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Preliminary report on mineral suspension in the North Atlantic, the Baltic and the Norway Seas in summer

ABSTRACT: During the cruise of the research ship r/v *Oceania* owned by the Institute of Oceanology of the Polish Academy of Sciences in Sopot a research on mineral suspension concentration and dispersion distributions was conducted. The research area included the western part of the Baltic Sea, the Danish Straits, the Norwegian Sea, the waters around Spitsbergen and the North Atlantic Ocean. Samples of water were collected from the surface layer. They were subjected to microscopic analysis. Measurements were done with a projection microscope (magnification 1000x) and using the Burker's table. After counting the particles dispersion distribution was determined. The largest concentration of mineral suspension was noted offshore in the Norwegian Sea and around Spitsbergen and the smallest in the central Atlantic Ocean.

Key words: North Atlantic, mineral suspension, concentration, dispersion distribution.

Introduction

Mineral suspension is a main component of sea water. It is composed of particles with diameters larger than 0.1mm (Ivanoff 1978). The particles are carried from the land into sea by rivers and wind or washed out from a continental shelf during storms. Particles of mineral suspension differ one from another in size, chemical constitution, structure and physical properties. They form different dispersion distributions. Concentration and dispersion distributions of suspension depend on many factors such as dynamical and geographical setting and capacity of their sources, distance from the observation point to the source, dynamics of water region and its depth, and dynamics of meteorological processes (Gurgul 1991, 1996, 1998).

Sea currents from coasts of North America and Scandinavia carry suspension particles in waters of the North Atlantic. Concentration and dispersion distributions of suspension in West Spitsbergen waters depend on dynamic processes: especially on coastal currents, waves and glacier melting. Mineral suspension gets into fiords with meltwaters and is dragged by wind (Wassman *et al.* 1996).

Mineral suspension becomes dissolved, oxidated and biologically etched, which changes its concentration, size and dispersion distributions (Horne 1972). In this way the mineral suspension and substances dissolved in water influence the main physical and chemical properties of sea water such as: density, heat properties, electric condition and others (Dera 1983; Gurgul 1996).

Size, cross-section and volume are important geometrical parameters of suspension. Many particles have surfaces of very developed irregular solids. It influences increase of chemical activity of suspension, so that it is subjected to chemical and biochemical processes in a deep sea.

Suspension concentration in water is a most important parameter. There are about 10^9 suspended particles per cubic metre, with diameters larger than 1 mm in a clear sea water. The concentration in littoral waters exceeds 10^{13} particles/m^3 (Jerlov 1976; Dera 1983). The suspension concentration changes with latitude. In the area between the English Channel and the King George Island the largest concentration of suspension appeared in the English Channel and the Drake's Passage while the smallest concentration occurred in the intertropics (Gurgul *et al.* 1993). A small concentration of mineral suspension in this region results from insignificant inflow of river water and a long distance from the land. Within a year cycle the highest concentration of suspension in waters of the Ezcurra Inlet (King George Island) was recorded in March 1989 during intensive melting of glaciers and high precipitation. The concentration was 1.4×10^{12} particles/m^3 (Gurgul 1993; Gurgul *et al.* 1995).

Mineral and organic suspension, oil emulsion and gas bubbles often form aggregates which may activate sedimentation or influence their shift to distant places (Kopeć *et al.* 1990; Gurgul 1993).

Dispersion distribution presents contents of suspension particles in a given sample per a given diameter or it illustrates distribution of probable occurrence of particles of a given size in the tested sample. In the open sea and at long distances from the land the dispersion distributions are "short". It means that the range of sizes is small. In other words there are almost the small particles only in a water sample. In river and coastal waters there is a long range of particle sizes and dispersion distributions are called "long". Dispersion distributions are very important while analysing the molecular, optical and acoustic processes.

The aim of this paper is a preliminary analysis of the size distributions and concentration of mineral suspension in the North Atlantic and West Spitsbergen waters.

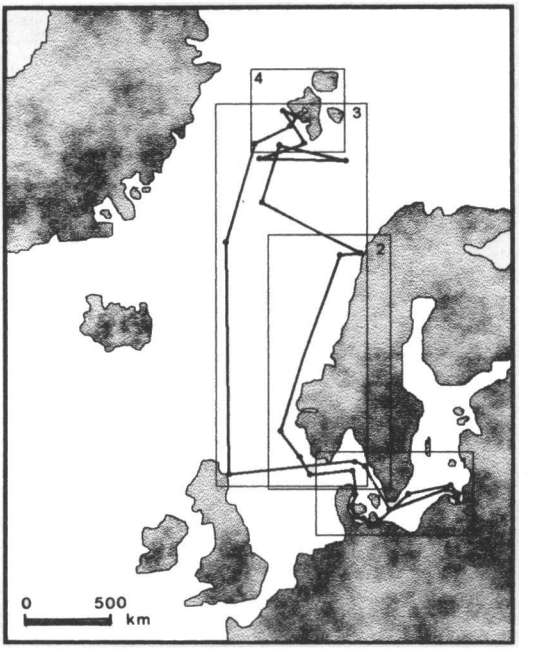

Fig. 1. The cruise of r/v *Oceania* (06.06–09.08.1996) from Gdańsk through the Danish Straits, the North Sea and the Norwegian Sea to West Spitsbergen; for polygons 1–4 see Figs 7–10.

Location and methods

The examination was carried out in June–August 1996 during a cruise of the research ship r/v *Oceania*, owned by the Institute of Oceanology of the Polish Academy of Sciences in Sopot. The research area included the North Atlantic Ocean and fiords of Spitsbergen (Fig. 1).

Water samples were collected from the surface layer to depth of 0.5 m. Simultaneously, the co-ordinates of measurement points were determined. Temperature of the collected water was measured with accuracy of ±0.1°C. Then the samples were subjected to microscopic analysis with the microscope MP-3 (magnification 1000x) using the Burker's table. Diameters of the investigated mineral suspension were determined with the Foret's method (Figs 2 and 3). Counted suspension particles were divided into classes which differed one from another by 1mm in diameter. The proper class of particles included those which satisfied the relation $d \in (u-1,u)$. Suspension particles of diameter 1 μm and larger were counted.

The Foret's method to measure the suspended particle diameters requires the co-ordinate system with a metric scale. The suspension is introduced onto the screen. One unit of scale under the magnification 1000 corresponds to 1 μm. The edges of a particle need to be tangent to the extreme parts of it. The approximated Foret's diameter is just the distance between the two farthest tangentials.

