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Modeling of fluid flow in microgap considering the boundary change of dynamic viscosity

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Summary. The flow of a viscous incompressible fluid in small gaps hydraulic devices and devices based on the hop boundary changes in viscosity. For the distribution model adopted dynamic viscosity was integrate the equations of fluid motion, whereby expressions are obtained for the velocity of the liquid height of the gap. The expressions for calculation of the fall capacity flow section are determined. Examples of the calculation of distributions velocity and falling bandwidth to a narrow gapare given.The estimation of the limits of applicability of classical approach to the calculation of viscous flow in micro gap is executed.

Key words: dynamic viscosity, fluid pressure, flow velocity distribution.

INTRODUCTION

Fluid flow in small gaps is of practical interest in connection with the tasks of hydraulic devices and appliances, tight joints moving pairs that often while providing execution of the guaranteed micron gap [6, 9, 11]. The classical approach to the calculation of the flow in the narrow gaps suggests permanence dynamical viscosity of the fluid over the cross section [10, 16, 17]. The flow of fluid to hydraulic systems common operating conditions is laminar flow and the calculation produced based on known relationships for a flat, concentric and eccentric cracks [5, 7].

However, according to the theoretical principles and the experimental data of several authors [3, 4] near the boundary of a solid body (up to several thousand angstrem $1A^0 = 10^{-4}$ mkm $= 10^{-10}$ m) viscosity mineral oil abruptly increased. Although the thickness of the layers, where the deviation is shown toughness, significantly less than the micro annulus (gap sizes in hydro apparatuses usually perform at least 10…15 microns), there is a great interest to the evaluation of hydro flow parameters, taking into account the marked effect [12, 14, 20].

PUBLICATION AND METHOD ANALYSIS

The magnitude of the shock affects kind of liquid material is a solid wall, temperature, and so on. Studying the properties of the boundary layers of liquids shows that this layer where the liquid phase is in a special state-characterized adhesive high degree of ordering molecules. One of the simplest consistent theories, take into account the effects of the near-wall flows, based on the continuum model with internal rotation nonpoint structural elements [1, 2, 8].

Despite the fact that the qualitative assessment boundary increase in viscosity in the vicinity of the solid surface is widely represented in the literature, quantitative characteristics for the evaluation of hydromechanical parameters of flow in microgap are absent [13, 18, 19].

OBJECTS AND PROBLEMS

A mathematical model of a viscous in compressible fluid in small gaps hydraulic devices and devices based on the hop boundary viscosity change and install according to the calculation of the fall capacity flow section are offered.

THE MAIN SECTION

Consider steady flow in a narrow gap formed by two parallel walls of then non-limited width. Choosing the x-axis along the flow axis y – perpendicular to the walls of the slot (Fig. 1), we write the equation of motion in compressible medium with variable dynamic viscosity [5, 7]:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u_x}{\partial y}\right), \tag{1}$$

where: μ – dynamic viscosity, $\partial p/\partial x$ – the pressure gradient, u_x – the rate of fluid flow.

We assume that the walls defining the slot are formed, in general, of different materials. This involves consideration of the introduction of various values of jumps viscosity near the solid surface, as well as different sizes of cross border-layers, which manifests change in viscosity. In this, the job is a consistent assignment the distribution of dynamic viscosity adjustment of the gap (Fig. 2):

$$\mu = \begin{cases} \mu_0 + \Delta\mu_1, & y < \Delta\delta_1 \\ \mu_0, & \Delta\delta_1 \le y \le \delta - \Delta\delta_2 \\ \mu_0 + \Delta\mu_2, & y > \delta - \Delta\delta_2, \end{cases} \tag{2}$$

where: δ – the size of the gap; $\Delta\delta_1$, $\Delta\delta_2$ – dimensions of boundary layers in each of the walls near which there is an increase in viscosity; μ_0 – dynamic viscosity (mostly liquid stream is the boundary layers); $\Delta\mu_1$, $\Delta\mu_2$ – the value of the border jumps viscosity.

