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ZERO-POINT THERMAL NOISE IN RESISTORS? A CONCLUSION

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Abstract

The main points of the UPoN-2018 talk and some valuable comments from the Audience are briefly summarized. The talk surveyed the major issues with the notion of zero-point thermal noise in resistors and its visibility; moreover it gave some new arguments. The new arguments support the old view of Kleen that the known measurement data "showing" zero-point Johnson noise are instrumental artifacts caused by the energy-time uncertainty principle. We pointed out that, during the spectral analysis of blackbody radiation, another uncertainty principle is relevant, that is, the location-momentum uncertainty principle that causes only the widening of spectral lines instead of the zero-point noise artifact. This is the reason why the Planck formula is correctly confirmed by the blackbody radiation experiments. Finally a conjecture about the zero-point noise spectrum of wide-band amplifiers is shown, but that is yet to be tested experimentally.

Keywords: Fluctuation Dissipation Theorem, resistors and fermionic versus bosonic systems, time-energy uncertainty principle versus location-momentum uncertainty principle.

1. Introduction

The key sentence of the talk about this, often heated, debate-topic was from Richard Feynman: *I would have rather questions that cannot be answered than answers that cannot be questioned!*

In today's world of political correctness, character assassinations, Facebook-likes, social-media followings, global warming debates and the frequent neglect in some leading scientific media, Feynman's truth-seeking encouragement indicates the way to improve originality, validity and value. Asking such questions was my main motivation to create the conference series of *Unsolved Problems of Noise* in 1996 and, since then, this motivation has grown even stronger because these types of hard challenges are so often missing nowadays.

While the summary of this talk is brief, the detailed presentation of the talk will be available at the UPoN-2018 website.

1.1. Thermal noise in resistors (Johnson noise)

Inspired by Johnson's original and careful breakthrough-experiments [1] that tested various materials, conductance mechanisms, phases (solid and liquid), temperatures and geometries,

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Nyquist [2] developed the statistical physical theory of the power density spectrum (noise spectrum) of the thermal noise voltage in resistors. In the classical-physical limit (low-frequencies, $f \ll kT/h$) a resistor R produces the familiar $S_u(f, T) = 4kTR$ result of Johnson, where k is the Boltzmann constant, h is the Planck constant, T is the absolute temperature and R is the resistance.

The inputs of Nyquist's treatment were Planck's blackbody radiation formula and the *Second Law of Thermodynamics* including (implicitly) the *Principle of Detailed Balance*. His generic result for the power density spectrum of a passive impedance $Z(f)$ in thermal equilibrium is given as (for details see Subsection 1.2):

$$S_{\mathrm{u,\,Nyq}}(f, T) = \mathrm{Re}\,[Z(f)]\,Q(f, T) = R(f)Q(f, T), \tag{1}$$

where, in thermal equilibrium, $Q(f, T)$ is a universal function of frequency and temperature which is independent of material properties, including phase (solid, liquid, *etc*.), geometry, and mechanism for electrical conduction.

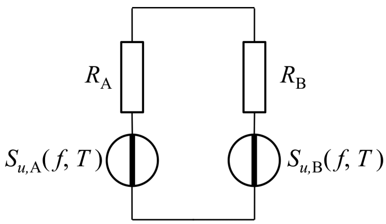

Fig. 1. A circuit with Johnson noise to discuss the Second Law and the zero net power flow between different resistors R_A and R_B with arbitrary materials, conductance mechanisms, geometries, structures and phases in thermal equilibrium. The voltage generators with power spectra $S_u(f, T)$ at frequency f and temperature T represent the Johnson noise.

It is very important to observe the *universality* of the function Q. The frequent negligence of this essential fact leads to *hidden perpetual motion machines* [3–5]. The power flow between two resistors at temperature T, in bandwidth Δf, is given as:

$$P_{A\to B}(f, \Delta f) = [Q_A(f, T) - Q_B(f, T)]\,\frac{R_A R_B}{(R_A + R_B)^2}\Delta f = 0. \tag{2}$$

Thus the following equation must hold to satisfy the Second Law of Thermodynamics:

$$Q_A(f, T) = Q_B(f, T) \tag{3}$$

for any chosen frequency f, and temperature T; any choice of materials including different materials A, B; and different conductance mechanisms, geometries, phases, structures, *etc.*

For example, it is a Second Law violation to assume that the Q function is different for resistors with fermionic or bosonic charge transport. That assumption would imply that there is a non-zero net power flow in a system in thermal equilibrium, at least in certain frequency ranges, see also Subsection 4.2. Such implication would be the direct violation of the Principle of Detailed Balance, too.

