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The accuracy of the local assessment of the bulk density of copper-silver deposits in the Legnica-Głogów Copper District and its impact on the valuation of ore resource and mining production

Introduction

Dry bulk density, understood as the mass of sample divided by the total volume including all types of rock voids, is a deposit parameter which is crucial to estimate mineral resources. In Polish literature on the subject, this term is used interchangeably with "volumetric" or "spatial" density, in American English literature as a "tonnage factor" while the European Standard EN 1936:2006 (Polish standard PN-EN 1936: 2010 Natural stone test methods) uses the term "apparent density". The most commonly used term in the literature is bulk density (e.g. Abzalov 2013, 2016; Arseneau 2014; Dominy et al. 2002; Makhuvha et al. 2014; Rossi and Deutsch 2014; Scogings 2015; Sinclair and Blackwell 2002).

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The accuracy of the mineral (ore) bulk density assessment is one of the factors determining the accuracy of the estimation of ore and metal resources in weight units. Resources are an important asset of mining companies and their incorrect estimation can lead to the failure of mining projects. An equally important issue is the proper accounting of production carried out in short periods of time (reconciliation of the forecasted ore resources and metal content to the values based on the actual output from the deposit).

A comprehensive review of potential sources of resource estimation errors, including those related to the bulk density assessment, was presented by Dominy et al. (2002). Despite the obvious importance of bulk density for the correct estimation of resources and metal content, the question of the accuracy of its assessment against the background of other resource parameters has not been thoroughly studied. Sometimes this parameter is overlooked or attracts less attention than other resource parameters and sampling parameters (Abzalov 2013) neglected (Scogings 2015) or even regarded as of secondary importance (Mucha et al. 2017). The reason for this is presumed to be a relatively small variability, much smaller than other deposit parameters, and, as a consequence, the high accuracy of estimating its average value for a small number (20–30) of measurements (Nieć et al. 2012a, b) made in different parts of the deposit. Rossi and Deutsch (2014) claim that within internally homogeneous geological domains (units), the number of bulk density measurements should not be less than 30 and for modeling this parameter should be from 100 to 1,000 depending on the type and size of the deposit. It should be remembered, however, that the use of the average bulk density for ore resources estimations is justified and gives good results when estimating the resources of the entire deposit or a substantial part of it sampled for bulk density, but may lead to erroneous estimates of resources in small parts of the deposit.

1. Research objective

The main purpose of the presented research was to analyze the range of possible differences between the constant values of bulk densities (hereinafter referred to as reference values) assigned during deposit documentation to the main lithological units and bulk densities of these units at the local observation scale, determined based on the results of experimental sampling of individual lithological units within the copper and silver deposits (Lubin, Polkowice-Sieroszowice, and Rudna) in Legnica-Głogów Copper District (LGCD) exploited by KGHM Polska Miedź SA (hereinafter referred to as KGHM). The local scale of the observation is meant as small parts of the deposit, roughly corresponding to the deposit area exploited on a monthly basis (area of 60,000–70,000 m^2).

2. The influence of the accuracy of the assessment of the ore bulk density on the accuracy of estimating Cu resources

The simplest, statistical method of estimating Cu resources is the arithmetic mean method according to which the amount of Cu resources in mass units is calculated as the product of the arithmetic mean values of three resource parameters identified at the sampling points of the deposit and the resource estimation area:

$$Q_{\mathrm{Cu}}\left[Mg\right] = \bar{z} \cdot \bar{\rho}_b \cdot \bar{M} \cdot F \cdot \frac{1}{100\%}$$

↳ $\bar{z}, \bar{\rho}_b, \bar{M}$ – arithmetic means, respectively: Cu content [%], ore bulk density [Mg/m^3], deposit thickness [m] at sampling points,
F [m^2] – the area of the deposit treated as a fixed size.

Assuming the independence of resource parameters, the relative standard error in the estimation of Cu resources is expressed by the formula (Sinclair and Blackwell 2002):

$$\varepsilon_R\left(Q_{\mathrm{Cu}}\right) = \sqrt{[\varepsilon_R(\bar{z})]^2 + [\varepsilon_R(\bar{M})]^2 + [\varepsilon_R(\bar{\rho}_b)]^2} = \sqrt{\left[\frac{v_z}{\sqrt{n_z}}\right]^2 + \left[\frac{v_M}{\sqrt{n_M}}\right]^2 + \left[\frac{v_{\rho b}}{\sqrt{n_{\rho_b}}}\right]^2}$$

↳ $\varepsilon_R(\bar{z}), \varepsilon_R(\bar{M}), \varepsilon_R(\bar{\rho}_b)$ – relative standard errors of estimation of mean values of resource parameters, respectively: Cu content, deposit thickness, ore bulk density,
v_z, v_M, $v_{\rho b}$ – coefficients of variation, respectively: Cu content, the deposit thickness, and ore bulk density,
n_z, n_M, n_{ρ_b} – the number of measurements, respectively: Cu content, thickness, and bulk density.

The correlation relationship between the Cu content and the ore bulk density, which should theoretically occur in the case of ore deposits (Nieć et al. 2012b; Arseneau 2014), turned out to be weak or very poor under LGCD deposit conditions, probably due to the variable porosity of the ore (Mucha et al. 2017). This makes it possible to consider the conditions of applicability of the formula for the value of the resource estimation error to be met.

For example, in a deposit area of 250 × 250 m (i.e. an area of 6.25 ha) mined approximately for one month about n = 100 samples are collected in mining and development excavations, with an average distance between samples of 25 m. Assuming typical values of the coefficients of variation of Cu content and deposit thickness equal to 50% ($v_z = v_M$ = 50%), and ore bulk density of 5% ($v_{\rho b}$ = 50%) (Table 1) for LGCD deposits, with simultaneous measurements of all resource parameters of $n = 100$, the relative standard errors of estimates of average resource parameters will be as follows:

$$\varepsilon_R(\bar{z}) = \varepsilon_R(\bar{M}) = \frac{50\%}{\sqrt{100}} = 5\%, \quad \varepsilon_R(\bar{\rho}_b) = \frac{50\%}{\sqrt{100}} = 0.50\%$$

and the relative standard error of the estimation of Cu resources will be:

$$\varepsilon_R(Q_{\mathrm{Cu}}) = \sqrt{5^2 + 5^2 + 0.5^2}\,\% = 7.09\%$$

Assuming an absolutely accurate assessment of the average ore bulk density (i.e. with zero error), the resource estimation error will be:

$$\varepsilon_R(Q_{\mathrm{Cu}}) = \sqrt{5^2 + 5^2 + 0^2}\,\% = 7.07\%$$

The comparison of both results confirms the view that there is no noticeable impact of the accuracy of the assessment of the average ore bulk density on the accuracy of the estimation of Cu resources. This approach was maintained when carrying out a small number of ore bulk density measurements ($n_{\rho_b} = 4$, as an exemple) while maintaining all previous assumptions ($n_z = n_M = 100$). This means that the number of bulk density measurements is only 4% of the measurements of the remaining resource parameters. Abzalov (2013), analyzing a number of deposits (mainly ore deposits), stated that the number of bulk density measurements ranged from 5% to 100% of the total number of metal content measurements.

For the adopted number of bulk density measurements, the standard relative error is:

$$\varepsilon_R(\bar{\rho}_b) = \frac{5\%}{\sqrt{4}} = 2.50\%$$

and the relative error of the standard estimation of Cu resources is:

$$\varepsilon_R(Q_{\mathrm{Cu}}) = \sqrt{5^2 + 5^2 + 2.5^2}\,\% = 7.50\%$$

The comparison of the error value for 100 and 4 measurements of ore bulk density allows us to state that the observed difference of 0.41% has low practical significance for the accuracy of the Cu resource estimates.

