



METROLOGY AND MEASUREMENT SYSTEMS

Index 330930, ISSN 0860-8229 www.metrology.pg.gda.pl



## A NEW APPROACH TO MEASUREMENT OF FREQUENCY SHIFTS USING THE PRINCIPLE OF RATIONAL APPROXIMATIONS

## Fabian N. Murrieta-Rico<sup>1</sup>), Vitalii Petranovskii<sup>2</sup>), Oleg Y. Sergivenko<sup>3</sup>), Daniel Hernandez-Balbuena<sup>4)</sup>. Lars Lindner<sup>3)</sup>

1) Posgrado en Física de Materiales, Centro de Investigación Científica y Educación Superior de Ensenada, Carretera Ensenada-Tijuana 3918. Zona Plavitas. 22860 Ensenada, B. C., México (🖂 fmurriet@cicese.edu.mx. +52 01 646 175 0500)

2) Universidad Nacional Autónoma de México, Centro de Nanociencias y Nanotecnología, Carretera Tijuana-Ensenada Km. 107, Pedregal Playitas, 22860 Ensenada, B. C., México (vitalii@cnyn.unam.mx)

3) Universidad Autónoma de Baja California, Instituto de Ingeniería, Calle de la Normal S/N y Blvd. Benito Juárez Col. Insurgentes Este Mexicali B. C., México (srgnk@uabc.edu.mx, lindner.lars@uabc.edu.mx)

4)Universidad Autónoma de Baja California, Facultad de Ingeniería, Calle de la Normal S/N y Blvd. Benito Juárez Col. Insurgentes Este Mexicali B. C., México (dhernan@uabc.edu.mx)

#### Abstract

When a frequency domain sensor is under the effect of an input stimulus, there is a frequency shift at its output. One of the most important advantages of such sensors is their converting a physical input parameter into time variations. In consequence, changes of an input stimulus can be quantified very precisely, provided that a proper frequency counter/meter is used. Unfortunately, it is well known in the time-frequency metrology that if a higher accuracy in measurements is needed, a longer time for measuring is required. The principle of rational approximations is a method to measure a signal frequency. One of its main properties is that the time required for measuring decreases when the order of an unknown frequency increases. In particular, this work shows a new measurement technique, which is devoted to measuring the frequency shifts that occur in frequency domain sensors. The presented research result is a modification of the principle of rational approximations. In this work a mathematical analysis is presented, and the theory of this new measurement method is analysed in detail. As a result, a new formalism for frequency measurement is proposed, which improves resolution and reduces the measurement time.

Keywords: frequency measurement, rational approximations, sensors.

© 2017 Polish Academy of Sciences. All rights reserved

### 1. Introduction

Frequency Domain Sensors (FDS) are input transducers, which change their frequency output when they are under stimulation of a physical variable. This change is known as a frequency shift. FDS are also known as frequency output sensors. In the last years, the technology standards require performing highly accurate measurements in a short time. Specifically, FDS applied to detection of chemicals and measurement of concentration are being actively researched [1-6]. It is well-known that a gas-sensing device for toxic gases requires a high sensor sensitivity and a quick response [7]. After a careful review of the literature, where we focused on applications of the frequency measurement devices, we have learned that sensors employed in detection of chemicals are very sensitive (1 Hz corresponds to 4.3 ng/cm<sup>2</sup> [8]), but they require a long time for measuring. Typically, hundreds of seconds.

The principle of rational approximations is a method of frequency measurement, where the time required for measurement depends on the standard and the measurand. In other words, the higher the unknown frequency value, the shorter the time required for measurement. One of the main advantages of this method (compared with other well-known techniques [9–11])

Article history: received on May 18, 2016; accepted on Aug. 28, 2016; available online on Mar. 10, 2017; DOI: 10.1515/mms-2017-0007.

www.czasopisma.pan.j

is a high speed of measurement without diminishing its accuracy. Also, it is insensitive to most common sources of uncertainty in time-frequency measurement systems [12].

In the preliminary works, our research group has explored the application of the principle of rational approximations for measurement of frequency shifts [13, 14]. So far our proposal requires knowing *a priori* the value of a sensor's output before occurring a frequency shift. For this reason and due to defects of fabrication, there is required measurement of a sensor's output before the frequency shift occurrence. In consequence, to measure a frequency shift at the sensor output, at least two measurements – in two different and separated time intervals – are required.

