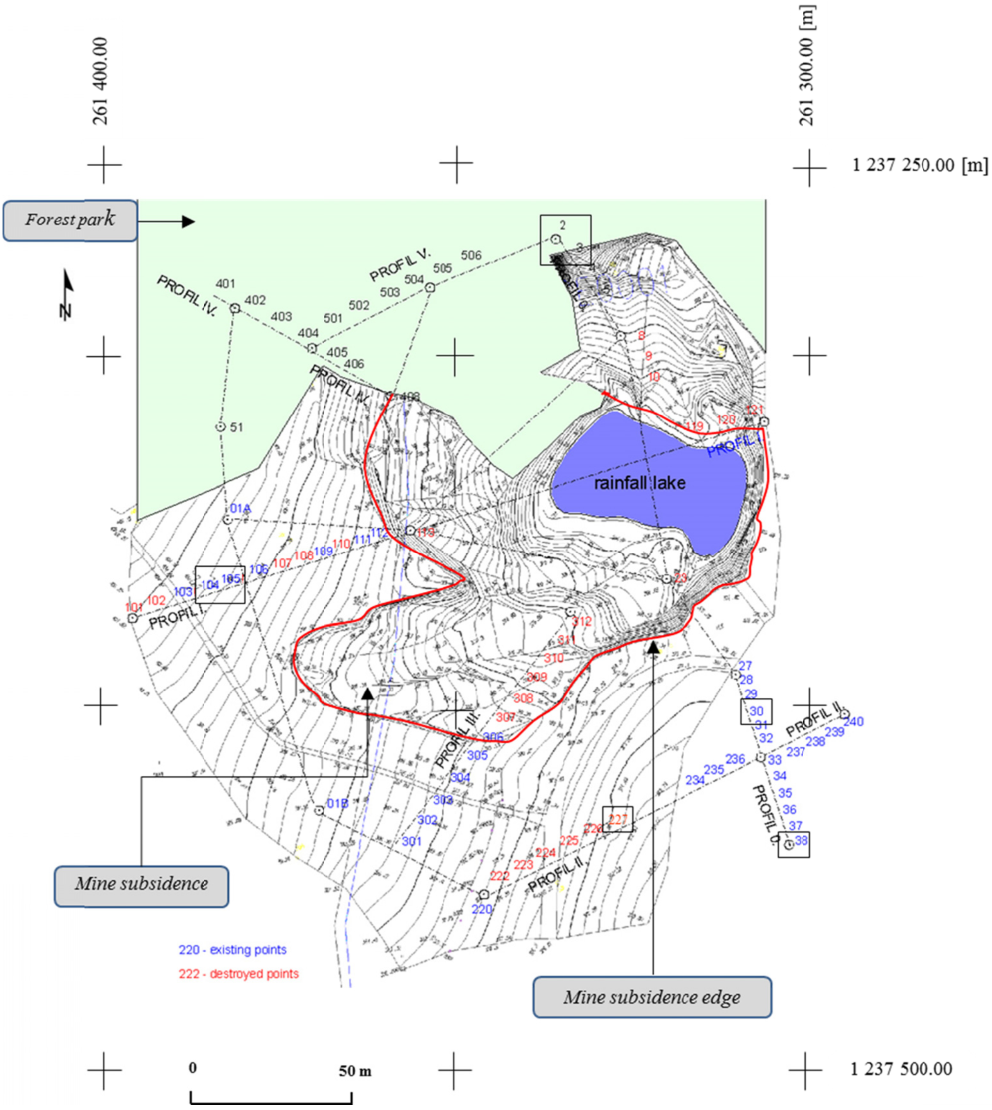

Fig. 2. Geodetic points of the Košice-Bankov monitoring station in ŠTS[2]; framed points – tested points (no.: 2, 3, 30, 38, 104, 105, 227)

The monitoring station project in the Košice-Bankov study case was designed and realised by the research staff of the Technical University of Košice in Slovakia in 1976. The first observed

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[2] ŠTS: State Trigonometric Network of the Slovak republic (in Slovak: Štátna trigonometrická sieť). Geodetic points of ŠTS represent GZ points (physical points of the Basic Point Field) that were taken from the original Czechoslovakian Trigonometric Network (ČSTS) and Czechoslovak Astronomical Geodetic Network (ČSAGS) from the territory of Slovakia and these points have designated position coordinates in the implementation of the Coordinate System of the Uniform Trigonometric Cadastral Network (S-JTSK).

data were taken from this monitoring station in the same year and each year the spring and autumn geodetic terrestrial and GPS measurements were realized. The monitoring station is situated in the earth surface in the Košice-Bankov mine region near by the shaft under the name – West shaft. The monitoring station is constructed from the geodetic network of the reference points (no.: 01A, 01B, 01C, 01D) and objective points (78 points) situated in geodetic network profiles (Fig. 2). Some of the reference points were destroyed by the subsidence processes.