3. Basic research material – experimental sampling

In the world-class Cu-Ag deposits of the Legnica-Głogów Copper District (LGCD), ore minerals occur in three main lithological units (series) of the Upper Permian (Zechstein) age: carbonate, shale and sandstone series (Piestrzyński ed. 2007).

To estimate the ore and metal resources within them, constant bulk density values are adopted, based on the results of previous studies of the LGCD deposits carried out at the stage of their exploration and documentation: 2.6 Mg/m^3 for the carbonate series, 2.5 Mg/m^3 for the shale series, and 2.3 Mg/m^3 for the sandstone series. These were treated as reference values and as a comparative standard for the results of bulk density measurements obtained as a result of experimental sampling described below.

Research conducted in the 1980s in mining excavations of LGCD deposits resulted in the determination of 18 individual lithological units within the main lithological units (Fig. 1). After standardizing their terminology in all LGCD mines in 2011, they were formally entered into the databases (Kaczmarek et al. 2014, 2017). The composition of individual lithological units in the vertical profile of the deposit is variable and can be associated with their location relative to the morphological structures observed in the top of sandstones ("Weissliegend"), which are: convex forms called elevations, flattened areas between neighboring elevations (so-called depressions or flats), and the smallest surfaces of the transition areas called elevation slopes.

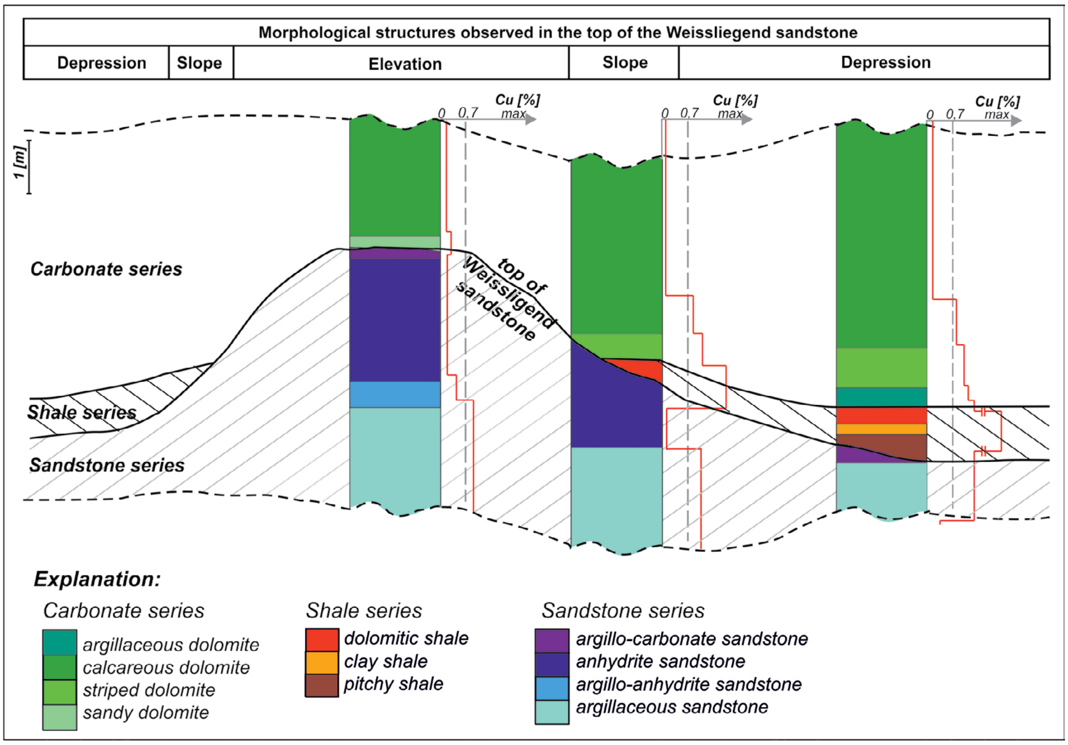

Fig. 1. Scheme of individual lithological units within main lithological units in the areas of elevation, elevation slopes and depression in the top of Weissliegendes Formation

Rys. 1. Schemat występowania wydzieleń szczegółowych w obrębie głównych serii litologicznych w obszarach elewacji, skłonów i depresji stropu białego spągowca

The bulk density analysis at the local observation scale were based on the results of special sampling of 10 out of 18 individual lithological units. Samples with an average mass of about 2 kg each were collected in mining excavations of the currently exploited Cu-Ag ore deposits: Lubin, Rudna, and Polkowice-Sieroszowice. In total, more than 1,600 samples were collected at 500 sampling points arranged irregularly in 3 deposits (Fig. 2).

The collected samples were subjected to the determination of bulk density, while the selected samples were subjected to total porosity and specific density analysis in the accredited Laboratory for Testing Rock Properties and Stone Products of the AGH University of Science and Technology in Kraków. The bulk density was determined using the wire basket method (Mucha et al. 2017), which is one of the hydrostatic (immersion) methods. According to the

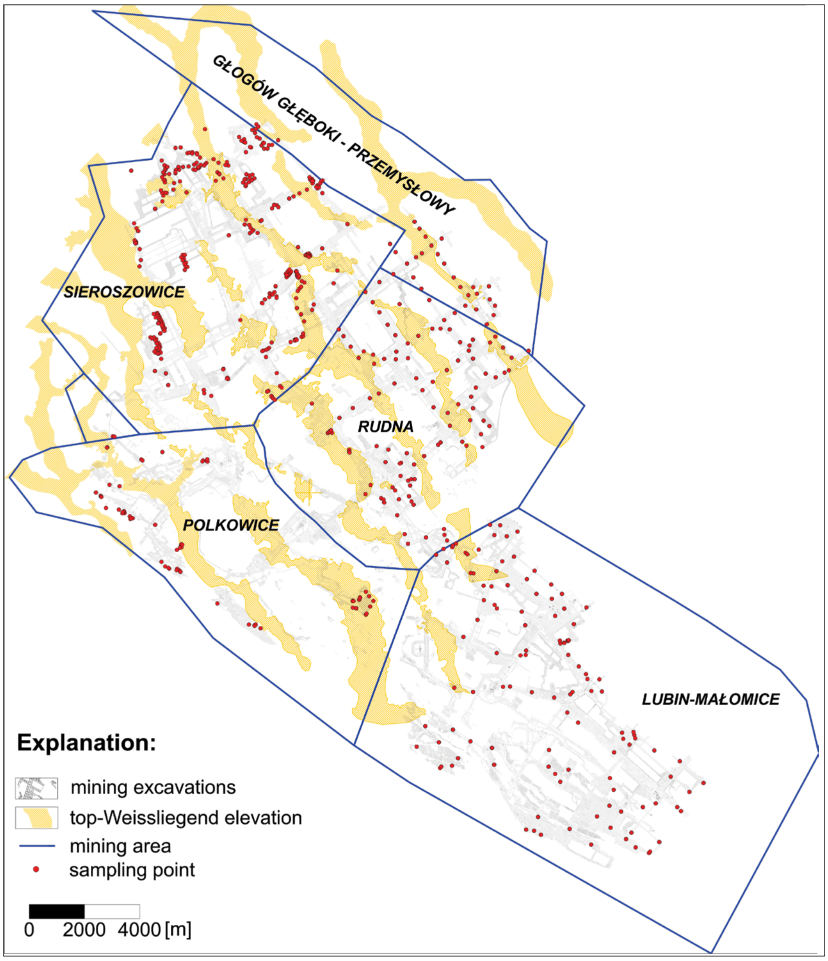

Fig. 2. Location of experimental sampling points for the determination of bulk densities of individual lithological units in the exploited deposits of LGCD

Rys. 2. Lokalizacja stanowisk opróbowania eksperymentalnego dla oceny gęstości objętościowej wydzieleń szczegółowych w eksploatowanych złożach LGOM

PN-EN 1936:2010 standard, bulk (apparent) density (ρ_b [Mg/m^3]) is expressed by the ratio of the mass of the dry specimen and its apparent volume and determined from the formula:

$$\rho_b = \frac{m_d}{m_s - m_h}\rho_{rh}$$

m_d – mass of the dry specimen [g],
m_s – mass of the saturated specimen [g],
m_h – mass of the specimen immersed in water [g],
ρ_{rh} – density of water at test temperature [Mg/m^3].