In this work, we show a modification of the rational approximations principle, and its measurement theory is expanded. As a result, after applying the principle of rational approximations, a frequency shift can be measured at the moment of its occurrence.

#### 2. Fast frequency measurements using the principle of rational approximations

The principle of rational approximations is a method of frequency measurement. It is based on the number theory, in particular – on a property of rational numbers: the mediant fractions [15]. The principle of rational approximations has many outstanding properties, including: its invariance to jitter, the accuracy limited only by the stability of a reference signal, and the measurement time decrease with the increase of the measurand value increases.

The frequency measurement using the principle of rational approximations is performed by comparing two signals: a reference one  $(S_0)$  whose frequency value is known  $(f_0)$  and a signal to measure  $(S_x)$  with an unknown frequency  $(f_x)$ . Both signals have corresponding periods  $T_0$ ,  $T_x$ . After a process of signal conditioning [13] the pulses in both signals must have the same pulse width  $(\tau)$ , which must be  $\tau \leq T_0/2$  [16, 17].

When the signals are compared, a pulse train of coincident pulses  $(S_x \& S_0)$  is generated. The frequency measurement process starts at the moment of the first coincidence of pulses. Also, simultaneously starts counting the pulses in  $S_x$ ,  $S_0$ . The value of  $\tau$  defines the duration of coincidences; this effect is analysed and reported in [16]. The numbers of pulses in  $S_x$ ,  $S_0$  are denoted by  $P_n/Q_n$ , where *n* is the number of coincidence (Fig. 1).

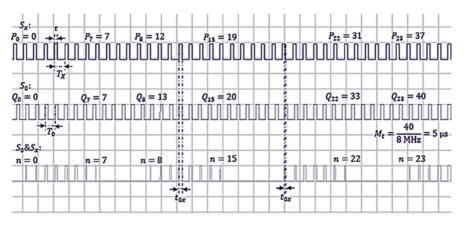


Fig. 1. Comparison of signals during the frequency measurement process, where  $f_x = 7.4$  MHz,  $f_0 = 8$  MHz, and  $\tau \approx 40$  ns.

In each coincidence, a fraction is formed  $(P_n/Q_n)$ , and an approximation to the measurand is obtained. The unknown frequency value is given by:

www.czasopisma.pan.pl

Metrol. Meas. Syst., Vol. 24 (2017), No. 1, pp. 45–56.

$$f_x = \frac{P_n}{Q_n} f_0 \tag{1}$$

or by the sum of all numerators and denominators that form the mediant fraction m:

$$f_x = f_0 \frac{\sum_m P_n}{\sum_m Q_n} \,. \tag{2}$$

The concept of mediant fraction is a well-known property in the number theory, and its application to frequency measurement is shown in (1) of [15]. The measurement time  $(M_t)$  is given by:

$$M_t = Q_n T_0 = \frac{Q_n}{f_0}.$$
(3)

For the frequency measurement process illustrated in Fig. 1, the best approximation to the measurand is obtained in the 23rd fraction, where the second perfect coincidence exists. As the concept of mediant states, all fractions between perfect coincidences are approximations to the measurand. For a continuous pulse counting process, the relative error ( $\beta$ ) in the frequency measurement process is defined as:

$$\beta = \frac{\left| f_x - \frac{P_n}{Q_n} f_0 \right|}{f_x} \,. \tag{4}$$

From the statements exposed in this section, we have reviewed the basics of the principle of rational approximations. In short, only by counting the pulses after the first coincidence, and in each posterior coincidence, from (2), the value of  $f_x$  is calculated. In the next section, the measurement process parameters will be examined and a modification of the principle of rational approximations will be proposed.

# 3. Simultaneous measurement of two frequencies using the principle of rational approximations

Determining an FDS frequency requires at least two measurements. In order to know how the sensor's output changes after a stimulus, measurement of the initial frequency is needed. This process requires two measurements, where the sensor output is measured twice – before and after the stimulus – in different time intervals. In this section, we introduce an original approach, not previously published, to measuring frequency shifts occurring in a given FDS under a stimulus.

When the FDS output has changed, there is a frequency shift of its frequency value. The last can be expressed as:

$$\Delta f = f_s - f_p, \tag{5}$$

where  $f_s$  denotes the initial frequency of the sensor; and  $f_p$  is the posterior frequency value. These signals have the corresponding periods  $T_s$  and  $T_p$ .

