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QUATERNION-BASED FILTERING FOR GYROLESS ATTITUDE ESTIMATION WITHOUT AN ATTITUDE DYNAMICS MODEL

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Abstract

Conventionally, the filtering technique for attitude estimation is performed using gyros or attitude dynamics models. In order to extend the application range of an attitude filter, this paper proposes a quaternion-based filtering framework for gyroless attitude estimation without an attitude dynamics model. The attitude estimation system is established based on a quaternion kinematic equation and vector observation models. The angular velocity in the system is determined through observation vectors from attitude sensors and the statistical properties of the angular velocity error are analysed. A Kalman filter is applied to estimate the attitude error such that the effect from the angular velocity error is compensated with its statistical properties at each sampling moment. A numerical simulation example is presented to illustrate the performance of the proposed algorithm.

Keywords: quaternion-based filtering, gyroless attitude estimation, angular velocity determination, Kalman filter.

1. Introduction

Attitude estimation is an important issue involved in many practical systems such as spacecraft, aerial vehicles, and robotic systems [1, 2]. With an appropriate attitude estimation algorithm, the orientation of a physical system in respect to a reference frame can be determined. For example, an attitude estimation algorithm is implemented on a star tracker to measure the attitude of a remote sensing satellite [3]. The high-precision attitude information is obtained by detecting small star spots in the *field of view* (FOV) of the star tracker [4]. The mathematical representation of attitude is diverse. One attitude representation that has been proven very useful is the attitude quaternion [5], which contains the minimal number of parameters necessary for the globally defined attitude and is recognized as the most valuable parameterization due to its computational efficiency and linear state propagation nature of the kinematic equation [6].

Numerous algorithms for determining the attitude quaternion of a physical system have been developed over the past few decades, following two main techniques that the closed-form algorithms determine the optimal attitude by solving the Wahba's problem [7, 8] and the Kalman

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filtering methods obtain the sequential quaternion estimate by minimizing the estimate error covariance [9, 10]. In the filtering methods for attitude quaternion estimation, the norm constraint of a quaternion should be tackled appropriately because the Kalman filter does not have the property of preserving the norm of the estimate result, and accordingly many filtering algorithms are proposed to overcome this difficulty, among which the multiplicative extended Kalman filter and unscented quaternion estimator are the most well-known representatives.

Filtering methods for attitude estimation are achieved based on the quaternion kinematic equation in which the knowledge of angular velocity is required for attitude propagation [11, 12]. A gyro is the preferred sensor to measure angular velocity, and in this regard, the attitude estimation with gyros for angular velocity information and attitude sensors for vector observations are extensively studied in the literature [13, 14]. A realistic model of rate integrating gyro is usually applied to construct the process equation, where the system state vector is augmented to include the random gyro drift. Although the gyro error is compensated by estimated gyro drift, a rate integrating gyro has an inevitable tendency to degrade or even fail due to its mechanical and electrical effects. For example, EAS Remote Sensing Satellite of European Space Agency had an orbital rescue because of gyro failure [15]. In addition, some small spacecraft may not carry gyros because of their high price and high power consumption, yet they need to determine their orientation for attitude control [16]. For these reasons, the gyroless attitude estimation has become an important research direction in recent years [17–19]. The basic idea of gyroless attitude determination is to estimate angular velocity based on attitude dynamics of a rigid body and correct attitude estimate error using measurements from attitude sensors. However, an attitude dynamics model may contain parameter uncertainties especially for complex spacecraft, due to difficulties in accurate modelling, fuel consumption, spacecraft docking and external disturbances, eventually leading to performance deterioration or instability of attitude estimation [20–22]. A simple approach is able to avoid parameter uncertainties in the dynamics model where the derivative of the angular velocity is regarded as white noise without solid knowledge about dynamics [23], but its estimation precision may be affected by variable angular velocity. Therefore, in order to improve the performance of a filtering algorithm in the aforementioned situation, we have developed a new method for attitude estimation without using a gyro or a dynamics model, which, to the best our knowledge, has not been intensively studied in the literature.

Based on the above discussion, in this paper, we investigate quaternion-based filtering for gyroless attitude estimation without an attitude dynamics model. The mathematical model of the attitude estimation system contains the attitude kinematic equation in terms of the attitude quaternion and vector observation models of attitude sensors. The angular velocity is determined dependent merely on the information from observation vectors. The attitude error is recursively estimated with a Kalman filter, whereas the angular velocity error is compensated by its covariance. The main contributions of this paper are as follows: 1) the addressed filtering problem is new, with both theoretical importance and practical significance; 2) only with observation vectors from attitude sensors, the angular velocity in the attitude kinematic equation is determined and the statistical properties of the angular velocity error are analysed for attitude estimation; 3) the simulation results illustrate the superiority of the proposed method.