The Košice-Bankov subsidence recorded its sudden (spontaneous) creation in time, especially during active mining activities. The residual deformations of the earth surface after the ending of mining were registered at the margin zones of the subsidence because due to the natural destruction of the points of the geodetic network of the monitoring station in the central part of the subsidence it was not possible to realize any geodetic measurements there. A number of cracks and breaks dangerous to the movement of people have been created at the edges of the subsidence. Figures 3 and 4 present the panoramic views to the Košice-Bankov mine subsidence from the south-west edge of this subsidence in 2001 and 2002 when the magnesite mine was abandoned for two up-to three years. In the case of the Košice-Bankov mine subsidence, this is a discontinuous type of deformation of the rock over the mined spaces.

Fig. 3. Košice-Bankov mine subsidence; panoramic view: autumn 2001

Fig. 4. Košice-Bankov mine subsidence; panoramic view: spring 2002

All surveying profiles of the Košice-Bankov monitoring station are deployed across and along the expected movements in the mine subsidence (Fig. 2). 3D data were firstly observed by 3D (positional and levelling measurements) terrestrial geodetic technology (since 1976) using classic optical geodetic theodolites and levelling devices for very high precision levelling, later total electronic surveying devices and also devices for GPS/GNSS technology (since 1997), i.e. Trimble 3303DR Total station, GPS: ProMark2 and GNSS: Leica Viva GS08. Periodic monitoring measurements are performed at the Košice-Bankov monitoring station twice a year (usually in spring and autumn) (Sedlák, 1998). In 1981, some points of this monitoring station were destroyed (defective) and again replaced in same year (points No.: 2, 3, 30, 38, 104, 105 and 227[3] on the profiles No.: 0, I and II). Also the reference points 01C and 01D were destroyed, but because these points had an in-depth stabilization (6 m), their reconstruction was not complicated. To defect of these points to occurred by some fit for felling works in close forest crop. The destroyed points were replaced very precision geodetic way according the origin coordinates.

3.2. Accuracy and quality assessment of the network

1D, 2D and 3D accuracy of the geodetic network points (the Košice-Bankov monitoring station) in the east Slovak region was appreciated by the global and the local indices. The global indices were used for the accuracy consideration of whole network, and they were numerically expressed. We used the variance global indices: $tr(\Sigma_{\hat{C}})$, i.e. a track of the covariance matrix $\Sigma_{\hat{C}}$ and the volume global indices and $\det(\Sigma_{\hat{C}})$, i.e. a determinant. The local indices were as a matter of fact the point indices, which characterize a reliability of the network points:

- mean 3D error $\sigma_p = \sqrt{\sigma^2_{\hat{X}_i} + \sigma^2_{\hat{Y}_i} + \sigma^2_{\hat{Z}_i}}$,
- mean coordinate error $\sigma_{XYZ} = \sqrt{\dfrac{\sigma^2_{\hat{X}_i} + \sigma^2_{\hat{Y}_i} + \sigma^2_{\hat{Z}_i}}{3}}$,
- confidence absolute ellipses or ellipsoids, which were used for a consideration of the real 2D or 3D in the point accuracy. We need to know the ellipsis constructional elements, i.e.: semi-major axis a, semi-minor axis b, bearing φ_a of the semi-major axis and ellipsoid flattening f, ($f = 1 - b/a$).

Characteristics of the network quality are mainly accuracy and reliability. Position accuracy of points can be expressed in addition to numerical also by graphical indicators of the network accuracy, which are the confidence curves and a confidence ellipse (confidence ellipsoids in 3D case). Ellipsoids determine a random space, in which the actual location of points will be lie with a probability $1\text{-}\alpha$, where α is chosen level of significance, according to which the ellipsoids are of different size. In geodetic practice the standard confidence ellipsoids are used for 3D space. Their design parameters can be derived either from of the cofactor matrix $\mathbf{Q}_L$ of the adjusted coordinates, which shall be these design parameters on the main diagonal, or from the coordinate covariance matrix of the coordinate estimations $\Sigma_{\hat{C}}$ of the determined points, which shall be them on the main diagonal.

All calculated data according to the presented specific theory about the deformation vector estimation in a case of any accepted changes in the geodetic network of the monitoring station are in Tables 1-5. In a general, Tables 1-5 are focussed on the accuracy and quality assessment of the geodetic network (Table 1: mean errors, Table 2: absolute confidence ellipse elements, Table 3: global indices, Table 4: local indices, Table 5: values of deformation vector). Tables 1-5 comprehend the adjusted mean errors of the individual coordinates, global and local 3D indices and their

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[3] The reference point No. 227 (profile II) was rebuilt instead of the point No. 226.

absolute confidence ellipsoid elements determining 3D accuracy of some chosen replaced points. The numbers in front of the back slash belong to year 1976 when geodetic measurements were started. The numbers after the back slash belong to 2014[4] when all geodetic measurements were finished. In 2007 the points No.: 2, 3, 30, 38, 104, 105 and 227 were re-stabilized due to small earth construction works needed to the preparation works for a future reclamation of the Košice-Bankov mining territory. The deformation vector values confirm possibility in the deformation vectors valuation according to the presented theory (Sedlák, 2004). However, the deformation vector values need not mean any displacement of the points. Despite the fact that the points of the geodetic network were adjusted in a common way according to the Gauss-Mark model, the deformation vector values can be loaded by accumulating measurement errors. Therefore, for their prominence testing it is required to carry out testing the deformation vector by the global and localization test of the congruence (see chapter 3.3). In the last surveying during the spring 2014 the deformation vectors on the tested points (No.: 2, 3, 30, 38, 104, 105 and 227) of the monitoring station were ranged from 9.7 mm (point No. 227) to –8.0 mm (point No. 30) (Table 5, Fig. 5). 3D mean errors were ranged from 14.1 to 37.5 mm (year 2014), and the mean coordinate errors were from 13.8 to 54.9 mm (year 2014) (Table 4). In autumn 2014 all points of the Košice-Bankov monitoring station were destroyed by the reclamation works, i.e. the reference points were removed and the object points were backfilled by the secondary imported soil from various land building and excavation works in and around of the city of Košice.