Two sensors of the same kind can be measured simultaneously, and the difference among them can be used to calculate the corresponding frequency shift. If the measured values of sensors are  $f_{xs}$  and  $f_{xp}$ , they can be calculated from measurements using the principle of rational approximations:

$$f_{xs} = \frac{P_{ns}}{Q_{ns}} f_0, \tag{6}$$

www.czasopisma.pan.pl

F. N. Murrieta-Rico, V. Petranovskii, et al.: A NEW APPROACH TO MEASUREMENT ...

$$f_{xp} = \frac{P_{np}}{Q_{np}} f_0. \tag{7}$$

The signal coincidence process of  $S_{xs} \& S_0$  and  $S_{xp} \& S_0$  is illustrated in Fig. 2. In this case  $f_{xs} = 4.7$  MHz,  $f_{xp} = 4.6$  MHz and  $f_0 = 8$  MHz. These values are chosen because  $f_{xs} = 4.7$ MHz is a common value in quartz crystal microbalances (QCMs) [18, 19]. According to the Sauerbrey equation [20], when the frequency is shifted, the frequency value in the QCM output decreases; this is the reason of choosing  $f_{xp} = 4.6$  MHz ( $f_{xp} < f_{xs}$ ). Also, until now, the principle of rational approximations states that if the reference frequency is closer to the measurand, the best approximations are obtained in a shorter time [16]. This is why the reference frequency is chosen to be  $f_0 = 8$  MHz – that is also a common value in quartz crystals used as the time reference.

An important remark about the graphs of Figs. 2, 3 is required. Simultaneous observing all required parameters of six signals is complicated, because the oscilloscopes (even in simulations) have maximum four channels. In our analysis it is important to evaluate the behaviour of signals from the beginning of measurement process. This enables to evaluate how factors, like the phase or amplitude, affect the frequency measurement process. For the analysed cases, the input signals are digitalized, and the amplitude has just discrete values, but for very narrow pulses the rising and falling times of the pulses could affect the pulse shape. Even when an alternative is to use a logic state analyser (LSA), for our purposes we need to observe specific time stamps, where the rising and falling times of pulses are known, without the sampling time, more like in an analogue analysis. This enables to evaluate the coincidence time of pulses  $(t_{0x})$ . For these reasons, in this work we use the analogue analysis available in SPICE simulations.

For the measurement time observed in Fig. 2, in both cases the number of counted pulses and the number of obtained fractions are the same. But the measurand value is different; this characteristic changes the rate of occurrence of coincidences. In other words, even when the P and Q values increase at the same rate – because  $f_{xs}$  and  $f_{xp}$  have a difference of 4.6 ns, that is greater than the pulse width [16] – the corresponding fractions  $(P_n/Q_n)$  have different values. According to the classic theory of time-frequency metrology [11], a better approximation to the measurand will be obtained if the measurement time is increased.

The measurement of  $f_{xs}$  and  $f_{xp}$  is done using the same frequency standard  $(f_0)$ . In consequence, the corresponding measurement time is given by:

$$M_{IS} = Q_{IS}T_0 = \frac{Q_{IS}}{f_0},$$
 (8)

$$M_{lp} = Q_{np}T_0 = \frac{Q_{np}}{f_0}, \qquad (9)$$

respectively with sub-indexes s and p for the starting and posterior frequency values.

The fundamental assumption of the principle of rational approximations is the existence of coincidences. If two regular pulse trains are continuously compared, a coincidence pulse train is generated. In the literature there is reported that such a comparison requires the use multiplication of functions modelling each signal (in Fig. 1, the functions modelling  $S_x$ ,  $S_0$  [16]. The main novelty of this work is focused on simultaneous comparison of three signals: the original signal without a frequency shift  $(S_{xs})$ , the reference signal  $(S_0)$ , and the signal after an unknown frequency shift  $(S_{xp})$ , where  $f_s$ ,  $f_0$ ,  $f_{xp}$  are their respective frequency values.



Metrol. Meas. Syst., Vol. 24 (2017), No. 1, pp. 45-56.