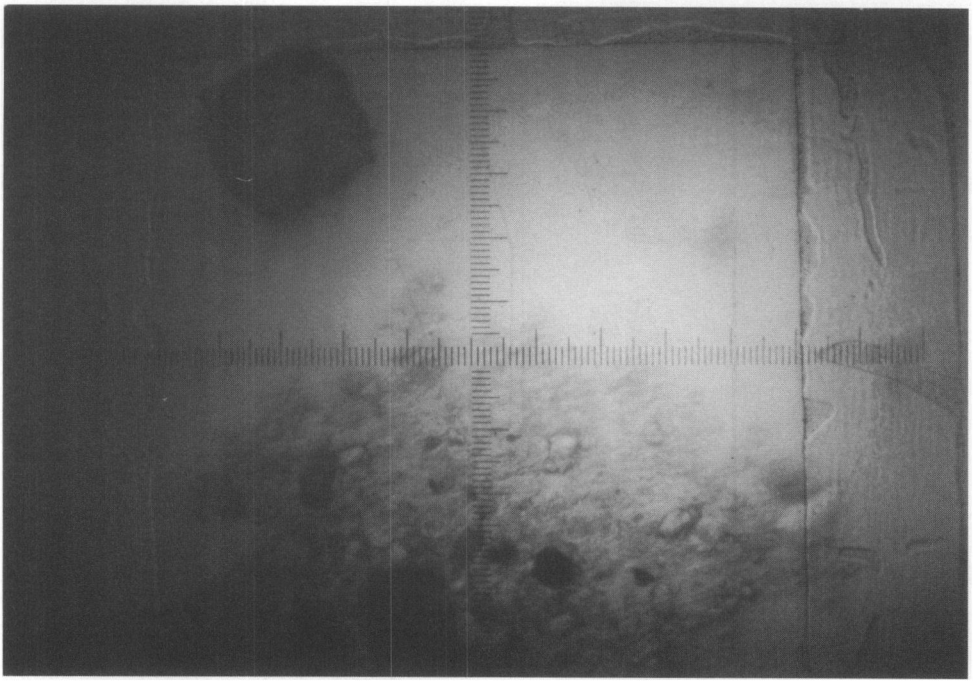

Fig. 2. Microscopic image of mineral particles (magnification 1000x) that form a crystal. Edges are regular and surfaces are almost of the same colour. Organic particles are irregular in shape and their colour is varying.

As it has been mentioned in the introduction, there is mineral and organic suspension in the water. They differ from each other in shape, colour and structure. Particles of mineral suspension possess angular edges. Equal width plates are of uniform colour in many cases. If a width changes without overlapping of a couple of crystal layers, then there is a sudden change of colour and the refraction index. Organic suspended particles have irregular, jagged, tangled and extended shapes which makes their colour variable and their boundaries very complicated.

The concentration of the suspension matter in particular classes was also counted. Dispersion distributions were obtained by counting from 50 fields on a screen. A detailed description of the measurements is presented in Gurgul and Stochmal (1985).

Results

A small range of dispersion distributions is characteristic for "short" distributions. In the samples of water collected at the stations in profiles I, II and III (Fig. 7) mainly "short" distributions appeared. This type of dispersion distributions oc-

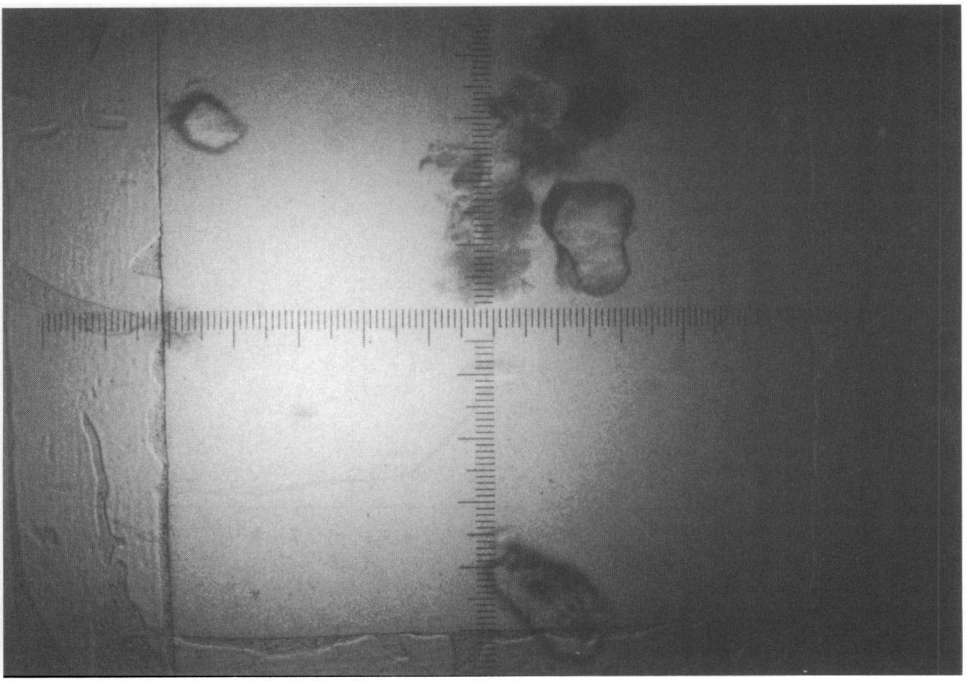

Fig. 3. Microscopic image of a mineral particle. A scale unit is 1 μm. In order to measure a Feret diameter a number of units from the left to the right edge of a particle is to be counted along the horizontal axis of the microscope screen.

curred mainly in waters of West Spitsbergen Current whereas "long" distributions were very rare. However, in the coastal region of West Spitsbergen "long" distributions occurred more often. It was caused by a flow of suspended matter from melting glaciers and in streams.

The greater number of "short" dispersion distributions of suspension had the range of their sizes from 1 to 6 μm, but in some distributions the range was even 1–4 μm. In "long" distributions the sizes of some particles exceeded several micrometers, but usually they ranged from 1 to 20 μm.

These investigations were carried out in different regions. The region of the Baltic Sea and the Danish Straits was one of them (Figs 5a, b). The points of water samples are marked on the map. All indicated values must be multiplied by 5×10^{11} particles/m^3 and it gives the total concentration of suspension in a cubic metre of water.

These results show that the largest concentration of mineral suspension was in the waters of the Gulf of Gdańsk ($253\times5\times10^{11}$ particles/m^3) and Skagerrak ($238\times5\times10^{11}$ particles/m^3). In the central part of the Baltic Sea and in the Danish Straits the concentration was smaller and it ranged from 50×10^{11} particles/m^3 to $98\times5\times10^{11}$ particles/m^3. The concentration was larger in coastal areas, shoals and

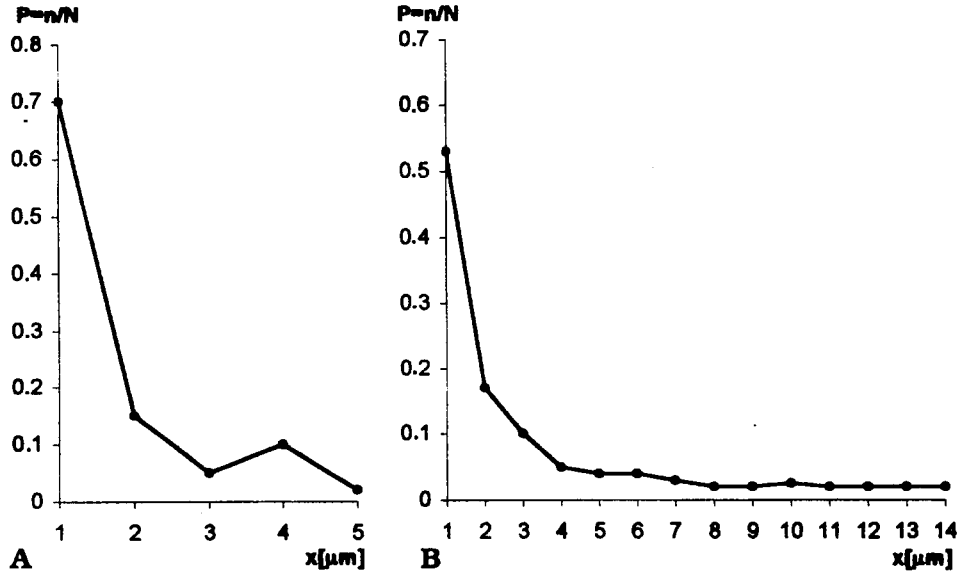

Fig. 4. Dispersion distributions of suspension: "short" (A), "long" (B).

in river mouths and it ranged from 10^{12} to 10^{14} particles/m^3. In this region the concentration of mineral suspension was noted to the north of Bornholm, in waters of the Gulf of Kiel and along the western coast of the Jutland Peninsula. A small concentration of suspension was observed in the eastern Kattegat and in the southern Skagerrak, and it ranged from 10^9 to 10^{11} particles/m^3.

