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A Novel Method for Determining Optimum Dimension Ratios for Small Rectangular Rooms

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A new method for determining optimum dimension ratios for small rectangular rooms has been presented. In a theoretical model, an exact description of the room impulse response was used. Based on the impulse response, a frequency response of a room was calculated to find changes in the sound pressure level over the frequency range 20–200 Hz. These changes depend on the source and receiver positions, thus, a new metric equivalent to an average frequency response was introduced to quantify the overall sound pressure variation within the room for a selected source position. A numerical procedure was employed to seek a minimum value of the deviation of the sound pressure level response from a smooth fitted response determined by the quadratic polynomial regression. The most smooth frequency responses were obtained when the source was located at one of the eight corners of a room. Thus, to find the best possible dimension ratios, in the numerical procedure the optimal source position was assumed. Calculation results have shown that optimum dimension ratios depend on the room volume and the sound damping inside a room, and for small and medium volumes these ratios are roughly 1 : 1.48 : 2.12, 1 : 1.4 : 1.89 and 1 : 1.2 : 1.45. When the room volume was suitably large, the ratio 1 : 1.2 : 1.44 was found to be the best one.

Keywords: room acoustics; small rooms; optimum dimension ratios; room impulse response; frequency room response.

1. Introduction

In room acoustics, enclosed spaces that have interior volumes in the range from a few cubic meters to a few hundred cubic meters are classified as small rooms (Kleiner, Tichy, 2014). Small sizes cause that room acoustics is dominated by wave behavior (Meissner, 2016a), thus, a smooth frequency response at low frequencies is important for the acoustic quality (Bistafa *et al.*, 2012). A shape of small rooms is usually rectangular, therefore, acoustic issues typical for such rooms stem from a flutter echo, amplification or attenuation of sound at certain frequencies and the unwanted sound coloration caused by strong early reflections (Sevastiadis *et al.*, 2010). These effects together with improper reverberation parameters (Meissner, 2016b; 2017) prevent the correct perception of sound in small rooms such as performance studios, studio control rooms, listening rooms and lecture rooms where speech, music, listening or recording is part of normal use.

In order to obtain smooth frequency response in rectangular rooms, attempts have been made to classify room's low frequency sound distribution with regards to its dimension ratios. Special metrics have been proposed for the optimal modal response and from these good dimension ratios have been found. Bolt (1946) suggested that differences between frequencies of the successive modes and a mean value in a certain frequency range is a measure of the regularity of modal distribution. Using this metric he found the ratio 1 : 1.25 : 1.6 to be the best one. Błaszak (2008) pointed out that in the Bolt's criterion the frequency range that should be taken into account is especially important. She used in the analysis a frequency range up to the Schroeder frequency and found the dimension ratio 1 : 1.2 : 1.4 as particularly recommended. The study of Sepmeyer (1965) has proved that to minimize the irregularity of modal response one of the following room ratios should be selected: 1 : 1.14 : 1.39, 1 : 1.28 : 1.54 and 1 : 1.6 : 2.33. Louden (1971) assumed that the distribution of modal frequencies is regular when it is very close to the modal spacing of three dimensional enclosures at high frequency. Such a condition led him to the dimension ratio of $1:1.4:1.9$. Investigations of Milner and Bernhard (1989) verified and extended the results of Louden. Using the finite element method they were able to reproduce the analytical results of Louden to five significant digits and for the room volume chosen by Louden they found the optimal dimension ratio as $1:1.186:1.439$.

The metrics based on the pressure frequency response take into account and require information on the source and receiver positions, as well as the damping properties of the room. Cox and D'Antonio (2001) and Cox *et al.* (2004) assumed that the source and the receiver are located in opposite room corners and calculated the deviation from a predicted frequency response and a flat frequency response in the frequency range 20–200 Hz. In the optimization procedure, room dimensions were changed to achieve the flattest possible frequency response and obtained results shown a dependence of the optimum ratios on the room volume. This method produces the optimum dimension ratio of $1:1.55:1.85$ for a room volume close to the one selected by Louden and the general ratio of $1:2.19:3$ which performs well for all room volumes studied. Another measure of overall frequency response flatness, termed the Variance of Spatial Average (VSA), was proposed by Welti and Devantier (2006). To determine the VSA, the mean sound pressure levels for a number of receiving positions are calculated for frequencies from the band of concern, and next the variance of this spatial average is computed in this frequency band. The VSA metric has been used to analyze the low-frequency performance of subwoofer-room systems (Welti, Devantier, 2006; Welti, 2012) and to examine the utility of the Bonello criteria (Bonello, 1981) for small room acoustics (Welti, 2009). Sarris (2011) studied and compared the various 'optimum' ratios that have been found till now and proposed a new method for determining good room ratios based on the variance of mean sound pressure index (Sarris, 2014). He found that the ratio of $1:2.19:3$ provided by Cox *et al.* (2004) performs acceptably well for small and larger room volumes. Rindel (2015) considered the global frequency response assuming source and receiver positions in room corners since this placement ensured that all modes were included in the frequency response. He used the modal energy analysis to calculate the frequency response and reverberation time in small and nearly rectangular rooms, but he did not report the optimized dimension ratios.