2. Gyroless attitude estimation system

2.1. Kinematic equation of attitude error

Before establishing the kinematic equation studied in this work, the attitude quaternion is presented for convenient explanation of the system model and attitude estimation. A quaternion is defined as:

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{q}_v \\ q_4 \end{bmatrix}, \tag{1}$$

with

$$\boldsymbol{q}_v = [q_1 \;\; q_2 \;\; q_3]^{\mathrm{T}}. \tag{2}$$

In order to parameterize the attitude, the quaternion is required to satisfy the 2-norm constraint $\|\boldsymbol{q}\| = 1$. Given the unit quaternion $\boldsymbol{q}$, the corresponding attitude matrix is obtained according to the relationship:

$$\boldsymbol{A}(\boldsymbol{q}) = \left(q_4^2 - \|\mathbf{q}_v\|^2\right)\boldsymbol{I}_3 + 2\boldsymbol{q}_v\boldsymbol{q}_v^{\mathrm{T}} - 2q_4[\boldsymbol{q}_v\times], \tag{3}$$

where $\boldsymbol{I}_3$ represents a 3×3 identity matrix and the cross-product matrix $[\boldsymbol{q}_v\times]$ is defined as:

$$[\boldsymbol{q}_v\times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}. \tag{4}$$

With the quaternion representation, the attitude kinematic model can be expressed by the following differential equation:

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})\boldsymbol{q}, \tag{5}$$

with

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -[\boldsymbol{\omega}\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^{\mathrm{T}} & 0 \end{bmatrix}, \tag{6}$$

where $\boldsymbol{\omega}$ is the angular velocity. And, correspondingly, the attitude matrix $\boldsymbol{A}(\boldsymbol{q})$ satisfies:

$$\dot{\boldsymbol{A}}(\boldsymbol{q}) = -[\boldsymbol{\omega}\times]\boldsymbol{A}(\boldsymbol{q}). \tag{7}$$

The discrete-time kinematic model of the attitude quaternion is given by [13]:

$$\boldsymbol{q}_{k+1} = \overline{\boldsymbol{\Omega}}(\boldsymbol{\omega}_k)\boldsymbol{q}_k, \tag{8}$$

with

$$\overline{\boldsymbol{\Omega}}(\boldsymbol{\omega}_k) = \begin{cases} \cos\left(\frac{1}{2}\|\boldsymbol{\omega}_k\|T\right)\boldsymbol{I}_4 + \begin{bmatrix} -[\boldsymbol{\Psi}_k\times] & \boldsymbol{\Psi}_k \\ -\boldsymbol{\Psi}_k^{\mathrm{T}} & 0 \end{bmatrix} & \|\boldsymbol{\omega}_k\| \neq 0 \\ \boldsymbol{I}_4 & \|\boldsymbol{\omega}_k\| = 0 \end{cases}, \tag{9}$$

where $\boldsymbol{\Psi}_k = \sin(\|\boldsymbol{\omega}_k\|T/2)\,\boldsymbol{\omega}_k/\|\boldsymbol{\omega}_k\|$ and T is the sampling period.

A straightforward way for attitude estimation is to regard the quaternion $\boldsymbol{q}$ as the estimated system state in the algorithm development, while it is not appropriate because of ignoring the unit norm constraint of attitude quaternion. The multiplicative quaternion error is utilized to tackle this problem, in which the constraint is kept to be within first-order [9]. For this reason, the attitude quaternion error is defined as:

$$\delta\boldsymbol{q} = \boldsymbol{q}\otimes\hat{\boldsymbol{q}}^{-1}, \tag{10}$$

where $\hat{\boldsymbol{q}}$ is the quaternion estimate and $\otimes$ indicates quaternion multiplication. Then the differential equation of attitude quaternion error can be yielded as follows based on (5), (6) and (10):

$$\delta\dot{\boldsymbol{q}} = \frac{1}{2}\left\{\begin{bmatrix}\hat{\boldsymbol{\omega}}\\0\end{bmatrix}\otimes\delta\boldsymbol{q} - \delta\boldsymbol{q}\otimes\begin{bmatrix}\hat{\boldsymbol{\omega}}\\0\end{bmatrix}\right\} + \frac{1}{2}\begin{bmatrix}\delta\boldsymbol{\omega}\\0\end{bmatrix}\otimes\delta\boldsymbol{q}, \tag{11}$$

with the angular velocity error $\delta\boldsymbol{\omega}$ defined as:

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \hat{\boldsymbol{\omega}}. \tag{12}$$

Using small angle approximation [24], the attitude quaternion error $\delta\boldsymbol{q}$ can be represented as:

$$\delta\boldsymbol{q} = [\phi/2 \;\; \theta/2 \;\; \psi/2 \;\; 1]^{\mathrm{T}}, \tag{13}$$

where ϕ, θ and ψ are the errors in roll, pitch and yaw angles, respectively. Substituting (13) into (11) and dropping the small second-order quantities, the continuous-time process equation of the gyroless attitude estimation system is obtained as:

$$\dot{\boldsymbol{x}} = -[\hat{\boldsymbol{\omega}}\times]\boldsymbol{x} + \delta\boldsymbol{\omega}, \tag{14}$$

where $\boldsymbol{x} = [\phi \;\; \theta \;\; \psi]^{\mathrm{T}}$ is the attitude error regarded as the system state.

