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Computing Impulse Response of Room Acoustics Using the Ray-Tracing Method in Time Domain

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This paper proposes a new approach for calculating the impulse response of room acoustics. Impulse response provides unique characterization of any discrete linear-time invariant (LTI) systems. We assume that the room is a linear time-invariant system and the impulse response is calculated simply by sending a Dirac Impulse into the system as input and getting the response from the output. Then, the output of the system is represented as a sum of time-shifted weighted impulse responses. Both mathematical justifications for the proposed method and results from simulation software developed to evaluate the proposed approach are presented in detail.

Keywords: room acoustics, acoustics simulation.

1. Introduction

Studies of acoustics simulation software have shown rapid growth during last 20 years. The main purpose of this software is to simulate the propagation of sound waves using a computer in order to get an idea about the room acoustics. During the last two decades, several simulation packages have been developed. Some of the most important applications are Odeon (Naylor, 1993), CATT Acoustics (Dalenbäck, 1993), DIVA (www.rhintek.com/cara/, 2003) and CARA (www.bksv.com/, 2003).

The process of software acoustical simulation has three main components; the first part is source conception, which deals with how the sound is emitted. The second part is the modeling method of room, which is the most important part of the sound simulation software and is also important to obtain the correct parameters. The third part is the modeling of the receiver.

In this study, we introduce an approach to calculate the impulse response of room acoustics. The proposed method will be mathematically proven and its correctness will be shown using simulation software. Our approach simply models the room as a linear time-invariant (LTI) system and calculates the impulse response by sending Dirac impulses into the system as input and getting the response as output. Room acoustics simulation software has been developed as well; it is called DAAD. The system provides a user-friendly interface to design rooms in 3D and to locate the receiver and the sound source at desired positions.

The remainder of the paper is organized as follows: Sec. 2 describes the basics of the ray-tracing method for calculating the impulse response of a room using our proposed approach. In Sec. 3, mathematical proof of our approach is presented. Section 4 gives the details of the simulation software developed to evaluate our approach. Section 5 gives the experimental results obtained from our simulation software and the last section, Sec. 6, concludes the paper and suggests an outlook on the future studies concerning this field and subject.

2. Ray-tracing and calculation of impulse response

Impulse response contains almost all acoustical information of a room. In acoustic simulation, calculation of the impulse response is the crucial part, and different methods are used to obtain the impulse response of a room. These methods can be classified into two main groups as shown in Table 1. Wave-based models are characterized very accurate results at single frequencies, thanks to creating very accurate models in relation to the architectural environment, where results in octave bands are usually preferred. Another problem of wave-based methods is applicability to modeling of low-frequency sound for small rooms (Rindel, 2000).

Table 1. Major methods used to calculate the impulse response of a room.

Ray-based methods	Wave-based methods
Ray-Tracing	Finite Element
Image Source	Boundary Element
Hybrid Method	

Another way of simulating sound propagation is based on sound particles traveling through sound rays, and is called the ray-based method. This approach is very successful for modeling high-frequency sound propagation in large rooms with a complicated structure. Basically, two methods exist, both based on sound particles: *Ray Tracing* and *Image Source*. Both methods have a problem that the wavelength or the frequency of the sound is not inherent in the model (Rindel, 2000). However, the pure ray-tracing method can be improved with respect to the number of reflections by directing the rays were directed into circular cones or pyramids; this method is called beam tracing (Arnost, 2000). However, they are still very time-consuming and in many cases the pure ray-tracing method is still the best choice. We have used the pure ray tracing method for this study.

In the ray-tracing method, a fixed number of particles is emitted in various directions with equal angles spherically from a given source point. These particles are traced along the rays and lose their energy at each reflection, according to absorption coefficients of the surfaces. When a particle hits a surface a new direction of ray is calculated according to Snell's laws, as shown in Fig. 1.

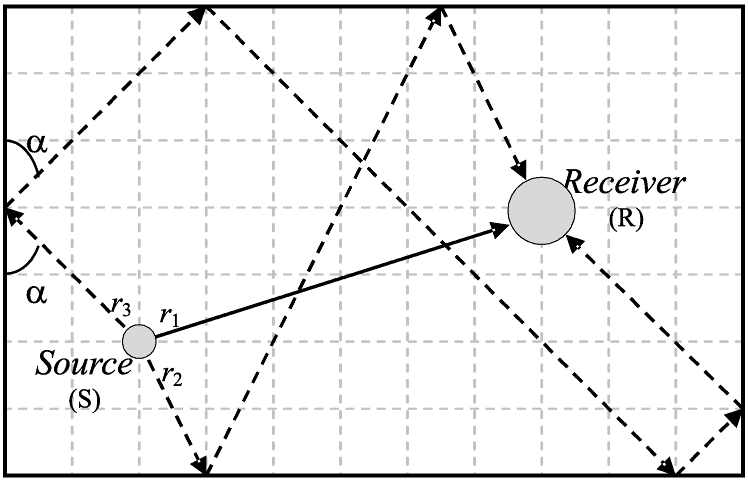

Fig. 1. Tracing three rays emitted from source point in 2D environment.