This paper presents a numerical method for finding acoustically good dimension ratios for a rectangular room. Such room, termed also in the literature as a rectangular prism, a rectangular parallelepiped, a cuboid, a cuboidal room or a shoebox room, represents the most popular type of rooms in buildings. A sketch of the rectangular room under study together with the associated coordinate system is shown in Fig. 1. The floor of the room is taken to be in the xy plane and the height along the z axis, and it is assumed that room dimensions satisfy the relations: $L_x \geq L_y \geq L_z$. A theoretical model is based on a modal description of the room impulse response. The level of sound damping included in the model is low which is inline with real room conditions where the low-frequency absorption is usually small. Using the room impulse response, a frequency response of a room is calculated and this response is used to determine variations of the sound pressure level inside a room over the assumed frequency range. Since these changes depend on the source and receiver positions, a metric equivalent to an average frequency response is introduced to quantify the overall sound pressure variation within the room for a selected source position. In the numerical procedure, the deviation of the sound pressure level response from a smooth fitted response is determined and the task of calculations is to find such room dimension ratios for which this deviation is minimal. Simulation data are shown to compare the effectiveness of the various dimension ratios in creating a flat frequency response. Based on these results, the optimum and nearly optimum dimension ratios are determined.

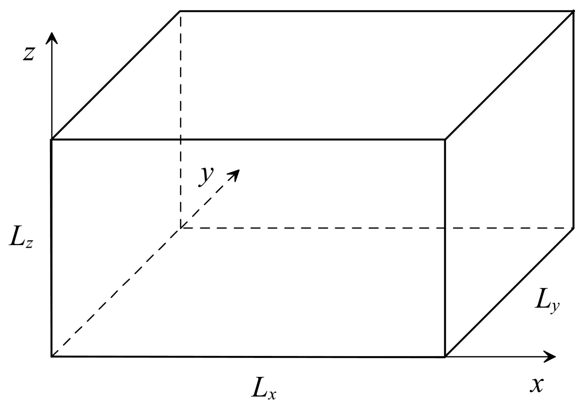

Fig. 1. A rectangular room under study together with the associated coordinate system. It is assumed that room dimensions satisfy the relations: $L_x \geq L_y \geq L_z$.

2. Low-frequency room response

In the low-frequency range, typical room dimensions are comparable with a length of sound wave. Therefore, in this frequency limit the method, which is most appropriate for describing the sound field inside a room, is based on the modal representation of the room impulse response (RIR). The RIR is very useful in room acoustics because a knowledge of the RIR function $h(\mathbf{r}', \mathbf{r}, t)$, describing the pressure response at the receiving point $\mathbf{r}$ to the time impulse at the point $\mathbf{r}' = (x', y', z')$, enables one to predict the indoor sound pressure $p(\mathbf{r}, t)$ for an arbitrary sound source. Indeed, by using the RIR function this pressure can be found from the following expression (Damelin, Miller, 2012)

$$p(\mathbf{r}, t) = \int\limits_V \int\limits_{-\infty}^{t} q(\mathbf{r}', \tau) h(\mathbf{r}', \mathbf{r}, t - \tau) \, \mathrm{d}\tau \, \mathrm{d}^3\mathbf{r}', \qquad (1)$$

where V is the room volume, $q(\mathbf{r}', \tau)$ represents the volume source term and $\mathrm{d}^3\mathbf{r}' = \mathrm{d}x' \, \mathrm{d}y' \, \mathrm{d}z'$ is the volume element. The RIR function is zero for $t < \tau$ because if an impulse occurs at τ, no effects of the impulse should be present at an earlier time. A method for finding the RIR function was presented in detail by Meissner (2016c) and the final result in the form

$$h(\mathbf{r}', \mathbf{r}, t) = c^2 \sum_{m=0}^{\infty} \frac{e^{-r_m t} \sin(\Omega_m t) \Phi_m(\mathbf{r}') \Phi_m(\mathbf{r})}{\Omega_m}, \qquad (2)$$

describes a low-frequency behavior of the room response and a chamber-like room behavior characterized by the zero-order room mode. In Eq. (2), c is the sound speed, the parameters $\Omega_m = \sqrt{\omega_m^2 - r_m^2}$ with positive values of m are the modal frequencies, ω_m are the natural frequencies and r_m are the modal damping factors given by

$$r_m = \frac{c}{2} \int\limits_S \gamma(\mathbf{r}_s) \Phi_m^2(\mathbf{r}_s) \, \mathrm{d}s, \qquad (3)$$

where γ is the specific wall conductance and $\mathbf{r}_s$ is a position coordinate on the surface S of room walls. It is assumed that γ is small because in the considered frequency range, typical materials covering room walls are characterized by a weak sound absorption (Kuttruff, 2009). The functions Φ_m occurring in Eqs. (2) and (3) represent the mode shape functions for rectangular rooms which fulfill the orthonormal property in the room volume V. Thus, at low frequencies, these functions can be approximated by

$$\Phi_m(\mathbf{r}) = \sqrt{\frac{\epsilon_{n_x} \epsilon_{n_y} \epsilon_{n_z}}{V}} \cos\left(\frac{n_x \pi x}{L_x}\right) \cdot \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right), \qquad (4)$$

where the index n_s $(s = x, y, z)$ is a non-negative integer and $\epsilon_{n_s} = 1$ for $n_s = 0$, and $\epsilon_{n_s} = 2$ for $n_s > 0$. The natural frequencies ω_m corresponding to these functions are the following

$$\omega_m = \pi c \sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}. \qquad (5)$$