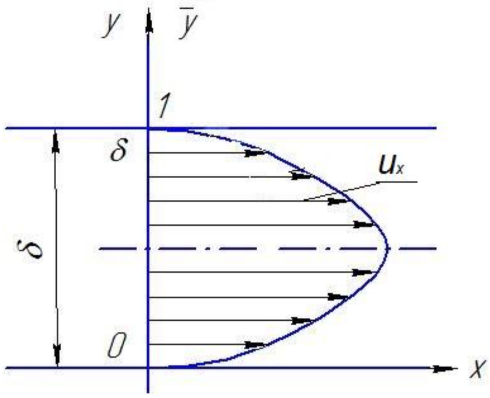

Fig. 1. Scheme of the flow in a narrow gap

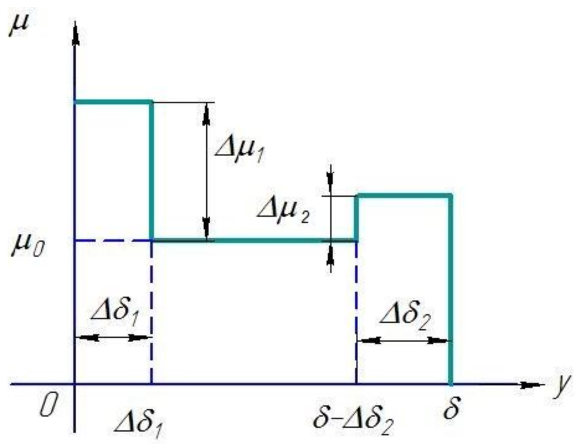

Fig. 2. Distribution of dynamic viscosity adjustment gap

In order to further simplify the mathematical calculations, we introduce the following dimensionless variables:

Dimensionless coordinates:

$$\bar{y} = y/\delta; \quad \bar{\delta}_1 = \Delta\delta_1/\delta; \quad \bar{\delta}_2 = \Delta\delta_2/\delta. \quad (3)$$

Dimensionless viscosity and its jump:

$$\bar{\mu} = \mu/\mu_0; \quad \bar{\mu}_1 = \Delta\mu_1/\mu_0; \quad \bar{\mu}_2 = \Delta\mu_2/\mu_0; \quad (4)$$

Dimensionless speed:

$$\bar{u} = \bar{u}_x/V_0, \quad (5)$$

where: u_0 – average speed in the classical calculation of flow in a narrow gap [5, 7]:

$$V_0 = -\frac{\delta^2}{12\mu_0}\frac{\partial p}{\partial x}, \quad (6)$$

Or in a form more convenient for engineering calculations:

$$V_0 = \frac{\Delta p \delta^2}{12\mu_0 l}, \quad (7)$$

where: Δp – pressure differential across the gap; l – length of the slot.

In view of (3…6), the equation (1) and distribution (2) take the following dimensionless form:

$$\frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{u}_x}{\partial \bar{y}}\right) = -12, \quad (8)$$

$$\mu = \begin{cases} 1+\mu_1, & y < \delta_1, \\ 1, & \delta_1 \le y \le 1-\delta_2, \\ 1+\mu_2, & y > 1-\delta_2. \end{cases} \quad (9)$$

Dimensionless viscosity distribution (9) is shown in Fig. 3.

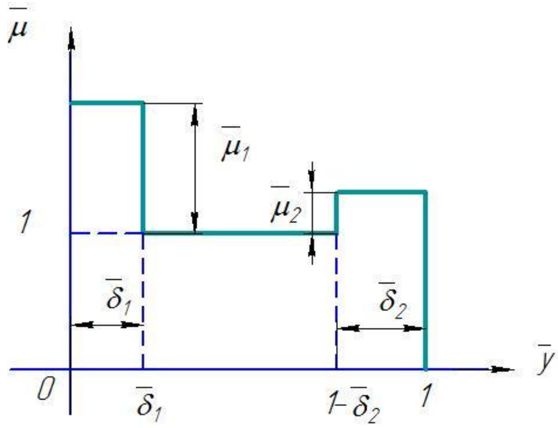

Fig. 3. The dimensionless distribution viscosity

To find the velocity distribution over the height of the slit separate in the gap (Fig. 4) three zones (0, 1, 2) corresponding to the constant values of the viscosity and integrate the equation (8) for each zone:

$$\bar{u}_1(\bar{y}) = -\frac{6}{1+\bar{\mu}_1}\bar{y}^2 + C_{11}\bar{y} + C_{12}, \quad (10)$$

$$\bar{u}_0(\bar{y}) = -6\bar{y}^2 + C_{01}\bar{y} + C_{02}, \quad (11)$$

$$\bar{u}_2(\bar{y}) = -\frac{6}{1+\bar{\mu}_2}\bar{y}^2 + C_{21}\bar{y} + C_{22}, \quad (12)$$

where: $C_{11}, C_{12}, C_{01}, C_{02}, C_{21}, C_{22}$ – integration constants.

