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## Optimization of the cut-off grade for underground polymetallic mines

### Introduction

The cut-off grade is the criterion that discriminates between ore and waste within a given mineral deposit (Taylor 1972). The mineralized material above the cut-off grade is considered as ore and, subject to access constraints, can be mined, while the material below the cut-off grade is regarded as waste and, depending upon the mining method used, is either left in its original situation or sent to a waste dump. Once the cut-off grade is determined, it provides a basis for the determination of resources and reserves of the deposit and consequently affects the service time of the mine. Simultaneously, it has a direct impact on the overall economic gain of a mining operation. Therefore, the cut-off grade is one of the most critical parameters in a mining operation.

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Most researchers have used break-even cut-off grade criteria to define ore as a material that will pay mining and processing costs, especially in China (Jia and Chen 2008; Wang et al. 2007). These methods are not optimal, but the mine planner often seeks to optimize the cut-off grade of ore to maximize the net present value (NPV). Choosing the best cut-off grade that maximizes the overall NPV has been a significant topic of research since the 1960s.

Lane's work has been regarded as the landmark in the general theory of cut-off grade optimization (Lane 1964, 1988). He developed a mathematical model that takes the maximization of the present value as the objective function and considers the capacity constraints of the mining, concentrating and refining stages as well as the capacity balancing between pairs of the three stages. Subsequently, several extensions of the original Lane's model were performed in different studies. Whittle and Wharton (1995a, b) added the idea of using opportunity cost to maximize the NPV of a project. They introduced two important pseudo costs: delay cost and change cost. Asad (2007) incorporated the commodity price and operating cost escalation into the basic Lane's model. Thus, the commodity price and operating costs do not remain constant during the life of the operation. Depending on the defined escalation rates, these economic parameters vary from one year to the next. Asad and Topal (2011) complemented the stockpiling option in Asad (2007). They presented the mathematical formula for the creation of stockpiles, described the strategy to reclaim the stockpile material after exhaustion of in-pit reserves, and compared the cut-off grade policies with and without stockpiling and demonstrated the benefits regarding the increase in NPV and life of the operation. Osanloo, Rashidinejad, and Rezai (2008) modified and improved the basic Lane's model by incorporating the environmental issues specific to the porphyry copper deposits. Apart from the traditional framework in basic Lane's model, this modified model accounts for separate waste dumps and tailing dams for acid and non-acid generating waste. Other relevant publications include Ahmadi (2018), Ahmadi and Shahabi (2018), Zarshenas and Saeedi (2017), and Yasrebi et al. (2015).

The contributions discussed above focused on the extension of Lane's model are applicable for the mineral deposits with a single economic mineral. Lane (1984), as well as Lane (1988), proposed a vital extension into the original model (Lane 1964), allowing for the cut-off grade calculation for mineral deposits with various economic minerals. Again, many studies then followed as an extension of Lane's model in two minerals case (Asad et al. 2016). Osanloo and Ataei (2003) introduced an equivalent grade factor in Lane's model and employed the golden section search method for defining the cut-off grade policy. Ataei and Osanloo (2003) presented an overview of the application of the genetic algorithm, golden section search method, grid search technique, and equivalent grades method for Lane's model, and shared the implementation of these methods in their computer program, along with a case study on a lead-zinc orebody. Cetin and Dowd (2016) described the general problem of cut-off grade optimization for multi-mineral deposits by means of genetic algorithms. To assess the performance of the genetic algorithms method, the grid search method and the dynamic programming method are used and compared with the results of the case of

genetic algorithms. Asad (2005) addressed the management of stockpiles in the case of two minerals, employed the grid search technique, and presented a step-by-step procedure in an algorithm implementing this extension in Lane's model.

It is noted that in the above-commented references, all of them are associated with the optimization of the cut-off grade in open pit mines with single or multiple economic minerals. After many years of research, the optimization of the cut-off grade for underground polymetallic mines remains, by and large, an unsolved problem. Poniewierski et al. (2003) pointed out that a key to determining an optimum cut-off grade in underground mine design is the ability to rapidly perform complex optimal mine layout designs combined with a rapid output of multiple potential schedules. Further industry experience is reported in Hall and Stewart (2004). Gu et al. (2010) advised dividing an underground deposit of a part of it into "decision units" and selecting the optimal cut-off grade by applying a forward dynamic programming based algorithm with an objective function of maximizing the overall NPV.

In the current existing literature, it can be seen that there is similar potential for the optimization of the cut-off grade between open pit mines and underground mines, but the optimization in underground mines is more complicated compared with open pit mines. That is because of the different nature of operation between these two kinds of mining strategies. More specifically, in an open pit mine, once the pit is determined, everything within the pit shell is removed and is either sent to the mill or discarded as waste. In an underground mine, however, how to exploit the deposit depends on the enormous number of constraints that are inherently interactional. If the mining method is selected, it will lead to the corresponding technical and economic parameters, the layout of stopes as well as complex development & transportation system. Apart from this, since mineral the grade varies with location to a certain extent, in almost all of the deposits, the optimal adopted cut-off grade policy could be different because of different mining methods under the premise of obtaining the maximum net present value, even in the same underground deposit. Additionally, the uneven grade distribution of different mineral resources in underground polymetallic mines adds more difficulty to optimize the cut-off grade.