Choosing the appropriate receiver volume is not a trivial problem in room acoustics and improper receiver sizes may lead to multiple, diminished or faulty reception. Furthermore, it has been proved by Lehnert (1992) that using a fixed-size receiver model produces systematic errors. Up to date, several receiver models have been proposed. In a study, Xiangyang *et al.* recommended a model for a spherical receiver and showed that it outperforms any previous models in this subject (Xiangyang *et al.*, 2003). In this approach, the receiver volume is a function of the number of rays, the distance between the sound source and the receiver and the volume of the room. This method is explained as follows:

$$V_{\text{reciever}} = w \cdot d_{SR} \cdot \sqrt{\frac{4}{N}}, \tag{1}$$

where V_{reciever} is the volume of receiver, d is the distance between the sound and the receiver, N is the number of rays and V_{room} is the volume of the room. The weight factor, w, is defined as follows:

$$w = \log_{10}(V_{\text{room}}). \tag{2}$$

In this study, we assume that the room is a linear time-invariant (LTI) system. The impulse response is calculated simply by sending Dirac impulses into system as input and getting the response from the output. Since any input signal can be represented by a sum of weighted impulses, the output of the system is represented by a sum of time-shifted weighted impulse responses.

It is assumed that rays carry Dirac Impulses when they are emitted from the source. This Dirac Function scales according to the absorption coefficient of the reflecting surfaces and shifted from the source to receiver as time progresses. For a single ray, say ray n, impulse response of the room for an arbitrary receiver location and for the j-th octave band in time t_n is calculated as follows:

$$h_n^j(t) = (-1)^{\psi-1}\left(\prod_{i=S}^{R} g_i^j\right)\cdot h_S\left(t-\sum_{i=S}^{R}\frac{d_i}{v_{\text{sound}}}\right)$$
$$= \begin{cases} 0 & \text{if } t \neq t_n, \\ (-1)^{\psi-1}\left(\prod\limits_{i=S}^{R} g_i^j\right)\cdot \delta\left(t_n-\sum\limits_{i=S}^{R}\frac{d_i}{v_{\text{sound}}}\right) & \text{else,} \end{cases} \tag{3}$$

where t_n is the time instant of the arrival of ray n from the source point S to the receiver R. Index i is used to indicate the i-th segment of ray n's path from the source to the receiver, and ψ represents the total number of path segments of ray n. For a direct ray, traveling directly from source to receiver without any reflections, $\psi = 1$. The total distance from the source to the receiver for ray n is $\sum d_i$ where d_i represents the length of the i-th segment. The impulse response at location S is h_S and the reflection coefficients of the surface where i-th segments reflected for j-th octave band is g_i^j. Meanwhile, for a direct ray, $i = 1$, it is defined as, $g_1^j \equiv 1$. To sum up, the impulse response of the room for the j-th octave band, $h^j(t)$ is represented as h_n^j for $t = t_n$.

Hereafter, this paper will confine the formulation of discrete time-analysis. Hence, $h^j(t)$ will be represented by a set of discrete values: $h^j(t_k)$s. M, which we define as the maximum value of k, equal to or less than N. The reason for that is that various rays may arrive to the receiver in time intervals corresponding to the same t_k value. For such time instances, the Dirac response at the receiver for j-th octave band entry $h_k^j(t = t_k)$ is defined as follows:

$$h_k^j \triangleq h^j(t_k) = \max_{n\in I_k}\left[h_n^j(t_n)\right], \tag{4}$$

where

$$I_k \triangleq \{n|t_k - \Delta/2 < t_n \leq t_k + \Delta/2\}. \tag{5}$$

For each octave band, Eq. (4) is calculated for all t_k. Note that each surface may have different material having different coefficient for each octave bands.