1.2. Nyquist's result for Johnson noise in the quantum domain

The detailed result of Nyquist's derivation of the function $Q(f, T)$ is:

$$Q(f, T) = 4hfN(f, T), \tag{4}$$

where the Plank number $N(f, T)$ gives the mean number of quanta in a given frequency mode of a linear-harmonic oscillator in thermal equilibrium:

$$N(f, T) = [\exp(hf/kT) - 1]^{-1}. \tag{5}$$

Note, the same exponential cut-off, as given by (5), characterizes Planck's blackbody radiation formula: a blackbody (with unity emissivity) radiates in each polarization direction with a power spectral intensity $I(f)$ given as:

$$I(f) = \frac{4\pi h f^3}{c^2} N(f, T), \tag{6}$$

In accordance with (3)–(5), the full-spectrum Johnson noise formula, by Nyquist [2], is as follows:

$$S_{\text{u, Nyq}}(f, T) = R(f)4hf\,[N(f, T)]. \tag{7}$$

1.3. The zero-point Johnson noise claim

Callen and Welton [6] (see also [7–9]) attempted to create a fundamental first-principles derivation of (7), however they had to make a number of extra assumptions, which we shall not discuss here. Their result, which is called *Fluctuation-Dissipation Theorem* (FDT), claims that the power density spectrum $S_{\text{u,FDT}}(f, T)$ of the voltage noise generated by a resistor differs from Nyquist's formula, (7), by an additive correction, see (8) and Fig. 2. This correction is a

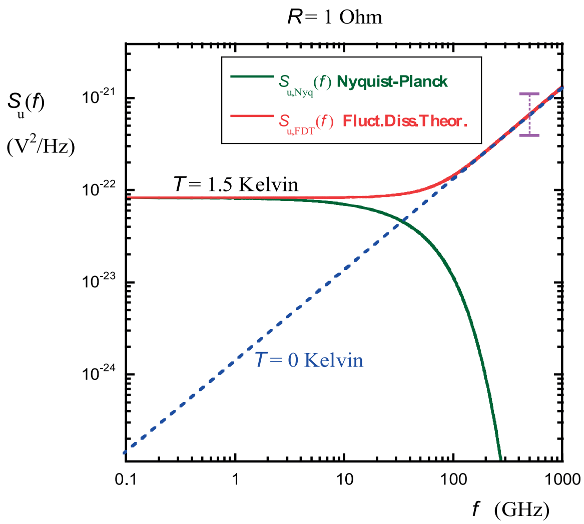

Fig. 2. Plots of Nyquist's (7) and the Fluctuation Dissipation Theorem, (8), at both 1.5 Kelvin, and zero temperatures. At zero Kelvin, Nyquist's result is zero at all frequencies thus it is not shown here. The error bar at the right is a rough indication of the relative scattering of data in the Koch, *et al.* experiments [10], in the high-frequency end, see Subsection 2.1.

constant 0.5 added to the Planck number, similarly to Planck's formula for the energy in a linear harmonic quantum oscillator, where the same additive constant represents the zero-point energy, the "half-quantum":

$$S_{\mathrm{u,FDT}}(f,T) = R(f)4hf\left[N(f,T) + 0.5\right]. \tag{8}$$

2. Supporting arguments about the existence of zero-point thermal noise

2.1. Zero-point energy in quantum oscillators (but not in their blackbody radiation)

University quantum physics courses teach Planck's derivation of the linear harmonic oscillator. Planck's famous result is the energy eigenvalues of the oscillator with resonance frequency f:

$$E = hf(n + 0.5), \tag{9}$$

where n is the quantum number, $n = 0, 1, 2, \ldots$. When the oscillator is coupled with a thermal reservoir and it is in thermal equilibrium, the number n is fluctuating and its mean value is the Planck number given below:

$$\langle n\rangle \equiv N(f,T) = \left[\exp(hf/kT) - 1\right]^{-1}. \tag{10}$$

Thus, the mean energy in the oscillator is:

$$\langle E\rangle = hf\left[\langle n\rangle + 0.5\right] = hf\left[N(f,T) + 0.5\right]. \tag{11}$$

In the zero temperature limit, $N(f,T) \to 0$ thus only the zero-point energy remains. Comparing Nyquist's (4) with Planck's (11), the similarity is striking and the natural question arises: If (11) has the additive 0.5 to the Planck number, why (4) does not have that? Should it be there, as the FDT's (8) is claiming, simply because of the existence of the zero-point energy?