Therefore, during the research on optimizing the cut-off grade for underground polymetallic mines, not only the time value of money considered in Lane's theory but also the dynamic nature of the cut-off grade related to space and mining methods should be taken into consideration. Also, with the continuous development of mining software, the application of the three-dimensional (3D) model makes it more convenient for the optimization of the cut-off grade in underground polymetallic mines.

This paper extends the utilization of Lane's theory to the optimization of the cut-off grade for underground polymetallic mines. Combined with the 3D visualization model of deposits, an optimization model of maximizing the overall economics is developed, and the objective function is expressed to one variable function of the cut-off grade by using the equivalent factors. The model simultaneously considers capacities of mining and mineral processing as constraints and the reasonable cut-off grade subjects to each constraint is

calculated by the golden section search method. Thus, the approach presented in this paper could be used to optimize the overall cut-off grade for a combination of mining-mineral processing underground mines with multi-metals.

## 1. Objective function

In a combined mining-mineral processing underground mine, there are typically two stages of operation: (i) the mining stage, in which units of various grades are mined up to some capacity; and (ii) the mineral processing stage, in which ore is milled and concentrated again up to some capacity. Each stage has its associated costs and limiting capacity. The relevant variables of the operation links for each mining unit are shown in Table 1.

Table 1. Related variables of the operation system

Tabela 1. Powiązane zmiennych systemu operacyjnego

Number	Operation stage	Production	Quantity	Variable costs	Capacity
			t	¥ · t <sup>-1</sup>	10 <sup>4</sup> t · a <sup>-1</sup>
1	Mining	Resource	1	<i>m</i>	<i>M</i>
2	Mineral processing	Concentrate	<i>x</i>	<i>h</i>	<i>H</i>

Here, *x* is the ratio of ore contained in a ton of resource; *m* and *h* are variable costs of mining and mineral processing, respectively; *M* and *H* are the production capacities of mining and mineral processing, respectively.

By considering costs and revenues in this operation, the formula for the cash flow arising from each unit of resource is determined by:

$$c = pxyg_k - xh - m - f\tau \quad (1)$$

- ↪ *p* – metal selling price,
- g* – cut-off grade applied to define ore,
- y* – the recovery rate in the mineral processing,
- g<sub>k</sub>* – average grade, which depends on the cut-off grade *g* and the recovery rate *y*,
- f* – fixed costs,
- τ* – time is taken to work through per unit of resource.

According to Lane's theory, the quite remarkable result is the following equation (Lane 1988):

$$\begin{cases} \frac{dV^*}{dR} = \max \{c - F\tau\} \\ F = \delta V^* - dV^*/dT \end{cases} \quad (2)$$

- ↪  $V^*$  – maximum present value at each stage during the operation,
- $R$  – amount of resource in a deposit,
- $c$  – cash flow arising from per unit of resource,
- $F$  – an opportunity cost,
- $\tau$  – time is taken to work through one unit of mineralized material,
- $\delta$  – discount rate,
- $T$  – the present time.

Eq. (2) implies that  $dV^*/dR$  must be a maximum at all points along an optimum strategy track. It means that the strategies must be chosen so that the decrement of each unit in  $R$  has the highest possible effect on  $V^*$ . This is intuitively obvious because summing all changes in  $V^*$ , corresponding to a sequence of decrements in  $R$ , all the way to zero will then give the highest total. Alternatively, in reverse, if an optimum strategy track is followed backward from  $R = 0$  in steps of  $r$ , then, if each step is taken in the direction of steepest ascent up  $V^*$ , the final point reached must be the highest possible.

The term  $F$  in Eq. (2) is an opportunity cost in economic parlance. The operation can be regarded as having a capital value of  $V$ . This is one interpretation of the present value. It is, in a sense, capital which incurs two penalties as a result of being tied up in operation. One is the interest that could have been earned were it deployed elsewhere; this is  $\delta V$ . The other is the decline in value as a consequence of deteriorating economic conditions; this is  $-dV^*/dT$ . Summarizing,  $F$  is an unknown value because it is dependent upon the future strategy. Since the unknown value appears in the equation thus, the iterative process must be used.

Then, substituting Eq. (1) into Eq. (2), the optimum exploitation strategy for maximizing the present value of operation based upon a finite resource can be determined at any stage by maximizing the expression:

$$v = pxyg_k - xh - m - (f + F)\tau \quad (3)$$

- ↪ where  $v$  is the rate of change of  $V^*$ , the optimum present value, with respect to resource usage ( $dV^*/dR$ ).

