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Research paper

Validation of European Gravimetric Geoid models in context of realization of EVRS system in Poland

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Abstract: In this study, several variants create and choose of a local quasi-geoid model in Poland have been considered. All propositions have a source in European Gravimetric Geoid models – EGG2008 and EGG2015, which are purely gravimetric models of reference surface. In the course of this work, each model has been analyzed in various ways: without any corrections, by parallel shifting of residuals, by the 7-parameter conformal transformation and by fitting residuals by 4- and 5-parameter trigonometric polynomials. Eventual corrections were based on points of national GNSS/levelling networks (EUVN, EUVN_DA, POLREF, EUREF and ASG-EUPOS eccentric points). As a final result of this study, a comparison of the accuracy of selected models has been carried out by RMSE statistics and maps showing spatial distribution of residuals and histograms. Validation has shown that the maximum achievable accuracy of the EGG models is approximately 2 cm for the ETRF2000 reference system and approximately 8 cm for ETRF89. In turn, fitting with the use of different mathematical methods results in an improvement of the standard deviation of residues to the level of 1.3–1.4 cm. The conclusions include an evaluation of considerations for and against the use of models based only on EGG realizations and, on the other hand, fitted to the points of Polish vertical network. Its usefulness is strictly connected with needs of the definition of up to date quasi-geoid model for the new realization of heights system in Poland, based on EVRF2007 frame.

Keywords: quasi-geoid model, height frame, height anomaly, GNSS/levelling

1. Introduction

The PL-EVRF2007-NH frame is a new defined Polish height reference frame, fully consistent with the European Vertical Reference Frame (EVRF2007) established in 2012 as the part of the national spatial reference system in Poland (Regulation of the Council of Ministers, 2012). EVRF2007 was recommended for the Pan-European use by the EUREF Sub-Commission in 2008. The frame was determined by the adjustment of geopotential numbers (Sacher et al., 2008) of the precise levelling networks of European states, indirectly referring to the vertical level of Normaal Amsterdams Peil (NAP) in Amsterdam (Ihde et al., 2008). The main advantage of using EVRF2007 for the Polish vertical control network is the use of height differences determined during the fourth national levelling campaign (1999-2003), while the current PL-KRON86-NH frame is based on measurements from the years of 1974–1982. Official work on the EVRF2007 frame was finished in 2008. The standard deviation of 1 km levelling is around 1.11 kGal·mm and of datum points height equals to 16.05 kGal·mm over the area of the Europe (Łyszkowicz and Bernatowicz, 2014). According to the European INSPIRE directive and a corresponding regulation of the Polish Council of Ministers, the new height system should be adapted by the end of 2019 (INSPIRE Data Specification on Coordinate Reference Systems, 2014). Till this date an obligatory model of local quasi-geoid is PL-geoid-2011 model, which realizes only aging PL-KRON86-NH height reference frame in relation to ETRF89 and ETRF2000 realizations.

The main purpose of this work was to verify the accuracy of European Gravimetric (quasi)Geoid 2008 and 2015 on the territory of Poland and to test various methods of fitting these models into GNSS/levelling data. Results of this study can help in developing local quasi-geoid model dedicated to new PL-EVRF2007-NH height frame, model dedicated to practical geodetic applications e.g. GNSS levelling.

2. Quasi-geoid models

The issues of quasi-geoid and geoid modelling over the area of Europe was entrusted to the Sub-Commission 2.4a of the International Association of Geodesy, Gravity and Geoid in Europe. Previously this research was carried out by the European Gravity and Geoid Project (EGGP), which was established in 2003 on the basis of research conducted in the Institute of Geodesy in Hannover. So far, three official Pan-European quasi-geoid models, covering the area of Poland, were published:

- EGG97 – European Gravimetric Geoid 1997 (Institute of Geodesy in Hannover);
- EGG2008 – European Gravimetric Geoid 2008 (EGGP);
- EGG2015 – European Gravimetric Geoid 2015 (IAG).

The computation algorithm used to determine models of European Gravimetric quasi-Geoid is based on the modelling of disturbing potential on the Earth surface, then used to determine height anomalies (quasi-geoid undulations).

The EGG2008 model (Denker, 2013) was published in 2008. The basic computational strategy in this model is consistent with the strategy used for model EGG97

(Denker and Torge, 1998). The modelling of a disturbing potential was based on the RCR (remove-compute-restore) technique, on the basis of present global geopotential models. To model the impact of topography, terrain reductions were calculated using RTM technique (Residual Terrain Model). Main computations related to the disturbing potential were carried out by means of spectral combination technique (SCT). To match finally newly created gravimetric model of quasi-geoid in the EVRF2007 vertical frame, the constant ζ_{0i} was estimated by the comparison of empirical height anomalies ($\mathrm{h}^{\mathrm{P-HEVRF2007}}$) at points of the EUVN_DA network. The expected value was $\zeta_{0i} = 0.302$ m. The computed surface of quasi-geoid model was moved in the vertical plane by rounded value $\zeta_0 = +0.300$ m, so that the final EGG2008 model would be consistent with the vertical frame EVRF2007. Unfortunately, the reference frame of ellipsoidal heights h^{P} used to calculate values of $h^{P-HEVRF2007}$ was not given in ETRS89 realization, which should be considered as a reference frame for the height anomalies in Europe. However, it is assumed that the ITRF1996 frame was for epoch 1997.4, which corresponds to the epoch of measuring campaign for the main EUVN points.

The EGG2015 (Denker, 2015) is presented as the latest realization of the European quasi-geoid model. The height anomaly values refer to GRS80 and the zero tide system, as recommended by IAG. The GOCO05S model and terrestrial gravity data with the standard deviation of gravity differences equal to 0.2 mGal were used to compute the disturbing potential. Furthermore, the values are compatible with the European Vertical Reference Frame 2007 (EVRF2007). As in the case of EGG2008, a vertical constant translation $\zeta_0 = +0.305$ m has been made by averaging differences between model and GNSS/levelling results on EUVN_DA points. The evaluation of EGG2015 by GNSS and levelling data indicates an accuracy of 1–2 cm on a national basis, and 2–4 cm at continental scales (Denker, 2015).