After determining the bulk density, the Cu content in the samples was also determined. Finally, the results of determination of bulk density of 1,462 samples were considered reliable, including 350 from the Lubin deposit, 404 from the Rudna deposit, and 708 from the Polkowice-Sieroszowice deposit. The reason for the rejection of many measurements was the difficulty in reliable determination of the bulk density in samples collected from the selected individual lithological units; this particularly applies to clay and pitchy shale, characterized by low strength of the rock material and, as a result, disintegrating in water during the test.

4. Methods and results

The results of bulk density measurements in samples taken from 10 individual lithological units were subjected to a basic statistical analysis, which included a determination of the minimum and maximum values in each data set, the arithmetic mean, and the coefficient of variation (Table 1). Insignificant, relative variability of the bulk density of individual lithological units in LGCD deposits is confirmed by low coefficients of variation of up to 6% (Table 1). It should be noted, however, that the difference of extreme values of bulk density determinations in individual lithological units is significant and most often is 0.4–0.6 Mg/m^3 and reaches a maximum of 0.83 Mg/m^3 (argillaceous sandstone in the Rudna deposit).

The arithmetic means of the bulk density of individual lithological units ($\overline{\rho}_{bi}(E)$) determined during experimental sampling were compared with the reference values of bulk density ($\rho_{bM}(R)$) of the main lithological units; their relative differences (ε_{Ri}) were calculated using the following formula:

$$\varepsilon_{Ri}[\%] = \frac{\overline{\rho}_{bi}(E) - \rho_{bM}(R)}{\rho_{bM}(R)} \cdot 100\%$$

$\overline{\rho}_{bi}(E)$ – arithmetic mean of bulk density measurements in the "i" individual unit based on experimental sampling,
$\rho_{bM}(R)$ – the reference bulk density for the main lithological unit to which a given individual lithological unit belongs.

Table 1. Statistics of bulk densities $\rho_b(E)$ of samples collected during experimental sampling of selected individual lithological units in Cu-Ag LGCD deposits (Lubin, Rudna, Polkowice-Sieroszowice) and relative differences of bulk densities (ε_{Ri}) determined on the basis of experimental sampling and reference values used in KGHM for the main lithological units

Tabela 1. Statystyka gęstości objętościowej $\rho_b(E)$ prób pobranych w ramach opróbowania eksperymentalnego z wytypowanych wydzieleń litologicznych szczegółowych w złożach LGOM (Lubin, Rudna, Polkowice-Sieroszowice) oraz ich różnice względne odniesione do wartości referencyjnych dla serii głównych (ε_{Ri})

Deposit	Main lithological units	Individual lithological units	$\rho_b(E)$					$\rho_{bM}(R)$	ε_{Ri}
			N	$\rho_b(E)_{min}$	$\rho_b(E)_{max}$	$\bar{\rho}_b(E)$	CV		
				[Mg/m^3]			[%]	[Mg/m^3]	[%]
Lubin	Carbonate series	Striped dolomite	15	2.21	2.72	2.61	5.0	2.6	0.3
		Argillaceous dolomite	33	2.33	2.77	2.63	3.0		1.3
		Calcareous dolomite	22	2.38	2.76	2.65	3.3		2.0
	Shale series	Dolomitic shale	27	2.21	2.86	2.52	4.3	2.5	0.8
		Clay shale	1	2.54	2.54	2.54	–		1.4
		Boundary Dolomite	34	2.43	2.80	2.70	3.0		8.2
	Sandstone series	Argillaceous sandstone	134	2.05	2.71	2.25	5.9	2.3	–2.0
		Carbonate sandstone	82	2.08	2.69	2.53	4.9		10.0
		Anhydrite sandstone	2	2.57	2.67	2.62	2.8		14.0
Rudna	Carbonate series	Striped dolomite	73	2.56	2.81	2.69	1.9	2.6	3.3
		Argillaceous dolomite	28	2.43	2.73	2.62	2.5		0.8
		Calcareous dolomite	59	2.55	2.86	2.73	2.4		4.8
	Shale series	Dolomitic shale	33	2.35	2.75	2.54	3.8	2.5	1.4
		Clay shale	12	2.21	2.66	2.38	4.7		–4.8
		Pitchy shale	1	2.21	2.21	2.21	–		–11.4
	Sandstone series	Argillaceous sandstone	162	2.06	2.89	2.33	6.0	2.3	1.4
		Carbonate sandstone	21	2.31	2.76	2.64	3.6		14.6
		Anhydrite and argillo-anhydrite sandstone	15	2.63	2.76	2.71	1.3		17.8
Polkowice-Sieroszowice	Carbonate series	Striped dolomite	195	2.55	2.83	2.70	2.1	2.6	3.8
		Argillaceous dolomite	38	2.43	2.77	2.66	2.2		2.3
		Calcareous dolomite	86	2.57	2.85	2.73	2.1		5.0
	Shale series	Dolomitic shale	202	2.23	2.79	2.59	3.7	2.5	3.6
		Pitchy shale	6	2.21	2.45	2.32	3.6		–7.2
	Sandstone series	Argillaceous sandstone	64	2.02	2.68	2.35	6.0	2.3	2.2
		Carbonate sandstone	102	2.20	2.83	2.55	4.1		10.9
		Anhydrite sandstone	15	2.32	2.79	2.66	5.2		15.7

$\rho_b(E)$ – bulk density determined on the basis of experimental sampling, N – the number of samples, $\rho_b(E)_{min}$ – minimum value, $\rho_b(E)_{max}$ – maximum value, $\bar{\rho}_b(E)$ – arithmetic mean, CV – coefficient of variation; $\rho_{bM}(R)$ – reference bulk density used in KGHM for the main lithological units, ε_{Ri} – bulk density differences determined on the basis of experimental sampling (for the individual lithological units) and reference values used in KGHM for the main lithological units.

The relative differences of arithmetic means of bulk density of individual lithological units related to reference values of bulk density depend on the type of individual lithological unit (Table 1). The highest of them, ranging from 14 to 18%, relate to anhydrite or argillo-anhydrite sandstone, whose share in Cu resources, however, is marginal and does not exceed 0.2%. High differences were also found for carbonate sandstone (10–15%), whose share in Cu resources is also low (<1.4%). The individual lithological units most abundant in Cu (striped dolomite, calcareous dolomite, argillaceous dolomite, dolomitic shale, clay shale, and argillaceous sandstone) show smaller differences in mean bulk densities compared to reference values while their absolute values do not exceed 5%. Therefore, it can be assumed that in general, in the case of Cu-Ag LGCD deposits (or their substantial parts), the impact of errors in the determination of the bulk density of the main lithological units will not have a significant impact on the accuracy of the estimation of mineral and metal resources.

Then, the homogeneity of the bulk density of individual lithological units and Cu content within the main lithological units (Table 2) and the homogeneity of these parameters in individual lithological units for the three analyzed deposits (Table 3) were examined using the Games-Howell test (Games and Howell 1976). The advantage of this test comparing mean parameter values simultaneously for all pairs of data sets is the lack of requirements for equal parameter variance and equality of data sets. All statistical calculations and Games-Howell tests were performed using STATGRAPHICS Centurion XVII software (Statpoint Technologies, Inc.).

The results of the Games-Howell test indicate that, with the exception of the two main units (carbonate and shale series in the Lubin deposit), the bulk densities of the individual lithological units do not form homogeneous groups within the main units in which they occur (Table 2). Individual lithological units form 2 or 3 separate groups of homogeneous sets of bulk density determinations. The Cu content is more homogeneous. In 4 out of 9 main lithological units considered, the Cu contents in individual lithological units form homogeneous groups that do not show statistically significant differences in respect to the average content of this metal (Table 2).