In application, it is advantageous to operate in discrete time and the following discrete-time process equation is given based on the discretization of (14):

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_k\boldsymbol{x}_k + \boldsymbol{\Gamma}_k\delta\boldsymbol{\omega}_k\,, \tag{15}$$

where:

$$\begin{aligned}\boldsymbol{F}_k &= \exp\{-[\hat{\boldsymbol{\omega}}_k\times]T\}\\ &= \boldsymbol{I}_3 - \frac{[\hat{\boldsymbol{\omega}}_k\times]}{\|\hat{\boldsymbol{\omega}}_k\|}\sin(\|\hat{\boldsymbol{\omega}}_k\|T) + \frac{[\hat{\boldsymbol{\omega}}_k\times]^2}{\|\hat{\boldsymbol{\omega}}_k\|^2}[1-\cos(\|\hat{\boldsymbol{\omega}}_k\|T)],\end{aligned} \tag{16}$$

$$\begin{aligned}\boldsymbol{\Gamma}_k &= \int_0^T \exp\{-[\hat{\boldsymbol{\omega}}_k\times]t\}\,dt\\ &= T\boldsymbol{I}_3 - \frac{[\hat{\boldsymbol{\omega}}_k\times]}{\|\hat{\boldsymbol{\omega}}_k\|^2}[1-\cos(\|\hat{\boldsymbol{\omega}}_k\|T)] + \frac{[\hat{\boldsymbol{\omega}}_k\times]^2}{\|\hat{\boldsymbol{\omega}}_k\|^2}\left[T - \frac{\sin(\|\hat{\boldsymbol{\omega}}_k\|T)}{\|\hat{\boldsymbol{\omega}}_k\|}\right].\end{aligned} \tag{17}$$

And, if $\|\hat{\boldsymbol{\omega}}_k\| = 0$, then $\boldsymbol{F}_k = \boldsymbol{I}_3$ and $\boldsymbol{\Gamma}_k = T\boldsymbol{I}_3$.

2.2. Measurement model

The attitude sensors considered here are any sensors whose measured quantities depend on the directions of some objects in the body frame, such as a star tracker, sun sensor or lunar sensor.

Let there be N observation vectors obtained from attitude sensors at a sampling moment k. Assuming that the observation vectors in the body frame are related to the corresponding vectors in the inertial reference frame with an attitude matrix $\boldsymbol{A}(\boldsymbol{q}_k)$, the vector observation model is given by:

$$\widetilde{\boldsymbol{b}}_{i,k} = \boldsymbol{A}(\boldsymbol{q}_k)\boldsymbol{r}_i + \boldsymbol{v}_{i,k}\,, \quad i = 1,\ldots,N, \tag{18}$$

where $\widetilde{\boldsymbol{b}}_{i,k}$ is the ith observation vector in the body frame, $\boldsymbol{r}_i$ is the ith reference vector in the inertial frame, $\boldsymbol{v}_{i,k}$ is the measurement noise. Usually, the measurement noise of an attitude sensor is white noise. For example, the measurement noise of a star tracker is caused by the centroid extraction error which is uncorrelated at different sampling moments. Therefore, we assume that $\boldsymbol{v}_{i,k}$ is white noise satisfying $\mathrm{E}\{\boldsymbol{v}_{i,k}\} = 0$ and $\mathrm{E}\{\boldsymbol{v}_{i,k}\boldsymbol{v}_{i,k}^{\mathrm{T}}\} = \sigma_i^2 \boldsymbol{I}_3$. Moreover, the observation vectors are obtained either from different stars in the FOV of a star tracker or from other attitude sensors at the sampling moment k, and each process of acquiring the observation vectors is independent. For this reason, the measurement noise also satisfies $\mathrm{E}\{\boldsymbol{v}_{i,k}\boldsymbol{v}_{j,k}^{\mathrm{T}}\} = 0$, $i \neq j$.

Based on the definition of the attitude quaternion error and the small angle approximation, the vector observation model (18) can be rewritten as:

$$\begin{aligned}
\widetilde{\boldsymbol{b}}_{i,k} &= \boldsymbol{A}\left(\delta \boldsymbol{q}_k \otimes \hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_i + \boldsymbol{v}_{i,k} \\
&= \boldsymbol{A}\left(\delta \boldsymbol{q}_k\right)\boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_i + \boldsymbol{v}_{i,k} \\
&\approx \left(\boldsymbol{I}_3 - [\boldsymbol{x}_k \times]\right)\boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_i + \boldsymbol{v}_{i,k} \\
&= \boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_i + \left[\boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_i \times\right]\boldsymbol{x}_k + \boldsymbol{v}_{i,k}\,,
\end{aligned} \tag{19}$$

where $\boldsymbol{x}_k$ is the system state in the process equation (15) and $\hat{\boldsymbol{q}}_{k|k-1}$ is the one-step prediction of the attitude quaternion at the sampling moment $k-1$.