The zero-order room mode, which is called the Helmholtz mode, has the modal indices n_x, n_y, n_z equal to zero. The natural frequency ω_0 for this mode is zero and the mode shape function has the form $\Phi_0(\mathbf{r}) = 1/\sqrt{V}$. Thus, the Helmholtz mode does not represent a typical resonant mode because for this mode the room behaves like a pressure chamber. The RIR function for the Helmholtz mode differs from those for other modes because the zero-order component in series in Eq. (2) can be written as

$$h_0(t) = \frac{c^2(1 - e^{-2r_0 t})}{2r_0 V}, \qquad (6)$$

where the damping factor r_0 is given by

$$r_0 = \frac{c}{2V} \int\limits_S \gamma(\mathbf{r}_s) \, \mathrm{d}s. \qquad (7)$$

Other types of acoustic modes of the rectangular room are as follows: the axial modes featured by one non-zero modal index, the tangential modes characterized by two non-zero modal indices and the oblique modes with all non-zero modal indices.

The frequency response of the room is defined as the frequency spectrum of the sound pressure signal $p(\mathbf{r}, t)$ received at the observation point $\mathbf{r}$, when the room is excited by a point source with a flat power spectrum. As is well known, the power spectral density is perfectly flat for the impulse excitation, thus the volume source term in Eq. (1) is assumed to have the form

$$q(\mathbf{r}', \tau) = Q\delta(\mathbf{r}' - \mathbf{r}_0)\delta(\tau), \qquad (8)$$

where $\mathbf{r}_0 = (x_0, y_0, z_0)$ determines the source position and the parameter Q, which depends on the source power W, is given by $Q = \sqrt{8\pi\rho c W}$ (Kinsler *et al.*, 2000), where ρ is the air density. From a mathematical point of view, the frequency response is equivalent to a Fourier transform of $p(\mathbf{r}, t)$. Thus, inserting Eq. (8) into Eq. (1) one can obtain

$$\begin{aligned} P(\mathbf{r}, \omega) &= \mathcal{F}[p(\mathbf{r}, t)] = \int\limits_{-\infty}^{\infty} p(\mathbf{r}, t) e^{-\mathrm{j}\,\omega t} \, \mathrm{d}t \\ &= Q \int\limits_{0}^{\infty} h(\mathbf{r}_0, \mathbf{r}, t) e^{-\mathrm{j}\,\omega t} \, \mathrm{d}t \\ &= Qc^2 \sum_{m=0}^{\infty} \frac{\Phi_m(\mathbf{r}_0) \, \Phi_m(\mathbf{r}) \, e^{\mathrm{j}\phi_m(\omega)}}{\sqrt{(\omega_m^2 - \omega^2)^2 + 4r_m^2\omega^2}}, \end{aligned} \qquad (9)$$

where $\mathcal{F}$ is the Fourier transform and the phase $\phi_m(\omega)$ is determined by

$$\phi_m(\omega) = \tan^{-1}\left(\frac{2r_m\omega}{\omega^2 - \omega_m^2}\right). \qquad (10)$$

Equation (9) shows that in the low-frequency range where individual modes are well separated from each other, the room response will contain spectral peaks at frequencies of these modes. A magnitude of these peaks depends on the damping of energy in modes represented by the modal damping factors r_m, where a smaller damping leads to more intense peaks. The factors r_m depend on absorbing properties of a room and in further analysis it is assumed that room walls are characterized by the absorption coefficients α_i, where $i = 1, 2...6$. The absorbing properties of a whole room is described by the mean absorption coefficient α given by

$$\alpha = \frac{1}{S}\sum_{i=1}^{6} \alpha_i S_i, \tag{11}$$

where S is the surface of all room walls and S_i is the area of the i-th wall. The relation between α_i and the conductance γ_i is somewhat complex because it is expressed by (Kuttruff, 2009)

$$\alpha_i = 8\gamma_i \left[1 + \frac{\gamma_i}{1+\gamma_i} - 2\gamma_i \ln\left(1 + \frac{1}{\gamma_i}\right)\right]. \tag{12}$$

Therefore, when α_i is given, the calculation of γ_i requires the use of a numerical procedure.

3. Numerical method and calculation results

In the low-frequency range, spacings between room modes on the frequency axis are large causing substantial peaks and dips in the frequency response. In order to eliminate this acoustical flaw, the frequency response of a room should be smooth. Of course, this is not possible considering the range of sound damping in typical rooms, therefore, the numerical method is based on finding such room dimensions that will provide the flattest possible frequency response. The basis of numerical procedure is a magnitude of P, i.e., the amplitude spectrum of room response which depends on the source and receiver positions $\mathbf{r}_0$ and $\mathbf{r}$, as shown by Eq. (9). Consequently, to employ the concept of optimum room dimensions one needs to know in advance exact locations of source and receiving points. In practice, the source position $\mathbf{r}_0$ is usually well recognized, however, the receiver location $\mathbf{r}$ is not precisely defined because, as a rule, there are several receiving positions inside the room. Therefore, it is suggested that for design purposes it is sufficient to consider the amplitude spectrum of a mean sound pressure representing the root mean square pressure inside the room volume V defined by

$$P_{\mathrm{av}}(\omega) = \left[\frac{1}{V}\int_V P(\mathbf{r},\omega)P^*(\mathbf{r},\omega)\,\mathrm{d}^3\mathbf{r}\right]^{1/2}, \tag{13}$$

where an asterisk indicates the complex conjugate. Since the integration with respect to $\mathbf{r}$ is performed over the whole room space, the spatial averaging includes all possible receiving points. After inserting Eq. (9) into Eq. (13) and using the orthonormal property of the functions Φ_m, the formula for the pressure P_{av} can be found as

$$P_{\mathrm{av}}(\omega) = Qc^2\left[\frac{1}{V}\sum_{m=0}^{\infty}\frac{\Phi_m^2(\mathbf{r}_0)}{(\omega_m^2-\omega^2)^2+4r_m^2\omega^2}\right]^{1/2}. \tag{14}$$