The resulting impulse responses for all eight octave bands, h_k, are superposition of all h_k^j for all octave bands:

$$h_k = h(t_k) = \bigcup_{j=1}^{8} h_k^j. \tag{6}$$

The union operator here represents the resulting effects of all octave bands. The total impulse response of the room at the receiver will be

$$h(t) = \sum_{i=1}^{M} h_i(t).$$

The impulse responses are transformed into the frequency domain and represented as follows:

$$H(\omega) = \mathfrak{F}\{h(t)\} = \mathfrak{F}\left\{\bigcup_{j=1}^{8} h^j(t)\right\}. \tag{7}$$

Finally, we have frequency responses for each octave band. It needs to be summed together as in Eq. (8) in order to get a final impulse response of the room.

$$H(\omega) = \sum_{j=1}^{8} H^j(\omega), \tag{8}$$

where $H^j(\omega)$ is defined as the DFT of $h^j(t)$ which is trimmed (i.e. multiplied by) a filter function for octave bands to clip the unrelated frequencies from respective frequency response: As $\omega_{\min}$ and $\omega_{\max}$ are lower and upper frequency limits of the desired octave band, respectively, the filter function in frequency (ω) domain is

$$B^j(\omega) = \begin{cases} 1 & \text{if } \omega_{\min}^j < \omega < \omega_{\max}^j, \\ 0 & \text{if not.} \end{cases} \tag{9}$$

As a result, the room frequency response, $H(\omega)$ is obtained. Equation (9) is elaborated in the next section.

In order to clarify how our approach works, let us consider a trivial example room as seen in Fig. 1 with a single source and receiver. The north and south walls are covered with a material with a reflection coefficient of 0.6, and east and south walls' reflection coefficients are 0.4 for the 500 Hz octave band. Table 2 depicts the computed values: The distance column shows the total distance each ray has travelled, assuming each grid segment in Fig. 1 represents a distance of one meter. The next column represents the time interval (in seconds) between the departure of a ray from the source and the arrival at the destination. The amplitude column shows the amplitude value of respective rays as they arrive at the receiver.

Table 2. Calculation of IR of the room given in Fig. 1.

Ray n	Distance	Time Elapsed [sec]	Amplitude
r1	**6.32**	0.0186	**1**
r2	2.24+7.83+3.35 = **13.42**	0.0395	1×0.4×0.4 = **0.36**
r3	2.83+4.24+9.90+1.41+4.24=**22.63**	0.0666	1×0.6×0.4×0.6×0.4 = **0.0576**

Figure 2 shows the impulse response of a room given in Fig. 1. In this example, only tree rays are taken into consideration and are subject to simulation. In a more realistic scenario, the typical number of rays should be at least several tens of thousands.

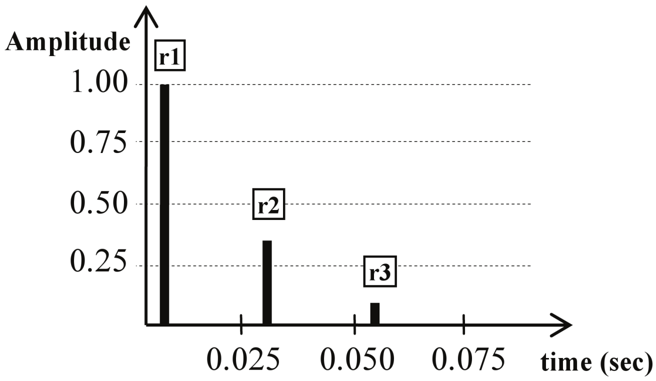

Fig. 2. Impulse response of the example room based on three rays.

3. Mathematical justification of method

First, let us tackle the problem in single step: By definition, the impulse response at the receiver location resulting from the source signal is

$$r(t) = h(t) * s(t) = \int_0^\infty s(\tau)h(t-\tau)\,\mathrm{d}\tau \tag{10}$$

and, by definition, when

$$s(t) = \delta(t). \tag{11}$$

We will compute $h(t)$ numerically using the ray-tracing method. This requires time-domain analysis (numerical) of the room's components (e.g. walls) for different frequencies of octave bands. In general, a room's time-domain response at the receiver location is a superposition of signals coming from the source after several reflections from the walls. Let us assume that N rays arrive to the receiver after such reflections. In this case, the response for an arbitrary time instant is as follows:

$$r(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} c_n \delta(t - d_n), \tag{12}$$

where

$$c_n^j = \prod_{i=1}^{\Psi} g_i^j \tag{13}$$

and its value depends upon the frequency for different octave bands. Hence, by using superscript for indication of the referenced octave band, we can introduce the symbol c_n^j for the j-th octave band. The total distance traveled by ray n is

$$d_n = \frac{\sum\limits_{i=1}^{\Psi} L_{n,i}}{v_{\text{sound}}}, \tag{14}$$

g_i^j is the reflection coefficient of the surface that ray n is reflected by the i-th step of its path. $L_{n,i}$ is the length of the i-th step (segment) in ray n's path. The Fourier transform of $r(t)$, in general, may be given as

$$R(\omega) = \sum_{n=0}^{N} c_n(\omega) e^{-j\omega d_n} = \sum_{n=0}^{N} R_n(\omega), \tag{15}$$

where $j^2 = -1$ and d_n, as mentioned above, is the total length of ray n's path from source to the receiver. For the n-th ray (ray n), the partial impulse response is as follows:

$$\begin{aligned} r_n(t) &= \mathfrak{F}^{-1}\left\{c_n(\omega) e^{-j\omega d_n}\right\} \\ &= \mathfrak{F}^{-1}\left\{c_n^1 e^{-j\omega d_n} B(\omega - \omega_1) + c_n^2 e^{-j\omega d_n} B(\omega - \omega_2) + \ldots\right\}. \end{aligned} \tag{16}$$