Considering that  $\tau$  is related to the constrain capacity, two cases arise depending on which of the two capacities is the limiting factor.

- ◆ If the mining capacity ( $M$ ) is the limiting factor, then the time  $\tau$  is given by

$$\tau = 1/M \quad (4)$$

- ◆ If the mineral processing capacity ( $H$ ) is the limiting factor, then the time  $\tau$  is controlled by the concentrate:

$$\tau = 1/H \quad (5)$$

Substituting Eq. (4) and Eq. (5) into Eq. (3) yields:

$$v_m = pxyg_k - xh - m - (f + F)/M \quad (6)$$

$$v_m = pxyg_k - xh - m - (f + F)/H \quad (7)$$

This means that, for any cut-off grades, it is possible to calculate the corresponding  $v_m$  and  $v_h$ . According to the determination of  $\tau$ , the graphs of two forms of  $v$  as an objective function of the cut-off grade,  $g$ , are similar; convex upwards, with a single maximum. This maximum corresponds to the limiting economic cut-off grade ( $g_m$  or  $g_h$ ) for the component concerned. It is illustrated in Figure 1(a).

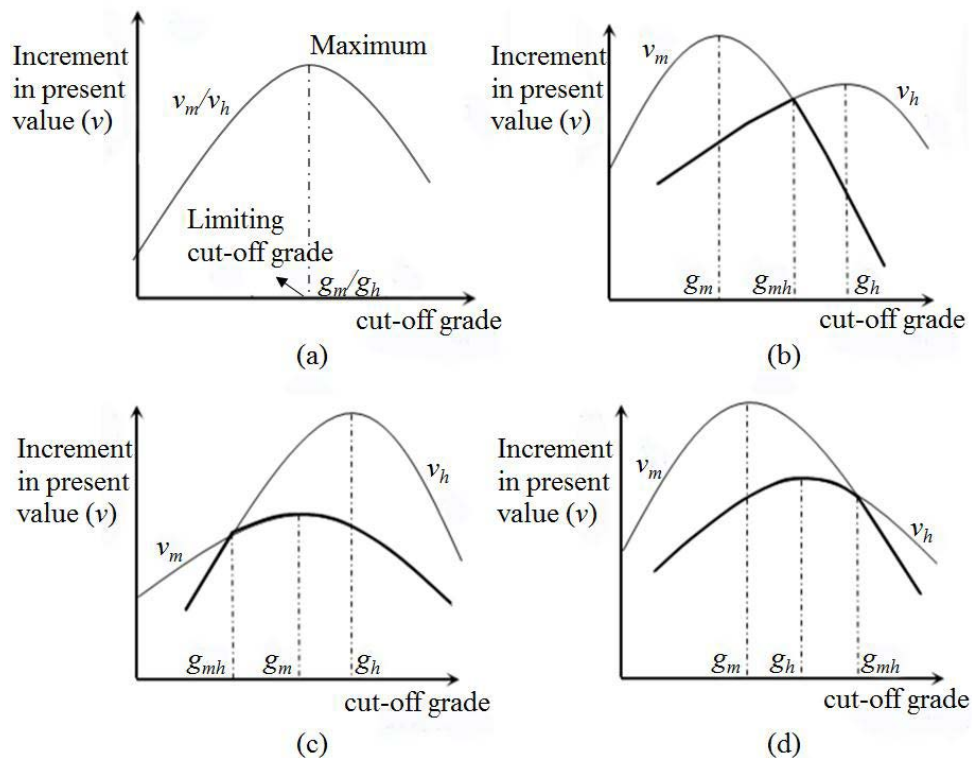


Fig. 1. Increment in the present value versus the cut-off grade (single or two component)

Rys. 1. Przyrost wartości aktualnej a wartość graniczna (jeden lub dwa składniki)

If the graphs for two forms of  $v$  are superimposed, then the intersection of two graphs is given by:

$$v_m = v_h$$

This means that the point of intersection corresponds to the balancing cut-off grade,  $g_{mh}$ .

Taking the maximization of marginal economics as a principle, the controlling capacity is always that corresponding to the least of the two equations. Therefore:

$$\max v = \max[\min(v_m, v_h)] \quad (8)$$

In other words, the feasible form of  $v$ , at any cut-off grade, is always the lower of the two curves. This is shown as a bold line in Figure 1(b). Further, it can easily be seen that for values of cut-off less than  $g_{mh}$ , the mineral processing is limiting and that for values above  $g_{mh}$  the mining is limiting. Thus, the maximum feasible value of  $v$  in the figure occurs at the point of intersection  $g_{mh}$ .

However, this is not always so; two other cases must also be examined, which can be illustrated as shown in Figure 1(c) and 1(d), respectively.