Utilizing the pressure P_{av}, the sound pressure level L_{av} can be determined as

$$L_{\mathrm{av}}(\omega) = 20\log\left[P_{\mathrm{av}}(\omega)/P_0\right], \tag{15}$$

where $P_0 = 2\cdot 10^{-5}$ Pa is the reference sound pressure. Based on changes in the sound pressure level L_{av} within a given low-frequency band, the best-fit curve L_c is calculated using the quadratic polynomial regression and this curve is applied to model the desired flat shape of a pressure level frequency response. The task of the numerical procedure is to seek such dimension ratios $S_x = L_x/L_z$ and $S_y = L_y/L_z$ for which the deviation of $L_{\mathrm{av}}(\omega)$ from $L_c(\omega)$ defined by

$$D = \left\{\frac{1}{\omega_u-\omega_l}\int_{\omega_l}^{\omega_u}\left[L_{\mathrm{av}}(\omega)-L_c(\omega)\right]^2\,\mathrm{d}\omega\right\}^{1/2} \tag{16}$$

is minimal, where ω_l and ω_u are lower and upper limits of the considered frequency band and $L_c(\omega) = A\omega^2 + B\omega + C$, where the coefficients A, B and C are determined via the regression method. Note that these ratios correspond only to a single source location because at other source locations optimum dimension ratios may be different.

A block diagram of the numerical procedure is shown in Fig. 2. At the beginning of the numerical procedure one should set input data such as: the room volume V, the position $\mathbf{r}_0$ and the power W of the source and the mean absorption coefficient α. In the numerical procedure the volume V is assumed to change from 50 to 300 m^3 with the step-size of 25 m^3. The source position coordinates x_0, y_0 and z_0 may vary within the limits: $0\le x_0\le L_x/2$, $0\le y_0\le L_y/2$, $L_z/2\le z_0\le L_z$, because the mode shape function Φ_m squared represents the even function with respect to $L_x/2$, $L_y/2$ and $L_z/2$. The sound source is assumed to have the power W of 10^{-4} W. In the calculation method, the mean absorption coefficient α is especially important because when α is known, the absorption coefficients α_i can be determined and subsequently, the specific wall conductances γ_i can be found. There is only one requirement: one needs to know in advance the relation between the absorption coefficients α_i. For example, if $\alpha_i = \alpha_j$ for $i \neq j$, the absorbing material on each room wall provides the same sound damping. The variable parameters in the numerical procedure are the dimension

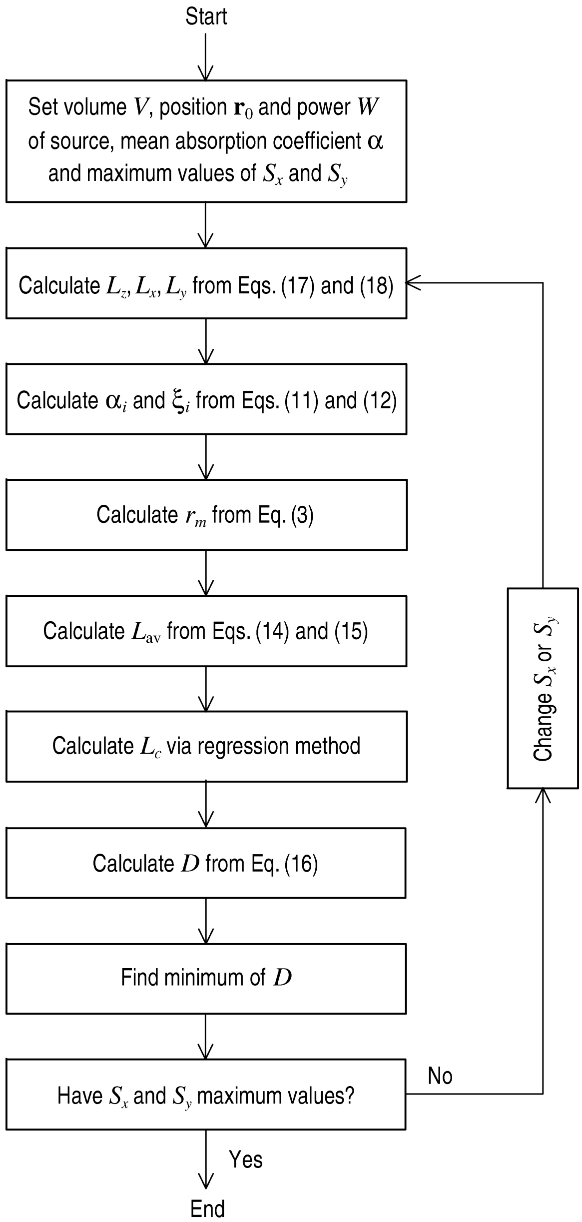

Fig. 2. Numerical procedure for finding optimum dimension ratios S_x and S_y for assumed input parameters: the room volume V, the position $\mathbf{r}_0$ and the power W of the source and the mean absorption coefficient α.