Here, B represents the frequency domain filter function, and π represents its inverse Fourier transform; a detailed proof is provided in Appendix A.

In Fig. 3, $H(\omega)$ is depicted using a thick curved line. Center frequencies of each octave-band are shown on the ω axis as (ω_1, ω_2, ω_3, ω_4, ω_5, ω_6, ω_7, ω_8). In each octave band section of $H(\omega)$ is approximated using a constant value. Hence, the actual $H(\omega)$ may be put into a piecewise constant form. The dashed horizontal line segments show these constant values, H_i. With this assumption we may write the following:

$$H(\omega) = \sum_{i=1}^{8} H(\omega) B(\omega - \omega_i) = \sum_{i=1}^{8} H_i(\omega),$$

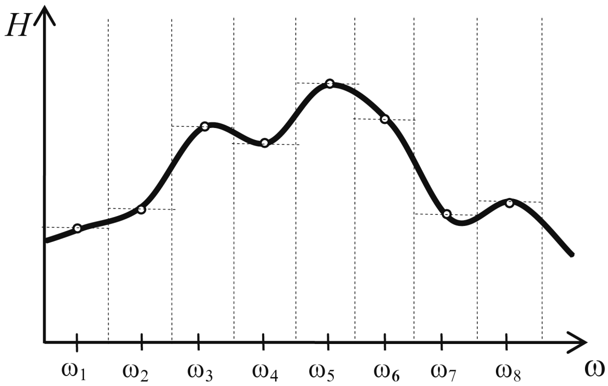

Fig. 3. Transfer function and its segmentations into octave bands.

where B is defined as given in Eq. (9). On the other hand;

$$r(t) = h(t) * \delta(t) \Leftrightarrow R(\omega) = H(\omega) \cdot 1.$$

Therefore;

$$R(\omega) = H_1(\omega) \cdot 1 + H_2(\omega) \cdot 1 + \ldots = H(\omega)B(\omega - \omega_1) \cdot 1 + \ldots$$

or

$$r(t) = \mathfrak{F}^{-1}\left\{H(\omega)B(\omega-\omega_1)\right\} + \ldots$$

$$= \int_0^t h(\tau)\pi(t-\tau)e^{j\omega_1(t-\tau)}\,\mathrm{d}\tau + \ldots$$

$$r(t) = \int_0^t h(\tau)\pi(t-\tau)\sum_{\ell=1}^{M} e^{j\omega_\ell(t-\tau)}\,\mathrm{d}\tau + \ldots . \tag{17}$$

We have evaluated each term on the right-hand side of Eq. (17) as $r_m(t)*B(t)e^{-jwt}$ by taking numerical integration.

4. Application

Figure 4 shows the entire process in a nutshell, including both calculation of the impulse response of a room and auralization. In our approach, to hear the sound of a modeled room, the given sound wave is simply convolved with the impulse response of the room, $h_R(t)$. The resulting sound signal is the sound emitted from the source as heard at the receiver location. The block diagram shown in Fig. 4 may be translated to a software implementation as shown in Algorithm 1.