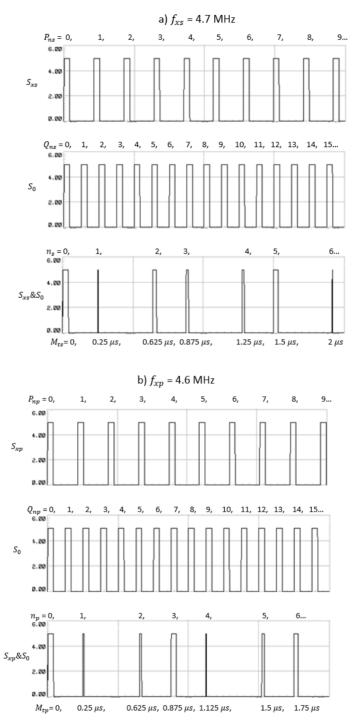


Fig. 2. Comparison of signals during the frequency measurement process, where  $f_{xs} = 4.7$  MHz,  $f_{xp} = 4.6$  MHz,  $f_0 = 8$  MHz, and  $\tau \approx 40$  ns.



Any signal comparison process has its measurement time associated with specific values of  $(f_x, f_0, \tau)$ . From (8), (9):

$$M_{ts} = Q_{ns} \frac{M_{tp}}{Q_{np}},\tag{10}$$

if  $M_{ts} = M_{tp}$ , then  $Q_{ns} = Q_{np}$ . The last relationships have sense, if we understand that the signal coincidence process  $T_0$  is constant in  $S_{xs} \& S_0$  and  $S_{xp} \& S_0$ . In consequence, during measurement in simultaneous comparison of three signals, the *P*-values  $(P_{ns}, P_{np})$  are the only "truly" independents in continuous pulse counting (Fig. 3).

In the principle of rational approximations, after the first coincidence (when  $M_t = 0$ ) of pulses from input signals, all the future coincidences are approximations to the measurand. When there is a comparison of three input signals, the measurement process starts when there is a coincidence of pulses from three input signals ( $S_{xs} \& S_0 \& S_{xp}$ ). It is worth remembering that for proper functioning of the principle of rational approximations, in order to have the same pulse width, all input signals must be conditioned. Following functioning of the principle of the rational approximations, after the n – coincidence, the next, (n + 1) – coincidence is an approximation to the measurand. But in this case, rather than measuring the frequency value of a signal, the difference between frequencies of two signals ( $S_{xs}, S_{xp}$ ) is measured. Basing on the previous analysis, the measured frequency difference between two signals ( $S_{xs}, S_{xp}$ ) is defined as the frequency shift. (5) becomes:

$$\Delta f = f_{xs} - f_{xp}. \tag{11}$$

Since this proposal aims to measure the frequency shift occurring in a sensor's output, two sensors are required. Ideally, if two sensors of the same kind and operation range were identically fabricated – an example is measurement of the frequency shift from a QCM while it is loaded [18, 19] – they would have the same frequency value before a stimulus. The frequency of one sensor without a stimulus is the starting frequency ( $f_{xs}$ ). On the other hand, the sensor frequency after stimulation is its posterior frequency ( $f_{xp}$ ). Based on the last statements, an approximation to measurement of the frequency shift can be performed by using two sensors. One of them will be not stimulated, whereas the other one will be under a stimulus, which leads to a meaningful frequency shift. This approach is similar to the idea of differential measurement [21].

If the frequency values of starting and posterior frequencies are measured, after the first coincidence of pulses from the three input signals, they can be used for calculating the frequency shift:

$$\Delta f = \frac{P_{ns}}{Q_{ns}} f_0 - \frac{P_{np}}{Q_{np}} f_0, \qquad (12)$$

$$\Delta f = \left(\frac{P_{ns}}{Q_{ns}} - \frac{P_{np}}{Q_{np}}\right) f_0 \tag{13}$$

for any n > 0. According to the principle of rational approximations, after the first coincidence the pulses of input signals are continuously counted. Any coincidence of the pulses of three input signals ( $S_{xs}$ ,  $S_{xp}$ ,  $S_0$ ), is an approximation to  $\Delta f$ .