Another region where the concentration of suspension was examined was the north-western part of the Atlantic Ocean (Fig. 6). Samples of water collected from the Norwegian Sea had the same concentration of mineral suspension as those taken from the Baltic Sea. The concentration ranged from $90\times5\times10^{11}$ to $950\times5\times10^{11}$ particles/m^3. Higher concentration was noted in the southern part of the sea. The highest concentration was in waters surrounding the fiords in the southern part of the Scandinavian Peninsula i.e. about $116\times5\times10^{11}$ particles/m^3. A considerably smaller concentration was found in the water samples collected at the northeastern coast of Iceland. The highest concentration of suspension in waters of the Atlantic Ocean was $7\times5\times10^{11}$ particles/m^3. A larger concentration was in the water samples collected at the junction of the North Sea and the Norwegian Sea and near the Bear Island. Large changes in concentration caused by dynamics of sea waters, the inflowing waters from melting glaciers and fiords were observed in a coastal area of the Scandinavian Peninsula (profile I). In waters of the open sea a varying concentration was insignificant (profile II) (Fig. 1).

In waters of the Atlantic Ocean the investigations of mineral suspension were conducted in three regions i.e. in West Spitsbergen (1P), North Atlantic to Spitsbergen (2P) and Greenland Sea (3P).

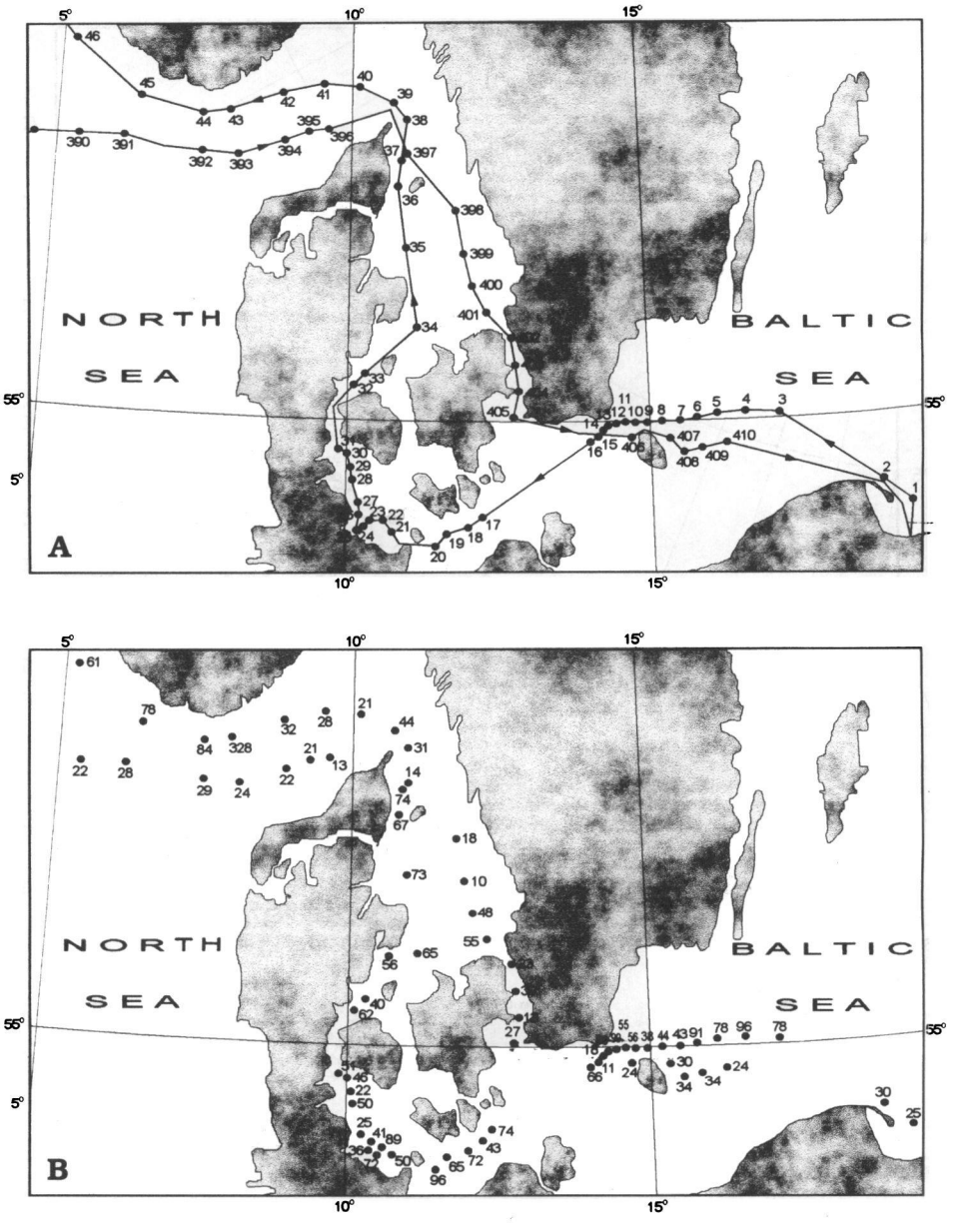

Fig. 5. **A.** Approximate cruise of *Oceania* from the Baltic Sea, Danish Straits and the North Sea. The numbers indicate sampling sites. **B.** Measurement sites and present amount of mineral suspensions in 1 m^3 (the figures should be multiplied by 5×10^{12} units/m^3).

In the region 2P (Fig. 10) there were considerable changes in concentration of mineral suspension. The largest was $157\times5\times10^{12}$ units/m^3, and the smallest $6\times5\times10^{12}$ units/m^3. It means that the difference between those extreme values is