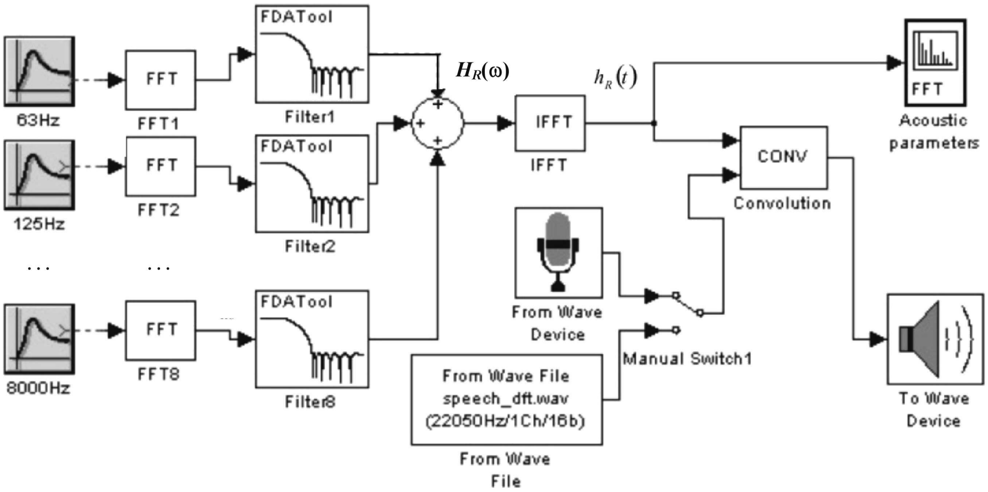

Fig. 4. Block diagram of impulse response of a room and auralization.

Algorithm 1. Pseudocode of calculations shown in Fig. 3.

```
Function Calc_Freq_Response(N)
   for(m=1,8,++)                   //octave band
      for(i=1,N,++)                //i^th ray, total N ray
         calculate c_i             //Equation #13
         calculate d_i             //Equation #14
         calculate C_i
      endfor i
      calculate r_m(t)
      calculate r_m(t)*π(t)e^{-jωt}
      add
   endfor m
Endfunction
```

In order to show how our approach to establishing the impulse response of a room works, simulation software has been developed. The package is called DAAD (Dokuz Eylul Architectural Acoustic Design) (Ozgur *et al.*, 2004). The system determines the characteristics of a room using the ray-tracing method and allows users to listen to the sound affected by room acoustics at the receiver location. The system was developed by using Microsoft Visual C++ 6.0, OpenGL 1.2, and Intel C++ Compiler under the Windows NT platform. DAAD has a very powerful user interface facilitating the design of 3D architecture. The main part of the 3D design subsystem is called the Room Modeling Tool (RMT). RMT provides a complete solution for architectural acoustics design and allows acousticians to easily modify the room object and its materials. It also provides tools to place all the objects (such as walls, surfaces, etc.) and easily set the absorption coefficients. For this reason, RMT is an integral part of DAAD and it is more flexible than many other 3D drawing applications. As shown in Fig. 5, it has some outstanding features which are unique to RMT, such as opening more than one room at once and viewing the room from many cameras simultaneously. RMT can also import DXF files, which makes it possible to import existing architectural drawings. The DAAD system is capable of calculating many of the major room acoustic parameters defined in ISO 3382. Moreover, it also accepts an aechoic sound file as a parameter and lets the user listen to the sound deformed by the respective room according to the calculated acoustical parameters (Ozgur, 2000).

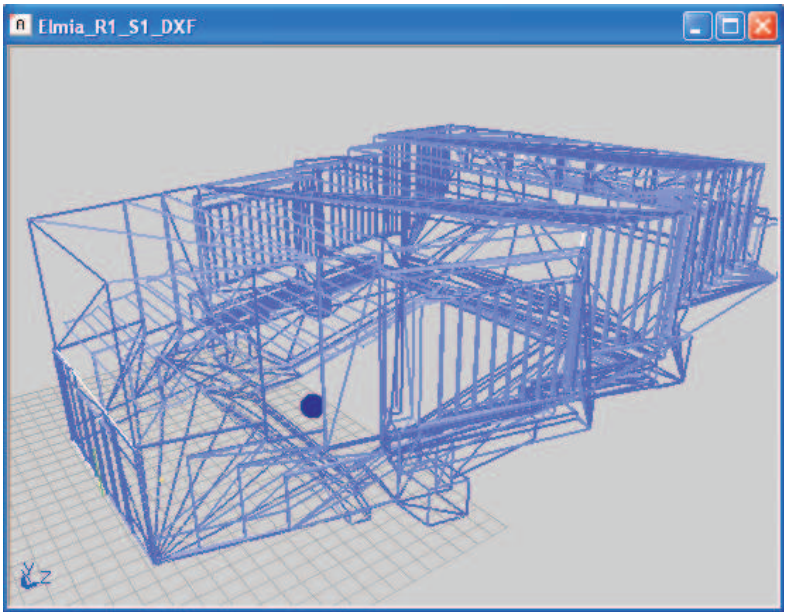

Fig. 5. Wire-frame representation the Elmia Hall.

Rindel states that "it can be extremely useful for the acousticians to see a mapping of spatial distribution of acoustical weak spots can be easily localized and appropriate countermeasures can be taken" (Rindel, 2000). Beyond this idea, it is also useful to see a volumetric display of a room with respect to some calculated acoustical parameters. This helps us realize a walk-through in room, or even a fly-through for potential applications of acoustic simulations.