It is noteworthy that, with the exception of the carbonate series in the Lubin, Rudna, and Polkowice-Sieroszowice deposits, the distinguished homogeneous groups of Cu content and bulk density are different.

The results of testing the homogeneity of the determination of bulk density and Cu content in individual lithological units of the three deposits are also interesting (Table 3). The tests were carried out for the five individual lithological units most abundant in copper, excluding clay and pitchy shale for which data sets were too small. With the exception of argillaceous dolomite, the bulk densities of the other individual lithological units and Cu contents do not form homogeneous sets within all three deposits as shown by the Games-Howell test.

In addition, as in the previous analysis (Table 2), the groups of homogeneous sets for both parameters are different. This suggests a lack of correlation between bulk density and Cu content. This observation confirms the conclusions resulting drawn from the correlation

Table 2. Results of the Games-Howell test* determining the homogeneity of the bulk density and Cu content of individual lithological units in LGCD deposits (Lubin, Rudna, and Polkowice-Sieroszowice) within main lithological units (based on experimental sampling)

Tabela 2. Wyniki testu Gamesa-Howella badania jednorodności gęstości objętościowych wydzieleń litologicznych szczegółowych i zawartości w nich Cu w złożach: Lubin, Rudna i Polkowice-Sieroszowice w obrębie głównych wydzieleń litologicznych

Deposit	Main lithological units	Individual lithological units	Bulk density			Cu content		
Lubin	Carbonate series	Striped dolomite	x			x		
		Argillaceous dolomite	x			x		
		Calcareous dolomite	x			x		
	Shale series	Dolomitic shale	x			x		
		Boundary dolomite	x				x	
	Sandstone series	Carbonate sandstone		x		x		
		Argillaceous sandstone	x			x		
		Anhydrite sandstone	x	x		x		
Rudna	Carbonate series	Striped dolomite	x			x		
		Argillaceous dolomite		x			x	
		Calcareous dolomite			x			x
	Shale series	Dolomitic shale		x		x		
		Clay shale	x			x		
	Sandstone series	Carbonate sandstone	x			x		
		Argillaceous sandstone		x		x		
		Anhydrite sandstone			x	x		
Polkowice-Sieroszowice	Carbonate series	Argillaceous dolomite	x			x		
		Striped dolomite		x			x	
		Calcareous dolomite			x			x
	Shale series	Pitchy shale	x			NI	NI	NI
		Dolomitic shale		x		NI	NI	NI
	Sandstone series	Argillaceous sandstone	x			x		
		Carbonate sandstone		x		x	x	
		Anhydrite sandstone			x		x	

* – Test significance level: 0.05; NI – not included in the study; homogenous groups are identified by columns of **x**'s.

Table 3. Results of the Games-Howell test* determining the homogeneity of the bulk density and Cu content of LGCD deposits (Lubin, Rudna, and Polkowice-Sieroszowice) in the five most abundant individual lithological units

Tabela 3. Test Gamesa-Howella badania jednorodności gęstości objętościowych i zawartości Cu złóż: Lubin, Rudna i Polkowice-Sieroszowice w pięciu wytypowanych, ważniejszych zasobowo wydzieleniach litologicznych szczegółowych

Deposit parameter	Deposits	Individual lithological units									
		Argillaceous sandstone		Dolomitic shale		Striped dolomite		Calcareous dolomite		Argillaceous dolomite	
Bulk density [Mg/m^3]	Lubin	x		x		x		x		x	
	Rudna		x	x		x	x		x	x	
	Polkowice-Sieroszowice		x		x		x		x	x	
Cu content [%]	Lubin	x		x		x	x	x		x	
	Rudna		x		x	x		x	x	x	x
	Polkowice-Sieroszowice	x			x		x		x		x

* – Test significance level: 0.05; homogenous groups are identified by columns of **x**'s.

and regression analyses, which showed a weak or very weak correlation of both parameters in the majority of individual lithological units, most likely due to the masking effect of the variable porosity of the mineral (Mucha et al. 2017).

The data on the thickness and bulk density of individual lithological units were interpolated in 250 × 250 m blocks using the geostatistical ordinary kriging procedure (Clark and Harper 2008; Journel and Huijbregts 1978). The geostatistical method was previously used to optimize the sampling interval for the purposes of assessing the bulk density of mineral of selected deposits (Abzalov 2013) and modeling the variability of bulk density of individual lithological units of the Cu-Ag Polkowice-Sieroszowice deposit belonging to the LGCD (Paszek and Wasilewska-Błaszczyk 2017). The ordinary kriging procedure takes the location of sampling points relative to each other and the calculation block, the size and shape of the block, and the structure of the variation of the estimated parameter as a function of the distance between sampling points expressed by means of variogram models into account.

The empirical omnidirectional bulk density variograms of 6 individual lithological units with matched geostatistical (spherical) models are presented in Fig. 3 while the equations of the models are shown in Table 4.

They reveal a different level and style of variability of the examined resource parameter (Fig. 3). For each individual lithological unit, the non-random component of bulk density variation (U_N) is marked with different strength (from 19 to 60%, Table 4). Its share is usually moderate for striped dolomite, argillaceous dolomite, dolomitic shale, and carbonate

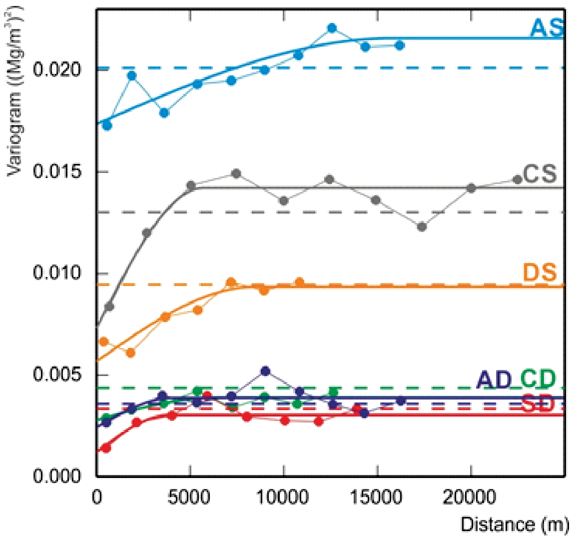

Fig. 3. Empirical variograms and theoretical models of bulk density of individual lithological units (based on experimental sampling)
SD – striped dolomite, AD – argillaceous dolomite, CD – calcareous dolomite,
DS – dolomitic shale, AS – argillaceous sandstone, CS – carbonate sandstone

Rys. 3. Semiwariogramy empiryczne gęstości objętościowej dla wydzieleń litologicznych szczegółowych wraz z dopasowanymi modelami teoretycznymi (na podstawie opróbowania eksperymentalnego)

Table 4. Parameters of geostatistical models fitted to variograms of bulk density of individual lithological units in Cu-Ag LGCD deposits

Tabela 4. Parametry modeli geostatystycznych dla semiwariogramów gęstości objętościowej dla wydzieleń litologicznych szczegółowych w złożu Cu-Ag LGOM

Individual lithological units	Parameters of geostatistical models				U_N
	Model type	a	C_0	C	
		[m]	[(Mg/m^3)^2]	[(Mg/m^3)^2]	[%]
Striped dolomite (SD)	Spherical	3 912	0.0012	0.0018	60.2
Argillaceous dolomite (AD)	Spherical	4 052	0.0024	0.0015	37.4
Calcareous dolomite (CD)	Spherical	6 486	0.0028	0.0011	28.5
Dolomitic shale (DS)	Spherical	8 464	0.0057	0.0037	39.3
Carbonate sandstone (CS)	Spherical	5 716	0.0073	0.0069	48.6
Argillaceous sandstone (AS)	Spherical	15 794	0.0174	0.0042	19.6

a – variogram range, C_0 – the nugget effect (variance of the random variation component), C – variance of the non-random variation component, U_N – the maximum share of the non-random component $U_N = [C/(C_0 + C)] \cdot 100\%$.

sandstone and low for calcareous dolomite and argillaceous sandstone. The variogram ranges vary from about 4 to nearly 16 km but in each case is larger than the mean distance between the experimental sampling points.