TABLE 1

Mean errors (year 1976 / year 2014)

Point	**m_X, [mm]**	**m_Y, [mm]**	**m_Z, [mm]**
2	15.7 / 16.4	32.9 / 45.5	12.5 / 72.4
3	14.8 / 34.3	27.2 / 58.9	30.5 / 69.1
30	21.1 / 25.6	26.5 / 24.1	45.5 / 32.7
38	16.6 / 14.9	16.3 / 8.1	20.1 / 18.4
104	18.2 / 41.3	34.1 / 69.0	55.4 / 78.0
105	28.2 / 31.6	17.1 / 21.1	9.9 / 17.8
227	20.0 / 16.9	8.5 / 4.7	10.9 / 10.8

TABLE 2

Absolute confidence ellipse elements (year 1976 / year 2014; $a = 0.05$)

Point	**a_i, [mm]**	**b_i, [mm]**	**φ_{a_i}, [gon]**	**f**
2	49.9 / 51.0	5.9 / 5.1	172.303 / 172.695	1.8818 / 0.9
3	40.8 / 32.5	12.3 / 3.7	172.704 / 179.151	0.6985 / 0.8862
30	43.0 / 45.9	18.2 / 21.5	160.340 / 160.058	0.5767 / 0.5316
38	23.5 / 28.1	21.8 / 22.4	40.966 / 41.203	0.0723 / 0.2028
104	47.5 / 79.2	24.0 / 9.9	211.146 / 217.148	0.4947 / 0.875
105	42.8 / 42.4	15.3 / 17.6	370.337 / 370.624	0.6425 / 0.5849
227	28.8 / 25.2	8.1 / 10.9	19.634 / 19.781	0.7188 / 0.5675

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[4] Deformation survey on the Košice-Bankov monitoring station without the reclamation works intervention was finished in the autumn 2014.

TABLE 3

Global indices (year 1976 / year 2014)

Rank $rk(\Sigma_{\hat{C}})$	**Track** $tr(\Sigma_{\hat{C}})$, **[mm^2]**	**Determinant** $\det(\Sigma_{\hat{C}}).\ 10^{25}$	**Average mean error** $\sigma_{\hat{C}_{pr}}$, **[mm]**	**Norm** $nor(d_{\hat{C}})$, **[mm]**
14/14	7041.901 / 7041.054	2.869 / 2.869	22.428 / 22.971	124.218 / 126.155

TABLE 4

Local indices (year 1976 / year 2014)

Point	**Mean 3D error** σ_p, **[mm]**	**Mean coordinate error** σ_{XYZ}, **[mm]**
2	36.4 / 37.5	25.7 / 18.8
3	30.9 / 28.4	21.8 / 24.5
30	33.9 / 33.0	23.9 / 23.0
38	23.3 / 26.7	16.5 / 13.8
104	38.6 / 14.1	27.3 / 54.9
105	32.9 / 27.4	23.3 / 19.9
227	21.7 / 22.5	15.3 / 17.4

TABLE 5

Deformation vector values (year 2014)

$d\hat{\mathbf{C}}$ [mm]	**Point**						
	2	3	30	38	104	105	227
	2.4	–2.9	–8.0	6.7	–4.0	0.6	9.7

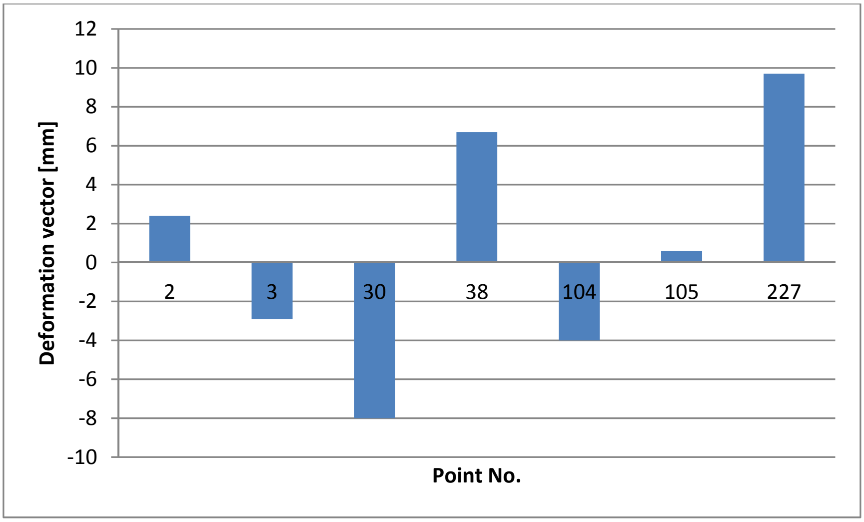


Fig. 5. Graphical representation of the deformation vectors on the tested monitoring station points; year 2014