To determine the constants of integration consider the following boundary conditions:

$$\bar{u}_1(0) = 0; \; \bar{u}_2(1) = 0, \quad (13)$$

and entering as shown in Fig. 4 yet unknown speed:

$$\bar{u}_1^* = \bar{u}_1(\bar{\delta}_1) = \bar{u}_0(\bar{\delta}_1), \tag{14}$$

$$\bar{u}_2^* = \bar{u}_2(1-\bar{\delta}_2) = \bar{u}_0(1-\bar{\delta}_2). \tag{15}$$

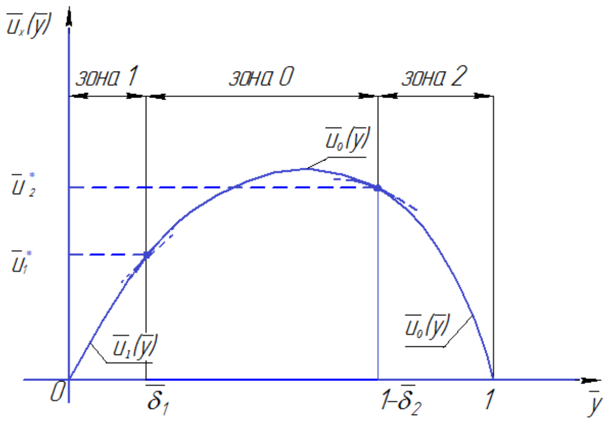

Fig. 4. Determination of the velocity distribution in a narrow gap

In view of (13…15) to obtain the values of the constants, which, after substituting (10…12) have:

$$\bar{u}_1(\bar{y}) = \frac{6}{1+\bar{\mu}_1}\bar{y}(\bar{\delta}_1-\bar{y}) + \bar{u}_1^*\frac{\bar{y}}{\bar{\delta}_1}, \tag{16}$$

$$\bar{u}_0(\bar{y}) = \bar{u}_1^* + 6\bar{\delta}_1^2 - 6\bar{y}^2 + \frac{\bar{u}_1^* - \bar{u}_2^* + 6\left(\bar{\delta}_1^2 - (1-\bar{\delta}_2)^2\right)}{\bar{\delta}_1 - (1-\bar{\delta}_2)}(\bar{y}-\bar{\delta}_1), \tag{17}$$

$$\bar{u}_2(\bar{y}) = \frac{6}{1+\bar{\mu}_2}(1-\bar{y})(\bar{\delta}_2 - 1 + \bar{y}) + \bar{u}_2^*\frac{(1-\bar{y})}{\bar{\delta}_2}. \tag{18}$$

Since the functional dependence of the rates must be continuous and smooth, then to find the velocity using the condition that the derivative of the velocity at points "stitching" velocity profiles of different zones (Fig. 4):

$$\left.\frac{d\bar{u}_1}{d\bar{y}}\right|_{\bar{\delta}_1} = \left.\frac{d\bar{u}_0}{d\bar{y}}\right|_{\bar{\delta}_1}, \tag{19}$$

$$\left.\frac{d\bar{u}_2}{d\bar{y}}\right|_{1-\bar{\delta}_2} = \left.\frac{d\bar{u}_0}{d\bar{y}}\right|_{1-\bar{\delta}_2}. \tag{20}$$