2.1. PL-geoid-2011 – official Polish quasi-geoid model

The obligatory PL-geoid-2011 quasi-geoid model was created for PL-KRON86-NH height frame, relating to a reference tide gauge in Kronstadt. It is based on fitting height anomalies from fully solution of EGM2008 geopotential model into data acquired from GNSS/levelling measurements on 570 control points of Polish height networks (Kadaj, 2013) The fit has been made by 7-parameter Helmert transformation and then local Hausbrandt post-transformation correction at reference points. This procedure has been conducted in order to retain the original height anomalies on the control points. In the result, as their residuals were reduced to zero by Hausbrandt correction, this differences were redistributed on interpolated points in the proportion to the inverse of the square of the distance from the fitting point to adjust. This kind of redistribution is based on the inverse distance interpolation (Ligas and Banasik, 2014). To summarize, this model is not strictly gravimetric but adapted to the vertical datum and depended on errors in a national height network. Moreover, the use of all 570 points of Polish

GNSS/levelling network as control points makes the model difficult to validate, because Hausbrandt correction reduces residuals to zero to obtain the best fit of the model. As it was mentioned in Introduction, the expiry date of the PL-geoid-2011 model is by the end of 2019, afterwards PL-EVRF2007-NH frame will be the only valid height frame in Poland.

The different realisation of the geoid, mainly based on the Stokes formulae and on the gravimetric data utilisation was presented by Łyszkowicz (Łyszkowicz, 2012) and Kryński (Kryński, 2007). The latest work in the aspect of gravimetric geoid computation for the territory of Poland was delivered by Szelachowska (Szelachowska and Kryński, 2014). In this paper, in a proposed quasi-geoid model GDQM-PL13, the discrepancy between model and measured height anomalies is estimated at 18 mm for satellite / levelling network (EUVN, ASG-EUPOS) and 22 mm for POLREF network. Similar results were obtained by Kuczyńska-Siehień in 2016 (Kuczynska-Siehien et al., 2016), using the modified by KTH LSMSA Stokes approach. In this work, the ASG-EUPOS network was used for fitting and validation, and matching error has the value of 22 mm for 7-parameter trigonometric polynomial. These results represent currently the best achievable accuracy of geoid models computed by gravimetric methods when comparing with the GNSS/levelling height anomalies (measured as the differences of the ellipsoidal and normal heights) defined on the base multifunctional networks points in Poland.

3. Levelling and satellite networks

In this research, three Polish satellite-levelling networks were taken into consideration: EUVN with densification points EUVN_DA, POLREF and ASG-EUPOS. Only ASG-EUPOS network has been created by GNSS measurements, previous networks were created using only with GPS measurements. It was necessary for chosen points to have normal heights determined in PL-EVRF2007-NH and ellipsoidal coordinates in ETRF89 and ETRF2000 (epoch 2011.0) frames as they are official realizations of ETRS89 systems valid in Poland.

3.1. EUVN+EUVN_DA networks

European Vertical Reference Network (EUVN) on the continent has been operating since 1997. In Poland, 11 localizations have been chosen as EUVN sites – 4 GNSS permanent sites (BOR1, JOZE, LAMA, BOGI), 5 stations of UELN network, one EUREF site in Rozewie and one new point nearby a tide gauge in Swinoujscie. From 2003 to 2009 a network densification action was carried out and called EUVN_DA. 52 points were selected in Poland, including sites belonging to the former 1st class of vertical control network (Keyneres et al., 2010). In this study, a total of 40 EUVN and EUVN_DA network sites were used, due to the destruction of the 12 points since 1997.

These are the same 40 stations that took part in the campaign of the ETRF2000 realization on the new Polish base networks consisting of POLREF, ASG-EUPOS and EUVN points, done at the end of 2010. The accuracy of height anomalies obtained on EUVN points from GNSS/levelling measurements is estimated at ±2 cm (Kryński and Figurski, 2006).

3.2. POLREF network

Polish Reference Network, abbreviated to POLREF, was created in 1995 as a densification of EUREF-POL control network. 348 evenly distributed geodetic points measured by GPS techniques formed the "national first-order satellite network", and since then this network has perform as the basic horizontal network. The providers assessed the original ellipsoidal height accuracy of about 1 cm in the relation to starting point, lower nowadays due to the network age and possible destabilization of points. In (Kryński and Figurski, 2006) the accuracy of POLREF height component was estimated at 3–4 cm. From the set of 348 sites, only 319 were used in this study. Many points had no information about normal or ellipsoidal height, or their destruction was discovered during 2010 campaign (ETRF2000). Moreover, in a priori studies, POLREF points were considered to be less accurate from the reason of the normal height differences measured by technical spirit levelling technique, linked in some cases unfortunately even to the III class vertical network. From this reason, the computed height anomaly has less precision, estimated at ±5 cm level.

3.3. ASG-EUPOS network

Polish Active Geodetic Network (ASG-EUPOS) was founded in 2008. Now it consists of 103 permanent stations, fifteen of which are included to the EPN (European Permanent Network) network as fundamental satellite base in Poland. Each station has at least two eccentric points linked with precise geometric leveling to the existing base vertical network that allow transferring normal height to the exact location of the GNSS antenna (Bosy et al., 2008). Every set (the centre and two eccentric points) has ellipsoidal coordinates determined with the uncertainty less than 1 cm. However, it is worth mentioning that only eccentric points have normal heights measured by the technique of precise levelling, hence this set of points has been taken into account in this study.

Comparing maps with spatial distribution of the considered sites leads to the conclusion that these three networks provides a good coverage on the territory of Poland (Figure 1).