3.3. Global test of the congruence

Significant stability, respectively instability of the network points is rejected or not rejected by verifying the null-hypothesis H_0 respectively, also other alternative hypothesis (Lehmann & Romano, 2005; Sedlák, 1998)

$$H_0 : d\hat{\hat{\mathbf{C}}} = 0; \quad H_\alpha : d\hat{\hat{\mathbf{C}}} \neq 0 \tag{18}$$

H_0 expresses an insignificance of the coordinate differences (deformation vector) between epochs $t_{(0)}$ and $t_{(i)}$.

To testing can be use e.g. test statistics T_G for the global test

$$T_G = \frac{d\hat{\hat{\mathbf{C}}}\mathbf{Q}_{d\hat{\hat{C}}}^{-1}\, d\hat{\hat{\mathbf{C}}}^T}{k\,\bar{s}_0^2} \approx F\left(f_1, f_2\right) \tag{19}$$

where $\mathbf{Q}_{d\hat{\hat{C}}}$ is the cofactor matrix of the final deformation vector $d\hat{\hat{\mathbf{C}}}$, k is the coordinate numbers entering into the network adjustment ($k = 3$ for 3D coordinates) and $\bar{s}_0^2$ is the posteriori variation factor common for both epochs $t_{(0)}$ and $t_{(i)}$.

The critical value T_{KRIT} is searched in the tables of F distribution (Fisher–Snedecor distribution) tables according to the degrees of freedom $f_1 = f_2 = n - k$ or $f_1 = f_2 = n - k + d$, where n is number of the measured values entering into the network adjustment and d is the network defect at the network free adjustment. Through the use of methods MINQUE is $s_0^{2_{t(0)}} = s_0^{2_{t(i)}} = \bar{s}_0^2 = 1$ (Lehmann & Romano, 2005; Sedlák, 1998). The test statistics T should be subjugated to a comparison with the critical test statistics T_{KRIT}. T_{KRIT} is found in the tables of F distribution according the network stages of freedom.

Two occurrences can be appeared:

1. $T_G \leq T_{KRIT}$: The null-hypothesis H_0 is accepted. It means that the coordinate values differences (deformation vectors) are not significant.
2. $T_G > T_{KRIT}$: The null-hypothesis H_0 is refused. It means that the coordinate values differences (deformation vectors) are statistically significant. In this case we can say that the deformation with the confidence level α is occurred.

Table 6 presents the global testing results of the geodetic network congruence.

TABLE 6

Test statistics results of the geodetic network points of the Košice-Bankov monitoring station

Point	$T_{G(i)}$	< ≤ >	*F*	Notice
2	1.297	<	3,724	deformation vectors are not significant
3	3.724	≤		
30	3.501	<		
38	3.724	≤		
104	2.871	<		
105	1.403	<		
227	2.884	<		

4. Mine subsidence in GIS for mining landscape reclamation

The Geographic Information System (GIS) as an information system for collecting, storing, analyse and management of the geodata and the relevant properties was used for 3D virtual visualization of changes in the Košice-Bankov mine subsidence and surrounding area. From the territory of the Košice-Bankov magnesite mine, during the last years of the magnesite mining (the 1990s of last century) several GIS projects were created. These, however, focused on data processing from 3D visualization of mining works and adjoining quarry (which is already not in operation more than fifty years), further on data processing of geological and tectonic ratios of the mining, profitability of magnesite deposit, or an impact of mining activity on the environment (Blišťan, 2003). Despite the long-lasting almost 40-year geodetic measurements of surface deformations in the Košice-Bankov mining area, no relevant GIS project was created from the area of mine subsidence.

The first 3D models of the gradual creation of the Košice-Bankov mine subsidence from the data of periodical geodetic measurements at the site of the subsidence and its surroundings in the GIS processing environment were realized only after the end of the magnesite mining process at the beginning of the 21st century (Sedlák, 2004). Due to the termination of the magnesite mining and the subsequent reclamation of the entire mining area of Košice-Bankov did not create a coherent uniform GIS project (GIS Košice-Bankov project) with the structure of complex geodata (geographic data, geodetic data, geological data, tectonic data, mining operational and technical data, etc.). Upon the request of the self-government of the city of Košice for providing a set of geoinformation data for the implementation of the project of gradual reclamation of the mining area of Košice-Bankov predetermined the creation of the GIS Košice-Bankov project exclusively from the locality of the mine subsidence and the surrounding area.