Interpolation of the bulk density and thickness of individual lithological units was based on the results of experimental sampling, and the results of routine sampling of the deposit, respectively (Fig. 4). Eight of the closest sampling points were used for the interpolation of the parameter value in each blocks. Based on the block maps of the thickness and bulk density of the individual lithological units, the weighted mean bulk densities for the main lithological units (Fig. 5) and the economic deposit (Fig. 6) were determined (with the thickness of individual lithological units as the weights). The mean bulk density for the economic deposit was calculated in similar manner based on the reference values used in KGHM (Fig. 6). In the case of clay and pitchy shale, no bulk density block maps were made due to the low number of data sets from experimental sampling. For these individual lithological units, the arithmetic mean bulk density values given in Table 1 was used in calculation in each node of the grid.

Exemplary block maps of bulk density for carbonate and argillaceous sandstones (Fig. 4) clearly show the scale of local variability of bulk density within the individual lithological units. The wide range of the discussed parameter for individual lithological units (Table 1, Fig. 4) translates into a large variation in average bulk densities for the main lithological

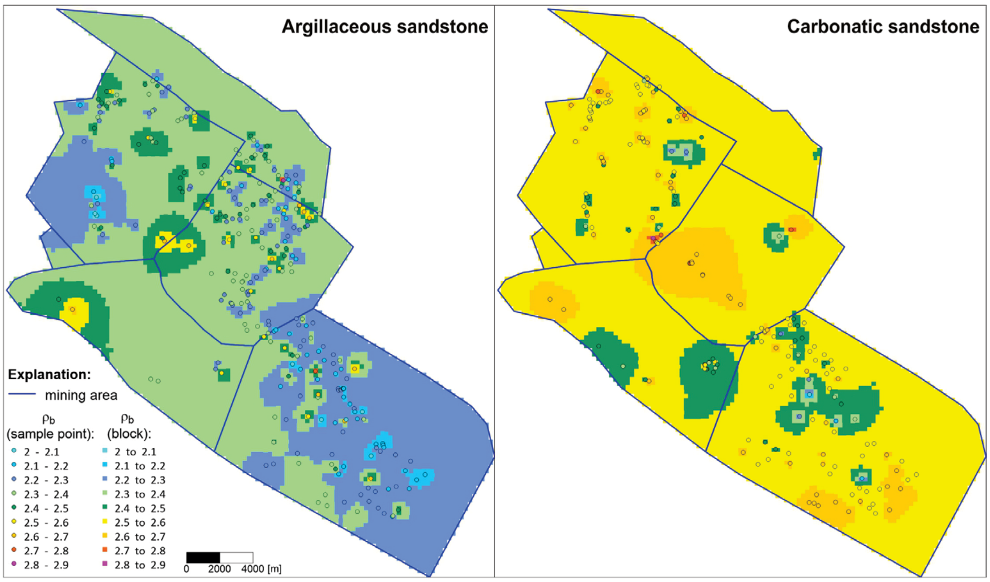

Fig. 4. An indicative block maps (250 × 250 m blocks) of the bulk density (ρ_b) of argillaceous and carbonate sandstone in the Cu-Ag LGCD deposits based on experimental sampling

Rys. 4. Orientacyjna mapa blokowa gęstości objętościowej (ρ_b) piaskowca ilastego i węglanowego w złożach Cu-Ag LGOM wykonana na podstawie opróbowania eksperymentalnego units (Fig. 5) and the economic deposit (Fig. 6). The mean bulk density within the economic deposit calculated in 250 × 250 m blocks based on the results of experimental sampling has a much larger range of values than the corresponding mean calculated based on reference values of bulk density used in KGHM (Fig. 6).

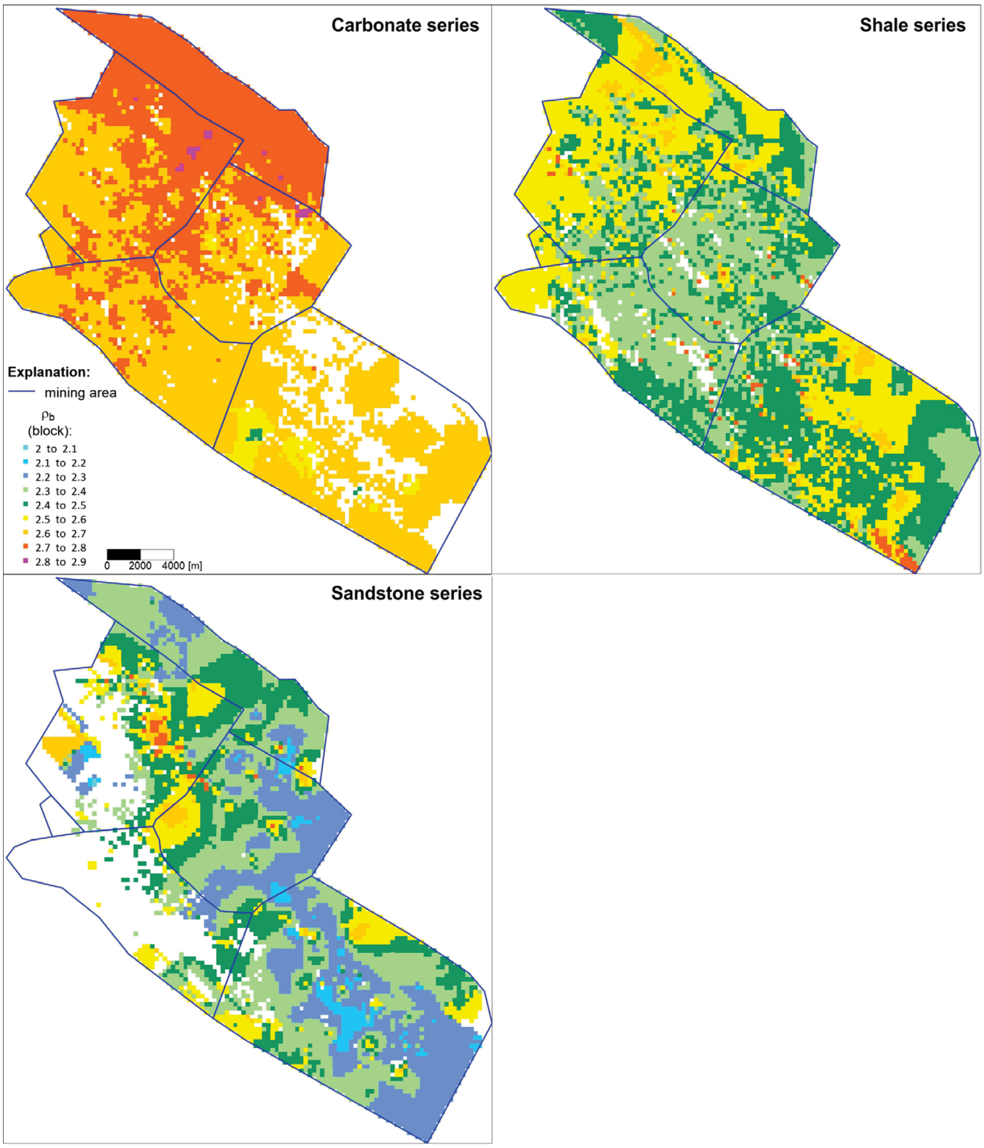

Fig. 5. An indicative block map (250 × 250 m blocks) of the bulk density (ρ_b) of carbonate, shale, and sandstone series in the Cu-Ag LGCD deposits based on experimental sampling