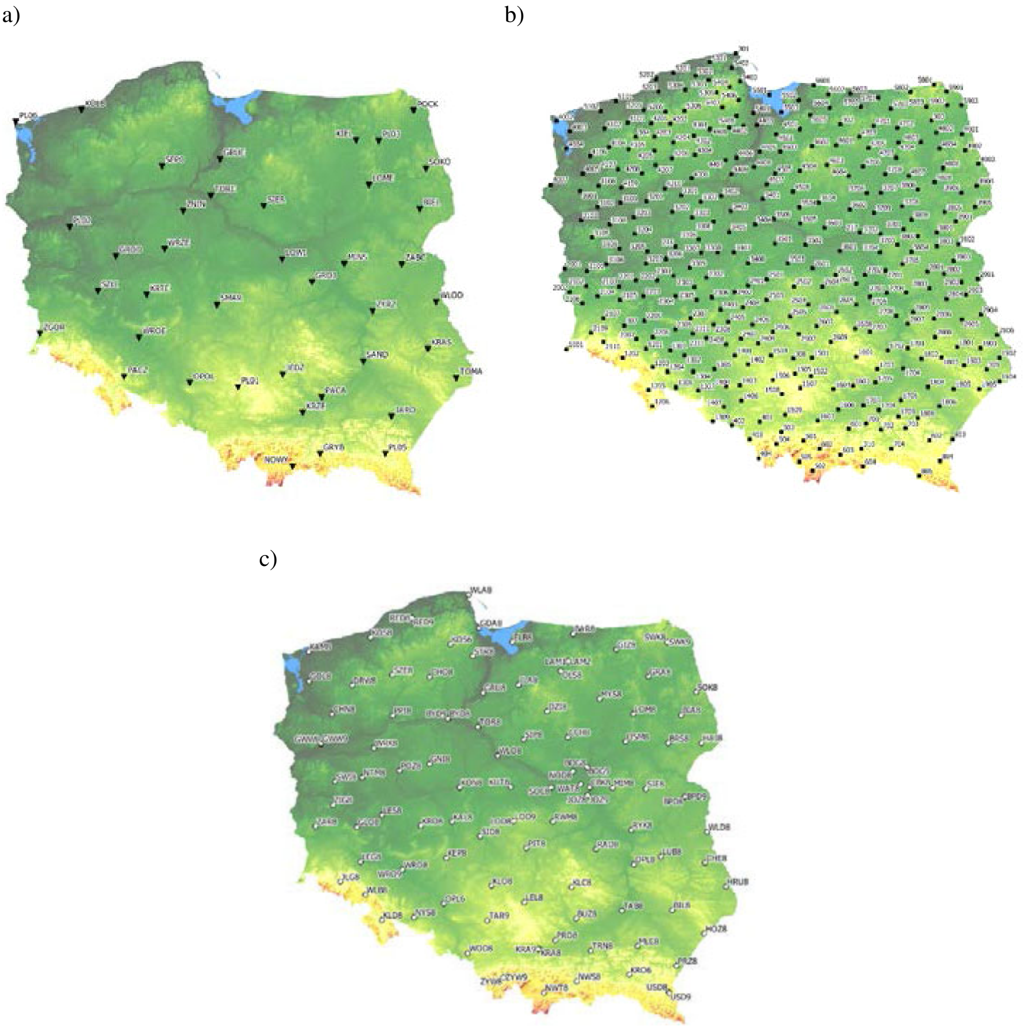

Fig. 1. Spatial distribution of considered GNSS levelling networks: a) EUVN, b) POLREF, c) ASG-EUPOS eccentric points

4. Approaches and results

In this study, the two sets of GNSS/levelling points were considered: the first containing all networks (EUVN+ASG+POLREF, abbreviated to E+A+P), and the second one with only EUVN, EUVN_DA and ASG-EUPOS points (EUVN+ASG, abbreviated to E+A) as they are believed to be more accurate according to previously mentioned data sets specification. The total number of points qualified for the analysis reaches 468. This is less than the set which have been used to create the PL-geoid-2011 due to the exclusion of the centres of ASG-EUPOS stations at which normal heights were computed, not determined by direct levelling measurements.

4.1. Validation of purely European Gravimetric Geoid 2008 and 2015 in the context of different Polish ETRF realizations

The validation of EGG requires the determination of quasi-geoid model heights at selected points. To interpolate them, an application EGGPRO was used (Heiner Denker personal contact). Geodetic coordinates $(\varphi\lambda \mathrm{h})_{\mathrm{ETRF2000}}$ and $(\varphi\lambda \mathrm{h})_{\mathrm{ETRF89}}$ were taken as input data to interpolate $(\varphi\lambda)$ and compute modelled height anomalies.

The EVRF2007 frame is expressed in a so-called zero-tide system that maintains a part of permanent tides of the Earth. In turn, the ellipsoidal coordinates of points in ETRS are determined in a tide-free system. To obtain ellipsoidal heights in zero-tide system, it was necessary to add a correction imposed by EVRS conventions (Mäkinen and Ihde, 2009) (Eq. (1)):

$$dh_{PT}^{el} = 60.34 - 179.01\sin^2\varphi - 1.82\sin^4\varphi \quad [\mathrm{mm}] \tag{1}$$

This correction dh_{PT}^{el} does not depend on time, but only on geodetic latitude φ. Its value for Poland varies from −4.3 cm for the southern part and −6.0 cm for the northern area. Hence, including of this correction is significant, because the variation of this correction equal to 1.7 cm could be noticeable on the territory of Poland as a tilt of zero-tidal geoid model to non-tidal ellipsoid.

In next step, quasi-geoid heights computed for analyzed models were compared to empirical height anomalies $(\mathrm{h}^{\mathrm{ETRF2000/89}} - \mathrm{H}^{\mathrm{EVRF2007}})$. In Table 1 and Table 2, the results of validation for each network are presented, in both reference frames that express ellipsoidal heights – ETRF2000 and ETRF89. The comparison includes also both EGG realizations – 2008 and 2015 – and fulfils the condition of zero-tide heights.

Table 1. Statistics of differences between height anomalies obtained from EGG models and GNSS/levelling data with ellipsoidal heights in ETRF2000 frame

ref. frame	ETRF2000					
network	EUVN		POLREF		ASG	
model	EGG2008	EGG2015	EGG2008	EGG2015	EGG2008	EGG2015
RMSE [cm]	2.3	2.1	2.4	2.3	2.0	1.9
STD [cm]	2.1	2.0	2.4	2.3	1.8	1.7
min [cm]	−5.1	−4.1	−9.6	−7.9	−5.3	−5.3
max [cm]	4.6	4.9	10.7	10.1	3.2	3.5
avg [cm]	−0.9	−0.8	−0.1	−0.1	−0.9	−0.8
median [cm]	−0.9	−0.7	−0.3	−0.3	−1.1	−0.8

When comparing consistency of differences between height anomalies obtained from EGG models and GNSS/levelling data for different ellipsoid reference frames it is clearly visible that:

– standard deviation (STD) values are lower for ETRF2000 realization, from 1.8 to 2.4 cm, for ETRF89 they are between 2.8 and 3.3 cm, as well as root mean squared error (RMS) values,
– averages of residuals for ETRF2000 realization are below 1 cm while for ETRF89 are even larger than 7 cm.

