

Mathematical Simulation of Reinforced Concrete Beams with the External Reinforcement in the Form of Steel Cut and Stretchy Sheet

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Summary. The results of bearing capacity, deformability and fracture toughness of reinforced concrete beams with the external reinforcement in the form of steel cut and stretchy sheet, obtained due to the conducting of the experiment and mathematical simulation which were made of concrete of C40/50 class are given in the article. Mathematical simulation of beam structures is done on the basis of the deformation model which allows to conduct calculations of the unified methodological positions of different elements with diverse configuration of cross section and reinforcement as well as take into consideration elastic and plastic properties of concrete and reinforcement, assessing the actual stress-strain state of sections of reinforced concrete elements at different loading levels, including ultimate one. The deformation model is based on the actual diagrams use of concrete and reinforcement materials deformation and conditions of efforts balance in the normal section and hypothesis of flat sections. The theoretical value of bearing capacity and deformability, obtained as a result of the mathematical simulation was compared to the experimental data. The satisfactory coincidence of the mathematical calculation of bearing capacity, deformability, fracture toughness and experimental data gives an opportunity to use the algorithm not only for beam structures with bar reinforcement but also for beam structures with the external reinforcement in the form of steel cut and stretchy sheet.

Key words: mathematical simulation, bearing capacity, deformability, fracture toughness, beam structure, steel cut and stretchy sheet, deformation model.

INTRODUCTION

The determination algorithm of bearing capacity of reinforced concrete elements at normal sections requires the step-by-step method of the successive loads. The iteration process of the relevant deformations in the basic areas is implemented at every stage of this method.

The deformation model allows conducting calculations of the unified methodological positions of different elements with diverse configuration of cross section and reinforcement as well as take into consideration elastic and plastic properties of concrete and reinforcement, assessing the actual stress-strain state of sections of reinforced concrete elements at different loading levels, including ultimate one [1-3].

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

A significant contribution to the problems solution of the stress-strain state research which are the deformation theory basis, is done under the guidance of Ukrainian eminent scientists of State Enterprise "State Research Institute of Building Constructions": O.B.Holyshev [4], A.M.Bambura [5-7], V.Ya.Bachynskiy [8]. The first recommendations as to updated calculation of reinforced concrete elements taking into account the full compression chart appeared due to their developments

The works of Ye.M.Babych [9], A.M.Bambura, V.M.Barabash [10], A.Ya.Barashykov, V.Ya.Bachynskiy, Z.Ya.Bliharshkiy, V.V.Bilozir [11], A.A.Hvozdiev, O.B.Holyshev, V.S.Dorofeyev, F.Ye.Klymenko [12], A.S.Zaliesov [13], Yu.V.Zaitsev, D.V.Kochkariov, L.L.Lemys, P.B.Mitrofanov [14], A.M.Pavlikov, I.E.Prokopovych, S.I.Rohoviy, O.V.Semko, L.I.Storozhenko, I.A.Uzun, E.D.Chyhladze [15], L.M.Fomytsi, O.L.Shahin, A.A.Shkurupii [16], R.A.Shmyh, V.S.Shmukler, O.F.Yaremenko [17], Ye.A.Yaremenko, Ye.A.Yatsenko and others are devoted to the improvement issue of deformation model of reinforced concrete structures calculation which is based on the actual deformation charts " $\sigma - \varepsilon$ " of concrete and reinforcement.

OBJECTIVES

The purpose of our research is to conduct experimental investigations of the reinforced concrete beams with the external reinforcement in the form of steel cut and stretchy sheet, to develop algorithm of the bearing capacity, deformability and fracture toughness calculation of such structures under the loading on the basis of the deformation model and to compare the data.

THE MAIN RESULTS OF THE RESEARCH

The experimental concrete samples of C40/50 class (Fig. 1), section of 135×270 mm of the total length 2300 mm and the calculation span of 2000 mm (B-1, B-1*; * – beams-analogues) were produced for the beams work study with the external reinforcement in the form of steel cut and stretchy sheet [18-19]. The research of reinforced concrete beams, reinforced by steel cut and stretchy sheet was done on the special stand, where the loading was performed by two symmetrical concentrated forces, applied on the top of the beam pattern. Fig.2.

$$\sigma_c = f_{ck} \sum_{k=1}^5 a_k \eta^k, \quad (3) \quad \text{where: } \eta = \frac{\varepsilon_{ct}}{\varepsilon_{ctu}}$$

where: $\eta = \frac{\varepsilon_c}{\varepsilon_{c1,ck}}$.

$$\varepsilon_{ctu} = \frac{2f_{ctm}}{E_{ck}}$$

One should have the deformation chart of concrete in tension in order to find M_{crc} . We accept the chart « $\sigma_{ct} - \varepsilon_{ct}$ » for the tension in the following form:

$$\sigma_{ct} = f_{ctm} \sum_{k=1}^5 a_k \eta^k \quad (4)$$

This chart is accepted similarly as at the compression but ε_{ctu} is taken in accordance with paragraph 6.1 of this standard.