Rys. 5. Orientacyjne mapy blokowe gęstości objętościowej (ρ_b) serii węglanowej, łupkowej i piaskowcowej w złożu Cu-Ag LGOM wykonane na podstawie opróbowania eksperymentalnego

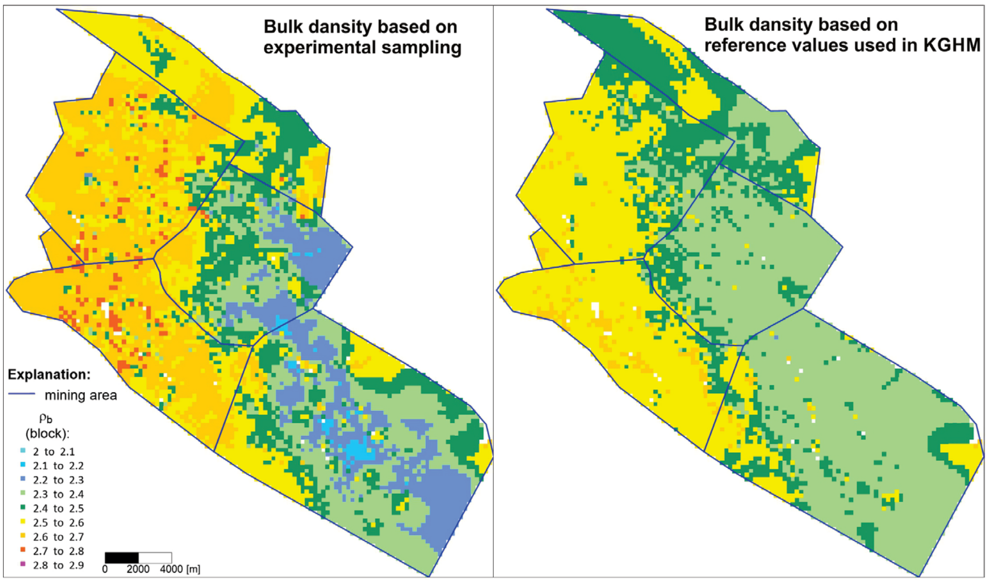

Fig. 6. An indicative block map (250 × 250 m blocks) of the weighted average bulk density (ρ_b) in the Cu-Ag LGCD economic deposits based on experimental sampling and reference bulk densities used in KGHM

Rys. 6. Orientacyjne mapy blokowe średniej ważonej gęstości objętościowej (ρ_b) w złożach Cu-Ag LGOM wykonane na podstawie wyników opróbowania eksperymentalnego i wartości referencyjnych gęstości objętościowej stosowanych w KGHM Polska Miedź SA

The comparison of bulk density measurements of the main lithological units determined by means of experimental sampling ($\rho_{bM}(E)$) with the reference values used in the mining plants of KGHM ($\rho_{bM}(R)$) were made using relative differences (ε_{R_M}) determined for each 250 × 250 m block from the following formula:

$$\varepsilon_{R_M} = \frac{\rho_{bM}(E) - \rho_{bM}(R)}{\rho_{bM}(R)} \cdot 100\%$$

The relative differences in bulk densities of the deposit (ε_{R_D}) were determined in a similar manner, i.e. on the basis of experimental sampling and reference values using the following formula:

$$\varepsilon_{R_D} = \frac{\rho_{bD}(E) - \rho_{bD}(R)}{\rho_{bD}(R)} \cdot 100\%$$

↳ $\rho_{bD}(E)$, $\rho_{bD}(R)$ – weighted averages of the thickness of the main lithological units; bulk densities of the deposit estimated on the basis of experimental sampling and reference values, respectively.

The relative differences in the bulk density assessments of the main lithological units and the deposit determined on the basis of experimental sampling and reference values in 250 × 250m blocks are clearly revealed by the block maps presented in Figure 7 and the statistics summarized in Table 5.

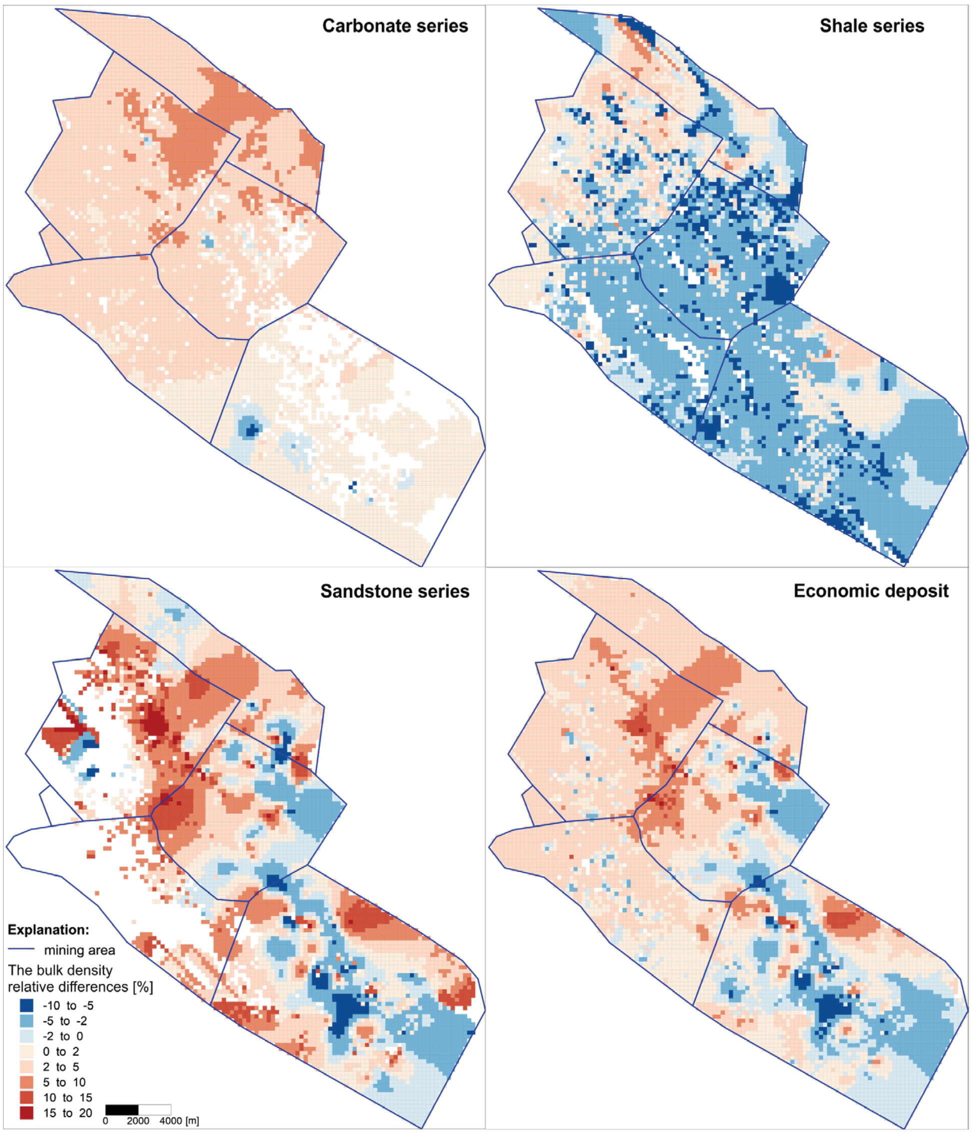

Fig. 7. The block maps (250 × 250 m blocks) of the bulk density relative differences [%] for the main lithological units and economic deposit determined on the basis of experimental sampling and reference values used in KGHM (the Cu-Ag LGCD deposits)

Rys. 7. Orientacyjne mapy blokowe (bloki 250 × 250 m) różnic względnych gęstości objętościowej [%] w głównych seriach litologicznych i złożu bilansowym określonych na podstawie opróbowania eksperymentalnego i wartości referencyjnych stosowanych w KGHM Polska Miedź SA (złożach Cu-Ag LGOM)