Table 2. Statistics of differences between height anomalies obtained from EGG models and GNSS/levelling data with ellipsoidal heights in ETRF89 frame

ref. frame	ETRF89					
network	EUVN		POLREF		ASG	
model	EGG2008	EGG2015	EGG2008	EGG2015	EGG2008	EGG2015
RMSE [cm]	8.4	8.2	7.3	7.3	8.0	8.0
STD [cm]	3.1	3.0	3.3	3.2	2.8	2.7
min [cm]	−13.9	−12.9	−15.2	−15.2	−14.1	−13.5
max [cm]	0.3	0.7	1.1	2.9	−1.6	−0.8
avg [cm]	−7.8	−7.6	−6.5	−6.6	−7.5	−7.5
median [cm]	−7.8	−7.4	−6.3	−6.6	−7.5	−7.6

And when comparing the different EGG realizations it can be noticed:
– no significant differences between the models calculated for both ETRF2000 and ETRF89 frames are observed, around 1 mm lower RMS and STD in favour of EGG2015,
– interestingly, it might seem that EGG2008 model fits better to the newer realization of ETRS (ETRF2000 epoch 2011.0) was officially established in Poland since 2012.

Finally, the estimation of similarity between each data sets results in:
– the lowest residues in ETRF2000 for ASG-EUPOS points (with STD equal to 1.8 cm), since it is the newest network,
– the worst performance of POLREF points, as its estimated STDs are 2.4 and 3.3 cm for EGG2008, 2.2 and 3.2 cm for EGG2015, which are the maximal obtained values of standard deviation in every case,
– the absolute value of minimum and maximum residuals in ETRF2000 are below 5 cm for EUVN and ASG-EUPOS points, and even more than 10 cm for POLREF points.

Spatial distribution of residuals and the corresponding histograms are presented in Figures 2, 3, 4 and 5.

Figure 2 discloses that:
– EGG2015 has better consistency in the Western Poland – close to border with Germany – than EGG2008, due to the update of gravimetric data in the latest realization,

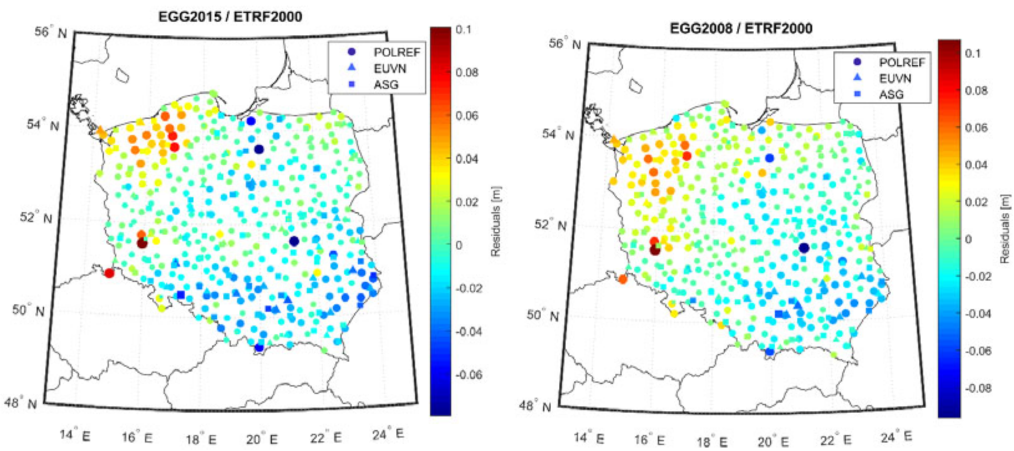

Fig. 2. The EGG2008 (left) and EGG2015 (right) models compared to GNSS/levelling quasigeoid heights, for ETRF2000 frame

- in both cases, i.e. for EGG2008 and EGG2015, there are some noticeable outliers randomly distributed over the area of Poland – these are the points of mostly POLREF network that could have been moved or damaged, this case shows an additional advantage of gravimetric model: a possibility of validation of GNSS/levelling network consistence,
- there are some POLREF points in the North Western Poland that have distinctive positive residuals. This phenomenon is even better noticeable for EGG2015, and could be connected with systematic errors in local determination of normal or ellipsoidal heights.

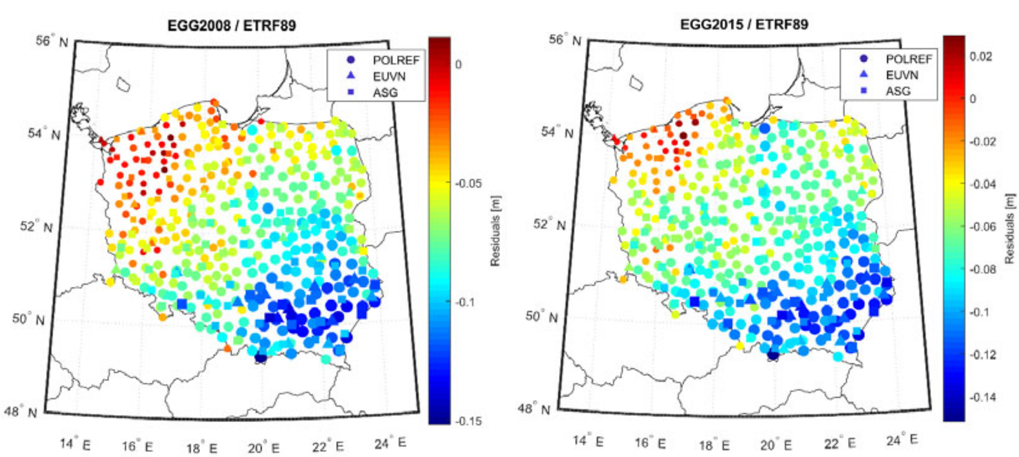

Fig. 3. The EGG2008 (left) and EGG2015 (right) models compared to GNSS/levelling quasigeoid heights, for ETRF89 frame

Both Figure 2 and Figure 3 show:

– a kind of slope of the residuals in the direction from south-east (positive res.) to north-west (negative res.), which may be due to systematic errors in adjusting levelling heights in PL-EVRF2007-NH.
– the systematic inclination is even clearer in the case of ETRF89 frame for both EGG realizations, so outliers are very difficult to be noticed. The only way to evaluate the consistency within data sets is to reduce residuals by the so-called parallel shifting. More details in Section 4.