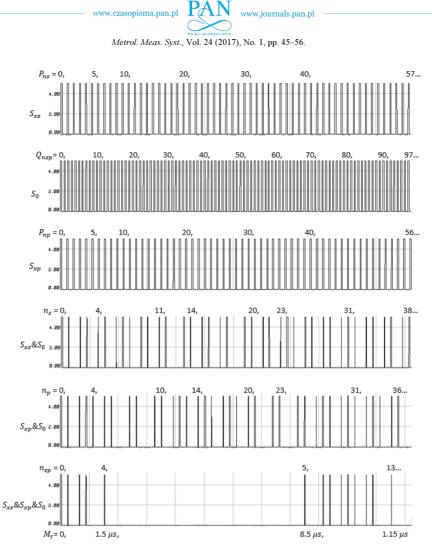


Fig. 3. Simultaneous comparison of three signals.

Since the reference frequency is the same when measuring both  $f_{xs}$ ,  $f_{xp}$ :

$$\frac{f_{xs}}{f_0} - \frac{f_{xp}}{f_0} = \frac{P_{ns}}{Q_{ns}} - \frac{P_{np}}{Q_{np}},$$
(14)

there is a value in the measurement time, where  $Q_{ns} = Q_{np}$ . At this moment, measurement of  $f_{xs}$  and  $f_{xp}$  can be done for calculating the corresponding frequency shift ( $\Delta f$ ). (10) and (14) show a property of the physical phenomenon of signal comparison. For the ratios  $f_{xs}/f_0$ ,  $f_{xp}/f_0$ there is a measurement time where  $M_{tp} = M_{ts}$ . Knowing  $f_0$ , when  $M_{tp} = M_{ts}$  counts of the reference pulses in both comparison processes give  $Q_{ns} = Q_{np}$ . This property explains why simultaneous comparison of three input signals enables to measure the signal frequency immediately after the shift. Such a behaviour is illustrated in Fig. 3. The generality of this formalism is illustrated by (14). For any given value of  $f_{xs}$ ,  $f_{xp}$  there is a measurement time, during which the frequency shift is calculated.

The nature of signal comparison enables to understand the effect of comparing three signals. It is mainly reflected in the measurement time needed for achieving measurement results. As stated before, measurement of  $\Delta f$  requires measuring separately  $f_{xs}$  and  $f_{xp}$ . These

operations require at least a total measurement time of:  

$$M_{tsp} = M_{tp} + M_{ts} + O_t, \qquad (15)$$

where the operation time  $(O_t)$  defines the time required for switching from measurement of  $f_s$  to  $f_p$ .  $O_t$  includes the time used by the operator of the frequency measurement instrument, or – in the case of automatic switching – the time required by the microcontroller. With the herein proposed approach the time required for measuring a frequency shift is given by:

$$M_{tfs} = Q_{ns}Q_{np}T_0. \tag{16}$$

The last equation shows the principal strength of the presented formalism. For the case of  $M_{tsp}$ , both signals  $(f_{xs}, f_{xp})$  need to have enough time for being approximated, which depends directly on  $Q_{ns}$ ,  $Q_{np}$ . In the traditional approach to the principle of rational approximations,  $Q_{ns} \neq Q_{np}$ . On the other hand, since  $M_{tfs}$  uses simultaneous comparison of three signals, the Q – values increment at the same rate; in other words:  $Q_{ns} = Q_{np} = Q_{nsp}$ , where nsp is the number of coincidence of three pulses of  $S_{xs}$ ,  $S_{xp}$ ,  $S_0$ , and  $Q_{nsp}$  is the count value of the pulses in  $f_0$  for coincidences  $S_{xs} \& S_0 \& S_{xp}$ . One implication of these statements is that  $M_{tfs} < M_{tsp}$ . The last property shows how simultaneous comparison of three signals enables to obtain a better approximation than that of two signals at a different time. As a result, the frequency shift is calculated from (13), for any coincidence nsp > 0, by:

$$\Delta f = \left(\frac{P_{ns} - P_{np}}{Q_{nsp}}\right) f_0 \quad , \tag{17}$$

and the measurement time is defined as:

$$M_t = M_{tfs} = Q_{nsp}T_0. \tag{18}$$

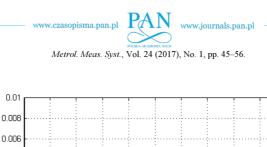
As it has been shown in previous works [12–16], the time required for measuring depends on overlapping of existing pulses at the same time. The likelihood of two overlapped pulses (generated independently) is greater than overlapping of three pulses, which are also generated independently. This leads to a longer time required for measuring than that for three signals.