The GIS Košice-Bankov project to reclamation of whole mining landscape of Košice-Bankov is based on the following decision points: basic and easy observed geo-data presentation, basic database administration and wide information availability (Sedlák, 2004). The best viable solution is to execute GIS project as the free Open Source (OS) application available on the Internet. The general facility feature is free code and data source viability through the HTTP (HyperText Transfer Protocol) and FTP (File Transfer Protocol) located on the project web pages. Inter among others features range simple control, data and information accessibility, centralized system configuration, modular stuff and any OS platform depending on PHP (Hypertext Preprocessor), MySQL (My-Structured Query Language) and ArcIMS (Arc Internet Map Server) ports. PHP channels are involved in other Internet services because they contain dynamic libraries of some Internet protocols, i.e., HTTP, FTP, POP3 (Post Office Protocol), SMTP (Simple Mail Transfer Protocol), LDAP (Lightweight Directory Access Protocol), SNMP (Simple Network Management Protocol), NNTP (Network News Transfer Protocol), etc. Network based application MySQL is in a present time the most preferred database system on Internet and it was applied also on the deformation survey outputs from the Košice-Bankov monitoring station. The complex planimetry and altimetry of the Košice-Bankov mine subsidence site from the detailed tachometric measurements were processed in the ArcIMS web map server environment (Fig. 6A). ArcIMS is a GIS that is designed to serve maps across the Internet. The map created from the mining area of Košice-Bankov in the ArcIMS environment allows to select its individual layers and to add in the required attributes of geodata. The ArcView software tool was used to further enable the

GIS application in the visualization of spatial information from the area of the Košice-Bankov mine subsidence territory (Fig. 6B). ArcView as a desktop GIS application offers simple access to geographic data and includes an intuitive user interface, elaborate contextual system and comprehensive documentation, enabling relatively simple and fast work with geographic data. The MicroStation CAD (Computer-aided design) software was also used to create a 3D model from the Košice-Bankov mine subsidence. The software (MicroStation V8) very operatively enabled creation of 3D model of the Košice-Bankov mine subsidence (Fig. 6C). The GIS Košice-Bankov project with 3D models of the mine subsidence in the applications of multi-layer GIS was delivered to the reclamation plan of the Košice-Bankov mining territory for the self-government of the city of Košice.

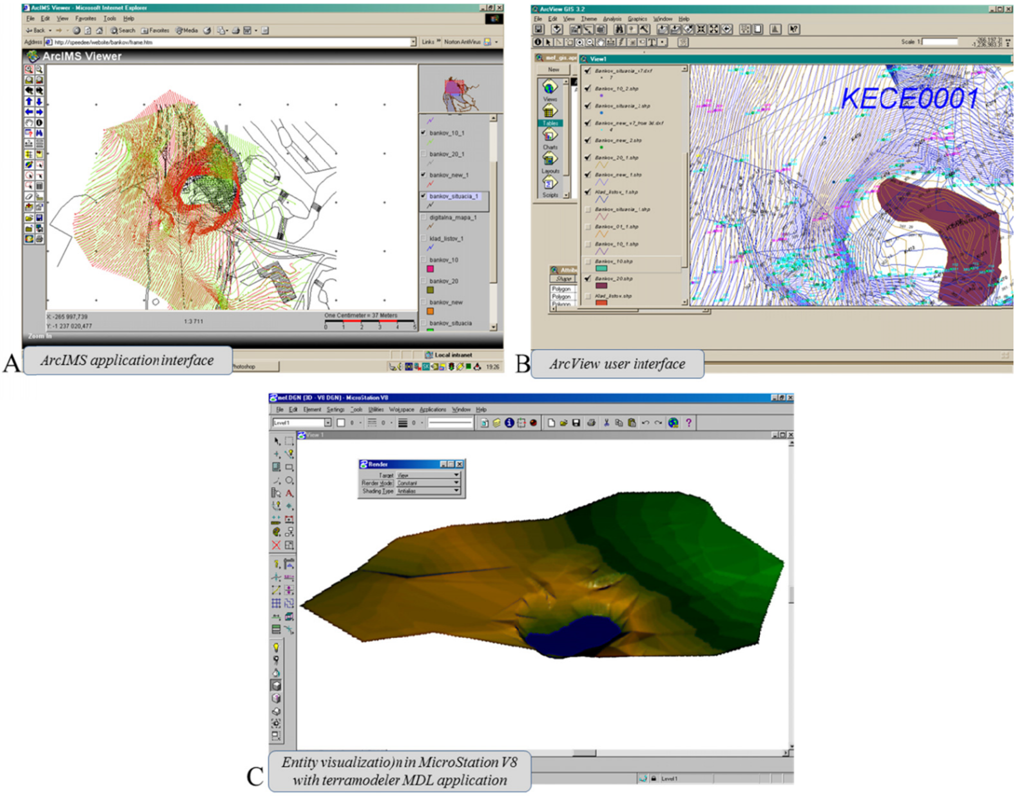
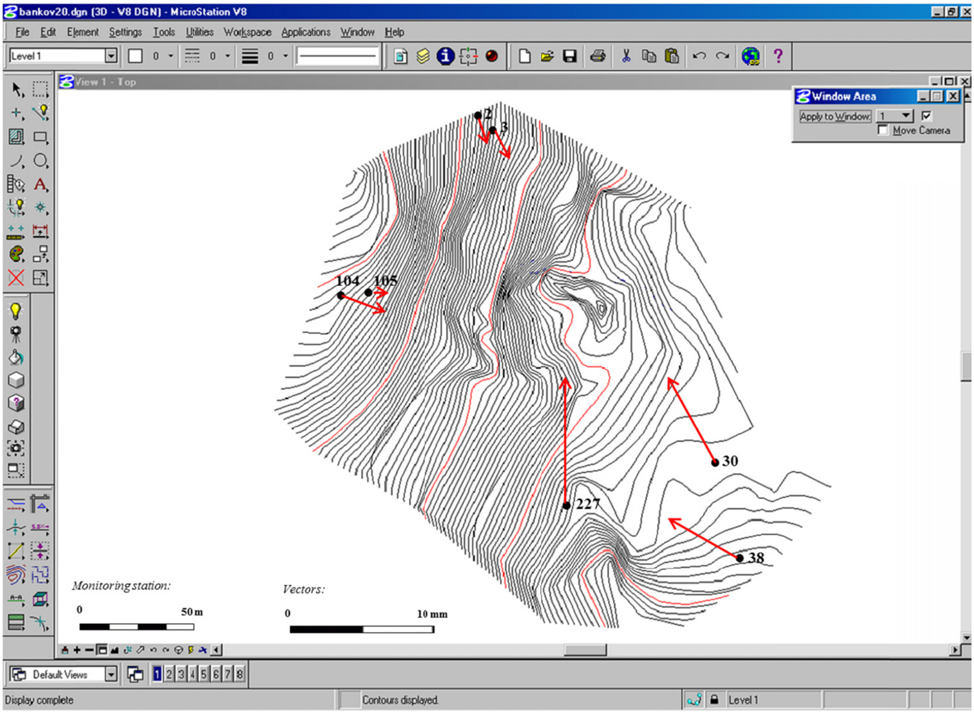