In this receiver-less model, we propose a model where no specific receiver location is required. This is because only a small modification on the single receiver model provides us with means to render the acoustical parameters for the entire volume. This is easily done by dividing the volume into equally-sized cubes bounded by a spherical receiver located in the center of the each cube.

Computationally, the most costly fraction of the ray-tracing model with a single receiver is the part given in Eq. (2). However, in this equation, number of receiver is not a factor of $h(t)$. It means that all $h^j(t)$ values for all receivers/cubes the ray passes through are simply calculated with using a voxelization algorithm, while $h^j(t)$ is calculated for a single ray. Moreover, this modification – filling the volume with identical receivers – does not add too much computational overhead. In other words, the computational cost is almost the same as for the single receiver model. This model leads many open research problems such as determination of the optimum grid size, or calculation of the unit receiver size (Sarigül, 2003).

5. Performance evaluation and comparison

The performance of our approach for the simulation of room acoustics was tested with the scope of Round Robin II (web page, 2010), with 16 participants from 9 different countries. The aim of the contest is to compare and prove their computer programs' efficiency on a specific object. The participants had to calculate the acoustic properties of the ELMIA concert hall in Jönköping, Sweden, and compare them with independent measurement results. The Round Robin contest allowed the manufacturers of software packages to check and improve their products using verified measurements. Moreover, the users of software for the simulation of room acoustics, i.e. acousticians, architects and civil engineers, were given the opportunity to carry out a practical test of the software they use. Table 3 shows the results of DAAD simulation software and the average of RR2 with respect to T30 and EDT values (ISO 3382, 2003), respectively. The row labeled "Measured" indicates the mean value of three physical measurements conducted by three different teams. The rows labeled RR2 indicate the average values for six octave bands; the standard deviation of each value has been shown with symbol of ±. Originally the the RR2 contest was held for six receiver positions for two given source. Here, we included only the S1R1 values, which represent the fist receiver position for the first sound source.

Table 3. DAAD results comparison with Round Robin II.

		Hz					
		125	250	500	1000	2000	4000
T30/s	Measured	2.22	2.30	2.02	2.02	2.00	1.78
	RR2	1.63±0.3	1.92±0.4	2.09±0.5	2.11±0.4	1.99±0.3	1.67±0.3
	DAAD	1.51	1.55	1.58	1.88	1.70	1.68
EDT/s	Measured	1.99	1.95	1.92	2.08	1.99	1.95
	RR2	1.57±0.4	1.83±0.4	2.00±0.4	2.00±0.4	1.86±0.4	1.56±0.4
	DAAD	1.48	1.55	1.58	1.69	1.60	1.55

The simulation results show that the DAAD results always stay within the range of RR2. In the current release, the application does not include the diffraction and scattering effect yet. The readers who want to get the complete set of results and to listen to the sample sound files with acoustic distortion may refer to the web page at http://aad.cs.deu.edu.tr/.

6. Conclusion

In this study, we have presented an approach to calculate the impulse response of a room using ray-tracing. In our approach, we assumed the room is a linear time-invariant system and the impulse response is calculated simply by sending Dirac impulses into the system as input and the getting the response from the output. Since any of the input signals can be represented by a sum of weighted impulses, the output of the system is represented by a sum of time-shifted weighted impulse responses. Both mathematical justifications for the proposed method and the results obtained using the developed simulation software are presented in detail.

To evaluate the approach, we used the DAAD architectural acoustic software for exact calculation of the impulse response, aimed at designing an amendable acoustic simulation program using basic modeling methods. DAAD is capable of calculating acoustical parameters given by ISO based on our approach as well as 3D modeling and visualization of the building. In DAAD, the user can hear wave signals emitted from arbitrarily selected source points as transformed by room acoustics. Moreover, DAAD is also capable of obtaining impulse responses for the entire space by dividing the space into small cubes. This new method also helps to visualize the entire space with respect to any acoustic parameters. Consequently, the application program has the main characteristics of an acoustic simulation program and makes it possible to calculate parameters by placing receivers and sources, or dividing the space into cubes.

The results obtained from simulation experimentation using DAAD has been compared to some benchmarks of well-known concert halls such as ELMIA, and has been found very close to real measurements. In longer term, we expect the DAAD project to lead us into new research in many dimensions, including scattering and diffraction effects, new visualization techniques and applications of acoustics.