Histograms of residues that represent EGG realizations compared to GNSS/levelling quasigeoid undulations are shown in Figure 4 and Figure 5. They have similar distribution of values, however a larger group of residues in the range of ± 1 cm is noted for EGG2015 model in the case of ETRF2000 ellipsoidal heights. For ETRF89 frame, a situation is slightly different because of residuals shifted from zero. When comparing ranges of residuals for each network, scales are narrower for EGG2008 model.

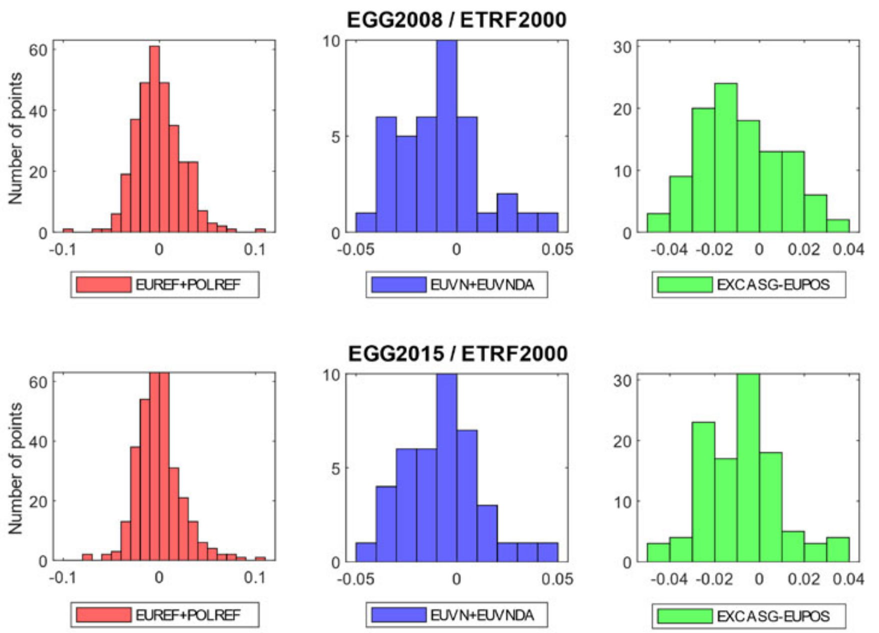

Fig. 4. Histograms of residuals of EGG realisations compared to quasigeoid undulations for ETRF2000 frame – EGG2008 (top) and EGG2015 (bottom)

Summarizing the conclusions in this section indicate a much better match between both EGG2008 and EGG2015 for the implementation of the ETRF2000 frame than ETRF89. The RMSE in this case are even three times smaller, and average values of height anomaly differences reach 7 cm in ETRF89 in comparison to -1 cm in ETRF2000. There is also a significant difference in the context of networks used for validation. In the case of the POLREF network, the discrepancies are characterized by greater spread than in other networks. These differences reach about 10-15% RMSE

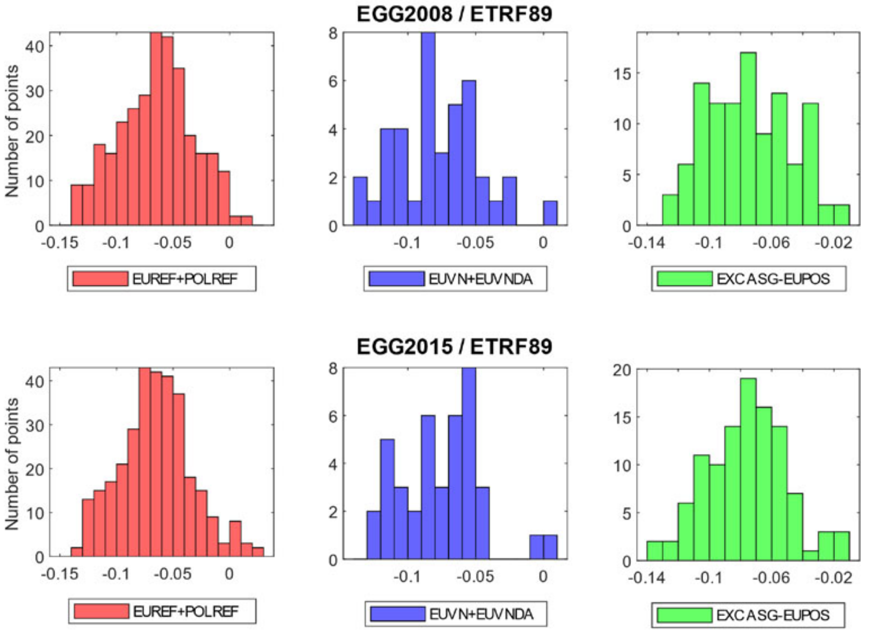

Fig. 5. Histograms of residuals of EGG realizations compared to quasigeoid undulations for ETRF89 frame – EGG2008 (top) and EGG2015 (bottom)

values in both compared quasi-geoid models and is the most significant in terms of the differences between (min – max) of residuals. In this statistic is two times bigger than for the other networks. Slightly better characteristics of the POLREF networks in the ETRF89 system result from the fact that the coordinates of the points newer networks (EUVN, ASG) have been transformed into the ETRF89 system using the POLREF network as linked points.