Table 5. The block map statistics (250 × 250 m blocks) of the differences in relative bulk density [%] for the main lithological units (ε_{R_M}) and economic deposit (ε_{R_D}) determined on the basis of experimental sampling and reference values used in KGHM

Tabela 5. Statystyka różnic względnych gęstości objętościowych map blokowych dla głównych serii litologicznych (ε_{R_M}) i złoża bilansowego (ε_{R_D}) określonych na podstawie opróbowania eksperymentalnego i wartości referencyjnych stosowanych w KGHM Polska Miedź SA

	Count of blocks	Minimum	Maximum	Median
		[%]		
Carbonate series	6 279	7.9	9.3	3.0
Shale series	7 129	–7.2	8.1	–1.9
Sandstone series	5 865	–11.0	19.4	1.5
Economic deposit	7 369	–7.7	17.9	1.6

The smallest differences, mostly positive, were recorded for the carbonate series. Large parts of the LGCD deposit area are characterized by a constant range of differences with a predominance of values from 0 to 5% and, to a smaller extent, from 5 to 10%. They confirm slightly higher assessments of the bulk density than indicated by the reference values.

The relative differences were different for the shale and sandstone series, where both negative and positive differences were observed. However, while the shale series is dominated by negative difference values suggesting overestimated actual bulk density by the reference values, the sandstone series is dominated by positive differences indicating their underestimation. Extreme differences in relative assessments for the shale series and sandstone series range from –7% to 8% and from –11% to 20%, respectively (Table 5).

The map of differences in relative assessments of the mineral bulk density within the boundaries of the economic deposit shows great similarities to the analogous map for the sandstone series, although with a slightly smaller range of variation in differences (Fig. 7).

5. Discussion

As it was demonstrated for all of Cu-Ag LGCD deposits (or their large parts) the diversity of estimates of average bulk densities of ores based on the results of experimental sampling and reference values is low (with a median not exceeding 3%) (Table 5). However, the presented research results indicate the possibility of a significant difference of both assessments at the local observation scale, depending on the type of main lithological units, ranging from 8% to nearly 20% (Table 5). This may have a noticeable influence on the correct estimation of ore and metal resources in small parts of deposits and, as a consequence, hinder the reconciliation of the planned and actual mining production. Proper determination of bulk density makes this resource parameter important for production planning and scheduling (Makhuvha et al. 2014; Scogings 2015).

In respect to the main lithological unites, the difference of bulk density is a consequence of a local lack of some individual lithological units or changes in the proportion of their thickness in the vertical profile of the deposit and the internal heterogeneity of the bulk density of individual lithological units, as shown by the Games-Howell test. The individual lithological units, determined macroscopically on the basis of internal similarity of lithological features do not guarantee homogeneity of bulk density determinations, which would ensure the reliability of the averaged values as representative of all LGCD deposits.

The issue of homogeneity of determinations can be associated with the concept of geological domains defined loosely as a deposit area within which the mineralization and lithological features of the host rocks are more similar to each other than outside this area (Glacken and Snowden 2001; Arseneau 2014) or, in a broader sense, with geometallurgical domains where similar mechanical properties of rocks, enrichment, floatability, and grindability of ores are expected (Kaczmarek et al. 2017). According to Arseneau (2014), each of the geological domains should be studied individually and bulk density values should be assigned to them separately. Their determination in the case of LGCD deposits, due to discontinuity of individual lithological units and their mutual thickness proportions in the main lithological units, is troublesome intensive. Reliable lithological and geochemical 3D models of the deposit would be useful for their determination. This task, under geological conditions of LGCD deposits, is difficult due to their high variability and relatively wide sampling interval in mining excavations (20–40 m) in relation to continuous sampling of the deposit in the vertical profile.

Conclusions

Data from the literature confirms the little known fact that the bulk density of the mineral, despite its relatively low variability, should be the subject of thorough analysis; particular attention should be given to careful sampling and a correct quantitative assessment, as well as other resource parameters. This approach becomes important in the case of accounting short-term production requiring reconciliation of the forecasted ore reserves and Cu content in small parts of the deposit to the actual mined reserves.

Under the conditions of Cu-Ag LGCD deposits, the adoption of fixed reference bulk density values for the main lithological units may locally lead to a significant understatement or overstatement of ore reserves estimates ranging from a few to several percent, making reconciliation of mined reserves more difficult. The ideal solution would be the determination of bulk density in each sample collected for Cu content assaying, which is unrealistic from an economic and organizational point of view, given that a large number of such samples, of the order of 200 thousands are collected every year. It is also unreliable to assign a bulk density value, calculated from the regression function linking this parameter to the Cu content, to each sample due to the weak correlation between these parameters in the majority of the individual lithological units in the discussed deposits.

Currently, with no systematic sampling for bulk density assessment, increasing the accuracy of its local estimates and at the same time estimates of ore reserves and Cu content, facilitating reconciliation of mining production against reserves can be achieved in two ways:

- by individually determining the bulk density of the main lithological units at each sample site based on the mean values of this parameter for individual lithological units determined experimentally using a weighted average of their thickness; this simple way takes the internal lithological composition of the main lithological units into account but its effectiveness is reduced by the heterogeneity and variability of the individual lithological units at the local scale,
- by direct determination the bulk density for each sampling point based on block maps of this parameter prepared for the main lithological units (Fig. 5); this method for determining bulk densities is associated with unavoidable errors of interpolation, which is the basis for the development of block maps.

The proposed methods based on the results of experimental sampling of individual lithological allow for estimating bulk densities of the main lithological units, which are much closer to the actual values than the reference bulk densities used for this purpose.

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REFERENCES

Abzalov, M.Z. 2013. Measuring and modelling of dry bulk rock density for mineral resource estimation. *Applied Earth Science* 122(1), pp. 16–29.

Abzalov, M.Z. 2016. *Applied Mining Geology*. Modern Approaches in Solid Earth Sciences 12. Springer International Publishing, 448 pp.

Arseneau, G.J. 2014. Estimating of bulk density for mineral resource reporting. *CIM. Journal* 5(2), pp. 95–100.

Clark, I. and Harper, W.V. 2008. *Practical Geostatistics 2000*. Geostokos (Ecosse) Limited, Scotland, 430 pp.

Dominy et al. 2002 – Dominy, S.C., Noppe, M.A. and Annels, A.E. 2002. Errors and uncertainty in mineral resource and ore reserve estimation: the importance of getting it right. *Explor. Mining Geol.* 11(1–4), pp. 77–98.

Games, P.A. and Howell, J.F. 1976. Pairwise Multiple Comparison Procedures With Unequal N's and/or Variances: A Monte Carlo Study. *Journal of Educational Statistics* 1(2), pp. 113–125.

Glacken, I.M. and Snowden, D.V. 2001. *Mineral Resource Estimation* [In:] *Mineral Resource and Ore Reserve Estimation* – The AusIMM Guide to Good Practice (Edwards, A.C. ed.), The Australasian Institute of Mining and Metallurgy, Melbourne, pp. 189–198.

Journel, A.G. and Huijbregts, C.J. 1978. *Mining Geostatistics*. London Academic Press, 600 pp.

Kaczmarek et al. – Kaczmarek, W., Rożek, R., Mrzygłód, M. and Jasiński, W. 2014. The detailed lithology in geological data base in KGHM (*Litologia szczegółowa w bazie danych geologicznych KGHM Polska Miedź SA*). *Górnictwo Odkrywkowe – Open Pit Mining* 55(2–3), pp. 86–91 (*in Polish with English abstract*).