For N observation vectors, the measurement model can be given by:

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k\,, \tag{20}$$

where:

$$\boldsymbol{y}_k = \begin{bmatrix} \widetilde{\boldsymbol{b}}_{1,k} - \boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_1 \\ \widetilde{\boldsymbol{b}}_{2,k} - \boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_2 \\ \vdots \\ \widetilde{\boldsymbol{b}}_{N,k} - \boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_N \end{bmatrix}, \qquad \boldsymbol{H}_k = \begin{bmatrix} \left[\boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_1 \times\right] \\ \left[\boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_2 \times\right] \\ \vdots \\ \left[\boldsymbol{A}\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_N \times\right] \end{bmatrix} \tag{21}$$

and $\boldsymbol{v}_k = \begin{bmatrix} \boldsymbol{v}_{1,k}^{\mathrm{T}} & \boldsymbol{v}_{2,k}^{\mathrm{T}} & \cdots & \boldsymbol{v}_{N,k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ is white noise with:

$$\boldsymbol{R} = \mathrm{E}\left\{\boldsymbol{v}_k \boldsymbol{v}_k^{\mathrm{T}}\right\} = diag\left(\sigma_1^2 \boldsymbol{I}_3,\ \sigma_2^2 \boldsymbol{I}_3,\ \ldots,\ \sigma_N^2 \boldsymbol{I}_3\right). \tag{22}$$

Remark 1. From (15) and (20), the process and measurement equations of the gyroless attitude estimation system are established. Unlike existing methods of gyroless attitude estimation, only the kinematic equation of attitude error is regarded as the process equation, which eliminates the dependency on an attitude dynamics model. On the other hand, the angular velocity plays an important role in the attitude propagation but has not been determined. It can be seen later that the angular velocity is calculated based on observation vectors.

3. Quaternion-based filtering for attitude estimation

3.1. Angular velocity determination

The least-squares approach is applied to determine the angular velocity in the kinematic (15) based on observation vectors from attitude sensors.

According to (18), the ideal observation vector can be written as:

$$\boldsymbol{b}_{i,k} = \widetilde{\boldsymbol{b}}_{i,k} - \boldsymbol{v}_{i,k} = \boldsymbol{A}(\boldsymbol{q}_k)\boldsymbol{r}_i \tag{23}$$

and the derivative of $\boldsymbol{b}_{i,k}$ is calculated using (7).

$$\dot{\boldsymbol{b}}_{i,k} = \dot{\boldsymbol{A}}(\boldsymbol{q}_k)\boldsymbol{r}_i = -[\boldsymbol{\omega}_k\times]\boldsymbol{A}(\boldsymbol{q}_k)\boldsymbol{r}_i = -[\boldsymbol{\omega}_k\times]\boldsymbol{b}_{i,k}\,. \tag{24}$$

On the other hand, the following equations are given by the Taylor expansion:

$$\boldsymbol{b}_{i,k+1} = \boldsymbol{b}_{i,k} + T\,\dot{\boldsymbol{b}}_{i,k} + \frac{T^2}{2}\ddot{\boldsymbol{b}}_{i,k} + \tau_1\,, \tag{25}$$

$$\boldsymbol{b}_{i,k-1} = \boldsymbol{b}_{i,k} - T\,\dot{\boldsymbol{b}}_{i,k} + \frac{T^2}{2}\ddot{\boldsymbol{b}}_{i,k} + \tau_2\,, \tag{26}$$

where τ_1 and τ_2 are higher-order terms. From (25) and (26), $\dot{\boldsymbol{b}}_{i,k}$ can be yielded as:

$$\begin{aligned}\dot{\boldsymbol{b}}_{i,k} &= \frac{1}{2T}\left(\boldsymbol{b}_{i,k+1} - \boldsymbol{b}_{i,k-1} + \tau_2 - \tau_1\right)\\ &\approx \frac{1}{2T}\left(\boldsymbol{b}_{i,k+1} - \boldsymbol{b}_{i,k-1}\right).\end{aligned} \tag{27}$$

It should be pointed out that (27) is the second-order approximation of $\dot{\boldsymbol{b}}_{i,k}$.
Substituting (27) into (24), we have:

$$\frac{1}{2T}\left(\boldsymbol{b}_{i,k+1} - \boldsymbol{b}_{i,k-1}\right) = -\left[\boldsymbol{\omega}_k\times\right]\boldsymbol{b}_{i,k}\,. \tag{28}$$

According to (23) and (28), the measurement equation of the angular velocity is obtained as:

$$\frac{1}{2T}\left(\widetilde{\boldsymbol{b}}_{i,k+1} - \widetilde{\boldsymbol{b}}_{i,k-1}\right) = \left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]\boldsymbol{\omega}_k + \boldsymbol{w}_{i,k}\,, \tag{29}$$

where:

$$\boldsymbol{w}_{i,k} = [\boldsymbol{\omega}_k\times]\,\boldsymbol{v}_{i,k} + \frac{1}{2T}\left(\boldsymbol{v}_{i,k+1} - \boldsymbol{v}_{i,k-1}\right) \tag{30}$$

and the covariance matrix of $\boldsymbol{w}_{i,k}$ is calculated as:

$$\begin{aligned}\boldsymbol{Q}_{k,k}^{(i)} &= \mathrm{E}\left\{\boldsymbol{w}_{i,k}\boldsymbol{w}_{i,k}^{\mathrm{T}}\right\}\\ &= [\boldsymbol{\omega}_k\times]\,\mathrm{E}\left\{\boldsymbol{v}_{i,k}\boldsymbol{v}_{i,k}^{\mathrm{T}}\right\}[\boldsymbol{\omega}_k\times]^{\mathrm{T}} + \frac{1}{4T^2}\mathrm{E}\left\{\boldsymbol{v}_{i,k+1}\boldsymbol{v}_{i,k+1}^{\mathrm{T}}\right\} + \frac{1}{4T^2}\mathrm{E}\left\{\boldsymbol{v}_{i,k-1}\boldsymbol{v}_{i,k-1}^{\mathrm{T}}\right\}\\ &= \sigma_i^2[\boldsymbol{\omega}_k\times][\boldsymbol{\omega}_k\times]^{\mathrm{T}} + \frac{\sigma_i^2}{2T^2}\boldsymbol{I}_3\,.\end{aligned} \tag{31}$$

Noticing the unknown $\boldsymbol{\omega}_k$ in (31), the accurate covariance matrix $\boldsymbol{Q}_{k,k}^{(i)}$ cannot be computed. Generally, the approximation $\|\boldsymbol{\omega}_k\|T \ll 1$ is valid for attitude estimation systems [10], and the covariance matrix can be simplified as:

$$\begin{aligned}\boldsymbol{Q}_{k,k}^{(i)} &= \sigma_i^2[\boldsymbol{\omega}_k\times][\boldsymbol{\omega}_k\times]^{\mathrm{T}} + \frac{\sigma_i^2}{2T^2}\boldsymbol{I}_3\\ &= \frac{\sigma_i^2}{2T^2}\left(2T^2[\boldsymbol{\omega}_k\times][\boldsymbol{\omega}_k\times]^{\mathrm{T}} + \boldsymbol{I}_3\right)\\ &\approx \frac{\sigma_i^2}{2T^2}\boldsymbol{I}_3 = \overline{\sigma}_i^2\boldsymbol{I}_3\,.\end{aligned} \tag{32}$$

With N_1 equations in (29), *i.e.* $i = 1, \ldots, N_1$, $N_1 \le N$, the weighted least-squares approach can be applied to determine the angular velocity, that is:

$$\hat{\boldsymbol{\omega}}_k = \frac{1}{2T}\boldsymbol{B}_k^{-1}\sum_{i=1}^{N_1}\overline{\sigma}_i^{-2}\left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]^{\mathrm{T}}\left(\widetilde{\boldsymbol{b}}_{i,k+1}-\widetilde{\boldsymbol{b}}_{i,k-1}\right), \tag{33}$$

where $\boldsymbol{B}_k = \sum_{i=1}^{N_1}\overline{\sigma}_i^{-2}\left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]^{\mathrm{T}}\left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]$. $N_1 \ge 2$ is required for the matrix $\boldsymbol{B}_k^{-1}$ to exist.

Substituting (29) into (33), the angular velocity error $\delta\boldsymbol{\omega}_k$ can be expressed by:

$$\delta\boldsymbol{\omega}_k = \boldsymbol{\omega}_k - \hat{\boldsymbol{\omega}}_k = -\mathrm{B}_k^{-1}\sum_{i=1}^{N_1}\overline{\sigma}_i^{-2}\left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]^{\mathrm{T}}\boldsymbol{w}_{i,k}. \tag{34}$$

Based on the relationship between $\delta\boldsymbol{\omega}_k$ and $\boldsymbol{w}_{i,k}$ in (34) and the statistical property of $\boldsymbol{w}_{i,k}$ in (32), we have the following covariance matrix of $\delta\boldsymbol{\omega}_k$:

$$\boldsymbol{Q}_{k,k} = \boldsymbol{B}_k^{-1}\sum_{i=1}^{N_1}\overline{\sigma}_i^{-4}\left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]^{\mathrm{T}}\boldsymbol{Q}_{k,k}^{(i)}\left[\widetilde{\boldsymbol{b}}_{i,k}\times\right]\boldsymbol{B}_k^{-1} \approx \boldsymbol{B}_k^{-1}. \tag{35}$$

3.2. Quaternion-based filtering method

The quaternion-based filtering framework is presented in this section for the gyroless attitude estimation issue. The angular velocity in the process (15) is determined by (33), while introducing the angular velocity error into the process equation. The covariance of the angular velocity error is analysed in (35) and can be used in the attitude estimation.

Based on the multiplicative extended Kalman filtering technique [9], the quaternion-based filtering framework for the gyroless attitude estimation without an attitude dynamics model is outlined as follows:

1) Initialization
Given the initial attitude quaternion estimate $\hat{\boldsymbol{q}}_{0|0}$, angular velocity estimate $\hat{\boldsymbol{\omega}}_0$, attitude error estimate $\hat{\boldsymbol{x}}_{0|0}$ and error covariance $\boldsymbol{P}_{0|0}$, the state and covariance are then propagated until measurements are made.
2) Time update
The angular velocity estimate $\hat{\boldsymbol{\omega}}_k$ is calculated by (33) and the $\boldsymbol{Q}_{k,k}$ is obtained from (35). The attitude quaternion $\hat{\boldsymbol{q}}_{k|k}$ is propagated to obtain the a priori quaternion estimate $\hat{\boldsymbol{q}}_{k+1|k}$ as:

$$\hat{\boldsymbol{q}}_{k+1|k} = \overline{\Omega}\left(\hat{\boldsymbol{\omega}}_k\right)\hat{\boldsymbol{q}}_{k|k}, \tag{36}$$

where $\overline{\Omega}$ is defined in (9).
Due to the operation of the attitude update at the sampling moment k, set $\hat{\boldsymbol{x}}_{k|k} = 0$, and then $\hat{\boldsymbol{x}}_{k+1|k} = 0$. The one-step prediction error covariance $\boldsymbol{P}_{k+1|k} = E\left\{\boldsymbol{x}_{k+1|k}\boldsymbol{x}_{k+1|k}^{\mathrm{T}}\right\}$ is calculated as:

$$\boldsymbol{P}_{k+1|k} = \boldsymbol{F}_k\boldsymbol{P}_{k|k}\boldsymbol{F}_k^{\mathrm{T}} + \boldsymbol{\Gamma}_k\boldsymbol{Q}_{k,k}\boldsymbol{\Gamma}_k^{\mathrm{T}}, \tag{37}$$

where $\boldsymbol{F}_k$ and $\boldsymbol{\Gamma}_k$ are defined in (16) and (17), respectively.
3) Measurement update
The attitude error estimate is updated as:

$$\hat{\boldsymbol{x}}_{k+1|k+1} = \boldsymbol{K}_{k+1}\boldsymbol{y}_{k+1}, \tag{38}$$

with the filter gain given by:

$$\boldsymbol{K}_{k+1} = \boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^{\mathrm{T}}\left(\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^{\mathrm{T}}+\boldsymbol{R}\right)^{-1}, \tag{39}$$

where $\boldsymbol{y}_{k+1}$, $\boldsymbol{H}_{k+1}$ and $\boldsymbol{R}$ are defined in (21)–(22), respectively.

The filtering error covariance $\boldsymbol{P}_{k+1|k+1} = E\left\{\boldsymbol{x}_{k+1|k+1}\boldsymbol{x}_{k+1|k+1}^{\mathrm{T}}\right\}$ is calculated as:

$$\boldsymbol{P}_{k+1|k+1} = (\boldsymbol{I}_3 - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\boldsymbol{P}_{k+1|k}. \tag{40}$$

4) Attitude update
The a posteriori quaternion estimate $\hat{\boldsymbol{q}}_{k+1|k+1}$ is yielded from the attitude error estimate $\hat{\boldsymbol{x}}_{k+1|k+1}$ and the a priori estimate $\hat{\boldsymbol{q}}_{k+1|k}$.

$$\hat{\boldsymbol{q}}_{k+1|k+1} = \begin{bmatrix} \frac{1}{2}\hat{\boldsymbol{x}}_{k+1|k+1} \\ 1 \end{bmatrix} \otimes \hat{\boldsymbol{q}}_{k+1|k}\,. \tag{41}$$

Remark 2. The filtering problem for gyroless attitude estimation without an attitude dynamics model is solved from the above scheme. It is worth pointing out that many important filtering algorithms have been developed for attitude estimation in recent years. Unfortunately, the existing results cannot be simply applied to the system (15) and (20), hindered by the requirement of gyro measurement or attitude dynamics modelling. In contrast, the process model (15) constructed only with the attitude kinematic equation, the angular velocity information obtained from (33) and (35), and the effect from angular velocity error compensated in the filtering algorithm are unique, which constitutes the main difference from the existing results. In conclusion, the proposed quaternion-based filtering framework provides an approach that complements the existing filtering techniques for attitude estimation.

For the sake of clarity, a flowchart of the proposed algorithm is shown in Fig. 1.

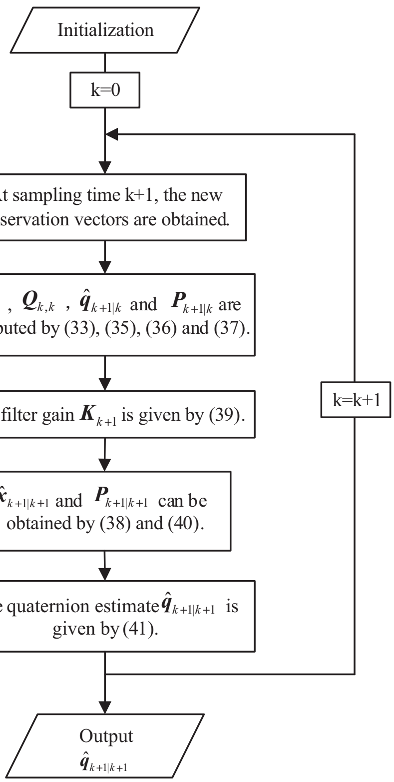

Fig. 1. A flowchart of the proposed algorithm.