Fig. 6. Visualization of the Košice-Bankov mine subsidence in various user interfaces

The individual layers of the GIS project include graphical and numerical data of the Košice-Bankov mine subsidence ad surrounding area, such as the development of 3D deformations from 1976 to 2014, numerical parameters (surface area, depth) of the subsidence, exact the subsidence boundary with delimitation of the edge-punched zones on the earth surface with the last ground passing deformations and zones without any ground deformations, 3D visualization (character and size) of the deformation vectors, especially at the edges of the subsidence, calculation of the secondary embankment material necessary to fill and reclamation of the subsidence, data-complex with the most suitable types of artificially planted forest stands, etc. The selected processing graphic and numerical procedures of geodata from the subsidence and surrounding area, supported by the ArcIMS map server and ArcView and MicroStation GIS software tools, enable multiple variability of 3D virtual visualization of the mapped mining area in the GIS project. Any operations with the GIS project database created from the Košice-Bankov mining territory require minimal knowledge of its process control, which is a simple manipulation process for the Land Planning Department of the city of Košice. One layer of the Košice-Bankov GIS project included models of the magnitude of the 3D deformation vectors of the mine subsidence. The data for these deformation vector models was processed in the software CAD file of MicroStation V8. Figure 7 presents a horizontal projection of 3D deformation vectors at selected test points of the Košice-Bankov Monitoring Station in the context of Table 5.



Display complete

Contours displayed.

Fig. 7. 3D deformation vectors of the tested points of the Košice-Bankov monitoring station realized by MicroStation V8; year 2014

A part of the GIS Košice-Bankov project was a layer of data necessary for reclamation of tailings piles, their area and volume parameters, which served as background data for their reclamation and the construction of solar collectors (solar park) at their original sites.

Given the fact that extraction of magnesite has been completed at the Košice-Bankov magnesite mine and these mine workings are abandoned since the end of the 90-years of the last century and whole mining territory of Košice-Bankov with the huge mine subsidence on the conclusions of the deformation investigations are stable, the self-government of the city of Košice adopted the plan for the reclamation of that mine landscape. Numerical and graphical presentation of long-term investigations on the Košice-Bankov deformation monitoring station with their successive test analyses of the deformation vectors confirmed stability of the mine subsidence and surrounding mining area. The mine subsidence and by mining activities devastated all surroundings around the mine plant of huge proportions (tailings piles, excavations and other mining earth-works, etc.) began gradually to backfill by a secondary imported soil. The reclamation works on the basis of the investigation geodetic deformation conclusions around the former mining area of Košice-Bankov began at the beginning of this century. Some final reclamation works were completed in summer 2016.

On the territory of the former extensive mine subsidence area the Košice-Bankov forest park was built as the environmental green-forest part of the urban recreation area of the city of Košice. The mine subsidence began gradually to backfill by imported natural material from many construction and earth-works in the city of Košice and surroundings of the city. Such sporadic embankment works took too long, i.e. more than years. After completion of the embankment and other earth-works the Košice-Bankov forest park was built on the territory of the former mine subsidence (Fig. 8). It was planted in particular birch trees. These trees are known by their unpretentiousness onto natural base and also by a rapid growth. Currently, birch grove constitutes by five to six years-old healthy tree.

Finalization of building the Košice-Bankov recreation area was completed in spring 2016. The former waste tailings piles and devastated surrounding area of the former mining plant were also reclaimed. Instead of the former tailings piles, solar collectors (solar park) were built to contribute to renewable energy for the city of Košice (Fig. 8).