In this Section, it was proposed a modification of the principle of rational approximations for application to measurement of frequency shifts. The next section is devoted to evaluating this theory.

### 4. Evaluation of measurement of frequency shifts

Measurement of frequency shifts uses simultaneous measurement of three signals, and – when the pulses of three signals are coincident - an approximation to the measurand is obtained (in this case  $\Delta f$ ). In this section, using Matlab, we evaluate the formalism proposed in Section 3.

According to (13), the measurement of  $\Delta f$  requires application of the principle of rational approximations for approximating the values of  $f_{xs}$  and  $f_{xp}$ . In Fig. 4, after using the (1) – (4), a relationship between the relative error and the measurement time is shown for  $f_{xs} = 4.7$  MHz – the data of Fig. 1. The duration of coincidences  $(t_{0x})$  – when  $f_{xs} = 4.7$  MHz – is illustrated in Fig. 2a. This shows different  $t_{0x}$  – values, which affect changing  $\beta$ , and – since counting of the pulses in  $S_{xs}$  is continuous –  $\beta$  decreases. According to (1), the best approximation occurs for  $P_{ns}/Q_{ns} = 47/80$  (Fig. 3) or  $M_{ts} = 1 \times 10^{-5}$  s (Fig. 4).



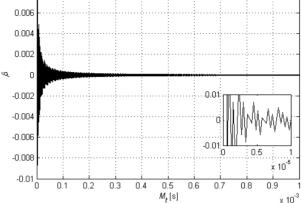


Fig. 4. The relative error in the frequency measurement process for  $f_{xs} = 4.7$  MHz.

On the other hand, Figs. 5a and 5c show the relative error when measuring  $f_{xp} = 4.6$  MHz and  $f_{xp} = 4.699$  MHz. For the cases presented in Fig. 2 and Fig. 3, after  $\Delta f = 100$  kHz, there are less pulses in  $S_{xn} \& S_0$  than in  $S_{xs} \& S_{xn}$ . This effect can be observed only after a long measurement time; this fact is illustrated by comparing Fig. 2 and Fig. 3. The reason of this decrement – in the number of coincidences – is that the time difference in periods of signals  $(T_{xs} - T_0 \text{ or } T_{xp} - T_0)$  becomes greater. The relative error in the measurement process is related to the duration of coincidence; if  $t_{0x} = \tau$ , there is the perfect coincidence and the best approximation to the measurand [16]. In a continuous signal comparison process, there are both partial ( $t_{0x} < \tau$ ) and perfect coincidences, the difference is in the way these coincidences appear (Figs. 1–3). This phenomenon affects the way of decreasing  $\beta$ , and it is illustrated in the zoomed boxes in Fig. 4 and Fig. 5a. If the frequency shift is smaller, there are less coincidences. An example is when  $\Delta f$  changes from 100 kHz to 10 kHz or  $f_{xp}$  changes from 4.6 MHz (Fig. 5a) to 4.699 MHz (Fig. 5c). After evaluating both  $\Delta f$  – values, the perfect coincidences exist in  $P_{np}/Q_{np} = 23/40$  for  $f_{xp} = 4.6$  MHz, and in  $P_{np}/Q_{np} = 4699/8000$  for  $f_{xp} = 4.699$  MHz. As we know, for obtaining the accurate approximation to the measurand during measurement,  $f_x$  and  $f_0$  must be stable within the measurement period – for the principle of rational approximations, this enables to obtain results comparable to those for the Allan variance [22]. The two fractions corresponding to the best approximations after the frequency shift require a measurement time of  $5 \times 10^{-6}$  s for  $P_{np}/Q_{np} = 23/40$  and  $1 \times 10^{-3}$  s for  $P_{np}/Q_{np} =$ 4699/8000, and such calculations are true only for the stationary values of  $f_{xp}$ .

The approach proposed in this work enables to measure the frequency shift "when it occurs"; the last was analysed in the previous section, and it is obtained only by simultaneous comparison of three signals  $S_{xs} \& S_0 \& S_{xp}$ . In this comparison, differences in periods of input signals  $(S_{xs}, S_{xp} \text{ and } S_0)$  – after their pulses coincide in time – generate different P, Q – values, which can be used for calculating  $\Delta f$  using (17). This is the main reason we could call this technique the principle of rational approximations for differential frequency measurements.