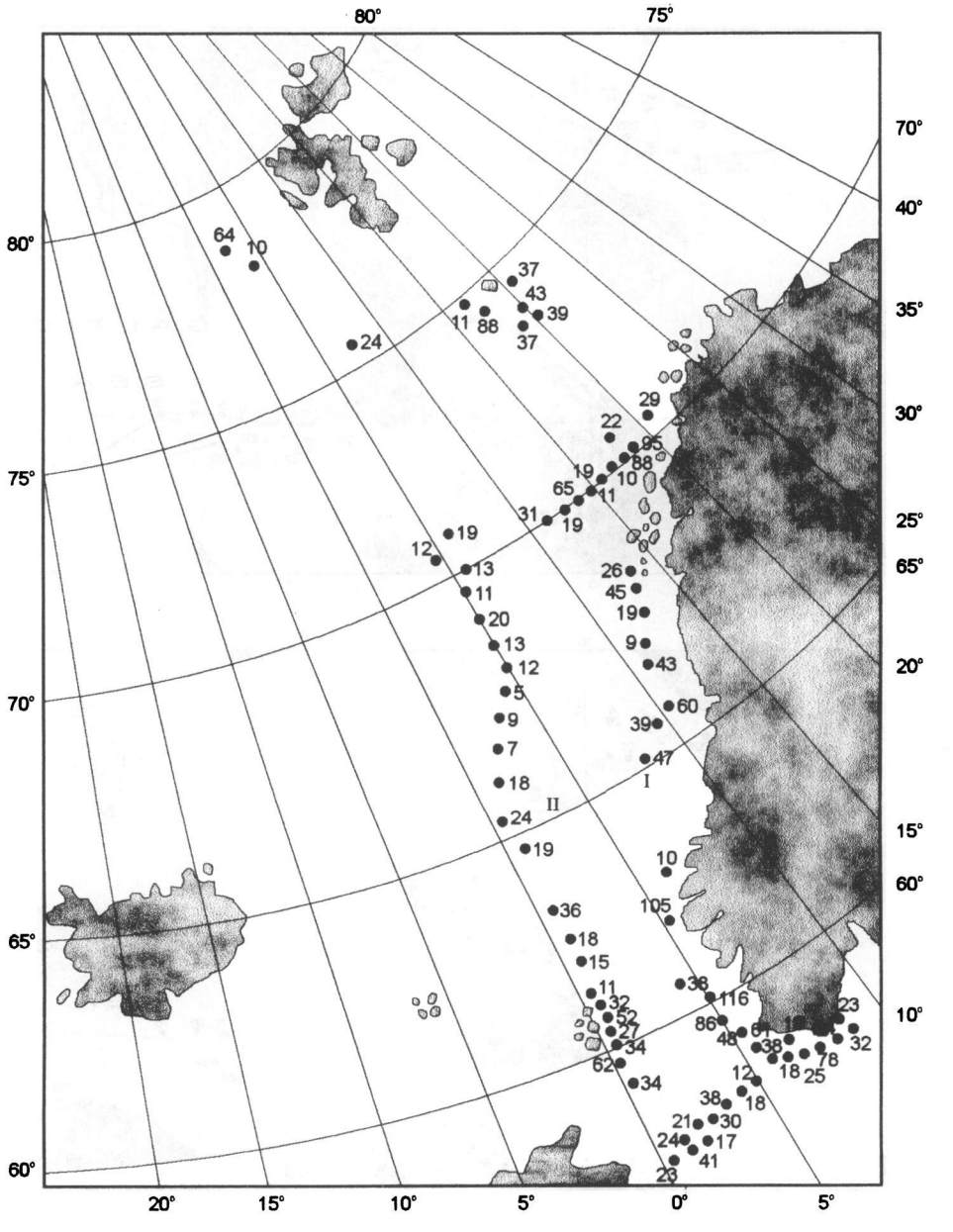

Fig. 6. Water sampling sites: the numbers (to be multiplied by 5×10^{12} units/m^3) refer to concentration of mineral suspension.

two grades. However, in the northern Atlantic a considerable change of concentration was found, while in the southern one a concentration was almost stabile.

In the region 3P (Fig. 9) the changes of concentration are almost insignificant. The biggest changes were recorded in the southwestern part. The smallest changes

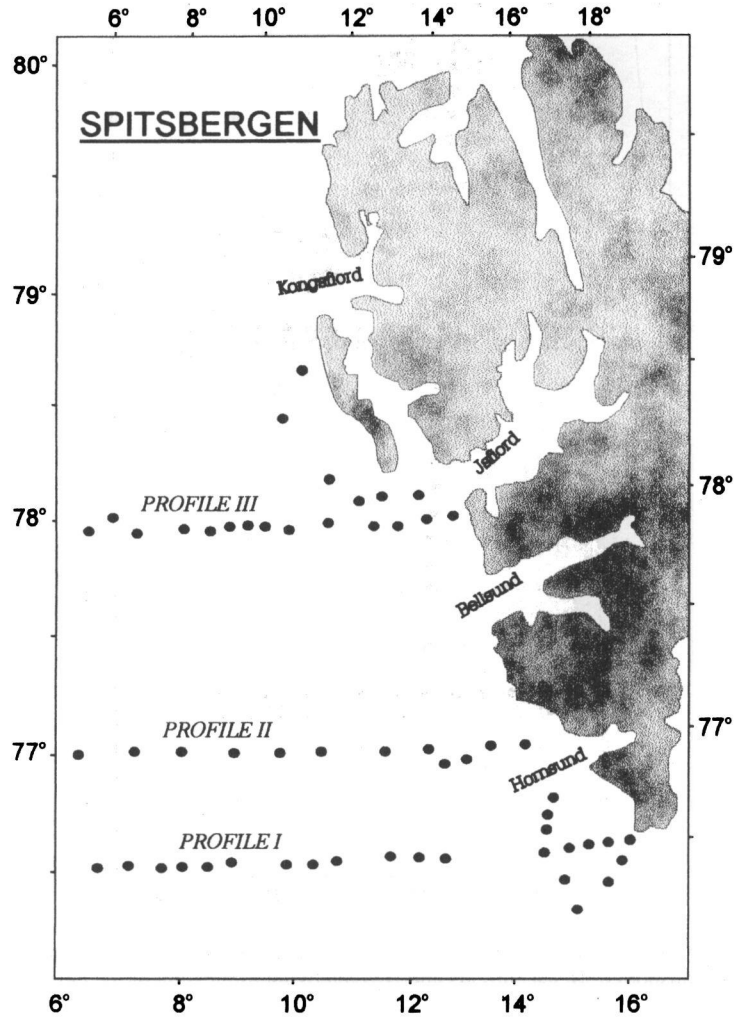

Fig. 7. Examined profiles (I – along 76°30'N, II – along 77°N, III – along 78°N) in the Spitsbergen Current (polygon 1 in Fig. 1) which flows from the south to the north along the western coast of Spitsbergen.

were measured in the central part of the region. Locations of the water mixing can be determined basing on concentration of suspension of different waters. Waters flowing into the southwestern part of the region 3P transport more particles of suspension in the surface layer than in the middle part (Fig. 9). The biggest gradients of concentration occurred in the south-west.

Research of the same type were also conducted in waters to the south of West Spitsbergen (Fig. 8). In this region the West Spitsbergen Current exists. Its waters are mixed with waters of the East Spitsbergen Current. The examination was carried out in three parallel profiles. In the regions adjacent to Spitsbergen there is a

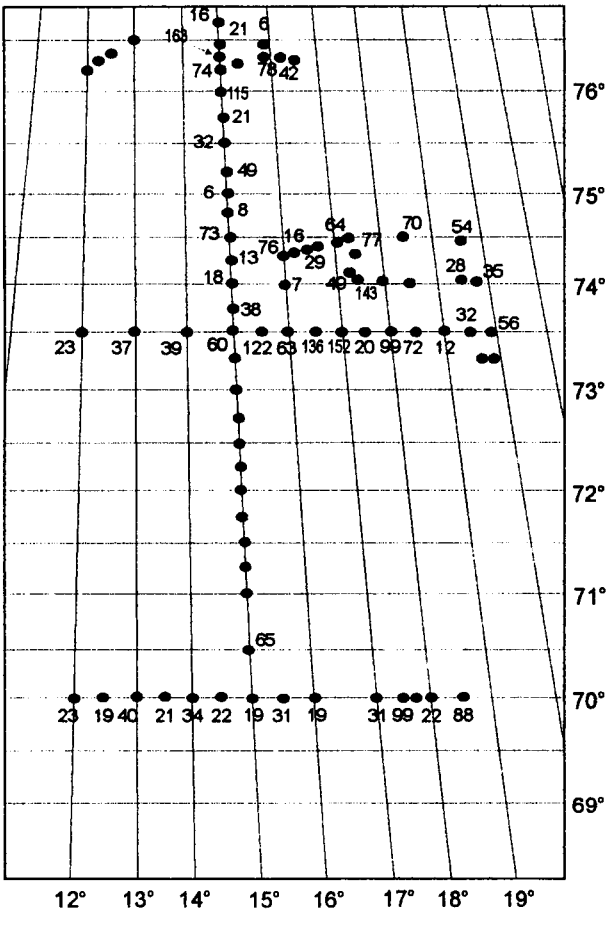

Fig. 8. Water sampling sites in the open sea (polygon 2 in Fig. 1). Suspension concentration depends on the amount transported by sea currents and is given by multiplication of the numbers by 5×10^{12} units/m^3.

high concentration of suspension. Far away from the shore the concentration decreases, whereas it increases in the west. To the north the concentration of suspension decreases. The changes of concentration suggest that the waters mix in a coastal zone of Spitsbergen and the southern part of region 1P (Fig. 10). The transport of suspended particles becomes smaller towards the north. In the southern part of the region there are the biggest gradients of concentration. A similar phenomenon is observed in the coastal area of Spitsbergen. The concentrations as well as its gradients decrease towards the north. The concentration of suspension is influenced by meltwater inflow.