5. Conclusions

Determination of the deformation vectors as the differences between the adjusted coordinate vectors obtained from two measured monitoring epoch in the geodetic networks is possible if the geometric observation network structure between the individual monitoring epochs is strictly saved. This research study presents the theory and practical outcomes about a possibility of the deformation vector solutions in the geodetic network of the monitoring station in a case of violation of the geodetic network structure during the period of monitoring movements of the earth's surface. The solved deformation vector affords reliable image about 3D changes of the geodetic network points in a frame of some specific deformation investigation, e.g. ground movements, mine subsidence, land-slides, dams, engineering constructions, buildings, or other building objects.

The presented theory of the specific deformation vector solution was verified at the selected tested points no. 2, no. 3, no. 30, no. 38, no. 104, no. 105 and no. 227. All tested points were restored during the period of deformation measurements (1976-2014) due to small earthworks in their vicinity. The largest deformation vector values were occurred on point no. 30 and no. 227 (Table 5, Fig. 5). The value of the deformation vector at point no. 30 was –8 mm and at point no. 227 was +9.7 mm (considering differences in the resultant 3D coordinates of the first measurement in 1976 and the last measurement in 2014). However, point no. 30 and no. 227 did not record high values in the other presented parameters, e.g. the mean errors, absolute confidence ellipse elements, local indicates of the geodetic points accuracy (Tables 1, 2 and 4). Due to the fact that

Fig. 8. Mine subsidence and surrounding (Košice-Bankov) after reclamation; panoramic view: summer 2016. Solar collectors in the places of the former tailings piles; afforestation (in the background) in the place of the former mine subsidence

the tested deformation vectors on these points (no. 30 and no. 227) were not significant according to the test statistics, we can declare these points as the static ones. The study case example confirmed availability and applicability of the presented theory on the deformation vector in a special occasion of deformation measurements at mine subsidence, where many violations in the geodetic structure of the monitoring station are occurred. Despite the validity of the verified presented method in solving specific deformation vector in the data non-homogeneous geodetic network points at the monitoring station may cause a distortion of the deformation effects in the monitoring territory. Therefore, maintaining data-homogeneity of the geodetic network structure should be a priority for whole periodicity of each deformation surveys.

Modelling the mine subsidence in GIS from the Košice-Bankov mining area was delivered to the self-government of the city of Košice to solution of the landscape planning to the future environmental rehabilitation of this abandoned old mining region such as the Košice-Bankov magnesite mine. Determination of the deformation vectors of the monitoring station in the undermined landscape of the abandoned magnesite mine of Košice-Bankov was important in delimitation and specification of the edges of the mine subsidence and the edge-punched zones of the subsidence with a lot of cracks and fissures. Very precise identification of the 3D position of such delimitation of the subsidence was a basic document for the plan preparation of the self-government of the city of Košice for the reclamation of the former mining area in Košice-Bankov as well as and the local ambient by mining activity affected landscape for a comprehensive revitalization and broadening recreational area in the suburban zone of the city of Košice. The self-government of the city of Košice has 3D model of the Košice-Bankov mine subsidence in GIS with possibilities of modelling natural and industrial disasters, which largely can be the helpful tools for many reclamation works in the landscape ecosystem restoration with the basic elements of safety measures against possible unforeseen and possible consequences of the former mining activities to protect the health and lives of people moving in the forest park in the former Košice-Bankov magnesite mine.

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References

Alehossein H., 2009. *Back of envelope mine subsidence estimation*. Aust. Geomech. **44** (1), 29-32.

Bauer R.A., 2006. *Mine Subsidence in Illinois: Facts for Homeowners*. Illinois State Geological Survey (Ed.), Circular 569. ISGS Publising, Champaign, Illinois. 20 pp.

Blachowski J., 2016. *Application of GIS spatial regression methods in assessment of land subsidence in complicated mining conditions: case study of the Walbrzych coal mine (SW Poland)*. Nat. Hazard., July 2016, **84** (2), 997-1014.

Blišťan P., 2003. *Project of the geographic information system for Bankov-Košice deposit.* In: Sborník vědeckých prací Vysoké Školy Báňské – Technické Univerzity Ostrava, Řada Hornicko-Geologická, **XLIX** (2), 87-94 (in Slovak).

Cai J., Wang J., Wu J., Hu C., Grafarend E., Chen J., 2008. *Horizontal deformation rate analysis based on multiepoch GPS measurements in Shanghai*. J. Surv. Eng. **134** (4), 132-137.

Can E., Mekik Ç., Kuşçu Ş., Akçın H., 2013. *Computation of subsidence parameters resulting from layer movements post-operations of underground mining*. J. Struct. Geol. **47** (Febr. 2013), 16-24.

Christensen R., 2011. *General Gauss-Markov models*. R. Christensen (ed.), Plane Answers to Complex Questions: The Theory of Linear Models. 4th edn. Springer, New York, 237-266.

Colorado Geological Survey, 2016. *Subsidence-Mine*. [online]. Colorado Geological Survey website, [cit. 26.09.2016]. Available at: <http://coloradogeologicalsurvey.org/geologic-hazards/subsidence-mine/>.