Figure 1(c) shows that when the balancing cut-off grade  $g_{mh}$  is less than  $g_m$ , the mining process is the bottleneck in the operation and  $g_m$  is the optimum cut-off grade. On the other hand, as Figure 1(d) shows, when  $g_{mh}$  is greater than  $g_h$ , the mineral processing is the bottleneck, and  $g_h$  is the optimum cut-off grade.

Hence, to find the overall optimal cut-off grade ( $G^*$ ), the following rule may be formulated for an underground operation limited by mining and mineral processing:

$$G^* = \begin{cases} g_m, & g_{mh} < g_m \\ g_h, & g_{mh} > g_h \\ g_{mh}, & g_m < g_{mh} < g_h \end{cases} \quad (9)$$

## 2. Determination of the optimal cut-off grade

### 2.1 Calculation of the comprehensive grade

The method of optimizing the cut-off grade for an underground operation mentioned above is applicable for the mineral deposits with a single economic mineral. For underground polymetallic mines, there is a need to introduce the average equivalent grade into Eq. (3). This means that  $g_k$  should be replaced with the average equivalent grade.

To calculate the average equivalent grade of ore based on equivalent factors and the average grade of each metal, the following equation can be used:

$$\alpha_t = \alpha + \sum f_i \cdot \alpha_i \quad (10)$$

In other words, the grade of metal  $i$  can be converted to the grade of the main metal by using the equivalent factor. In Eq. (10),  $\alpha_t$  is the average equivalent grade of ore,  $\alpha$  is the average grade of the main metal sent for mineral processing,  $\alpha_i$  is the average grade of metal  $i$  sent for mineral processing, and  $f_i$  is the equivalent factor of metal  $i$ .

The equivalent factor can be calculated using a valuation method (Sun et al. 2013) because the method considers not only the price factor but also the technical and economic indices such as production costs and recovery rates. The equivalent factor is equal to:

$$f_i = \frac{\varepsilon_i \cdot (U_i - C_i)}{\varepsilon \cdot (U - C)} \quad (11)$$

Where  $\varepsilon$  is the recovery of the main metal from the ore,  $\varepsilon_i$  is the recovery of metal  $i$  from the ore,  $U$  is the selling price of the main metal,  $U_i$  is the selling price of metal  $i$ ,  $C$  is the production cost of the main metal and  $C_i$  is the production cost of metal  $i$ .

## 2.2 Solution of the objective function

As previously mentioned, the objective function is unimodal, and the term  $F$  in Eq. (3) is an unknown value because it depends on the adopted cut-off grade policy. Then, in this study, elimination methods such as the dichotomous search method, the Fibonacci search method and the golden section search method are more applicable to be used to find optimum cut-off grades. These methods require only objective function evaluations and do not use the derivative of the function to see the optimal point. Osanloo and Ataei (2003) introduced the necessary steps of these methods and pointed out that the golden section search method is optimum. Thus, the solution of the objective function based on the golden section search method is chosen in this paper. Based on the research work of Osanloo and Ataei (2003), Figure 2 shows the flowchart for estimating optimal cut-off grades under different limiting factors by using the golden section search method.

More specifically, in the first step, it is assumed that the initial uncertainty space of cut-off grades  $[L, U]$  includes the optimal point and  $\varepsilon$  is the desired accuracy to determine the optimal point. In the next step, select two test points in the interval  $[L_k, U_k]$ , evaluate and compare the objective function at these test points, then a part of uncertainty space will be eliminated, where  $k$  is the number of times the uncertainty space updated. It is noted that



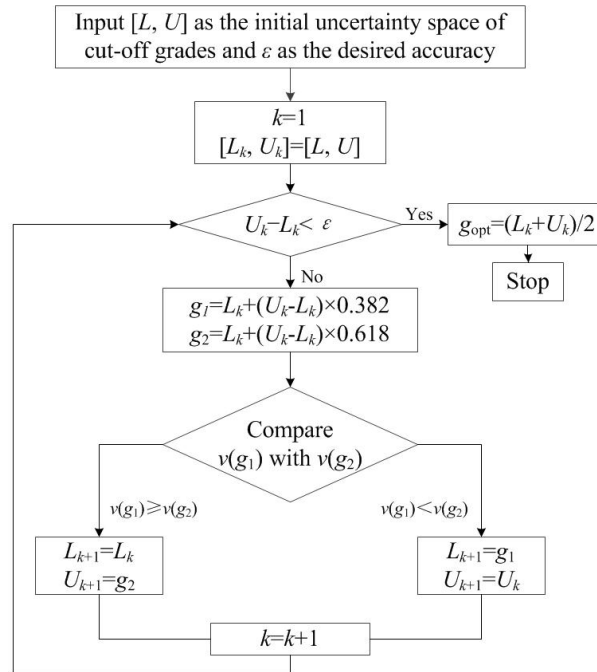


Fig. 2. Flowchart of the golden section search method (Osanloo and Ataie 2003)