Expressions (19, 20) give the following equation:

$$-\frac{6\bar{\delta}_1}{1+\bar{\mu}_1} + \frac{\bar{u}_1^*}{\bar{\delta}_1} = -12\bar{\delta}_1 + \frac{\bar{u}_1^* - \bar{u}_2^* + 6\left(\bar{\delta}_1^2 - (1-\bar{\delta}_2)^2\right)}{\bar{\delta}_1 - (1-\bar{\delta}_2)}, \tag{21}$$

$$-\frac{6\bar{\delta}_2}{1+\bar{\mu}_2} + \frac{12(1-\bar{y})}{1+\bar{\mu}_2} - \frac{\bar{u}_2^*}{\bar{\delta}_2} = -12(1-\bar{\delta}_2) + \frac{\bar{u}_1^* - \bar{u}_2^* + 6\left(\bar{\delta}_1^2 - (1-\bar{\delta}_2)^2\right)}{\bar{\delta}_1 - (1-\bar{\delta}_2)}, \tag{22}$$

Solving (21) and (22) we obtain:

$$\bar{u}_1^* = \frac{6\bar{\delta}_1^3\bar{\mu}_1 - 6\bar{\delta}_1^2\bar{\mu}_1}{1+\bar{\mu}_1} - \frac{6\bar{\delta}_1\bar{\delta}_2^2\bar{\mu}_2}{1+\bar{\mu}_2} + 6\bar{\delta}_1(1-\bar{\delta}_1), \tag{23}$$

$$\bar{u}_2^* = \frac{6\delta_2^3\mu_2 - 6\delta_2^2\mu_2}{1+\mu_2} - \frac{6\delta_1^2\delta_2\mu_1}{1+\mu_1} + 6\delta_2(1-\delta_2), \tag{24}$$

Substituting (23, 24) at (16, 18), we have:

$$\bar{u}_1(\bar{y}) = 6\bar{y}\left(1 - \frac{\bar{\delta}_2^2\bar{\mu}_2}{1+\bar{\mu}_2} + \frac{\bar{\delta}_1^2\bar{\mu}_1 - 2\bar{\delta}_1\bar{\mu}_1 - \bar{y}}{1+\bar{\mu}_1}\right), \tag{25}$$

$$\bar{u}_0(\bar{y}) = -\frac{6\bar{\delta}_1^2\bar{\mu}_1}{1+\bar{\mu}_1} + 6\bar{y}\left(1 - \bar{y} + \frac{\bar{\delta}_1^2\bar{\mu}_1}{1+\bar{\mu}_1} - \frac{\bar{\delta}_2^2\bar{\mu}_2}{1+\bar{\mu}_2}\right), \tag{26}$$

$$\bar{u}_2(\bar{y}) = 6(1-\bar{y})\left(\frac{\bar{y} + \bar{\delta}_2^2\bar{\mu}_2 - 2\bar{\delta}_2\bar{\mu}_2 + \bar{\mu}_2}{1+\bar{\mu}_2} - \frac{\bar{\delta}_1^2\bar{\mu}_1}{1+\bar{\mu}_1}\right). \tag{27}$$

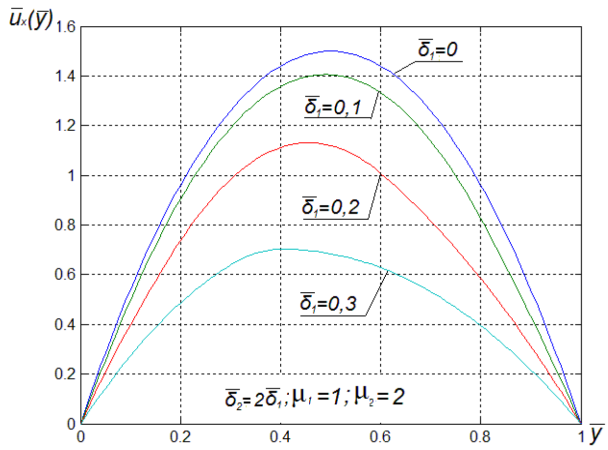

Fig. 5. The velocity distribution in the gap

Expressions (25 … 27) give a complete distribution of the dimensionless velocity on the size of the gap. Examples of velocity profiles calculated according to case $\delta_2 = 2\delta_1$ and $\mu_1 = 1$, $\mu_2 = 2$ shown in Fig. 5. As can be seen, flow velocity less than the speed determined by the classical calculation. Also, it is skewed, the maximum speed of the center of the gap at the uneven distribution of viscosity. We introduce the dimensionless flow rate in the gap:

$$\bar{Q} = \frac{Q}{Q_0} = \frac{\int_0^{\delta} u_x(y)dy}{V_0\delta}, \tag{28}$$

where: Q –the actual consumption in the gap, Q_0 – flow in the gap on the classical calculation.