ratios S_x and S_y. When S_x and S_y are selected, the unknown room dimensions are found from the following equations

$$L_z = \left(\frac{V}{S_x S_y}\right)^{1/3}, \tag{17}$$

$$L_x = S_x L_z, \qquad L_y = S_y L_z. \tag{18}$$

In the numerical procedure it is assumed that the dimension ratios S_x and S_y are from the range 1–4 and they change with the step-size of 0.01. The aim of calculations is to find such values of S_x and S_y for which the deviation D is minimal because it ensures the flattest possible frequency response. The value of D is computed in a frequency range with the limits $f_l = 20$ Hz and $f_u = 200$ Hz using the frequency step Δf of 0.1 Hz. Thus, a numerical version of Eq. (16) is the following

$$D = \left\{ \frac{1}{N} \sum_{n=0}^{N} \left[L_{\text{av}}(f_n) - L_c(f_n)\right]^2 \right\}^{1/2}, \tag{19}$$

where $N = (f_u - f_l)/\Delta f$ and $f_n = f_l + n\Delta f$. The computer program calculates the frequency response up to the frequency 250 Hz. This additional 50 Hz allows the residues from modes in the region 200–250 Hz to influence the frequency response below 200 Hz.

Numerical tests were carried out for rooms with uniformly distributed wall absorption. Exemplary computation results are shown in Fig. 3 to illustrate a typical dependence of D_m/D on the dimension ratios S_x and S_y, where D_m is a minimum value of D found for the assumed input data: the room volume V, the source position $\mathbf{r}_0$ and the absorption coefficient α. According to the notation method used in the literature, the dimension ratios will be presented in the form $1 : S_y : S_x$. Calculations discovered that this minimum was achieved for S_x and S_y equal to 2.16 and 1.54, respectively, and it is illustrated by the dot in Fig. 3. In order to show differences between optimal and not optimal cases, in Fig. 4 changes in the sound pressure level L_{av} corresponded to the frequency responses for optimum dimension ratios and for the ratios $S_x = 1$ and $S_y = 1$ (a cube-shaped room) are depicted. Dashed lines in this figure correspond to the fit curves L_c determined via the quadratic polynomial regression.

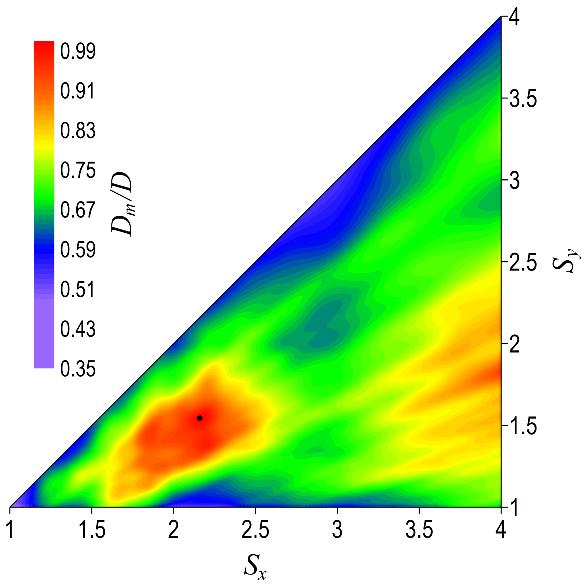

Fig. 3. Dependence of D_m/D on the dimension ratios S_x and S_y for the source position $(x_0, y_0, z_0) = (0.3L_x, 0.3L_y, 0.7L_z)$, the room volume V of 100 m^3 and the absorption coefficient α of 0.2. The dot indicates the optimum dimension ratio $1 : 1.54 : 2.16$.

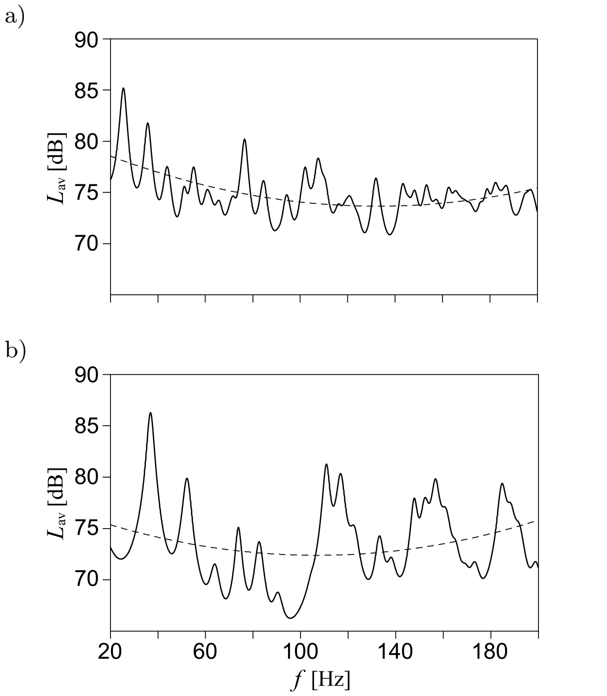

Fig. 4. Frequency dependence of the sound pressure level L_{av} for dimension ratios: a) $S_x = 2.16$, $S_y = 1.54$ and b) $S_x = 1$, $S_y = 1$, calculated for the following input data: $V = 100\ \mathrm{m}^3$, $(x_0, y_0, z_0) = (0.3L_x, 0.3L_y, 0.7L_z)$ and $\alpha = 0.2$. Dashed lines indicate the fit curves L_c computed via the quadratic polynomial regression.