Appendix A.
Details of mathematical justification

Detail mathematical analysis of Eq. (16) is provided in this appendix in order to give a more satisfactory justification of the method. Below, B shows the requency domains filter function, and π shows inverse Fourier transform of it. $\mathfrak{F}$ and $\mathfrak{F}^{-1}$ denotes Fourier and inverse Fourier transformation operations, respectively. The Dirac impulse response for the i-th ray can be written as multiplication in

ω domain for each octave band, and the result is the sum of all octave bands:

$$r_i(t) = \mathfrak{F}^{-1}\left\{c_i^1 e^{-j\omega d_i} B(\omega-\omega_1)\right\}\left\{+\mathfrak{F}^{-1}\left\{c_i^2 e^{-j\omega d_i} B(\omega-\omega_2)\right\}+\ldots\right\}.$$

By converting the multiplication operation in the ω domain to the convolution operation in time domain, we can reformulate as follows:

$$r_i(t) = \mathfrak{F}^{-1}\left\{c_i^1 e^{-j\omega d_i}\right\} * \mathfrak{F}^{-1}\{B(\omega-\omega_1)\} + \mathfrak{F}^{-1}\left\{c_i^2 e^{-j\omega d_i}\right\} * \mathfrak{F}^{-1}\{B(\omega-\omega_2)\}+\ldots$$

By using filter functions of the Dirac response we can write:

$$r_i(t) = \mathfrak{F}^{-1}\left\{c_i^1 e^{-j\omega d_i}\right\} * \pi(t)e^{-j\omega_1 t} + \mathfrak{F}^{-1}\left\{c_i^2 e^{-j\omega d_i}\right\} * \pi(t)e^{-j\omega_2 t}+\ldots,$$

$$r_i(t) = c_i^1\delta(t-d_i) * \pi(t)e^{-j\omega_1 t} + c_i^2\delta(t-d_i) * \pi(t)e^{-j\omega_2 t}+\ldots,$$

$$r_i(t) = \int_0^t c_i^1\delta(t-\tau-d_i)\pi(\tau)e^{-j\omega_1\tau}\,\mathrm{d}\tau + \int_0^t c_i^2\delta(t-\tau-d_i)\pi(\tau)e^{-j\omega_2\tau}\,\mathrm{d}\tau+\ldots,$$

$$r_i(t) = c_i^1\pi(t-d_i)e^{-j\omega_1(t-d_i)} + c_i^2\pi(t-d_i)e^{-j\omega_2(t-d_i)}+\ldots\,.$$

By taking summing over n;

$$r(t) = \sum_{i=0}^{I} r_i(t) = e^{-j\omega_1 t}\sum_{i=0}^{I} c_i^1\pi(t-d_i)e^{j\omega_1 d_i} + e^{-j\omega_2 t}\sum_{i=0}^{I} c_i^2\pi(t-d_i)e^{j\omega_2 d_i}+\ldots,$$

$$r(t) = \mathfrak{F}^{-1}\left\{\mathfrak{F}\left\{e^{-j\omega_1 t}\sum_{i=0}^{I} c_i^1\pi(t-d_i)e^{j\omega_1 d_i}\right\} + \mathfrak{F}\left\{e^{-j\omega_2 t}\sum_{i=0}^{I} c_i^2\pi(t-d_i)e^{j\omega_2 d_i}\right\}+\ldots\right\},$$

$$r(t) = \mathfrak{F}^{-1}\left\{\left\{\sum_{i=0}^{I}\mathfrak{F}\left\{c_i^1\pi(t-d_i)e^{j\omega_1 d_i}\right\}_{\omega\to\omega-\omega_1}\right\} + \left\{\sum_{i=0}^{I}\mathfrak{F}\left\{c_i^2\pi(t-d_i)e^{j\omega_2 d_i}\right\}_{\omega\to\omega-\omega_2}\right\}+\ldots\right\},$$

$$r(t) = \mathfrak{F}^{-1}\left\{\left\{\sum_{i=0}^{I} e^{j\omega_1 d_i} c_i^1 \mathfrak{F}\left\{\pi(t-d_i)\right\}_{\omega\to\omega-\omega_1}\right\} + \left\{\sum_{i=0}^{I} e^{j\omega_2 d_i} c_i^2 \mathfrak{F}\left\{\pi(t-d_i)\right\}_{\omega\to\omega-\omega_2}\right\} + \ldots\right\},$$

$$r(t) = \mathfrak{F}^{-1}\left\{\left\{\sum_{i=0}^{I} e^{j\omega_1 d_i} e^{-j\omega d_i} c_i^1 B\left(\omega-\omega_1\right)|\right\} + \left\{\sum_{i=0}^{I} e^{j\omega_2 d_i} e^{-j\omega d_i} c_i^1 B\left(\omega-\omega_2\right)|\right\} + \ldots\right\},$$

$$r(t) = \mathfrak{F}^{-1}\left\{\left\{\sum_{i=0}^{I} e^{-j\omega d_i} c_i^1 B\left(\omega\right)|_{\omega\to\omega-\omega_1}\right\} + \left\{\sum_{i=0}^{I} e^{-j\omega d_i} c_i^2 B\left(\omega\right)|_{\omega\to\omega-\omega_2}\right\} + \ldots\right\},$$