Cui X., Miao X., Wang J., Yang S., Liu H., Hu X., 2000. *Improved prediction of differential subsidence caused by underground mining.* Int. J. Rock Mech. & Min. Sci. **37** (4), 615-627.

Díaz-Fernández M.E., Álvarez-Fernández M.I., Álvarez-Vigil A.E., 2010. *Computation of influence functions for automatic mine subsidence prediction*. Comput. Geosci. **14** (1), 83-103.

Djamaluddin I., Mitani Z., Esaki T., 2011. *Evaluation of ground movement and damage to structures from Chinese coal mining using a new GIS coupling model*. Int. J. Rock Mech. Min. Sci. **48** (3), 380-393.

Donnelly L.J., Reddish D.J., 1994. *The development of surface steps during mine subsidence: "Not due to fault reactivation"*. Eng. Geol. **34** (3/4), 243-255.

Gene H., Golub G.H., Van Loan Ch.F., 2013. *Matrix Computations*. JHU Press, Baltimore.

Grecula P., Németh Z., Varga I., 1995: *Mineral deposits of the Slovak Ore Mountains.* Vol.1, Mineralia Slovaca Corporation, Bratislava (in Slovak).

Groß J., 2004. *The general Gauss-Markov model with possibly singular dispersion matrix*. Stat. Pap. **45** (3), 311-336.

Hu L.Y., 2000. *Gradual deformation and iterative calibration of Gaussian-related stochastic models*. Math. Geol, **32** (1), 87-108.

Jung H.C., Kim S.W., Jung H.S., Min K.D., Won J.S., 2007. *Satellite observation of coal mine subsidence by persistent scatterer analysis*. Eng. Geol. **92** (1), 1-13.

Knothe S., 1984. *Forecasting the influence of mining*. Śląsk Publishing House, Katowice. (in Polish).

Koníček P., Souček K., Staš L., Singh R., 2013. *Long-hole destress blasting for rockburst control during deep underground coal mining*. Int. J. Rock Mech. Min. Sci. **61** (July 2013), 141-153.

Kratzsch H., 1983. *Mine subsidence Engineering*. Springer-Verlag GmbH, Heidelberg.

Lehmann E.L., Romano J.P., 2005. *Testing Statistical Hypotheses*. 3rd edn. Springer, New York.

Li P.X., Tan Z.X., Deng K.Z., 2011. *Calculation of maximum ground movement and deformation caused by mining.* Trans. Nonferr. Met. Soc. China **21**, 562-569.

Lindgren F., Ruel H., Lindström J., 2011. *An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach*. J. R. N.A. Soc./B N.A. Meth. **73** (4), 423-498.

Lü W.C., Cheng S.G., Yang H.S., Liu D.P., 2008. *Application of GPS technology to build a mine-subsidence observation station.* J. China Univ. Min. Technol. **8** (3), 377-380.

Marschalko M., Fuka M., Treslin L., 2008. *Measurements by the method of precise inclinometry on locality affected by mining activity*. Arch. Min. Sci. **53** (3), 397-414.

Ng A.H., Ge L., Zhang K., Chang H.C., Li X., Rizos C., Omura M., 2011. *Deformation mapping in three dimensions for underground mining using InSAR – Southern highland coalfield in New South Wales, Australia*. Int. J. Remote Sens. **32** (22), 7227-7256.

Pinto G. et al., 2016. *Subsidence*. [online]. Illinois Department of Natural Resources website, [cit. 30.05.2016]. Available at: <https://www.dnr.illinois.gov/mines/AML/Pages/Subsidence.aspx>.

Reddish D.J., Whittaker B.N., 1989. *Subsidence: Occurrence, Prediction and Control*. Elsevier, Amsterdam.

Sedlák V., 1998. *Measurement and prediction of land subsidence above longwall coal mines, Slovakia.* W.J. Borchers (ed.), Land Subsidence/Case Studies and Current Research. U.S. Geological Survey, Belmont, 257-263.

Sedlák V., 2004. *Possibilities at modelling surface movements in GIS in the Košice depression, Slovakia*. RMZ – Mater. Geoenviron. **51** (4), 2127-2133.

Strzałkowski P., Scigala R., 2005. *The example of linear discontinuous deformations caused by underground extraction.* Trans. VŠB – Tech. Univ. Ostrava, Civil Eng., Ser. V (2), 193-198.

Wright P., Stow R., 1999. *Detecting mine subsidence from space*. Int. J. Remote Sens. **20** (6), 1183-1188.

Yang K.M., Xiao J.B., Duan M.T., Pang B., Wang, Y.B., Wang R., 2009. *Geo-deformation information extraction and GIS analysis on important buildings by underground mine subsidence*. In: Proceedings of the Inter. Conf. on Information Engineering and Computer Science – ICIECS 2009. IEEE (Ed.), Wuhan, 4 pp.

Yang K.M, Ma J.T., Pang B., Wang Y.B., Wang R., Duan M.T., 2012. *3D visual technology of geo-deformation disasters induced by mine subsidence based on ArcGIS engine*. Key Eng. Mater. **500**, 428-436.