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4. Numerical simulation

This section shows the performance of the proposed filtering algorithm. The simulation is performed using a star tracker to determine the attitude of a spacecraft in a low-Earth orbit. The spacecraft's x axis is pointed at the orbit velocity direction and y axis is pointed opposite to the orbit momentum vector at the initial moment, with the initial attitude quaternion given by:

$$\boldsymbol{q}_0 = [0 \;\; 0.7071 \;\; 0.7071 \;\; 0]^{\mathrm{T}}. \tag{42}$$

The star tracker's boresight is defined by its corresponding sensor z axis, which is assumed to be along the negative spacecraft body z axis. The star tracker outputs observation vectors of stars in the body frame simulated by:

$$\widetilde{\boldsymbol{b}}_i = \frac{1}{\sqrt{1+\widetilde{a}_i^2+\widetilde{b}_i^2}} \begin{bmatrix} -\widetilde{a}_i \\ -\widetilde{b}_i \\ 1 \end{bmatrix}, \tag{43}$$

where $\widetilde{a}_i$ and $\widetilde{b}_i$ are focal plane measurements. Their true quantities are denoted by a_i and b_i. Defining $\boldsymbol{\gamma}_i = [a_i \;\; b_i]^{\mathrm{T}}$, we obtain:

$$\widetilde{\boldsymbol{\gamma}}_i = \boldsymbol{\gamma}_i + \boldsymbol{\zeta}_i, \tag{44}$$

where $\boldsymbol{\zeta}_i$ is zero-mean Gaussian noise with the covariance given by [25]:

$$\boldsymbol{R}_i^{\mathrm{FOCAL}} = \frac{\sigma^2}{1+a_i^2+b_i^2} \begin{bmatrix} \left(1+a_i^2\right)^2 & (a_i b_i)^2 \\ (a_i b_i)^2 & \left(1+b_i^2\right)^2 \end{bmatrix}, \tag{45}$$

where $\sigma = 0.0053 \times (\pi/180)$ rad. The star tracker can sense up to 15 stars in a 7×7 deg field of view. The catalogue contains stars that can be sensed up to a magnitude of 6.0 (larger magnitudes indicate dimmer stars). Star images are taken at 1 s intervals. A plot of the number of available stars is shown in Fig. 2.

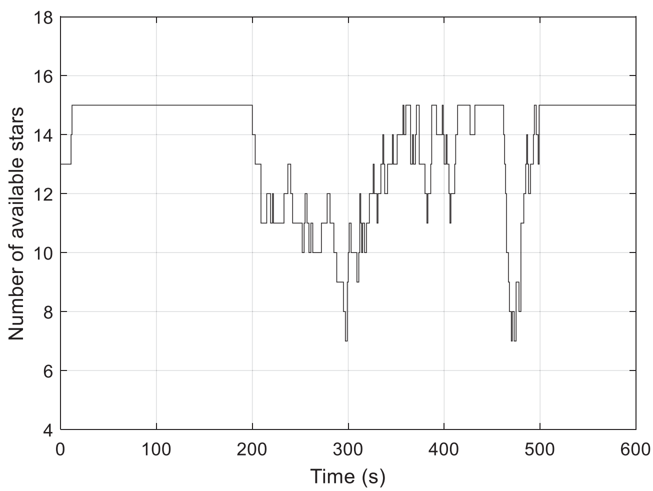

Fig. 2. Availability of stars.

Assume that the spacecraft is undergoing motion characterized by:

$$\boldsymbol{\omega} = \left[\begin{array}{ccc} 5.8148 \times 10^{-6} t & 0.0011 & 0.0052 \sin(0.0079t) \end{array}\right]^{\mathrm{T}} rad/s. \tag{46}$$

This true angular velocity is plotted in Fig. 3. The initial attitude estimate is given by its true value and the error covariance is set as $\boldsymbol{P}_{0|0} = 0.1\boldsymbol{I}_3$. The initial angular velocity estimate is also given by its true value. The new gyroless attitude estimation algorithm is compared with the method denoted by M2 in which the dynamics is modelled as [23]:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{\xi}, \tag{47}$$

where $\boldsymbol{\xi}$ is zero-mean white noise and the angular velocity $\boldsymbol{\omega}$ as the augmented system state is automatically updated along with the attitude quaternion by a Kalman filter.

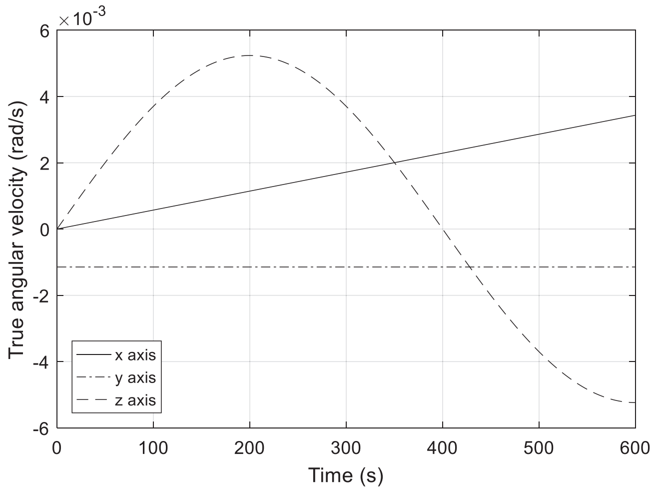

Fig. 3. The true angular velocity.