In the profile I there regions in which concentration of suspended particles is higher than 60×10^{11} particles/m^3 and it reaches even 80×10^{11} particles/m^3. The concentration on the almost whole part of the southern section exceeds 40×10^{11}

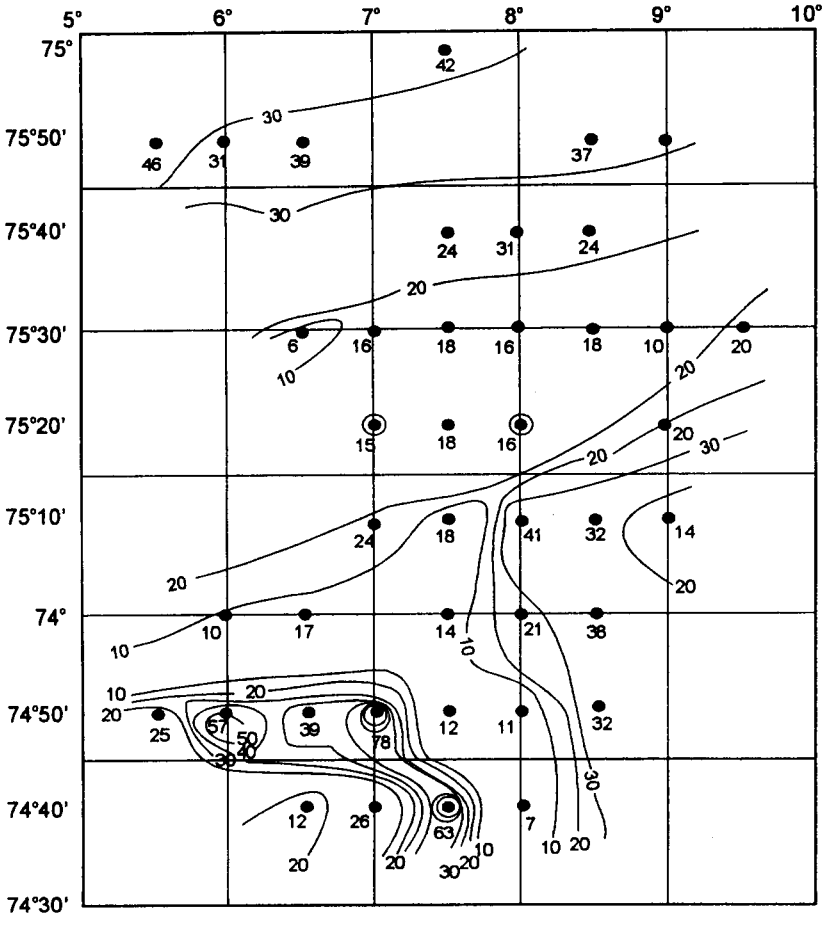

Fig. 9. Water sampling sites in the open sea (polygon 3 in Fig. 1). The distribution of the suspension concentration isolines allows to determining water types and their mixing (most intensive to the southwest). Concentration of mineral suspension is given by multiplication of the numbers by 5×10^{12} units/m^3.

particles/m^3. Towards the north the concentration decreases and in the profile III only in a single point exceeds 60×10^{11} particles/m^3. In other sampling sites it varies from 40 to 20×10^{11} particles/m^3. Sometimes it is even smaller. Decreasing of the concentration of mineral particles is considerably noticeable in the profile III. At the coast (Isfiord) the concentration exceeds 80×10^{11} particles/m^3. Several kilometres from the land the concentration decreases to less than 20×10^{11} particles/m^3.

Preliminary analysis of dispersion distributions of suspension shows that in the profile I there is not any noticeable superiority of a "short" distribution to a "long" one and *vice versa* (Fig. 11). It is seen easily from the profile II, where there are mainly "short" distributions. The same is recorded in the profile III. In both cases "long" distributions usually occur in a coastal zone.

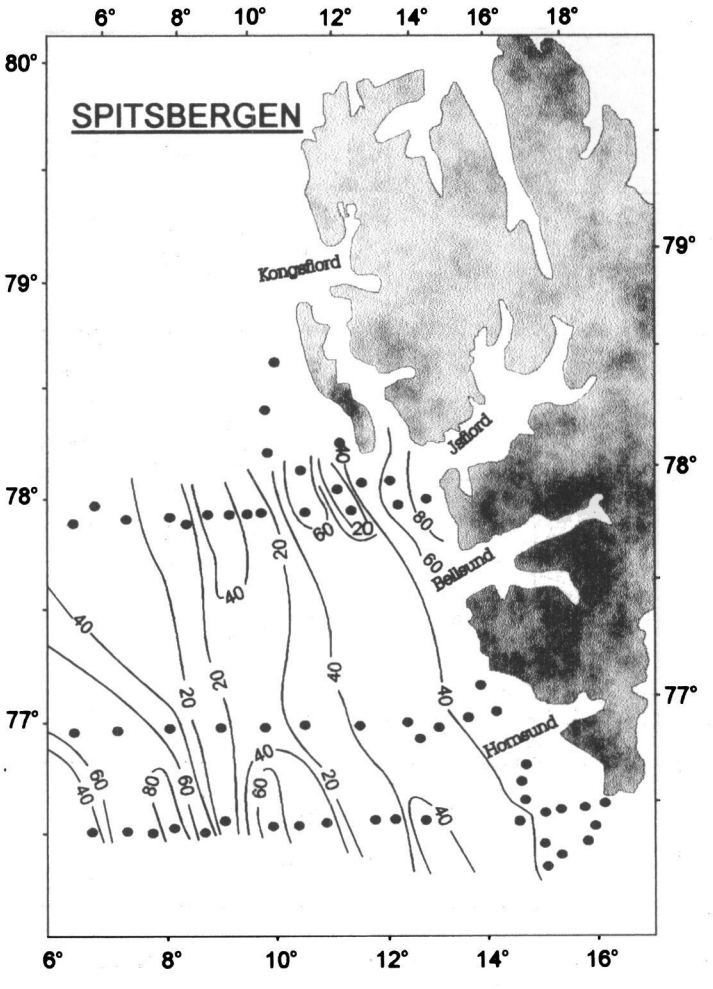

Fig. 10. Distribution of the suspension concentration isolines in waters of the West Spitsbergen Current (polygon 4 in Fig. 1). The numbers are to be multiplied by 5×10^{12} units/m^3. The biggest concentration of suspension occurs in the south, close to the polygon boundary.

Simultaneously, it can be claimed that dispersion distributions of suspension are usually geometric (Fig. 12) as it is in the profiles I and II. Only in single and rare cases there are modified geometric distributions. In the profile III both types of the distributions are recorded.