By the time of cracks formation, diagrams of deformation and stress are accepted in the following form (Fig. 3, b, c).

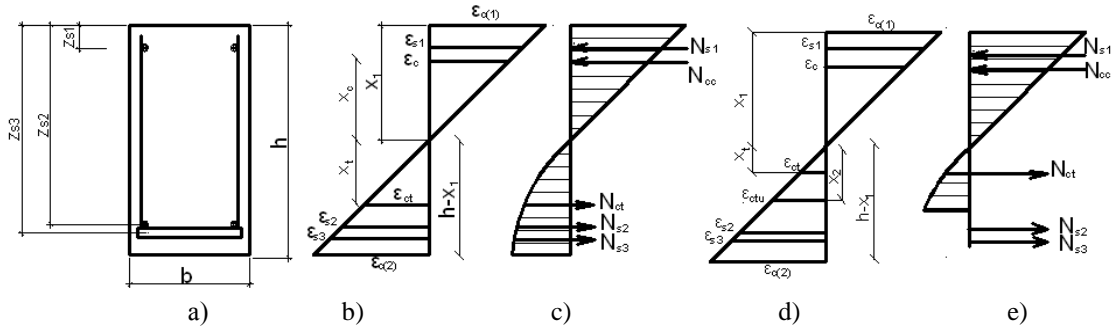


Fig. 3. The calculation of the bending elements as to the deformation model:

a) cross section; prior to the cracks formation moment; b) deformations diagrams; c) stresses diagrams; after cracks formation; d) deformations diagrams; e) stresses diagrams

The equation of equilibrium for reinforced concrete element, reinforced by steel cut and stretchy sheet can be recorded as following:

$$N_{cc} + N_{s1} - N_{ct} - N_{s2} - N_{s3} = 0, \quad (5)$$

N_{s1} – efforts in the compressed reinforcement;

N_{s2} – efforts in the bar reinforcement;

N_{s3} – efforts in steel cut and stretchy sheet.

Internal efforts in the compressed concrete:

$$N_{cc} = \int_0^{x_1} f_{ck} \sum_{k=1}^5 a_k \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h\varepsilon_{c1,ck}} \right)^k x_c^k b dx_c = \quad (6)$$

$$f_{ck} b \sum_{k=1}^5 \frac{a_k}{k+1} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h\varepsilon_{c1,ck}} \right)^k \left(\frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+1}$$

Efforts in the reinforcement:

$$N_{si} = \varepsilon_{si} E_{si} A_{si} = \frac{z_{si} (\varepsilon_{c(1)} + \varepsilon_{c(2)}) - \varepsilon_{c(1)} h}{h} E_{si} A_{si}, \quad (7)$$

Efforts in tensile concrete:

$$N_{ct} = \int_0^{h-x_1} f_{ctm} \sum_{k=1}^5 a_k \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)}) x_t}{h\varepsilon_{ctu}} \right)^k x_t^k b dx_t = \quad (8)$$

$$f_{ctm} b \sum_{k=1}^5 \frac{a_k}{k+1} \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)}) E_{ck}}{2f_{ctm} h} \right)^k \left(\frac{h\varepsilon_{c(2)}}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+1}$$

Moments are determined by formulas:

$$M_{c(c)} = \int_F \sigma_{c(t)} x_{c(t)} dF, \quad (9)$$

$$M_{si} = N_{si} (z_{si} - x_1). \quad (10)$$

In regard to the neutral axis, the moment is found:

$$M_{c(c)} = \int_0^{x_1} f_{ck} \sum_{k=1}^5 a_k \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h\varepsilon_{c1,ck}} \right)^k x_c^{k+1} b dx_c = \quad (11)$$

$$f_{ck} b \sum_{k=1}^5 \frac{a_k}{k+2} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h\varepsilon_{c1,ck}} \right)^k \left(\frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+2}$$

$$M_{ct} = f_{ctm} b \sum_{k=1}^5 \frac{a_k}{k+2} \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)}) E_{ck}}{2f_{ctm}} \right)^k \left(\frac{h\varepsilon_{c(2)}}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right), \quad (12)$$

$$M_{si} = \sigma_{si} A_{si} \left(z_{si} - \frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right). \quad (13)$$

Similarly, the bending moment for reinforced concrete element, reinforced by steel cut and stretchy sheet can be recorded in the following form:

$$M = M_{cc} + M_{ct} + M_{s1} + M_{s2} + M_{s3}. \quad (14)$$

The cracks formation moment is determined at $\varepsilon_{c(2)} = 2f_{ctm} / E_{ck}$. The experimental cracks formation

moment for pre-production samples B-1, B-1* is $M_{crc}^{ekcn} = 871,00 \text{ kN} \times \text{cm}$;

The theoretical cracks formation moment is $\dot{I}_{crc}^{SSU} = 792,82 \text{ kN} \times \text{cm}$.