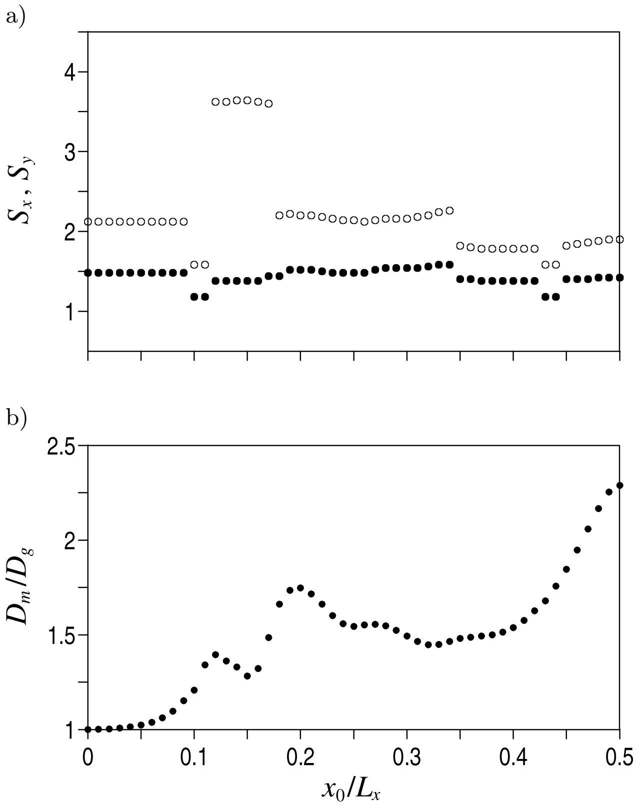

Fig. 5. a) Optimum dimension ratios S_x (white circles) and S_y (black circles) and b) the ratio D_m/D_g, for the source point moving along the half of the room diagonal defined as: $0 \le x_0/L_x \le 0.5$, $0 \le y_0/L_y \le 0.5$, $z_0/L_z = 1 - x_0/L_x$. The room volume V and the absorption coefficient α are set to 100 m^3 and 0.2, respectively.

The numerical procedure was performed for several positions of the sound source to find its optimal position for which the deviation D has the global minimum D_g. As it turned out, D reaches this minimum when the source is located at one of the corners of the room. That is because in these cases a square of the shape functions in Eq. (14) is constant for the same kinds of modes, i.e.

$$\Phi_m^2(\mathbf{r}_0) = \begin{cases} 1/V & \text{for Helmholtz mode,} \\ 2/V & \text{for axial modes,} \\ 4/V & \text{for tangential modes,} \\ 8/V & \text{for oblique modes.} \end{cases} \tag{20}$$

This regularity is proved by calculation results in Fig. 5 showing variations of the optimum dimension ratios S_x and S_y and the ratio D_m/D_g versus x_0/L_x for the room volume of 100 m^3 when the source point located at the room diagonal moves from the corner $(0, 0, L_z)$ to the room center. These data demonstrate that close to the room corner ($x_0/L_x \le 0.1$) the optimum dimension ratios do not change. However, if $x_0/L_x > 0.1$, there are ranges of x_0/L_x in which the ratios S_x and S_y are approximately constant and simultaneous jumps in value of these ratios are observed when there is a transition from one to the next range.

In the further numerical studies, calculations were carried out for the sound source located at one of the optimal positions, namely $(x_0, y_0, z_0) = (0, 0, L_z)$, to find the optimum room dimension ratios which are the best possible. Figures 6–8 show examples of numerical data that illustrate the mapped distribution of the ratio D_g/D as a function of S_x and S_y for three specific cases. Figure 6 shows the case when, for the ranges of S_x and S_y considered, there exists only one distinct maximum in the dependence of D_g/D on S_x and S_y. The dimension ratio $1:1.2:1.44$ corresponding to this maximum describes therefore optimal proportions of room dimensions. Figure 7 shows the case when there are two dimension ratios which are acceptable: the optimum dimension ratio $1:1.47:2.12$ (a dot denoted by 1) and the dimension ratio $1:2.55:3.44$ corresponding to the local minimum of the deviation D for which the ratio D_g/D is larger than 0.95 (a dot denoted by 2). The ratios satisfying this condition are fully usable, and therefore in the rest part of the paper they will be identified as nearly optimal. Figure 8 depicts calculation data where even three dimension ratios are plausible from practical viewpoint. A dot

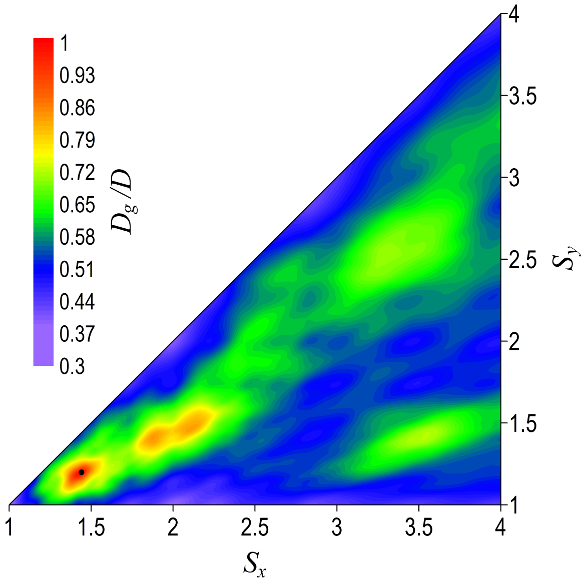

Fig. 6. Dependence of D_g/D on the dimension ratios S_x and S_y for the optimal source position $(x_0, y_0, z_0) = (0, 0, L_z)$, the room volume V of 300 m^3 and the absorption coefficient α of 0.2. The dot indicates the optimum dimension ratio 1 : 1.2 : 1.44.