The concentration of suspended matter and the probability of its occurrence in geometric dispersion distribution decreases exponentially with the increase of the diameters of the suspended particles. In the geometric modified dispersion distribution there is a local maximum concentration of the suspended particles. It means that there is the range of particle sizes in which there is the highest concentration of suspension whereas in the other ranges the concentration is smaller.

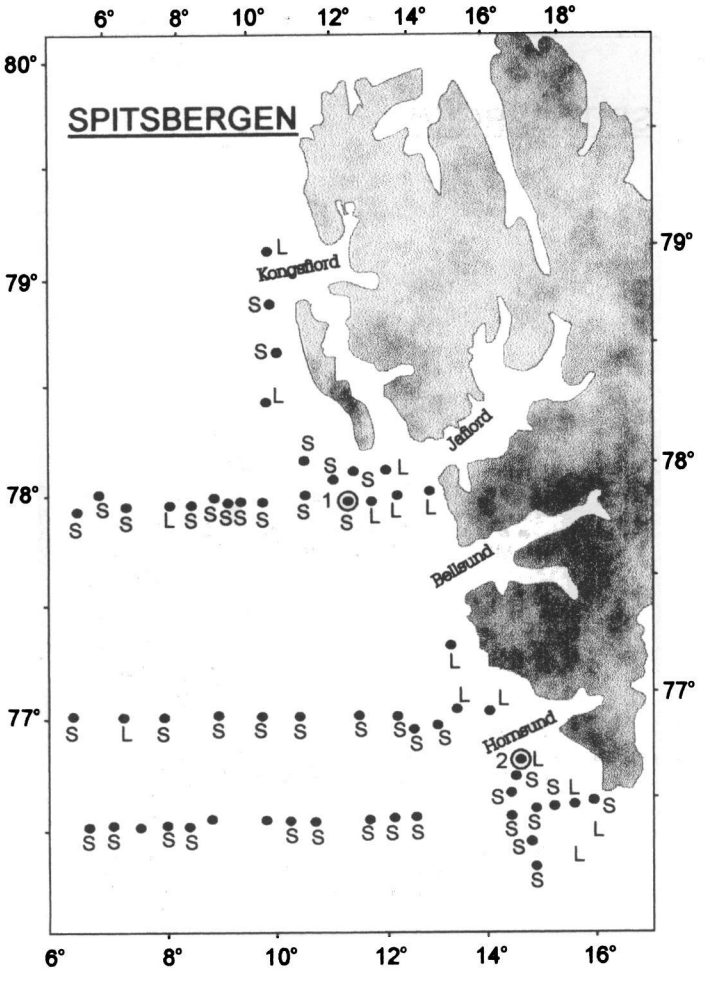

Fig. 11. Dispersion distributions extracted from microscopic measurements of mineral distribution divided into "short" (S) and "long" (L) distributions. "Short" distributions are characteristic for the open sea and refer to suspensions with particles to a few micrometers large. "Long" distributions are typical for coastal areas and refer to suspensions with particles up to 100 μm.

Conclusions

The author presented examination of suspension concentration in the waters of the North Atlantic and dispersion distributions in waters of the West Spitsbergen Current.

Suspension concentration and its horizontal gradient are the characteristics of sea waters. Larger concentration of mineral suspension occurred in coastal waters of Norway and Spitsbergen. It means that particles of mineral suspension are carried from the land or from melting glaciers by the rivers or meltwater streams. In

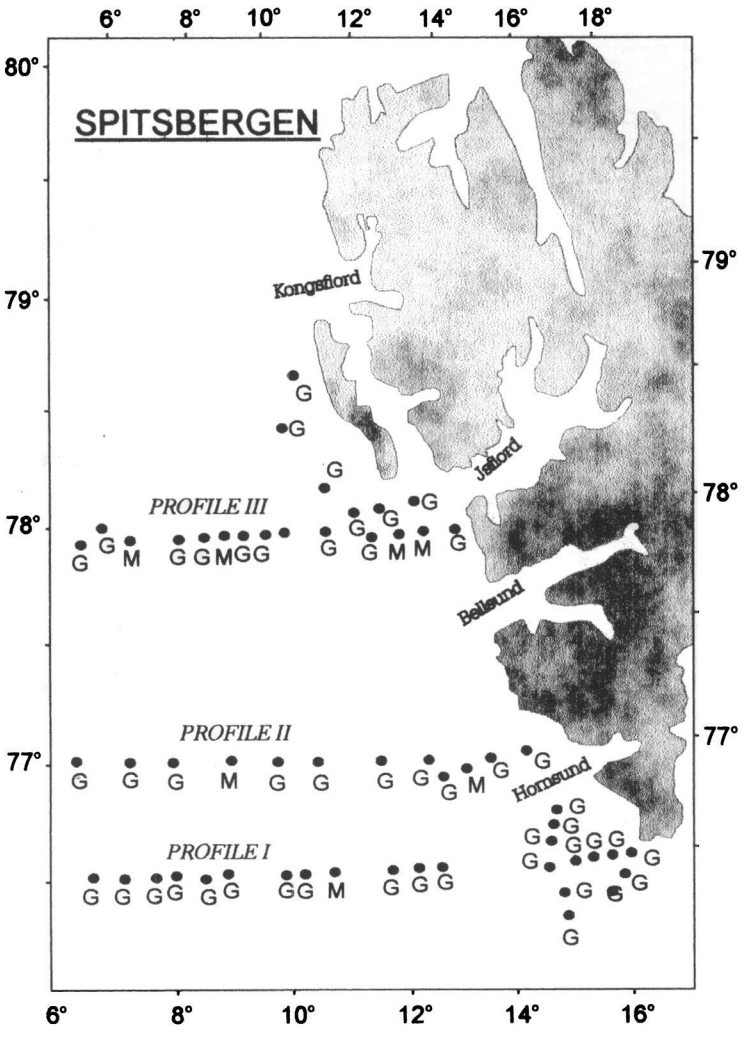

Fig. 12. Geometric distributions "G" are more frequent than the modified geometric distributions "M".

waters of West Spitsbergen the concentration of suspension is at least a grade larger than the concentration in waters of the West Spitsbergen Current. It is caused by inflow of meltwater from glaciers into the Spitsbergen fiords and then into the open sea. In order to explain this fully, similar invesigations should be conducted before glaciers melt and calve, and also at the turn of August, during the advanced ablation season.

Horizontal gradient of suspension concentration shows variety of sea waters, their origin and the fact that they mix. Based on surface distribution of the suspension concentration, these phenomena can be fully described using, for example, surface distribution of temperature and salinity of water.