Further interesting observation is the comparison of the results of testing both European models to local models determined in Poland (see 2.1). The obtained characteristics are surprisingly close to the accuracy of the geoid model GDQM-PL13, proposed as the last geoid model created for Poland. It shows clearly that such values are kind of glass ceiling of the purely gravimetric geoid models. The reason can lie in limited accuracy of gravimetric measurements bases in Poland as well as modest and heterogeneous set of GNSS/levelling networks.

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Further interesting observation is the comparison of the results of testing both European models to local models determined in Poland (see 2.2). The obtained characteristics are surprisingly close to the accuracy of the geoid model GDQM-PL13, proposed as the last geoid model created for Poland. It shows clearly that such values are kind of glass ceiling of the purely gravimetric geoid models. The reason can lie in limited accuracy of gravimetric measurements bases in Poland as well as modest and heterogeneous set of GNSS/levelling networks.

4.2. Fitting EGG models to GNSS/levelling data

In this work, the following methods for fitting residuals have been selected.

4.2.1. Parallel shifting by a constant (average) value

This method corresponds with approach made by H. Denker when fitting EGG2008 into EVRF2007 heights after EUVN_DA realization (Denker, 2013). This simple procedure consists in determining a parallel shift based on local $\zeta^{model} - \zeta^{(GNSS/lev)}$ differences according to the equation (2)

$$\Delta\zeta^{res} = \zeta^{model} - \overline{\left(\zeta^{model} - \zeta^{(GNSS/lev)}\right)} - \zeta^{(GNSS/lev)} \tag{2}$$

where ζ^{model} – EGG quasigeoid undulation, $\zeta^{(GNSS/lev)}$ – computed GNSS/levelling height anomaly (h-H), $\Delta\zeta^{res}$ – residual height anomaly.

The main advantage of this type of adjustment is very simple implementation and almost no interference in gravimetric shape of the quasigeoid.

4.2.2. 7-parameter conformal transformation (Burša-Wolf/Helmert 3D with least-squared method)

It is simple adjusting coordinates from start datum φ, λ, ζ^{model} to target datum φ, λ, $\zeta^{GNSS/levelling}$. This required the initial conversion of geodetic to Cartesian coordinates and then the return to the φ, λ, ζ system for the comparison of differences in the quasigeoid distance from the GRS'80 ellipsoid. In general, this method is not often used as a fitting procedure but this approach was used just because it corresponds with the algorithm for calculating the current quasigeoid model PL-geoid-2011 for PL-KRON86-NH frame (Kadaj, 2013).

4.2.3. 4-parameter and 5-parameter trigonometric polynomial

The method is based on an equation (Kotsakis and Sideris, 1999):

$$h_i^{el} - H_i^n - \zeta_i = a_i^T x + v_i \tag{3}$$

which describes most of geoid evaluation studies that treat about comparisons of models with GNSS/levelling data. The part $a_i^T x$ specifies any inconsistencies of data and systematic errors, v_i expresses possible random errors.

$$a_i^T x = x_0 + x_1 \cos\varphi_i \cos\lambda_i + x_2 \cos\varphi_i \sin\lambda_i + x_3 \sin\varphi_i \tag{4}$$

$$a_i^T x = x_0 + x_1 \cos\varphi_i \cos\lambda_i + x_2 \cos\varphi_i \sin\lambda_i + x_3 \sin\varphi_i + x_4 \sin^2\varphi_i \tag{5}$$

In a result, the correction surface, with coefficients from x_0 to x_3/x_4, that minimizes deviations over the entire area (depending on latitude and longitude of points) and maintains continuity of a function that expresses this surface is obtained.

This method was very well implemented in the MATLAB software. The calculations were computed in two iterations. The residuals ζ_i in exceeding $3 \times RMS$ were rejected from further consideration.

4.3. Validation of fitted EGG models

The presentation of the results of fitting models was prepared in a similar way as the presentation of uncorrected EGG models (see 4.1) and includes both implementations of the ETRS system used in Poland and both EGG models. A specified summary of statistics was created (see Table 3 and Table 4). It discloses RMS values for every method taken into consideration after carrying out the 2nd iteration. During the 1st iteration of fitting, only some points of POLREF network had the residuals that exceeded three times the RMS values – 6 out of 319 points for EGG2008, and 9 out of 319 points for EGG2015. The limitation of the set of matching points to the EUVN network and eccentric points

Table 3. RMS comparison for ETRF2000 frame after fitting (2nd iteration)

Ref. frame Method Model	ETRF2000			
	EGG2008		EGG2015	
Set of points	E+A+P	E+A	E+A+P	E+A
Parallel shifting [cm]	2.2	1.9	2.0	2.0
7- par. transformation [cm]	1.5	1.4	1.5	1.7
4- par. polynomial [cm]	1.5	1.3	1.4	1.4
5- par. polynomial [cm]	1.5	1.3	1.5	1.3
Raw differences [cm]	2.3	2.1	2.3	2.0

Table 4. RMS comparison for ETRF89 frame after fitting (2nd iteration)

	EGG2008		EGG2015	
Set of points	E+A+P	E+A	E+A+P	E+A
Parallel shifting [cm]	3.2	3.1	2.8	2.7
7- par. transformation [cm]	1.6	1.6	1.4	1.5
4- par. polynomial [cm]	1.6	1.6	1.3	1.4
5- par. polynomial [cm]	1.6	1.6	1.4	1.4
Raw differences [cm]	7.6	7.5	8.1	8.0

of ASG-EUPOS ($E + A$ set) reduced this number of "rejected" locations to zero, within few exceptions:

- parallel shifting for EGG2015 model and ETRF2000 reference frame: PL06 located in Swinoujscie excluded;
- 7-parameter transformation for EGG2008/both ETRF realisations, 4-par. and 5-par. polynomial for EGG2008/ETRF89: WLOD (ASG-EUPOS eccentric point located in Wlodawa, eastern Poland) excluded.

Nevertheless, it still proves that EUVN and ASG-EUPOS points are up to date and have better consistency with geoid models.

There is no doubt that fitting model into GNSS/levelling data can decrease both RMS and STD values – it can be noticed that RMS is the lowest for 4-parameter polynomial correction for both EGG models. However, it links the gravimetric quasi-geoid model with the results of GNSS/leveling measurements and makes the model dependent on possible local network errors.