It is easy to verify that the value is the correlation between the value of consumption, taking into account the specific boundary abrupt change in viscosity and flow values defined by the classical flow. The value can also be regarded as falling through put flow section microgap.

Using (3, 5) instead of (28) we have:

$$\bar{Q} = \int_0^1 \bar{u}_x(\bar{y})d\bar{y}, \qquad (29)$$

or, considering the integral (29) into zones (Fig. 4):

$$\bar{Q} = \int_0^{\bar{\delta}_1} \bar{u}_1(\bar{y})d\bar{y} + \int_{\bar{\delta}_1}^{1-\bar{\delta}_2} \bar{u}_0(\bar{y})d\bar{y} + \int_{1-\bar{\delta}_2}^{1} \bar{u}_2(\bar{y})d\bar{y}, \qquad (30)$$

Substituting (25…27) into (30) yields the following expression for determining the bandwidth drop flow section:

$$\bar{Q} = 1 - \bar{\mu}_1\bar{\delta}_1^2 \frac{3-2\bar{\delta}_1}{1+\bar{\mu}_1} - \bar{\mu}_2\bar{\delta}_2^2 \frac{3-2\bar{\delta}_2}{1+\bar{\mu}_2}. \qquad (31)$$

It should be noted that $\bar{\mu}_1, \bar{\mu}_2 \to 0$, or $\bar{\delta}_1, \bar{\delta}_2 \to 0$ expression (31) gives the $\bar{Q}$ =1 i.e. there is a clear limiting transition to the classical calculation of viscous in compressible fluid in a microgap.

A more general analysis of influence of an abrupt increase in viscosity can be done, if we consider the flow in a narrow gap between the walls of homogeneous materials. In this case we have a single value for the size of the border zone, and the magnitude of cross-border viscosity jump

$$\bar{\delta} = \bar{\delta}_1 = \bar{\delta}_2, \qquad (32)$$

$$\bar{\mu} = \bar{\mu}_1 = \bar{\mu}_2. \qquad (33)$$

In view of the emerging with the symmetry of the velocity distribution of dependence (25…27) record for half the size of the gap:

$$\bar{u}(\bar{y}) = \begin{cases} 6\bar{y}\left(1 - \dfrac{2\bar{\delta}\bar{\mu} + \bar{y}}{1+\bar{\mu}}\right), & 0 \le \bar{y} < \bar{\delta}, \\ 6\bar{y}(1-\bar{y}) - \dfrac{6\bar{\mu}\bar{\delta}^2}{1+\bar{\mu}}, & \bar{\delta} \le \bar{y} \le 0{,}5. \end{cases} \qquad (34)$$

Examples for the calculation of the velocity distributions at different values are shown in Fig. 6.

On the basis of (32) and (33) the following expression for the fall of the capacity flow section plane slit:

$$\bar{Q} = 1 - 2\bar{\mu}\bar{\delta}^2 \frac{3-2\bar{\delta}}{1+\bar{\mu}}. \qquad (35)$$

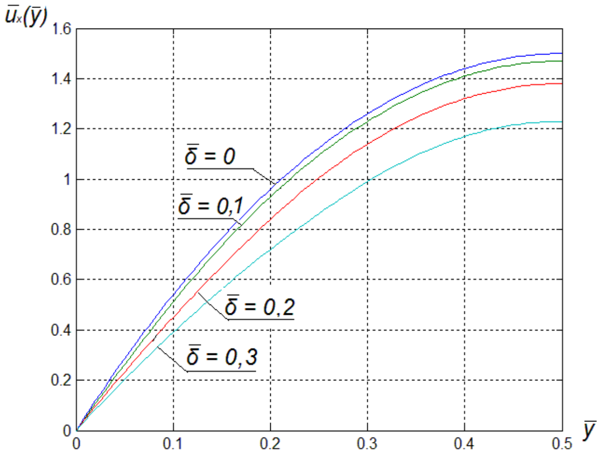

Fig. 6. The half-profile speed in the gap between the walls of homogeneous materials

Examples of calculation of the expression of falling through put flow cross section of the size of the border area at different values of the viscosity jump are shown in Fig. 7. Examples of calculation of the expression of falling through put flow section of the border jump viscosity at different sizes border zone shown in Fig. 8.