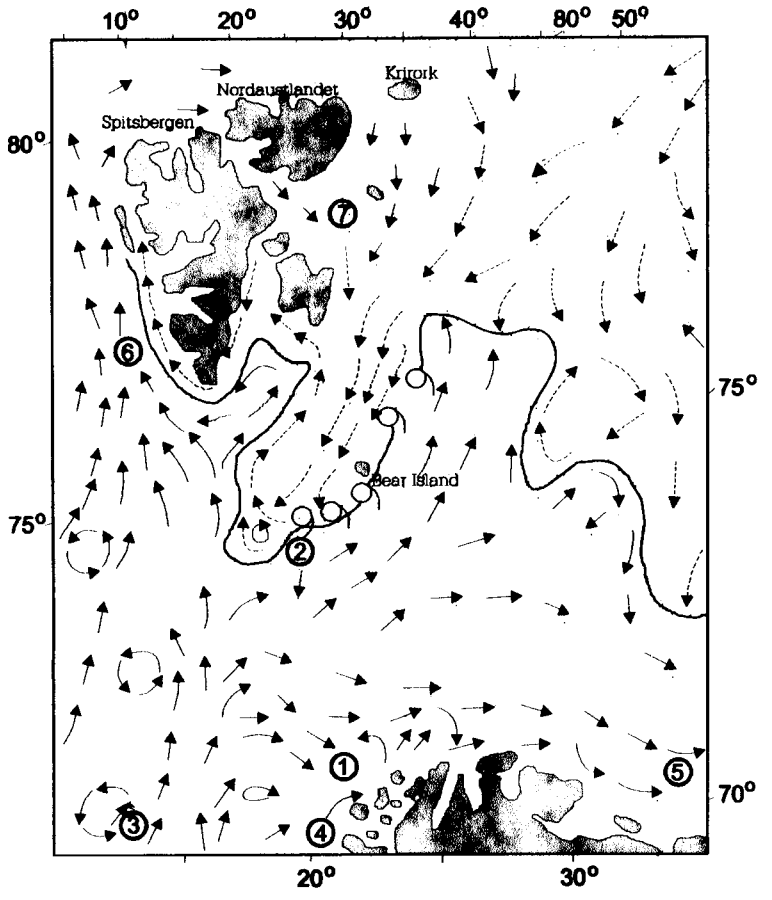

Fig. 13. Surface currents in the northeastern Atlantic: 1 – Tromsr Flaket; 2 – Bjrrnoyrenna, North Cape Current; 3 – Norwegian Atlantic Current; 4 – Norwegian Coastal Current; 5 – Murman Current; 6 – West Spitsbergen Current; 7 – East Spitsbergen Current (Den Norske Los, Arctic Pilot, 1990). Current directions and water mixing are indicated.

Analysis of structure and chemical composition of mineral suspension allows to find out where the suspension comes from and its transport path. Similar investigations should be carried out to the south of Spitsbergen (Fig. 13). After conducting investigations on the crosses including the West Spitsbergen Current at latitudes below 75°N and repeating on the ones presented in this paper, it will be possible to determine the transport of mineral suspension from the Norwegian Sea into the West Spitsbergen waters.

The problem of dispersion distributions has not been solved yet. There are a lot of mathematical descriptions and none of them is accurate. In the log-log method two linear functions are required. In this paper only a single one i.e. a function method of mathematical dispersion distribution was presented. A review of mathematical method of dispersion distributions deserves a separate paper.

Size distribution of suspension

The dispersion distributions of suspension are formed by repeatable mechanisms, so that they are described by different functions. Many publications give detailed description of the suspension (Gurgul 1988, 1991; Kopeć *et al.* 1990).

More variable dispersion distributions occurred, for example in a coastal area during storms while they were much more stable in the open sea. Dispersion distributions change with depth and can be divided into "long" and "short" distributions. "Long" dispersion distributions include those, a range of suspension of which (a, b) consists of more than a few classes. "Short" distributions are those which range (a, b) is up to a few classes of size. "Short" dispersion distributions occur in the open sea (Fig. 6a) and "long" appear in a coastal area during storms or when glaciers melt (Fig. 4b). The largest concentration of suspension is observed if "long" dispersion distributions occur.

In a value range of the size parameter (a, b) the concentration of suspension is described as (Gurgul and Kopeć 1985):

$$N_{\alpha\beta} = k(\alpha^{-c} - \beta^{-c}) = kc\int_{\alpha}^{\beta} x^{c-1}\,dx, \tag{1}$$

where: k – coefficient, x – size of suspension.

The equation (1) describes a continuous distribution of the size parameter of suspension. The size of a given suspension is a multiple of the smallest piece on a scale of a microscope. It leads to a formal discreteness of the distribution which is normally continuous. Therefore, discrete distributions:

$$P(x=n) = p_n = \frac{1}{f(\lambda)} a_{n-1}\lambda^{n-1} \qquad (n = 1,2,...) \tag{2}$$

where:

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \qquad a_n > 0 \qquad (n = 0,1,2,...), \tag{3}$$

$0 < \lambda < r$,

r – radius of discrepancy of series (b), were searched.

When *f(x)* does not depend on any additional numerical parameter, the equation (2) describes a one-parameter-family of distributions. Otherwise, coefficients of expansion (3) also depend on the parameters and the family (6) is polyparametral. Simple calculations of moments make distributions (3) very useful. After differentiating and multiplying by l we obtain:

$$\alpha_1 = \bar{x} = 1 + \lambda\frac{f'(\lambda)}{f(\lambda)} = 1 + \lambda\frac{d\ln f(\lambda)}{d\lambda}, \tag{4}$$

$$\alpha_2 = 1 + 3\lambda\frac{f'(\lambda)}{f(\lambda)} + \lambda^2\frac{f''(\lambda)}{f(\lambda)}, \tag{5}$$

$$\alpha_3 = 1 + 7\lambda \frac{f'(\lambda)}{f(\lambda)} + 6\lambda^2 \frac{f''(\lambda)}{f(\lambda)} + \lambda^3 \frac{f'''(\lambda)}{f(\lambda)}, \tag{6}$$

These relations allow estimating the parameters by the moment method. For parameters l and m the equations (4)–(6) are further simplified by replacing:

$$g(\lambda,\mu) = \frac{\delta \ln f\mu(\lambda)}{\delta\lambda}, \tag{7}$$

In order to determine parameters by the moment method, the system of equations (4)–(6) can be rewritten as:

$$\lambda g(\lambda,\mu) = \alpha_1^* - 1,$$

$$\lambda^2 \frac{\delta q}{\delta\lambda} = \alpha_2^* + 1 - \alpha_1^* - \alpha_1^{*2}, \tag{8}$$

where α_1^* and α_2^* are adequate moments in a sample.

The maximum likelihood method usually leads to the most complicated equations. Under the assumption that different realisations $k_1, k_2, ..., k_q$ of random variable x correspond to different sizes $r_{k1}, ..., r_{kq}$ and likelihood function is denoted by $L(\lambda,\mu)$, the relation for distributions of family (2) is obtained as follows:

$$\frac{\delta \ln L(\lambda,\mu)}{\delta\lambda} = \frac{1}{\lambda}\left(\sum_{i=1}^{q} r_{kq} k_i - n\right) - n \frac{\delta \ln f\mu(\lambda)}{\delta\lambda},$$

$$\frac{\delta \ln L(\lambda,\mu)}{\delta\mu} = \sum_{i=1}^{q} r_{ki} \frac{\delta \ln a_{ki-1}}{\delta\mu} - n \frac{\delta \ln f\mu(\lambda)}{\delta\mu}, \tag{9}$$

This gives the following system of equations that determine estimators:

$$\lambda \frac{\delta \ln f\mu(\lambda)}{\delta\lambda} = \alpha_1 - 1,$$

$$\sum_{i=1}^{q} r_{ki} \frac{d \ln a_{ki-1}}{d\mu} - n \frac{\delta \ln f\mu(\lambda)}{\delta\mu} = 0, \tag{10}$$

The equations (9) and (10) indicate that an estimator calculated by the moment method for a one-parameter distributions of family (2) can also be calculated by the maximum likelihood method. It means that it has good statistical properties.

Well-known distributions of the type (2) are geometric and Poisson's distributions. For geometric distribution function f(x) can be written as:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \; p_n = (1-\lambda)\lambda^{n-1} \quad (n = 1,2,...), \tag{11}$$

According to (8), we have:

$$\alpha_1^* = 1 + \frac{\lambda}{1+\lambda}, \quad \text{hence } \lambda = 1 - \frac{1}{\alpha_1^*},$$

where: λ – estimator obtained with the maximum likelihood method.