$$r(t) = \mathfrak{F}^{-1}\left\{\left\{B(\omega-\omega_1)\sum_{i=0}^{I} e^{j\omega_1 d_i} e^{-j\omega d_i} c_i^1\right\} + \left\{B(\omega-\omega_2)\sum_{i=0}^{I} e^{j\omega_2 d_i} e^{-j\omega d_i} c_i^2\right\} + \ldots\right\},$$

$$r(t) = \left\{\left\{\left(e^{-j\omega_1 t}\pi(t)\right) * \mathfrak{F}^{-1}\left\{\sum_{i=0}^{I} e^{j\omega_1 d_i} e^{-j\omega d_i} c_i^1\right\}\right\} + \left\{\left(e^{-j\omega_2 t}\pi(t)\right) * \mathfrak{F}^{-1}\left\{\sum_{i=0}^{I} e^{j\omega_2 d_i} e^{-j\omega d_i} c_i^2\right\}\right\} + \ldots\right\},$$

$$r(t) = \left\{\left\{\left(e^{-j\omega_1 t}\pi(t)\right) * \mathfrak{F}^{-1}\left\{\sum_{i=0}^{I}\left(c_i^1 e^{j\omega_1 d_i}\right) e^{-j\omega d_i}\right\}\right\} + \left\{\left\{\left(e^{-j\omega_2 t}\pi(t)\right) * \mathfrak{F}^{-1}\left\{\sum_{i=0}^{I}\left(c_i^2 e^{j\omega_1 d_i}\right) e^{-j\omega d_i}\right\}\right\}\right\} + \ldots\right\},$$

$$r(t) = \left(e^{-j\omega_1 t}\pi(t)\right) * \left\{\sum_{i=0}^{I} C_i^1(\omega_1)\delta(t-d_i)\right\} + \left(e^{-j\omega_2 t}\pi(t)\right) * \left\{\sum_{i=0}^{I} C_i^2(\omega_2)\delta(t-d_i)\right\} + \ldots,$$

where

$$C_i^1(\omega_1) = c_i^1 e^{j\omega_1 d_i} = c_i^1 \left(\cos(\omega_1 d_i) + j\sin(\omega_1 d_i)\right),$$

$$C_i^2(\omega_2) = c_i^2 e^{j\omega_2 d_i} = c_i^2 \left(\cos(\omega_2 d_i) + j\sin(\omega_2 d_i)\right),$$

$$\vdots$$

This concludes the mathematical justification of Eq. (16).

References

1. Arnost V. (2000), *Discrete Simulation of Sound Wave Propagation*, Proceedings of 34th Spring International Conference on Modeling and Simulation of Systems (MOSIS 2000), p. 241–246, ISBN 80-85988-44-5.
2. CARA (2003), Web Site, http://www.bksv.com/.
3. Dalenbäck B.-I. (1993), *CATT-Acoustic: Image source modeling augmented by ray tracing and diffuse reflections*, Applied Acoustics, **38**, 2–4, 350.
4. DIVA (2003), Web Site, http://www.rhintek.com/cara/.
5. ISO 3382 (2003), *Measurement of the reverberation time of rooms with reference to other acoustical parameters*, 2003.
6. Lehnert H. (1992), *Systematic Errors of the raytracing algorithm*, Algorithm Acoustics, **38**, 2–4, 207–221.
7. Naylor G.M. (1993), *ODEON – Another hybrid room acoustical model*, Applied Acoustics, **38**, 2–4, 131–143.
8. Ozgur E., Ozis F., Alpkocak A. (2004), *DAAD: A New Software for Architectural Acoustics Design*, Proceedings of the 33rd International Congress and Exposition on Noise Control Engineering (INTERNOISE'2004), Chech Rebublic.
9. Ozgur E. (2000), *Design and Development of an Architectural Acoustic Design Software*, M.Sc. Thesis, Dokuz Eylül University, Department of Computer Science and Engineering.
10. Rindel J.H. (2000), *The Use of Computer Modeling in Room Acoustics*, Journal of Vibroengineering, **3**, 4.
11. Round Robin-II (2010), Web Site, http://www.ptb.de/en/org/1/17/173/roundrob2_1.htm.
12. Sarigül Y. (2003), *Sound Modeling for Room Acoustic*, M.Sc. Thesis, Dokuz Eylül University, Department of Computer Science and Engineering.
13. Xiangyang Z., Ke'an C., Jincai S. (2003), *On the accuracy of the ray-tracing algorithms based on various sound receiver models*, Applied Acoustics, **64**, 433–441.