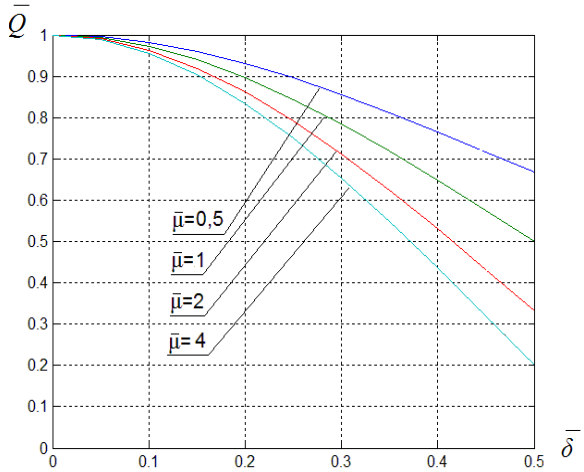

Fig. 7. The drop in capacity depending on the size of the boundary layer of higher viscosity

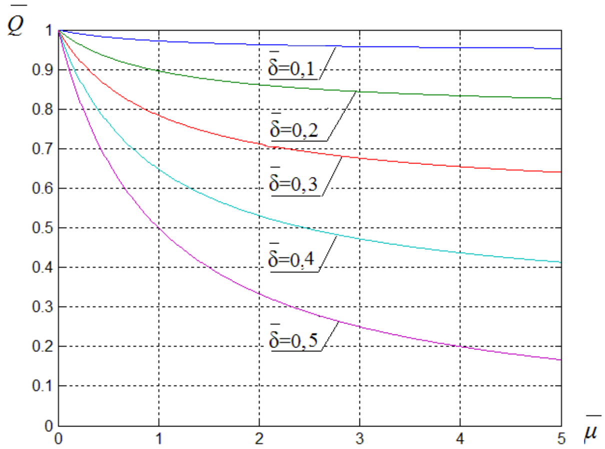

Fig. 8. The drop in capacity depending on the jump viscosity

Analysis of the results shows that the abrupt increase in the dynamic viscosity of the boundary clearly leads to a drop in bandwidth passage section microgap. Also, it can be assumed that the fall in consumption for a narrow gap is not more than 5%, and, as for the calculation of the flow are quite applicable known dependence [5, 7].

CONCLUSIONS

1. Investigations of a viscous incompressible fluid in a microgap hydraulic devices and devices based on a hopping boundary to increase the viscosity near the solid surface are conducted.

2. A narrow gap set according to the calculation of the fall capacity flow section is established.

3. A complete distribution of the dimensionless velocity on the size of the gap characterizes that flow velocity less than the speed determined by the classical calculation.

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МОДЕЛИРОВАНИЕ ТЕЧЕНИЯ ЖИДКОСТИ В МИКРОЗАЗОРЕ С УЧЕТОМ ГРАНИЧНОГО ИЗМЕНЕНИЯ ДИНАМИЧЕСКОЙ ВЯЗКОСТИ

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О. Кроль, В. Соколов

Аннотация. Рассмотрено течение вязкой несжимаемой жидкости в микрозазорах гидравлических устройств и аппаратов с учетом скачкообразного граничного изменения вязкости. Для принятой модели распределения динамической вязкости проинтегрированы уравнения движения жидкости, на основании чего получены выражения для скорости жидкости по высоте зазора. Установлены зависимости для расчета падения пропускной способности проходного сечения. Приведены примеры расчета распределений скорости и падения пропускной способности для плоской щели. Выполнена оценка границ применимости классического подхода к расчету течения вязкой жидкости в микрозазоре.

Ключевые слова: динамическая вязкость, жидкость, давление, расход, распределение скорости.