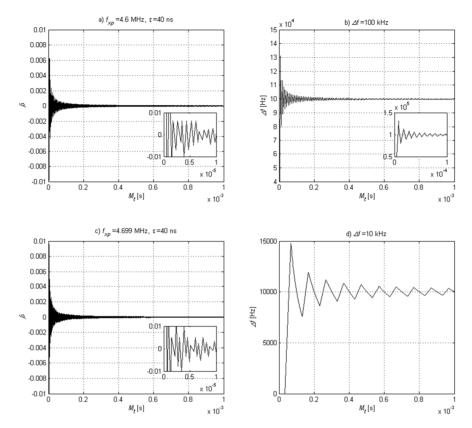


Fig. 5. The relative error in the frequency measurement process for  $f_{xp} = 4.6$  MHz (a);  $f_{xp} = 4.699$  MHz (c). Measurement of the frequency shift  $\Delta f = 100$  kHz (b);  $\Delta f = 10$  kHz (d).

In each coincidence  $(S_{xs} \& S_0 \& S_{xp})$ , the frequency shift value is calculated using (13). For the examined cases, the value of  $\Delta f$  is obtained in less than 1 ms. But another interesting property of our formalism is its clarity. The density of approximations is quite different for measuring  $\Delta f$  and for any "single" measurement of  $f_{xs}$  and  $f_{xp}$ . This phenomenon can be explained using Fig. 3. By comparing the number of pulses in  $S_{xs} \& S_0, S_{xp} \& S_0$  with that in  $S_{xs} \& S_0 \& S_{xp}$ , the variations in the number of coincident pulses is observed. Particularly, the lowest number of coincident pulses is observed in  $S_{xs} \& S_0 \& S_{xp}$ . This can be explained by the fact that comparison of three signals leads to a lower number of coincidences. In consequence, less approximations are observed when measuring  $\Delta f$ .

After the analysis shown in this section, it is clear that the principle of rational approximations for differential frequency measurements is a novel technique for application to frequency sources with dynamic values (in particular FDS), where the measurement time required for obtaining an approximation to the measurand is shorter than that in the traditional principle of rational approximations.

#### 5. Conclusions

Measurement of frequency shifts is required for sensors with high sensitivity. Well known frequency measurement techniques require more time for measuring if a greater accuracy is needed.



In this work, the theory of rational approximations is expanded by introduction of a new formalism for measuring frequency shifts. As a result, the principle of rational approximations is applied to differential frequency measurements, where  $\Delta f$  can be measured in a very short time, without diminishing its accuracy. The proposal of this work is to measure frequency shifts by comparing three signals simultaneously. (10) - (12) showed that such a comparison is possible. Also, a further analysis – presented in Section 4 – illustrated how the coincidence of pulses occurs when comparing three signals, in a similar way like in the "traditional" principle of rational approximations, where only two signals are compared.

Our analysis shows that – due to the phenomenon of coincidence of signals – the measurements could be improved by choosing an optimal frequency standard, as well as a pulse width value. This analysis can be easily implemented using the algorithms of [16] and (10) - (12) and (17), (18).