We will address this issue in the next sections in detail but, in advance, let us make a point of history of science: Planck was the one who deduced both (11) for the oscillator and, based on that, the spectral intensity $I(f)$ of emitted light in the blackbody radiation:

$$I(f) = \frac{4\pi f^2}{c^2} hfN(f,T), \tag{12}$$

which is the power density spectrum of radiated electromagnetic field (thermal noise) in one polarization direction. (Note: (12) is a rearranged form of (6) for an easier comparison of (7) and (11)). It is historically relevant to observe that while Planck created (11) he did not include the additive term 0.5 in (12) and kept only the N there. (12) has been verified by experiments and common experience (see Subsection 3.1.2). In conclusion, Planck himself was already facing our problem but he chose to follow the way seen later on in Nyquist's result.

2.2. Measurements of Johnson noise with Josephson junctions

In their seminal 1981 experiments, Koch, Van Harlingen and Clarke measured the Johnson noise of a resistor at 1.6 and 4.2 Kelvin by using heterodyne detection with a Josephson junction [10]. There is a strong scattering of data *in the range where zero-point noise dominates* (at the top, about −33% to +50% relative errors, see Fig. 2). However, in a log-log plot of [10], the scattered data convincingly follow the theoretical lines through the measured interval where the spectrum increases about 1 order of magnitude beyond the low-temperature $4kTR$ level.

Similar Josephson-junction based experimental confirmations with improvements have been around since, including the Hakonen Group's careful experiments with observing high-frequency sidebands [11]. On the practical side, the increased Johnson noise at quantum frequencies ($f \gg kT/h$) is an everyday experience in radio astronomy, where Josephson junctions are used as detectors.

Therefore, we conclude that the Fluctuation-Dissipation Theorem looks correct when a Josephson junction, *in the heterodyne detection mode*, measures the Johnson noise. Yet, since Heisenberg introduced the quantum uncertainty principle, we know that measurement statistics and noise can progressively be influenced by the fashion how the measurement is done, that is, by the instrumentation itself. For more details see the next sections.

One of the open, UPoN, questions is:

What would we see if we measure the Johnson noise with a real wideband (that is, non-heterodyne) voltage amplifier of flat amplification from near-to-DC up to the high-frequency cut-off?

For a conjecture about this question, see Section 5.

3. Former refutations of the existence of zero-point thermal noise

3.1. Experimental results not showing the zero-point term in thermal noise

3.1.1. The Voss-Webb experiment

Voss and Webb [12] analyzed the results of their careful experimental study [13] about the rate of Josephson junction switching from superconducting to normal conducting state at various temperatures. The switching occurs due to crossing the potential barrier and various excitation mechanisms were considered as trigger. The results were consistent only with thermal activation, that is, with Nyquist's (7). The measured switching rates were up to 9 orders of magnitude lower than the value implied by the FDT and its zero-point noise, see (8). We cite here the relevant part of their conclusion (omissions of the irrelevant parts are by us), which is in accordance with our own conclusion about the zero-point term in the FDT:

In conclusion, ... naive approaches (pair shot noise or zero-point Johnson noise in a "classical" Langevin equation) cannot be used to understand the limiting behavior as T approaches 0. Only a completely quantum-mechanical formulation (such as...) can give an adequate understanding of the experiments.