Rys. 2. Schemat blokowy metody wyszukiwania złotego odcinka

the ratio of the remaining length after the elimination process to the initial length in each dimension is called the reduction ratio, and the reduction ratio of the golden section method is invariable and equal to 0.618. Thus, the two test points selected each time are equal to  $g_1 = L_k + (U_k - L_k) \cdot 0.382$ ,  $g_2 = L_k + (U_k - L_k) \cdot 0.618$ , respectively. In the third step, compare the remaining length of the uncertainty space with the desired accuracy. If the remaining length of uncertainty space is larger than the desired accuracy  $\epsilon$ , step 2 will be repeated until the uncertainty space in each dimension is less than the desired accuracy. Lastly, the optimal point could be calculated according to the last renewal of the uncertainty space, which can be expressed as  $g_{opt} = (U_k + L_k)/2$ . The above process was solved using MATLAB software.

In summary, the optimization of the cut-off grade for underground polymetallic deposits can be carried out according to the following steps:

- ◆ Taking the constraints of mining and mineral processing capacities in an underground operation into consideration, a mathematical model of the cut-off grade optimization is constructed to maximize the NPV.
- ◆ With the help of a 3D model of deposits established using the SURPAC mining software, calculation of the cut-off grade is simplified by using equivalent factors.

- ◆ Based on the cut-off grade optimization model, using MATLAB as a tool and combining production practices in mining, the golden section search method is used to solve the objective function. Thus, the two limiting economic cut-off grades ( $g_m$  and  $g_h$ ) and the balancing cut-off grade ( $g_{mh}$ ) are calculated.
- ◆ Determining the overall optimal cut-off grade according to Eq. (9), and the analysis of grade index optimization is completed by estimating and comparing the NPV of the profits.

### 3. Case study

#### 3.1. Variables and data

Consider a polymetallic deposit in Tibet as an example. The main products of the deposit are concentrates of copper and molybdenum, and the main associated elements. Au and Ag are contained in the copper concentrate. The mining method used in the deposit is an empty stage field and subsequent backfilling mining. Its designed production indicators are shown in Table 2, and the associated costs, prices, production capacities and quantities are given in Table 3. The unit of the figure is Chinese RMB (¥), and the exchange ratio between USD and ¥ is 1:6.6 used in this paper.

Table 2. Designed production indicators

Tabela 2. Zaplanowane wskaźniki produkcji

Parameter		Unit	Quantity
Copper concentrate	Productivity	%	2.46
	Average grade (Cu)	%	22
	Average grade (Au)	$g \cdot t^{-1}$	4.06
	Average grade (Ag)	$g \cdot t^{-1}$	263.4
	Recovery (Cu)	%	88
	Recovery (Au)	%	45
	Recovery (Ag)	%	55
Molybdenum concentrate	Productivity	%	0.085
	Average grade (Mo)	%	47
	Recovery (Mo)	%	70

Table 3. Economic parameters for the polymetallic deposit

Tabela 3. Parametry ekonomiczne dla złoża polimetalicznego

Parameter	Unit	Quantity
Mining capacity	10 <sup>4</sup> t/y	640
Mineral processing capacity	10 <sup>4</sup> t/y	600
Mining cost	¥t <sup>-1</sup>	144.85
Mineral processing cost	¥t <sup>-1</sup>	67.44
Fixed costs	10 <sup>4</sup> ¥/y	47,494
The service life of the deposit	a	40
Price (Cu)	¥t <sup>-1</sup>	53,500
Price (Mo)	¥t <sup>-1</sup>	200,000
Price (Au)	¥g <sup>-1</sup>	208
Price (Ag)	¥g <sup>-1</sup>	3
Discount rate	%	9

### 3.2. Application of the proposed method to optimize the cut-off grade

Firstly, according to Eq. (10) as well as original data as shown in Tables 2 and 3, the equivalent factors for converting the grades of other metals into the copper grade are as follows:

$$f_{Mo} = \frac{\varepsilon_{Mo} \cdot (U_{Mo} - C_{Mo})}{\varepsilon_{Cu} \cdot (U_{Cu} - C_{Cu})} = \frac{70\% \cdot (200000 - 103643.277)}{88\% \cdot (53500 - 27724.577)} = 2.97$$

$$f_{Au} = \frac{\varepsilon_{Au} \cdot (U_{Au} - C_{Au})}{\varepsilon_{Cu} \cdot (U_{Cu} - C_{Cu})} = \frac{45\% \cdot (208 - 107.789) \cdot 10^4}{88\% \cdot (53500 - 27724.577)} = 0.2$$

$$f_{Ag} = \frac{\varepsilon_{Ag} \cdot (U_{Ag} - C_{Ag})}{\varepsilon_{Cu} \cdot (U_{Cu} - C_{Cu})} = \frac{55\% \cdot (3 - 1.555) \cdot 10^4}{88\% \cdot (53500 - 27724.577)} = 0.0034$$

Now, using the equivalent factors and the average grade of each metal, the equivalent copper grade is calculated:

$$\alpha_t = \alpha_{Cu} + 2.97\alpha_{Mo} + 0.2\alpha_{Au} + 0.0034\alpha_{Ag} \quad (12)$$

Thus, according to Eq. (12), the 3D economic model of the deposit is improved based on the existing resource model by using SURPAC mining software. Figure 3 shows the distribution of the equivalent copper grade, and the equivalent copper grades of the different copper intervals are calculated in Table 4.