After cracks formation we will take into consideration the gradual reduction of the height of tensile concrete zone in accordance with the proposals, designed by V.V.Bilozir [11]. Figure 3 shows:

$$\frac{\varepsilon_{c(2)}}{h - x_1} = \frac{2f_{ctm}}{E_{ck} x_2}. \quad (15)$$

$$\text{hence: } x_2 = \frac{2f_{ctm}(h-x_1)}{\varepsilon_{c(2)}E_{ck}} \quad (16)$$

$$\text{Otherwise: } \frac{\varepsilon_{ctu}}{x_2} = \frac{\varepsilon_{ct}}{x_t} \quad (17)$$

$$\text{hence: } \varepsilon_{ct} = \frac{\varepsilon_{ctu}x_t}{x_2} \quad (18)$$

After permutations and transformations we get:

$$\varepsilon_{ct} = \frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})x_t}{h} \quad (19)$$

$$x_2 = \frac{2f_{ctm}h}{E_{ck}(\varepsilon_{c(1)} + \varepsilon_{c(2)})} \quad (20)$$

Efforts in the tensile zone (with the increase of $\varepsilon_{c(1)}$ and $\varepsilon_{c(2)}$) will gradually decrease and be equal to:

$$N_{ct} = \int_0^{x_2} f_{ctm} \sum_{k=1}^5 a_k \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})x_t}{h\varepsilon_{ctu}} \right)^k b dx = \quad (21)$$

$$f_{ctm} b \sum_{k=1}^5 \frac{a_k}{k+1} \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})E_{ck}}{2f_{ctm}h} \right) \left(\frac{2f_{ctm}h}{E_{ck}(\varepsilon_{c(1)} + \varepsilon_{c(2)})} \right)^{k+1}$$

The moment component of the tensile zone:

Output data for Mathematical Simulation Conducting is provided in Table 1.

Table 1. Output data for Mathematical Simulation Conducting

Concrete				Steel Cut and Stretchy Sheet				Bar Reinforcement 2Ø6 A240C			
Class	$f_{ck,prism}$ MPa	f_{ctm} MPa	$E_{ck} \times 10^3$ MPa	area A_{sp} cm ²	f_{yk} MPa	$E_s \times 10^5$ MPa	ε_{ud}	area A_s , cm ²	f_{yk} MPa	$E_s \times 10^5$ MPa	ε_{ud}
C40/50	36	2,76	36,49	1,92	271	2,05	0,025	0,566	316	2,01	0,023

$f_{ck,prism}$ – design value of concrete strength on compression at 28 days;

f_{ctm} – design value of concrete strength on axial tension;

$E_{ck} \times 10^3$ – module of concrete elasticity;

A_{sp} – area of cross section of steel cut and stretchy sheet;

f_{yk} – design value of reinforcement resistance;

E_s – module of reinforcement elasticity;

ε_{ud} – ultimate relative elongation deformation.

The comparison of the experimental deflections values with theoretical values, calculated by the deformation model is provided in Fig. 3 and Table 2.

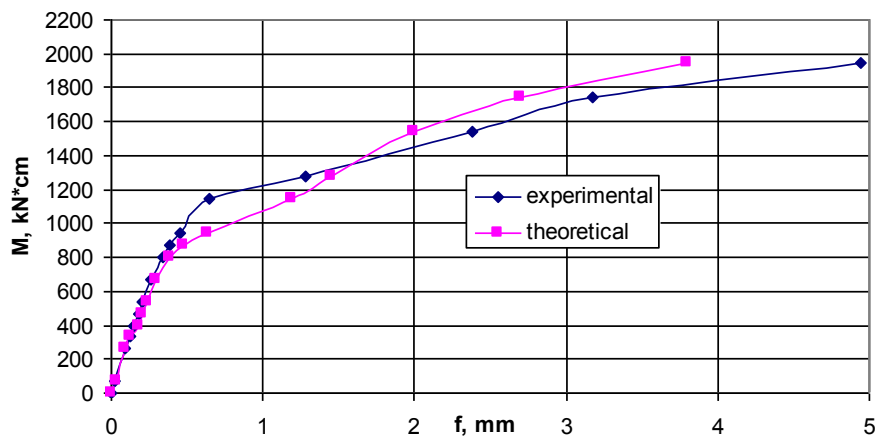


Fig. 3. Experimental and theoretical deflections of the pre-production samples

$$M_{ct} = f_{ctm} b \sum_{k=1}^5 \frac{a_k}{k+2} \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})E_{ck}}{2f_{ctm}h} \right)^k \quad (22)$$