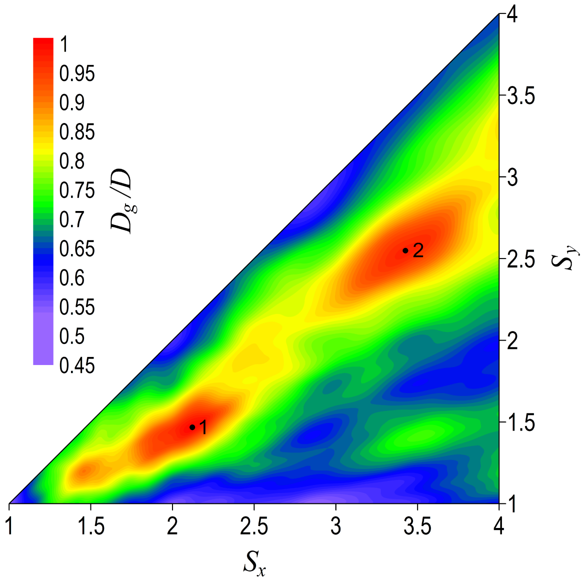

Fig. 7. Dependence of D_g/D on the dimension ratios S_x and S_y for the optimal source position $(x_0, y_0, z_0) = (0, 0, L_z)$, the room volume V of 50 m^3 and the absorption coefficient α of 0.2. A dot denoted by 1 indicates the optimum dimension ratio 1 : 1.47 : 2.12, whereas a dot denoted by 2 corresponds to the nearly optimum dimension ratio 1 : 2.55 : 3.44 $(D_g/D = 0.975)$.

with the number 1 indicates the optimum dimension ratio 1 : 1.2 : 1.45, whereas dots denoted by 2 and 3 correspond to the nearly optimum dimension ratios 1 : 1.4 : 1.88 and 1 : 1.47 : 2.1. Such order of nearly optimum dimension ratios results from the fact that the

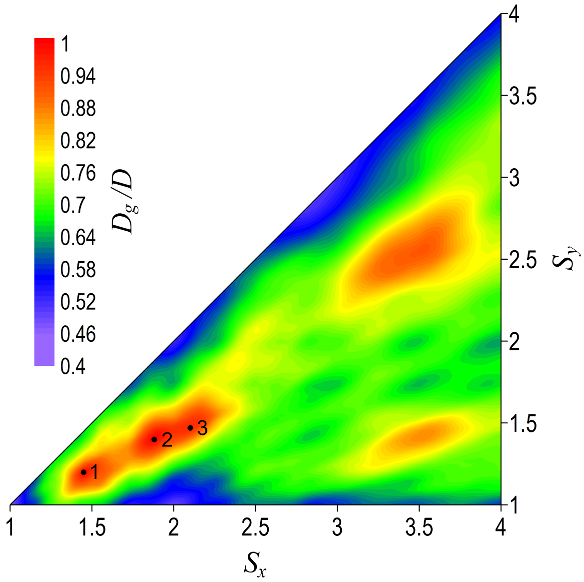

Fig. 8. Dependence of D_g/D on the dimension ratios S_x and S_y for the optimal source position $(x_0, y_0, z_0) = (0, 0, L_z)$, the room volume V of 150 m^3 and the absorption coefficient α of 0.2. A dot denoted by 1 indicates the optimum dimension ratio 1 : 1.2 : 1.45, whereas dots denoted by 2 and 3 correspond to the nearly optimum dimension ratios 1 : 1.4 : 1.88 $(D_g/D = 0.998)$ and 1 : 1.47 : 2.1 $(D_g/D = 0.964)$.

value of D_g/D for the first ratio is greater than for the second one.

In Table 1 the most important calculation results are collected to illustrate the impact of the room volume V and the absorption coefficient α on the optimum and nearly optimum room dimension ratios. These data show that for the ranges of V and α assumed in numerical tests there exist only three different optimum dimension ratios. To facilitate the identification of these ratios, they are denoted by yellow, green and cyan colors. Since parameters S_y and S_x slightly change for these ratios, the mean value $1 : \overline{S}_y : \overline{S}_x$ is defined where $\overline{S}_y$ and $\overline{S}_x$ represent arithmetic means of S_y and S_x, respectively. The first dimension ratio is indicated by the yellow color. This ratio is identified as optimal for the smallest room volumes: 50–100 m^3 and partially 125 m^3, and it is characterized by the mean value 1 : 1.48 : 2.12. The second ratio, denoted by the green color, is recognized as optimal for the room volume 125 and 150 m^3 only and it has the mean value 1 : 1.4 : 1.89. The third ratio is optimal for a wide range of room volumes: 150–300 m^3, and 1 : 1.2 : 1.44 is its mean value. As may be noted, depending on the room volume and the absorption coefficient, the above mentioned ratios can also represent nearly optimum dimension ratios. The last two ratios obtained via the proposed numerical method are roughly 1 : 2.52 : 3.41 and 1 : 1.45 : 3.54, and they are only nearly optimum dimension ratios.