Using a more generalized wording, we arrive at a similar conclusion [4, 5]: During the measurement of thermal noise in the quantum limit, that is, when the quantum system (the resistor) is coupled with a classical physical (or mixed classical-quantum) object, that is to a measurement system, the noise generation is influenced by this measurement system, for example, according to Heisenberg's uncertainty principle. The FDT's suggestion, that is, assuming a physical, *objectively (independently of the measurement system) existing* noise generator in the resistor, as suggested by (8), may be attractive for easy electrical engineering design but it is unphysical in the quantum (zero temperature) limit. Using such a hypothetical quantum noise source in calculations with the response function of the measurement system can lead to wrong predictions. One of the clear-cut experimental proofs of this conclusion is the comparison of the results of the Koch *et al.*, experiment [10] with the Voss and Webb experiment [12]: The *same physical system*, but different measured quantity: One [10] "proves" the FDT, the other [12] disproves it by 9 orders of magnitude, in the low-temperature limit.

3.1.2. Blackbody radiation: the zero-point term is lacking

There is no mathematical reason to expect the zero-point energy for the thermal noise of a resistance but neglect it for thermal radiation noise, as they have the same mathematical foundation via $N(f, T)$.

However, as we have already mentioned above, the zero-point term is lacking from the thermal radiation. A simple experiment, closing our eyes and seeing darkness is a hard proof for that: With zero-point radiation, instead of darkness, we would be blinded by a light that is more intensive than the radiation density of the sun's surface [4].

3.2. Theoretical and conceptual objections against the existence of zero-point thermal noise in thermal equilibrium

In this section we briefly list some of the theoretical arguments against the zero-point Johnson noise and skip many others, such as [14–16]. For a more detailed survey, see [4].

3.2.1. Ground states do not radiate

The term responsible for the claimed zero-point noise is the zero-point energy. However even though zero-point motion can be detected by experiments that inject energy in the system, such as Raman measurements, the ground state does not emit energy. For example, electrons in stable (ground state) atomic shells do not radiate even though their motion and energy are considerable.

3.2.2. Perpetual motion machines

The assumption of the existence of zero-point noise in the "objective" Callen-Welton fashion [6–9], as a "generator" that is included in the resistor, see (8), makes various perpetual motion machines possible [4, 17–19]. Moving capacitor-plate engines, and resistor-antenna systems combined with standard blackbody radiation would be able to violate the Second Law of Thermodynamics and create net electrical power from the zero-point noise.

3.2.3. Fermi-Dirac statistics

The assumption of noise current at zero temperature would violate the Fermi-Dirac statistics of electrons [5], which is the foundation of the solid-state physics of electrical transport in metals and semiconductors.

3.2.4. Uncertainty principle as instrumental artifact

Kleen [18] pointed out that the measurement results in the Koch *et al.* experiments [10] can be deduced from the time-energy uncertainty principle: a finite-bandwidth voltage or current amplifier has a time inaccuracy due its bandwidth. Energy in a voltage amplifier is related to the square of voltage (across unavoidable capacitances) and in a current amplifier – to the square of current (in unavoidable inductances). Thus, the energy uncertainty leads to voltage and current noises. The corresponding energy uncertainty in the narrow-band (resonant) detection in the Koch *et al.* experiments [10] leads to the seeming existence of the 0.5 zero-point term in (8), however it is only an experimental artifact because measurements avoiding the time-energy uncertainty relation problem may not have zero-point noise.

Indeed, the force-in-capacitor scheme proposed in (4) is free from the time-energy uncertainty relation problem, and the incorrect assumption of the validity of (8) leads to a perpetual motion machine, which supports Kleen's point. See Section 4 for more.

Even though the quantum noise of amplifiers is an involved topic and the results depend on fine details, see [20–22], it is obvious that Kleen is basically correct. An open question is the situation in a real wide-band amplifier with a flat-amplification bandwidth ranging from DC to a high cut-off frequency. For a conjecture, see Section 5.

4. New arguments against the general existence of zero-point thermal noise concept

4.1. No zero-point noise in blackbody radiation: location-momentum uncertainty principle

The uncertainty relation in an optical spectrometer is not based on the time-energy relation. It is based on the location-momentum uncertainty. The location of measured photons when reflected from the optical diffraction grating is related to the spatial resolution of the grating. The resulting momentum uncertainty of the observed photons will be an uncertainty of the momentum (wavelength in an Einstein – de Broglie fashion). The effect is not a zero-point thermal noise but the ultimate linewidth resolution of the optical spectrum.