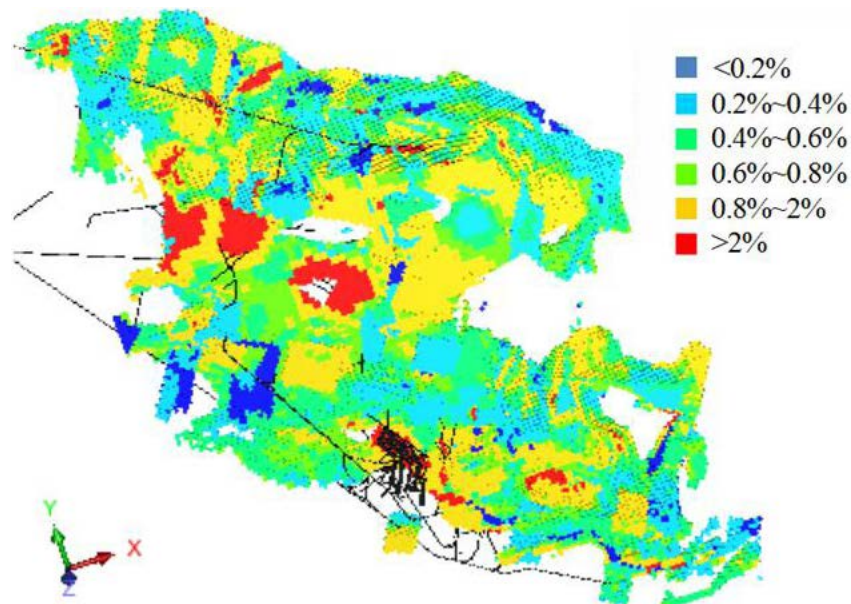


Fig. 3. Distribution of equivalent copper grade containing development system

Rys. 3. Rozkład ekwiwalentnych wartości miedzi w badanym systemie

Table 4. Equivalent copper grades of different copper intervals

Tabela 4. Ekwiwalentne wartości miedzi o różnych interwałach

Copper grade (%)	Average grade				Equivalent copper grade (%)	Tonnage (million tons)
	Cu (%)	Mo (%)	Au (g/t)	Ag (g/t)		
<0.1	0.057	0.082	0.055	1.090	0.315	13.80
0.1~0.2	0.155	0.060	0.143	3.245	0.372	15.02
0.2~0.3	0.251	0.046	0.129	5.287	0.432	26.30
0.3~0.4	0.351	0.044	0.215	7.961	0.550	28.35
0.4~0.5	0.449	0.042	0.223	9.881	0.653	28.10
0.5~0.6	0.548	0.045	0.254	11.801	0.772	23.03
0.6~0.7	0.648	0.046	0.294	13.401	0.890	18.19
0.7~0.8	0.748	0.048	0.322	14.764	1.007	16.52
0.8~2	1.165	0.047	0.476	21.808	1.473	70.65
>2	2.767	0.035	1.220	46.069	3.273	9.07
In total	0.709	0.048	0.321	13.750	0.962	249.03

After that, the optimal equivalent cut-off grades of copper under the constraints of mining production and mineral processing capacities respectively as well as the balancing equivalent cut-off grade of copper were calculated by using the golden section search method, and the results are shown in Figure 4.

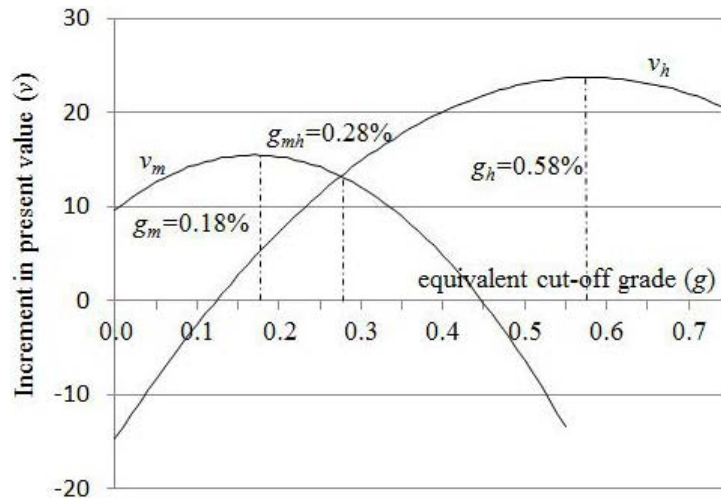


Fig. 4. Results of optimization cut-off grade for the case study

Rys. 4. Wyniki oceny optymalizacji wartości granicznej dla studium przypadku

From Figure 4, we can know that the mining limiting equivalent cut-off grade of copper,  $g_m$  is 0.18%, mineral processing limiting the equivalent cut-off grade of copper,  $g_h$  is 0.58%, and the balancing equivalent cut-off grade of copper,  $g_{mh}$  is 0.28%. It is clear that  $g_m < g_{mh} < g_h$ , namely, the overall optimal equivalent cut-off grade of copper is 0.28% according to Eq. (9).