Table 1. Optimum and nearly optimum dimension ratios versus the room volume V and the absorption coefficient α.

V [m^3]	Absorption coefficient α				
	0.1	0.15	0.2	0.25	0.3
50	1 : 1.44 : 2.10 1 : 1.20 : 1.45 1 : 2.47 : 3.37	1 : 1.46 : 2.11 1 : 2.53 : 3.43	1 : 1.47 : 2.12 1 : 2.55 : 3.44	1 : 1.49 : 2.14 1 : 2.55 : 3.43	1 : 1.5 : 2.14
75	1 : 1.46 : 2.11 1 : 1.20 : 1.45 1 : 2.44 : 3.33	1 : 1.47 : 2.12 1 : 1.42 : 1.93 1 : 2.52 : 3.42	1 : 1.48 : 2.13 1 : 1.42 : 1.94 1 : 2.55 : 3.44	1 : 1.50 : 2.14	1 : 1.51 : 2.14
100	1 : 1.46 : 2.11 1 : 1.45 : 3.54 1 : 1.20 : 1.45	1 : 1.47 : 2.12 1 : 1.41 : 1.92	1 : 1.48 : 2.12 1 : 1.42 : 1.93	1 : 1.49 : 2.11 1 : 1.42 : 1.93	1 : 1.50 : 2.11 1 : 1.42 : 1.93
125	1 : 1.46 : 2.11 1 : 1.20 : 1.45 1 : 1.45 : 3.54	1 : 1.46 : 2.10 1 : 1.41 : 1.92 1 : 1.20 : 1.45	1 : 1.41 : 1.91 1 : 1.47 : 2.11	1 : 1.41 : 1.90 1 : 1.48 : 2.08	1 : 1.41 : 1.90 1 : 1.49 : 2.07
150	1 : 1.20 : 1.45 1 : 1.45 : 2.10	1 : 1.20 : 1.45 1 : 1.39 : 1.89 1 : 1.46 : 2.10	1 : 1.20 : 1.45 1 : 1.40 : 1.88 1 : 1.47 : 2.10	1 : 1.40 : 1.87 1 : 1.20 : 1.46	1 : 1.39 : 1.86
175	1 : 1.20 : 1.45	1 : 1.20 : 1.45	1 : 1.20 : 1.45	1 : 1.20 : 1.45	1 : 1.20 : 1.46 1 : 1.40 : 1.86
200	1 : 1.20 : 1.45	1 : 1.20 : 1.45	1 : 1.20 : 1.45	1 : 1.19 : 1.44	1 : 1.20 : 1.45
225	1 : 1.20 : 1.45	1 : 1.19 : 1.44	1 : 1.19 : 1.44	1 : 1.19 : 1.44	1 : 1.19 : 1.44
250	1 : 1.20 : 1.44	1 : 1.19 : 1.44	1 : 1.19 : 1.44	1 : 1.19 : 1.44	1 : 1.19 : 1.43
275	1 : 1.20 : 1.44	1 : 1.20 : 1.44	1 : 1.19 : 1.43	1 : 1.19 : 1.43	1 : 1.19 : 1.43
300	1 : 1.20 : 1.44	1 : 1.20 : 1.44	1 : 1.20 : 1.44	1 : 1.19 : 1.43	1 : 1.19 : 1.43

4. Summary and conclusions

A new method has been presented for determining optimum dimension ratios for rectangular rooms. A theoretical model is based on the exact description of the room impulse response which allowed an easy prediction of the frequency response of a room. This response accounts for the source and receiver positions within the room, therefore, a new metric equivalent to an average frequency response was introduced. This response quantifies the overall sound pressure variation inside the room and is representative of the evenness of the frequency response among the different receiving positions. Based on changes in this response within the assumed frequency band, a best-fit curve was calculated using the quadratic polynomial regression and this curve was used to model the desired shape of the frequency response.

The indicator of room quality was defined as the deviation of the actual average frequency response from a smooth fitted response and the requirement, that this deviation is minimal, was the criterion for optimum dimension ratios. The influence of source position on the irregularity of the frequency response has been examined and most smooth responses were obtained when the source was located at one of the eight corners of a room. In the final numerical procedure, the optimal source location was assumed to find the best possible room dimension ratios and their dependence on the room volume and the absorption coefficient.

Calculation results have demonstrated that for small and medium room volumes ($V \leq 150$ m^3) and the mean absorption coefficient α from the range 0.1–0.3 the optimum dimension ratios are 1 : 1.48 : 2.12, 1 : 1.4 : 1.89 and 1 : 1.2 : 1.45 on average. The first result matches quite well with the ratio 1 : 1.6 : 2.33 recommended by Sepmeyer (1965) whereas the second one is almost identical to that found by Louden (1971). The third result is very close to the dimension ratios 1 : 1.186 : 1.439 and 1 : 1.2 : 1.4 obtained by Milner and Bernhard (1989) and Błaszak (2008). It is worth noting that the change of one optimum ratio to the next one is gradual. Thus, the above mentioned optimum dimension ratios can also represent the nearly optimum dimension ratios. This finding is important from the practical viewpoint because an irregularity of the frequency response for the nearly optimum dimension ratio is at most 5% bigger than for the optimum one, thus, it represents a good alternative for this ratio. For rooms with the volume from 175 to 300 m^3, only the ratio 1 : 1.2 : 1.44 was found to be the best one, therefore, it is recommended for room design in the range of larger room volumes.

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