For the Poisson's method the function f(x) can be written as follows:

$$f(x) = e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad p_n = e^{-\lambda} \frac{\lambda^n}{n!}, \qquad (n = 0,1,2,...), \quad \lambda = \alpha_1^*, \tag{12}$$

Results of microscopic investigations proved that this distribution could not be used to describe dispersion distributions of suspension. Therefore, apart from geometric distribution, modified geometric and Poisson's distribution were considered.

$$f(x) = (1+x)e^{x}, \quad p_n = \frac{e^{-\lambda}}{1+\lambda} \lambda^{n-1} \frac{n}{(n-1)!}, \quad (n = 1,2,...), \tag{13}$$

$$\hat{\lambda} = \frac{1}{2}\left[\alpha_1^* - 3 + \sqrt{(\alpha_1^* - 1)^2 + 4}\right],$$

$$f(x) = \frac{e^{x}}{1-x}, \quad p_n = (1-\lambda)\lambda^{n-1} e^{-\lambda} \sum_{\nu=0}^{n=1} \frac{1}{\nu!}, \tag{14}$$

$$\hat{\lambda} = \frac{1}{2}\left[\alpha_1^* + 1 - \sqrt{(\alpha_1^* - 1)^2 + 4}\right]$$

The estimator $\hat{\lambda}$ is determined and it is positive for every $\alpha_1^* > 1$.

The distribution (11) is monotonic for every α_1^*, the distribution (13) for $\alpha_1^* <$ 1.833..., and the distribution (14) for $\alpha_1^* < 2.5$.

Other methods that describe dispersive distributions can be found in Zalewski (1974), Kopeć and Szuman (1986) and Gurgul (1991).

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Supplement

1. From definition:

$$\alpha^2 = \sum_{n=1}^{\infty} np_n = \frac{1}{p(\lambda)} \sum_{n=1}^{\infty} na_{n-1} \lambda^{n-1}$$

$$\lambda f(\lambda) = \lambda = \sum_{n=0}^{\infty} a_n \lambda^n = \lambda \sum_{n=1}^{\infty} a_{n-1} \lambda^{n-1} = \sum_{n=1}^{\infty} a_{n-1} \lambda^n$$

Differentiating:

$$f(\lambda) + \lambda f'(\lambda) = \sum_{n=1}^{\infty} na_{n-1} \lambda^{n-1} , \tag{1}$$

dividing by $f(\lambda)$:

$$1 + \lambda \frac{f'(\lambda)}{f(\lambda)} = \frac{1}{f(\lambda)} \sum_{n=1}^{\infty} na_{n-1} \lambda^{n-1} = \alpha_1$$

Multiplying by λ:

$$\lambda f(\lambda) + \lambda^2 f'(\lambda) = \sum_{n=1}^{\infty} na_{n-1} \lambda^n$$

Differentiating:

$$f(\lambda) + 3\lambda f'(\lambda) + \lambda^2 f''(\lambda) = \sum_{n=1}^{\infty} n^2 a_{n-1} \lambda^{n-1} , \tag{2}$$

dividing by $f(\lambda)$:

$$1 + 3\lambda \frac{f'(\lambda)}{f(\lambda)} + \lambda^2 \frac{f''(\lambda)}{f(\lambda)} = \alpha_2$$

Multiplying (2) by λ and differentiating, and then dividing by f:

$$\lambda f(\lambda)+3\lambda^2 f'(\lambda)+\lambda^3 f''(\lambda)=\sum_{n=1}^{\infty} n^2 a_{n-1}\lambda^n$$

$$f(\lambda)+f\lambda f'(\lambda)+6\lambda^2 f''(\lambda)+\lambda^3 f'''(\lambda)=\sum_{n=1}^{\infty} n^3 a_{n-1}\lambda^n$$

$$1+7\lambda\frac{f'(\lambda)}{f(\lambda)}+6\lambda^2\frac{f''(\lambda)}{f(\lambda)}+\lambda^3\frac{f'''(\lambda)}{f(\lambda)}=\alpha_3$$

2. Because $g(\lambda,\mu)\,f\mu(\lambda)=f(\lambda)$, so:

$$\frac{\delta g}{\delta\lambda}(\lambda,\mu)\,f\mu(\lambda)+g(\lambda,\mu)\,f\mu'(\lambda)=f\mu''(\lambda)$$

$$\frac{f''(\lambda)}{f\mu(\lambda)}=\frac{\delta g}{\delta\lambda}+g(\lambda,\mu)\frac{f\mu(\lambda)}{f\mu(\lambda)}=\frac{\delta g}{\delta x}(\lambda,\mu)+g^2(\lambda,\mu)$$

From formulae for moments:

$$\alpha_1^*=1+\lambda g(\lambda,\mu),\qquad \alpha_2^*-1=3\lambda g+\lambda^2 g^2+\lambda^2\frac{\delta g}{\delta\lambda}=3(\alpha_1^*-1)+(\alpha_1^*-1)^2+\lambda^2\frac{\delta g}{\delta\lambda}$$

$$\lambda^2\frac{\delta g}{\delta\lambda}=\alpha_2^*-1-3\alpha_1^{*2}+3-\alpha_1^{*2}+2\alpha_1^*-1=\alpha_2^*-\alpha_1^{*2}-\alpha_1^*+1$$

3. The maximum likelihood method (in discrete case) consists in determining parameters in the way so that likelihood of getting a sample of given constitution is the highest. Because of independence of variables x_n realising the sample, likelihood of getting the sample with realisations of variables $k_1, k_2, \ldots, k_q$ and sizes $r_{k1}, r_{k2}, \ldots, r_{kq}$ gives the following likelihood functions:

$$L(\lambda,\mu)=(p_{k1})^{r_{k2}}\ldots\ldots(p_{kq})^{r_{kq}}$$

$$\ln L(\lambda,\mu)=\sum_{i=1}^{\theta} r_{ki}\ln p_{ki}$$

From definition results that:

$$\ln L(\lambda,\mu)=\sum_{i=1}^{q} r_{ki}\left[-\ln f\mu(\lambda)+(k_i-1)\ln\lambda\right]$$

Extremum may be determined by comparing derivatives of function $\ln L(\lambda,\mu)$ with zero:

$$\frac{\delta\ln L(\lambda,\mu)}{\delta\lambda}=\sum_{i=1}^{q}-\delta_{ki}\frac{f\mu(\lambda)}{f\mu(\lambda)}+\frac{r_{ki}}{\lambda}(k_i-1)=\frac{\delta\ln f\mu(\lambda)}{\delta\lambda}\sum_{i=1}^{q}n_{ki}+\frac{1}{\lambda}\sum_{i=1}^{q}k_i r_{ki}-\frac{1}{\lambda}\sum_{i=1}^{q}r_{ki}$$

$$\frac{\delta\ln L(\lambda,\mu)}{\delta\lambda}=-n\frac{\delta\ln f\mu(\lambda)}{\delta\lambda}+\frac{1}{\lambda}\left(\sum_{i=1}^{q}k_i r_{ki}-n\right)$$

$$\frac{\delta\ln L(\lambda,\mu)}{\delta\lambda}=\sum_{i=1}^{q}\left(n_{ki}\frac{\delta\ln a_{ki}-1}{\delta\mu}\right)-r_{ki}\frac{\delta\ln f\mu(\lambda)}{\delta\mu}\Big)=\sum_{i=1}^{q}r_{ki}\frac{\delta\ln a_{ki}-1}{\delta\mu}-n\frac{\delta\ln f\mu(\lambda)}{\delta\mu}$$