In Fig. 6, raw EGG2008 residuals for ETRF2000 reference frame (on the left) are compared with residuals after adjusting by 4-parameter trigonometric polynomial (fitted to the EUVN and ASG points – E+A variant; on the right). The main difference is removing the NW-SE slope trend. Map of the fitted EGG2015 (Figure 7) shows that the

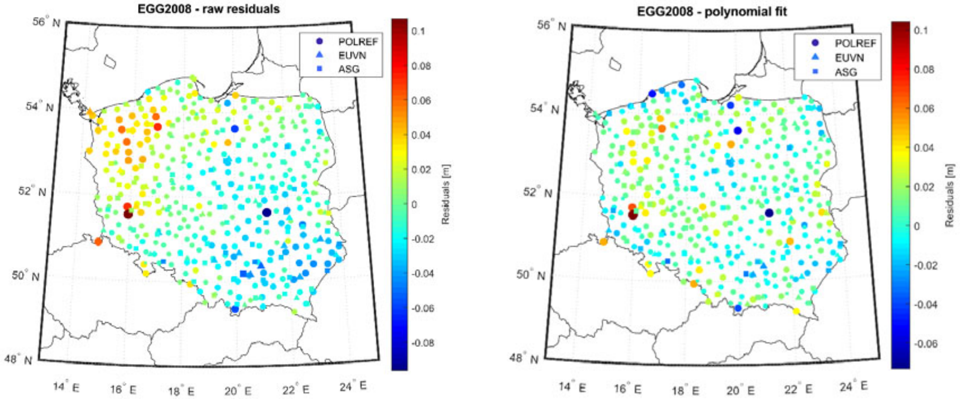

Fig. 6. Residuals of quasigeoid heights obtained from raw (left) and fitted with the use of the 4-parameter trigonometric polynomial (right) EGG2008 model and from GNSS/levelling data, for ETRF2000 frame

4-parameter polynomial has a smaller impact on reducing residuals in North Western Poland, probably just because of better "raw" consistency with GNSS/levelling data.

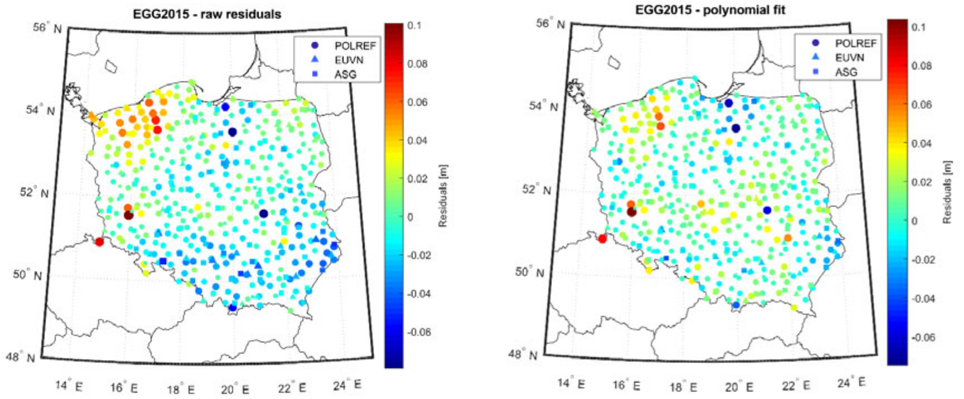

Fig. 7. Residuals of quasigeoid heights obtained from raw (left) and fitted with the use of the 4-parameter trigonometric polynomial (right) EGG2015 model and from GNSS/levelling data, for ETRF2000 frame

The analysis of histograms proves that polynomial method of fitting quasigeoid model systematizes the distribution of residues and locates most of points discrepancies in the range of 1 cm, even in a group of control points (POLREF, 1st column in Figures 8 and 9).

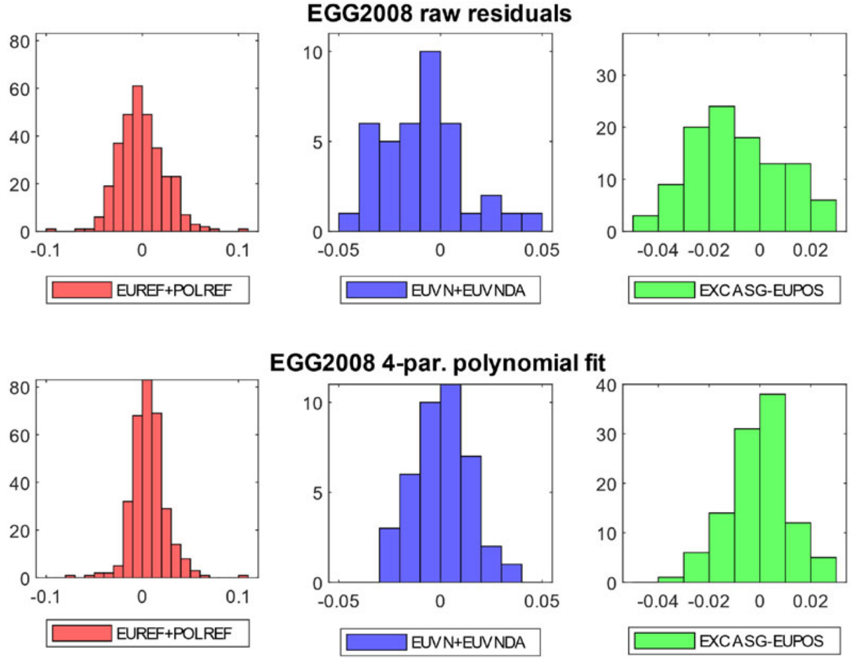

Fig. 8. Histograms of residuals – EGG2008 raw model (top) and EGG2008 fitted using 4-parameter polynomial (bottom) for ETRF2000