The angular velocity estimate error with the new method is shown in Fig. 4a. Clearly, the approach in (33) has the ability of angular velocity determination and the angular velocity estimate error is within its 3σ boundary. In addition, the error in Fig. 4a is smaller than that in Fig. 4b. It is reasonable that in order to ensure the convergence of angular velocity estimate error, the relatively large variances of ξ in (47) should be set to reflect error increases in the process of the angular velocity propagation under variable angular velocity for the reason that the zero-mean noise ξ in (47) implies that the angular velocity keeps constant in the propagation, which has an influence on the covariance of angular velocity estimate error and results in the larger 3σ boundary in Fig. 4b.

It can be seen from Fig. 5a that the predicted 3σ boundary does indeed bound the attitude estimate error which slightly increases at times when fewer stars are available. Therefore, the proposed quaternion-based filtering framework is effective for gyroless attitude estimation without an attitude dynamics model. This is due to the fact that we have made special efforts to establish the gyroless attitude estimation system, determine the angular velocity, analyse the properties of the angular velocity error and reduce the error effect in the filtering process. The 3σ boundary in Fig. 5a is about 20 μrad for x and y axes and 450 μrad for z axis, whereas in Fig. 5b is smaller than 30 μrad for x and y axes and 600 μrad for z axis. Therefore, the proposed method has better

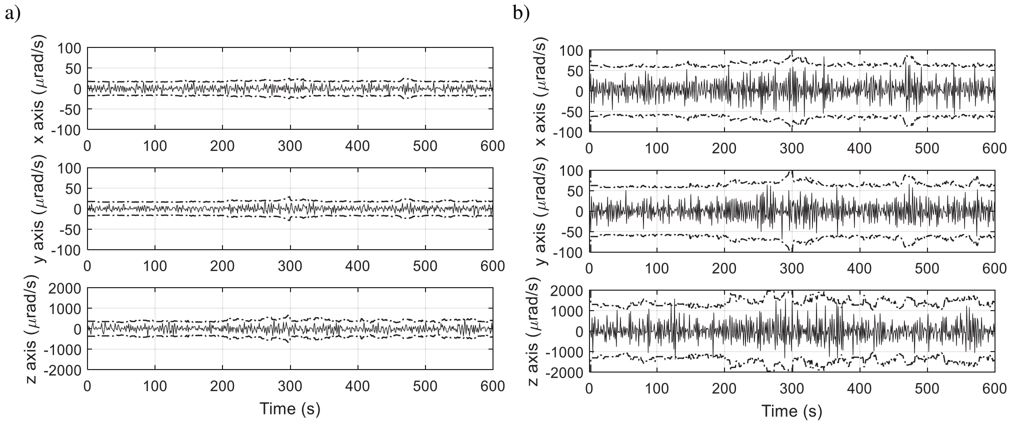

Fig. 4. The angular velocity estimate error and 3σ boundary. New method (a); M2 method (b).

performance than M2 because the angular velocity plays an important role in (8) and (15), and the smaller angular velocity estimate error in Fig. 4a brings a more accurate attitude estimate.

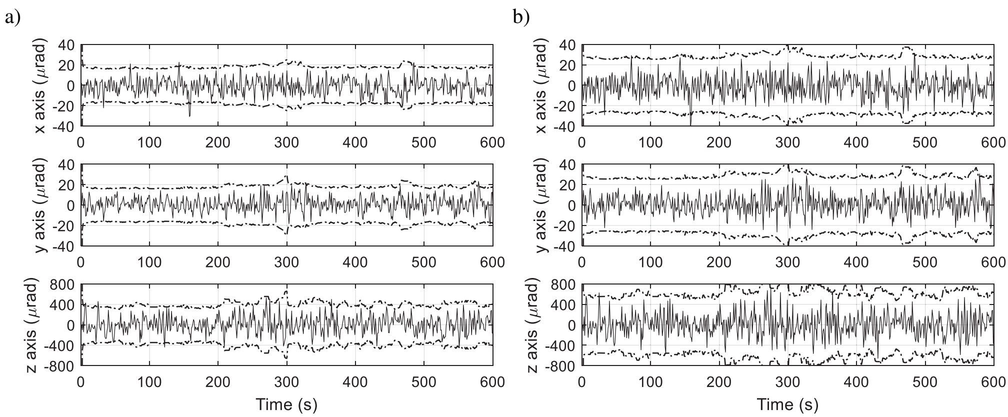

Fig. 5. The attitude estimate error and 3σ boundary. New method (a); M2 method (b).

5. Conclusions

In this paper, we have established the quaternion-based filtering framework for gyroless attitude estimation without an attitude dynamics model. The discrete-time kinematic equation of attitude error has been derived as the process model and the attitude sensor model has been used to construct the measurement equation. The angular velocity information has been obtained dependent only on the knowledge of observation vector measurements from attitude sensors, where the statistical properties of angular velocity error have been analysed. The multiplicative extended Kalman filtering technique has been used to reduce the effect from the angular velocity error and measurement noise and update the attitude quaternion estimate at each sampling moment. Finally, a simulation example has been given to illustrate the superiority of the proposed method.

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