### 3.3. Comparison with the current cut-off grade policy

At present, the cut-off grade strategy adopted by the deposit is that the equivalent cut-off grade of copper is 0.24%, and equivalent average grade is 0.996%. Since the service life of the deposit is 40 years, the quantities of product from mining and mineral processing is 6.34 million tons and 5.93 million tons, respectively.

Thus, the yearly revenue will be equal to:

$$\begin{aligned} \text{Yearly revenue} &= 5.93 \cdot 0.996\% \cdot 88\% \cdot 53500 \cdot 81\% \\ &= \text{RMB } 2,252.35 \text{ million} \end{aligned}$$

Where 81% means a discount rate of copper concentrate against copper metal, which is acquired based on the actual investigation. This is mainly due to the fact that the main

product of the polymetallic deposit in Tibet is concentrated of copper; whereas the value of copper concentrate is different from that of copper metal.

Besides, the yearly cost will be equal to:

$$\begin{aligned} \text{Yearly cost} &= 6.34 \cdot 144.85 + 5.93 \cdot 67.44 + 474.94 \\ &= \text{RMB } 1,793.21 \text{ million} \end{aligned}$$

Then, the yearly cash flow and NPV of the mining operation under the equivalent cut-off grade policy currently used was found to be:

$$2252.35 - 1793.21 = \text{RMB } 459.14 \text{ million}$$

$$\text{NPV} = 459.14 \cdot ((1 + 0.09)^{40} - 1) / (0.09 \cdot (1 + 0.09)^{40}) = \text{RMB } 4,939.13 \text{ million}$$

The estimate of NPV under the current cut-off grade strategy was compared with the estimate of NPV when the equivalent cut-off grade of copper was 0.28% through optimization. Table 5 shows the comparative analysis of economic effects before and after optimization of the equivalent cut-off grade of copper.

Therefore, the NPV of the mining operation is RMB 5,109.33 million under the policy of optimal equivalent cut-off grade of copper equal to 0.28%. Thus, if the mining operation

Table 5. Comparison of the economic effects of grade index before and after optimization

Tabela 5. Porównanie wskaźników efektów oceny ekonomicznej przed i po optymalizacji

Parameter	Unit	Current Strategy	Optimization Strategy
The equivalent cut-off grade of Cu	%	0.24	0.28
The equivalent average grade of Cu	%	0.996	1.020
Average grade of Cu	%	0.74	0.75
Average grade of Mo	%	0.05	0.05
Average grade of Au	g·t <sup>-1</sup>	0.33	0.34
Average grade of Ag	g·t <sup>-1</sup>	14.25	14.49
Mineral resources of Cu	10 <sup>4</sup> t	174.4	173.58
Mineral resources of Mo	10 <sup>4</sup> t	11.7	11.58
Mineral resources of Au	t	78.96	78.58
Mineral resources of Ag	t	3384.23	3369.31
Mine capacity	10 <sup>4</sup> t	634	621
Mill capacity	10 <sup>4</sup> t	593	580
Service life	a	40	40
Net cash flow	million yuan	459.14	474.96
Net present value	million yuan	4939.13	5109.33

is operated under the proposed method, the NPV will increase by RMB 170.2 million, compared with an NPV of mining operation under the current cut-off grade policy.

## Conclusions

One of the most critical aspects of mining is deciding which material in a deposit is worth mining and mineral processing and which material is waste. This decision-making is summarized by the cut-off grade policy. Compared with the open pit, although the underground mine design problem is more difficult, it has a similar potential for optimization. Therefore, this paper proposes an extension of Lane's theory to optimize the cut-off grade for underground polymetallic mines. The objective function is expressed as one variable function of the cut-off grade. For a combined mining-mineral processing underground polymetallic mine, it means that the main constraints include the production capacities of mining and mineral processing, and an equivalent grade of main metal is determined by using equivalent factors. Then, the optimization method of the golden section search is used to calculate reasonable cut-off grades corresponding to capacity constraints. The determined criterion for the overall optimal cut-off grade is put forward based on three candidate cut-off grades.