#### References

- [1] Filippov, P., Strizhak, P.E., Vlasenko, N.V., Kochkin, Y.N., Serebrii, T.G. (2014). Adsorption-Desorption Dynamics of Alcohols on H-Beta and H-CMK Zeolites Nanocrystallites Studied by Quartz Crystal Microbalance Method. *Adsorption Science & Technology*, 32(10), 807–820.
- [2] Afzal, N., Iqbal, A., Mujahid, Schirhagl. (2013). Advanced vapor recognition materials for selective and fast responsive surface acoustic wave sensors: A review. *Analytica Chimica Acta*, 787, 36 – 49.
- [3] Arnau, A. (2008). Review of Interface Electronic Systems for AT-cut Quartz Crystal Microbalance Applications in Liquids. *Sensors*, 8(1), 370–411.
- [4] Casteleiro-Roca, J.L., Calvo-Rolle, J.L., Meizoso-Lopez, M.C., Piñón-Pazos, A., Rodríguez-Gómez, B.A. (2014). New approach for the QCM sensors characterization. Sensors and Actuators A: Physical, 207, 1–9.
- [5] Ishii, R., Naganawa, R., Nishioka, M., Hanaoka, T. (2013). Microporous organic-inorganic nanocomposites as the receptor in the QCM sensing of toluene vapors. *Analytical sciences: the international journal of the Japan Society for Analytical Chemistry*, 29, 283–289.
- [6] Bhasker Raj, V., Singh, H., Nimal, A.T., Tomar, M., Sharma, M.U., Gupta, V. (2013). Effect of metal oxide sensing layers on the distinct detection of ammonia using surface acoustic wave (SAW) sensors. *Sensors and Actuators B: Chemical*, 187, 563–573.
- [7] Kikuchi, M., Shiratori, S. (2005). Quartz crystal microbalance (QCM) sensor for CH<sub>3</sub>SH gas by using polyelectrolyte-coated sol-gel film. *Sensors and Actuators B: Chemical*, 108(1), 564–571.
- [8] Bein, T., Mo, S., Mintova, S., Valtchev, V., Schoeman, B., Sterte, J. (1997). Growth of silicalite films on pre-assembled layers of nanoscale seed crystals on piezoelectric chemical sensors. *Advanced Materials*, 9(7), 585–589.
- [9] Kirianaki, N.V., Yurish, S.Y., Shpak, N.O. (2001). Methods of dependent count for frequency measurements. *Measurement*, 29(1), 31–50.
- [10] Kalisz, J. (2003). Review of methods for time interval measurements with picosecond resolution. *Metrologia*, 41(1), 17.
- [11] Johansson, S. (2005). New frequency counting principle improves resolution. *Frequency Control Symposium* and Exposition. Proc. of the 2005 IEEE International, 628–635.
- [12] Sergiyenko, O., Hernandez Balbuena, D., Tyrsa, V., Rosas Mendez, P.L.A., Rivas Lopez, M., Hernandez, W., Podrygalo, M., Gurko, A. (2011). Analysis of jitter influence in fast frequency measurements. *Measurement*, 44(7), 1229–1242.
- [13] Murrieta-Rico, F.N., Mercorelli, P., Sergiyenko, O.Y., Petranovskii, V., Hernández-Balbuena, D., Tyrsa, V. (2015). Mathematical modelling of molecular adsorption in zeolite coated frequency domain sensors. *IFAC PapersOnLine*, 48(1), 41–46.
- [14] Sergiyenko, O.Y. (2016). The mediant method for fast mass/concentration detection in nanotechnologies. *International Journal of Nanotechnology*, 13(1–3), 238–249.

- www.czasopisma.pan.pl

- [15] Hernandez Balbuena, D., Sergiyenko, O., Tyrsa, V., Burtseva, L., Rivas Lopez, M. (2009). Signal frequency measurement by rational approximations. *Measurement*, 42(1), 136–144.
- [16] Murrieta-Rico, F.N., Yu, O., Sergiyenko, Petranovskii, V., Hernandez Balbuena, D., Lindner, L., Tyrsa, V., Rivas-Lopez, M., Nieto-Hipolito, J.I., Karthashov, V.M. (2016). Pulse width influence in fast frequency measurements using rational approximations. *Measurement*, 86, 67–78.
- [17] Jansson, P.A. (1998). Deconvolution of Images and Spectra. New York: J. Wiley & Sons.
- [18] Yu, L., Ma, X., Wu, T., Ma, Y., Shen, D., Kang, Q. (2016). Monitor the Processes of Ice Film Disappearance under a Stimulant Convection Condition and Absorption Ethanol Vapor to Ice by a Quartz Crystal Microbalance. *International Journal of Electrochemical Science*, 11(4), 2595–2611.
- [19] Sasaki, I., Tsuchiya, H., Nishioka, M., Sadakata, M., Okubo, T. (2002). Gas sensing with zeolite-coated quartz crystal microbalances-principal component analysis approach. *Sensors and Actuators B: Chemical*, 86(1), 26–33.
- [20] Sauerbrey, G. (1959). Verwendung von schwingquarzen zur wägung dünner schichten und zur mikrowägung. Zeitschrift für physik, 155(2), 206–222.
- [21] Boyes, W. (2009). Instrumentation Reference Book. Butterworth-Heinemann.
- [22] Allan, D.W., Barnes, J.A. (1981). A modified "Allan variance" with increased oscillator characterization ability. *Thirty Fifth Annual Frequency Control Symposium IEEE*.