This argument supports Kleen's observation [18] that the observed zero-point noise in the Koch *et al.* experiment [10] is only an instrumental artifact. This is why other instruments, like optical spectrometers, or the same Josephson-junction instrumentation with a different observed quantity (Voss and Webb [12]) show the lack of zero-point thermal noise.

4.2. The FDT is for bosons not fermions

Igor Goychunk and Jerzy Luczka pointed out in the public discussions that the FDT is valid only for bosonic systems. That means the Callen-Welton [10] derivation is incorrect at its core because it talks about electrons (which are fermions) in resistors. All the commercial resistors are fermionic systems.

But, if we go further with this argument, we realize that there is a new unsolved problem: the parallel connection of fermionic and bosonic resistors (assuming that the second type exists). The two resistors must produce the same thermal noise spectral features; otherwise we have a new perpetual motion machine, see [3], (2) and Fig. 1.

5. Conclusion and still open problems

The correct formulation of the quantum term of the FDT requires taking account of the measurement instrument, too. The results of the Koch, *et al.* experiments [10], and similar ones since then, are a consequence of the time-energy uncertainty principle artifact. For more details, see Conclusions in [4].

However, there are still important unsolved problems. For example, the fermion-boson system, see Subsection 4.2.

Finally, we present a preliminary conjecture (estimation), see Fig. 3, of the *lower limit of the input voltage noise* spectrum of a wide-band voltage amplifier driven by an *RC* element in the zero-temperature limit. The estimation is based on the time-energy uncertainty principle and it is assumed that the *RC* cut-off frequency $(2\pi RC)^{-1}$ matches the amplifier's own (sharp) cut-off frequency. It is assumed that the spectrum analysis is carried out at the output of the amplifier, where the signal is already classical physical and, from that signal, the lowest limit of the equivalent input noise (the noise spectrum on the *RC*) is determined in the zero-temperature limit. The estimation is based on the energy-time uncertainty principle and details will be shown elsewhere. At the cut-off frequency, the zero-point noise spectrum would reproduce the FDT [6] and Koch, *et al.* [10] values. However, below the cut-off frequency, the noise has a flat spectrum and does not decrease linearly with frequency.

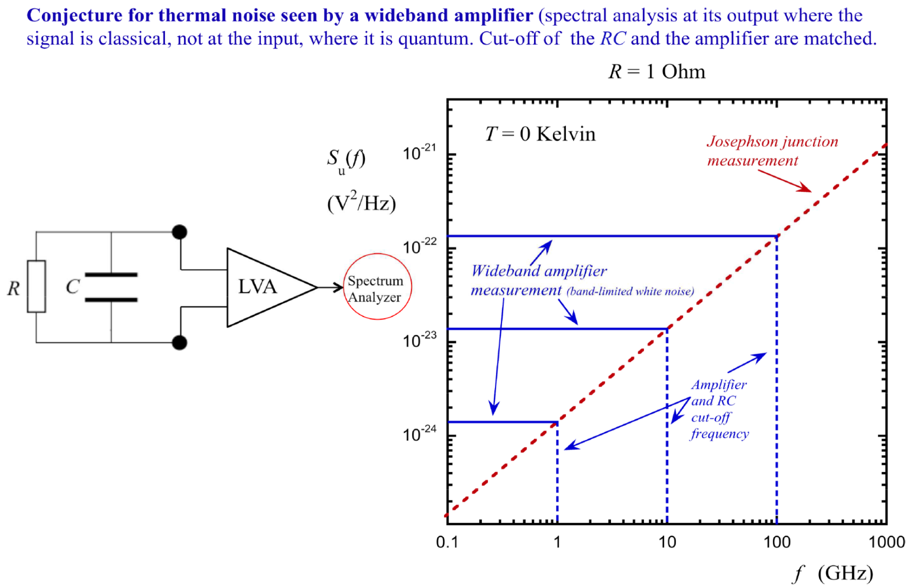

Fig. 3. A conjecture for an instrumental artifact of zero-point noise to be seen by a wide-band amplifier at various (sharp) cut-off frequencies. At the cut-off frequency, the zero-point noise spectrum would reproduce the FDT [6] and Koch, *et al.* [10] values. However, below the cut-off frequency, the noise has a flat spectrum and does not decrease linearly with the frequency but it stays high.

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