In the case study, a polymetallic copper deposit in Tibet is taken as an example to validate the approach proposed in this paper. The comparison results show the sensitivity of NPV with respect to the change of the cut-off grade. The optimal equivalent cut-off grade of copper, 0.28%, calculated by the presented method is superior and gives better NPV of RMB 5,109.33 million, compared to RMB 4,939.13 million under the current cut-off grade of 0.24%. Thus, the method proposed in this paper with no doubt provides a good idea for decision-makers to solve the practical problems in mining production and operation; it cannot only enhance the economic efficiency of enterprises on the premise of guaranteeing the rational utilization of mineral resources but can also provide scientific guidance for mine production planning.

It is noted that the optimization method proposed in this paper is suitable for selecting the overall optimum cut-off grade but not local blocks optimization for underground polymetallic mines. Apart from this, this paper does not consider the additional mining exploration project and the change of optimal cut-off grade as the deposit becomes depleted. Also, uncertainty in tech-economic parameters, especially, the uncertainties associated with mineral resources, metal price and costs of each production stage are not considered. Therefore, the developments of cut-off grade optimization in underground polymetallic mines considering the uncertainties of resources, metal price and production costs and allowing dynamic change of cut-off grade during the service of life are some of the areas for future research.

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**OPTIMIZATION OF THE CUT-OFF GRADE FOR UNDERGROUND POLYMETALLIC MINES****Keywords**

cut-off grade, marginal economics, optimization, underground polymetallic mines

**Abstract**

One of the most critical aspects of mine design is to determine the optimum cut-off grade. Despite Lane's theory, which aims to optimize the cut-off grade by maximizing the net present value (NPV), which is now an accepted principle used in open pit planning studies, it is less developed and applied in optimizing the cut-off grade for underground polymetallic mines than open pit mines, as optimization in underground polymetallic mines is more difficult. Since there is a similar potential for optimization between open pit mines and underground mines, this paper extends the utilization of Lane's theory and proposes an optimization model of the cut-off grade applied to combined mining-mineral processing in underground mines with multi-metals. With the help of 3D visualization model of deposits and using the equivalent factors, the objective function is expressed as one variable function of the cut-off grade. Then, the curves of increment in present value versus the cut-off grade concerning different constraints of production capacities are constructed respectively, and the reasonable cut-off grade corresponding to each constraint is calculated by using the golden section search method. The defined criterion for the global optimization of the cut-off grade is determined by maximizing the overall marginal economics. An underground polymetallic copper deposit in Tibet is taken as an example to validate the proposed model in the case study. The results show that the overall optimum equivalent cut-off grade, 0.28%, improves NPV by RMB 170.2 million in comparison with the cut-off grade policy currently used. Thus, the application of the optimization model is conducive to achieving more satisfactory economic benefits under the premise of the rational utilization of mineral resources.

**OPTIMALIZACJA WARTOŚCI GRANICZNEJ SUROWCA  
DLA PODZIEMNYCH KOPALŃ RUD POLIMETALICZNYCH****Słowa kluczowe**

ocena graniczna, krańcowa ekonomia, optymalizacja, podziemne kopalnie polimetaliczne

**Streszczenie**

Jednym z najbardziej istotnych aspektów projektowania kopalni jest określenie optymalnej wartości granicznej surowca. Pomimo teorii Lane'a, mającej na celu optymalizację wartości granicznej surowca poprzez maksymalizację wartości bieżącej netto (NPV), obecnie stosowanej w studiach planowania górnictwa odkrywkowego, natomiast jest ona mniej rozwinięta i stosowana w optymalizowaniu wartości granicznej surowca dla podziemnych kopalń rud polimetalicznych ze względu na

znaczne trudności. W artykule rozszerzono wykorzystanie teorii Lane'a poprzez propozycję modelu optymalizacji wartości granicznej surowca, zastosowanego do połączonych procesów górniczo-prze-róbczych podziemnych kopalń rud polimetalicznych. Za pomocą modelu wizualizacji 3D zasobów i wykorzystaniu ekwiwalentnych współczynników określono funkcję celu wyrażoną jako jedną zmienną funkcję oceny wartości granicznej. Następnie konstruowane są krzywe przyrostu wartości aktualnej w stosunku do wartości granicznej dotyczące różnych ograniczeń zdolności produkcyjnych, a rozsądna wartość graniczna odpowiadająca każdemu ograniczeniu jest obliczana za pomocą metody wyszukiwania złotego odcinka. Zdefiniowane kryterium globalnej optymalizacji oceny granicznej określa się poprzez maksymalizację efektów ekonomicznych. Jako przykład do sprawdzenia proponowanego modelu w studium przypadku wykorzystano podziemne złoża polimetaliczne miedzi w Tybecie. Wyniki pokazują, że całkowita optymalna ekwiwalentna wartość graniczna, 0,28%, poprawia NPV o 170,2 milionów juanów w porównaniu z obecnie stosowaną oceną graniczną. Tak więc zastosowanie modelu optymalizacyjnego sprzyja osiągnięciu bardziej satysfakcjonujących korzyści ekonomicznych przy założeniu racjonalnego wykorzystania zasobów mineralnych.