$$\left(\frac{2f_{ctm}h}{E_{ck}(\varepsilon_{c(1)} + \varepsilon_{c(2)})} \right)^{k+2}$$

Thus, after cracks formation we use the equations (5) i (14) with the substitution N_{ct} and M_{ct} in them, determined from equations (21) and (22).

For statically defined elements of the constant section which work under the beam chart, the deflection has been determined by the formula:

$$f = \frac{1}{r} k_m l^2 \quad (23)$$

where: k_m – coefficient, which is determined by table 5.5 [21];

l – design beam span [22].

Table 2. Experimental and design values of ultimate bending moments and deflections close to operating loading levels

Beam code	Experimental ultimate bending moment M_t^{exper} , kN × cm	Design moment as to State Standard of Ukraine (SSU) \dot{M}_t^{SSU} , kN × cm	$\frac{M_t^{SSU} - M_t^{exper}}{M_t^{exper}}$ %	Experimental operating moment M_{exper} , kN × cm	Experimental deflection f^{exper} , mm	Theoretical deflection F^{theor} , mm	$\frac{f^{theor} - f^{exper}}{f^{exper}}$ %
B-1	1657.50	1570.30	-5.23	1274.90	1.28	1.45	+13.2
B-1*	1657.50	1570.30	-5.23	1274.90	1.31	1.45	+10.6

As we understand (see Table 2), the developed mathematical model gives a positive result for the ultimate destruction moment and reinforced concrete beams with the external reinforcement in the form of steel cut and stretchy sheet and these deviations amount for 5,23 %, deflections – 10,6-13,2%, that is acceptable for engineering calculations [22].

CONCLUSIONS

1. Applied methods of pre-production beam samples making were used, accepted indicating instrument and the scheme of their location gave chance to obtain reliable research material for the description of stress-strain state of reinforced concrete beams with the external reinforcement in the form of steel cut and stretchy sheet from the beginning of loading to the destruction.

2. The developed deformation model allows analyzing the stress-strain state of reinforced concrete beams with the external reinforcement in the form of steel cut and stretchy sheet at all loading stages and provides the satisfactory coincidence (-5.23%) of ultimate bending moments determination.

3. Cracks formation moment, determined by State Building Standards B.2.6-98:2009, is smaller that the research one and the deviation amounts for 8.9%.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ
ЖЕЛЕЗОБЕТОННЫХ БАЛОК, С ВНЕШНИМ
АРМИРОВАНИЕМ В ВИДЕ СТАЛЬНОГО
ПРОСЕЧНО-ВЫТЯЖНОГО ЛИСТА

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Аннотация. В статье приведены результаты несущей способности, деформативности и трещиностойкости железобетонных балок с внешним армированием в виде стального просечно-вытяжного листа, полученных в результате проведения эксперимента и математического моделирования, которые изготавливались из бетона класса С40/50. Математическое моделирование балочных конструкций, осуществлено на основе деформационной модели, которая позволяет вести расчеты с единых методических позиций различных элементов с различной конфигурацией поперечного сечения и армирования, а также в полной мере учесть упруго-пластические свойства бетона и арматуры, оценивая фактический напряженно-деформированное состояние сечений железобетонных элементов на разных уровнях загрузки, включая предельный.

Деформационная модель основывается на использовании реальных диаграмм деформирования материалов бетона и арматуры, а также в условиях равновесия сил в нормальном сечении и гипотезе плоских сечений. Теоретические значения несущей способности и деформативности, полученные в результате математического моделирования было сравнимо с экспериментальными данными. Удовлетворительная сходимость математического расчета несущей способности, деформативности и трещиностойкости и экспериментальных данных, дает возможность использовать представленный алгоритм не только для балочных конструкций со стержневым армированием но и для балочных конструкций с внешним армированием в виде стального просечно-вытяжного листа.

Ключевые слова: математическое моделирование, несущая способность, деформативность, трещиностойкость, балочная конструкция, стальной просечно-вытяжной лист, деформационная модель.