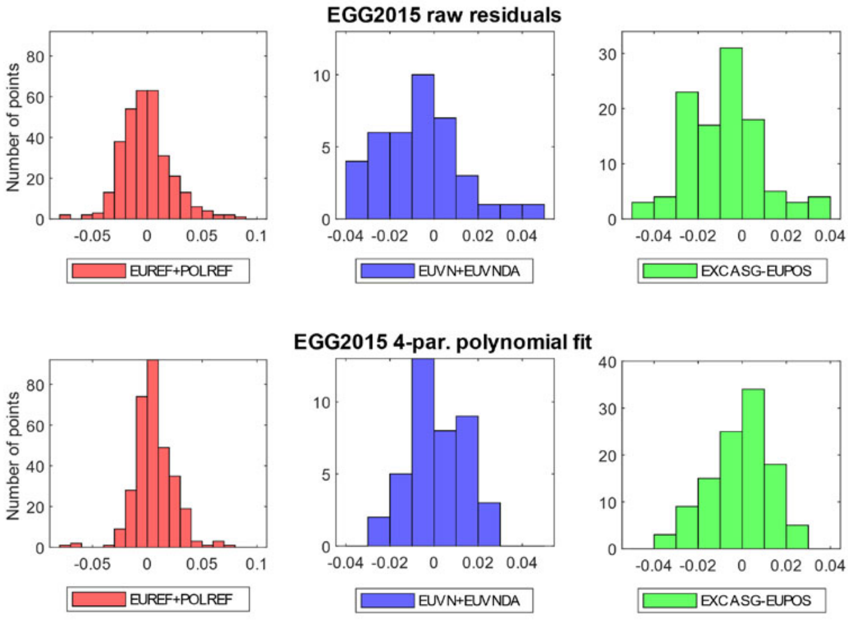

Fig. 9. Histograms of residuals – EGG2015 raw model (top) and EGG2015 fitted using 4-parameter polynomial (bottom) for ETRF2000 frame

The knowledge of spatial distribution of residuals have led the authors to the analysis of the maps more insightfully. It turned out that EGG models could be useful as a GNSS/levelling network validators. Figure 10 shows that increased residuals located

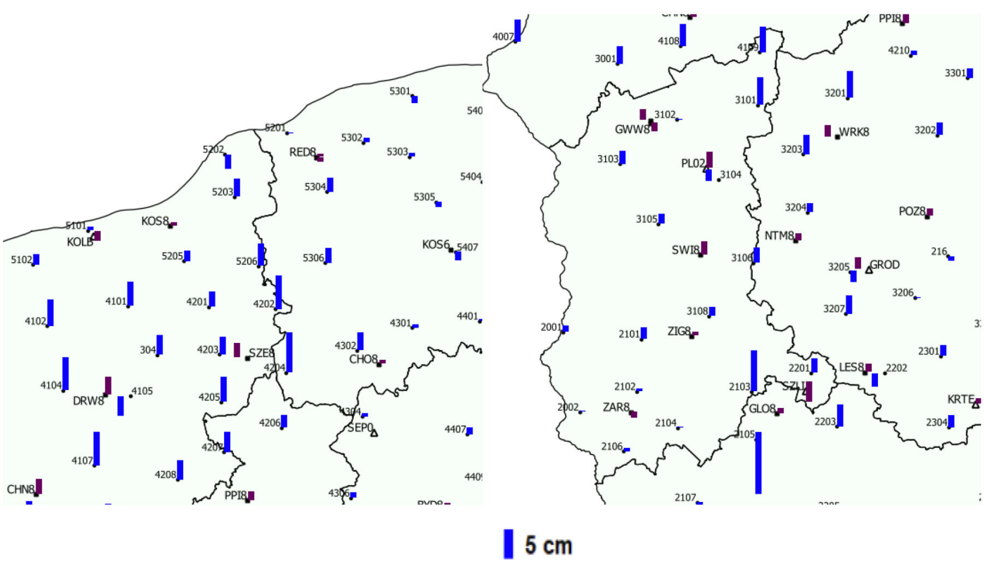

Fig. 10. An exemplary detection of local GNSS/levelling network errors with EGG2008 model

e.g. on the borders of the voivodeship can be noticed immediately without fitting the model into GNSS/levelling height anomalies. These errors may be accidental (e.g. displacement of a point, incorrect height entered to catalogues) or systematic (related to local adjusting of the 2nd and lower order levelling networks performed in the first stage independently in the small areas).

5. Conclusions

Both EGG2008 and EGG2015 seem to be very consistent with GNSS/levelling data in ETRF2000 frame, with the standard deviation below 2 cm. They can be a good choice for the new quasi-geoid model for PL-EVRF2007-NH height frame on the territory of Poland, they also gives the desired compromise between geoid continuity (in continental scale) and the relative accuracy of the model. Fitting modelled height anomalies into GNSS/levelling sites apparently improves the relative accuracy of the model (to max. 1.3 cm), however it makes the model dependent on possible systematic errors of realization vertical datum and ETRF frame.

All values of the errors, without corrections and after various fitting are the same or even better than agreement of different quasi-geoid realization on territory of Poland. It makes clear to say that 2 cm level now can be seen as contemporary limit of accuracy of purely gravimetric geoid, mainly due to Polish gravimetric data errors and inconsistency, and 13 mm after fitting to existing GNSS/levelling base networks. In our opinion, a fitted model should not be too restrictive and local in order to avoid losing the valuable property of the gravimetric geoid – the independence of geometric method and thus the ability to detect local errors in levelling and satellite networks.

On the basis of the achieved results, some conclusions can be drawn, i.e.:

- differences between model and empirical height anomalies have a clear slope in the direction from the south-east to the north-west, while in central Poland with almost-zero values. Probably this is due to systematic errors in adjusting levelling measurements in the EVRF2007 system, it is possible that it was referred to the point that generated the inclination of the entire structure;
- there are many points suspected of gross error – 2104, 2705 (POLREF), PRO8, PACA (ASG-EUPOS) in the ETRF2000 or 217 (Borowa Góra, EUREF) in the ETRF89 frame. They can result from, for example, errors in transferring normal heights, physical displacements of a point or catalogue mistakes;
  - EGG2015 has better compatibility with GNSS/levelling data in the west of Poland, and it is up-to-date, so that can be recommended as the base of new Polish quasi-geoid model.

Moreover, reducing normal and ellipsoidal heights to the zero-tidal system is very important in the context of assessing consistency of the datum. Incoming levelling campaign in Poland might be a good reason to improve results with respect to European vertical network and adopt new gravimetric quasi-geoid model based on EGG realization.

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