Kaczmarek et al. 2017 – Kaczmarek, W., Twardowski, M. and Wasilewska-Błaszczyk, M. 2017. Practical aspects of lithological types modelling in Cu-Ag ore LGOM deposit (Legnica-Głogów Copper District) (*Praktyczne aspekty modelowania litologicznych typów rud w złożach Cu-Ag LGOM (Legnicko-Głogowskiego Okręgu Miedziowego)*). *Biul. Państw. Inst. Geol. – Geological Bulletin of the Polish Geological Institute* 468, pp. 209–226 (*in Polish with English abstract*).

Makhuvha et al. 2014 – Makhuvha, M., Arellano, R.M. and Harney, D.M.W. 2014. Determination of bulk density, methods and impacts, with a case study from Los Bronces Mine, Chile. *Applied Earth Science (Trans. Inst. Min. Metall. B)* 123(3), pp. 196–205.

Mucha et al. 2017 – Mucha, J., Wasilewska-Błaszczyk, M., Auguścik, J. and Paszek, M. 2017. The volumetric density of ore – resource parameter of secondary importance? (the Polkowice-Sieroszowice Cu-Ag deposit, Lubin-Głogów Mining District) (*Gęstość przestrzenna rudy – parametr zasobowy drugiego planu? (Złoże Cu-Ag Polkowice–Sieroszowice, Legnicko-Głogowski Okręg Miedziowy)*). *Biul. Państw. Inst. Geol. – Geological Bulletin of the Polish Geological Institute* 468, pp. 227–236 (*in Polish with English abstract*).

Nieć, M. ed. 2012a. *Methodology of geological reporting of mineral deposits. Pt. 3. Sampling of mineral deposits* (*Metodyka dokumentowania złóż kopalin stałych. Część 3. Opróbowanie złóż kopalin*). Kraków: Publishing House MEERI PAS, 128 pp. (*in Polish*).

Nieć, M. ed. 2012b. *Methodology of geological reporting of mineral deposits. Pt. 4. Resource estimation* (*Metodyka dokumentowania złóż kopalin stałych. Część 4. Szacowanie zasobów*). Kraków: Publishing House MEERI PAS, 241 pp. (*in Polish*).

Paszek, M. and Wasilewska-Blaszczyk, M. 2017. The variability of volumetric density of the selected individual lithological units in the Cu-Ag deposit in the Lubin-Głogów copper district and its influence on the resources. *International Multidisciplinary Scientific GeoConference: SGEM: Surveying Geology & Mining Ecology Management* 17(1.1), pp. 679–686.

Piestrzyński, A. ed. 2007. *Monograph of KGHM Polska Miedź SA (Monografia KGHM)* Wrocław: KGHM CUPRUM Sp. z o.o. CBR, second edition, 1080 pp. (*in Polish*).

PN-EN 1936:2010 *Metody badań kamienia naturalnego – Oznaczanie gęstości i gęstośc objętościowej oraz całkowitej i otwartej porowatości* (PN EN 1936:2010 *Natural Stone Test Methods – Determination Of Real Density And Apparent Density, And Of Total And Open Porosity*). Warszawa: PKN.

Rossi, M.E. and Deutsch C.V. 2014. *Mineral Resource Estimation*. Springer, 332 pp.

Scogings, A. 2015. *Bulk density of industrial minerals: Reporting in accordance with the 2007 SME Guide*. *SME Mining engineering*, pp. 1–10.

Sinclair, A.J. and Blackwell, G.H. 2002. *Applied Mineral Inventory Estimation*. Cambridge University Press, 381 pp.

STATGRAPHICS® Centurion XVII, User Manual 2014. Statpoint Technologies, Inc., 311 pp.

THE ACCURACY OF THE LOCAL ASSESSMENT OF THE BULK DENSITY OF COPPER-SILVER DEPOSITS IN LEGNICA-GŁOGÓW COPPER DISTRICT AND ITS IMPACT ON THE VALUATION OF ORE RESOURCE AND MINING PRODUCTION

Keywords

bulk (apparent) density, copper and silver ore, resources, Legnica-Głogów Copper District (LGCD), statistics

Abstract

In the world-class Cu-Ag deposits of the Legnica-Głogów Copper District (LGCD), constant bulk density values are adopted to estimate the ore and metal resources within them based on the results of previous studies of the LGCD deposits carried out at the stage of their exploration and documentation: 2.6 Mg/m^3 for the carbonate series, 2.5 Mg/m^3 for the shale series, and 2.3 Mg/m^3 for the sandstone series. The main purpose of research was to analyze the range of possible differences at local scale of observation between constant values of bulk densities (hereinafter referred to as reference values) assigned during deposit documentation to the main lithological units and bulk densities of these units determined based on the results of experimental sampling of individual lithological units within the exploited copper and silver deposits (Lubin, Polkowice-Sieroszowice and Rudna).

In general, when it comes to Cu-Ag LGCD deposits (or their large parts), the relative diversity of estimates of average bulk densities of ores based on the results of experimental sampling (more than 1,600 samples from different individual lithological units were collected at 500 sampling points in mining excavations) and reference values is low (with a median not exceeding 3%). The results of studies indicate, however, that the application of reference bulk densities at the local observation scale may result in significant underestimation (up to nearly 20%) or overestimation (up to 11%) of real bulk densities of the main lithological units. This may have a noticeable impact on the correct estimation of ore and metal resources in small parts of deposits and, as a consequence, hinder the reconciliation of the planned and actual ore mining production.

DOKŁADNOŚĆ LOKALNEJ OCENY GĘSTOŚCI OBJĘTOŚCIOWEJ RUD Cu-Ag LGOM I JEJ WPŁYW NA OSZACOWANIE ZASOBÓW ZŁOŻA I ROZLICZANIE PRODUKCJI GÓRNICZEJ

Słowa kluczowe

gęstość objętościowa, rudy miedzi i srebra, zasoby, Legnicko-Głogowski Okręg Miedziowy (LGOM), statystyka

Streszczenie

W światowej klasy złożach Cu-Ag LGCD (the Legnica-Głogów Copper District) do oszacowań zasobów rudy i metali w obrębie trzech głównych serii litologicznych stosuje się stałe wartości gęstości objętościowej przyjęte na podstawie wyników wcześniejszych badań przeprowadzonych na etapie

ich rozpoznania i dokumentowania: 2,6 Mg/m^3 dla serii węglanowej, 2,5 Mg/m^3 dla serii łupkowej oraz 2,3 Mg/m^3 dla serii piaskowcowej. Zasadniczym celem badań była analiza zakresu możliwych różnic w lokalnej skali obserwacji między stałymi wartościami gęstości objętościowych przypisywanymi w trakcie dokumentowaniu złóż głównym seriom litologicznym (traktowanych jako wartości referencyjne) oraz gęstościami objętościowymi tych serii wyznaczonymi na podstawie wyników specjalnego opróbowania eksperymentalnego wydzieleń litologicznych szczegółowych w obrębie eksploatowanych złóż Cu-Ag LGCD (Lubin, Polkowice-Sieroszowice i Rudna).

W skali całych złóż Cu-Ag LGCD względne zróżnicowanie ocen średnich gęstości objętościowych kopaliny dokonanych na podstawie wyników opróbowania eksperymentalnego (około 1600 prób z różnych wydzieleń litologicznych szczegółowych na 500 stanowiskach opróbowań w wyrobiskach górniczych) i wartości referencyjnych jest małe, z medianą nieprzekraczającą 3%. Wyniki przeprowadzonych badań wskazują, że przy stosowaniu wartości referencyjnych w lokalnej skali obserwacji może dochodzić do znaczącego niedoszacowania (do blisko 20%) lub przeszacowania (maksymalnie do 11%) rzeczywistych gęstości objętościowych głównych serii litologicznych. Może to mieć już zauważalny wpływ na poprawność oszacowania zasobów rudy i metali w niewielkich partiach złóż i w konsekwencji utrudniać rozliczenie prognozowanych zasobów rudy i Cu z wielkościami stwierdzanymi w wydobytym urobku.