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STUDIES OF MASONRY STRUCTURE TECHNICAL WEAR IN MINING AREAS

BADANIA ZUŻYCIA TECHNICZNEGO KONSTRUKCJI MUROWANYCH NA TERENACH GÓRNICZYCH

The paper presents the results of assessment studies of the time course for technical wear in masonry buildings located in the area of mining-induced ground deformations. By using fuzzy inference system (FIS) and the "if-then" rule, corresponding language labels describing actual damage recorded in structure components were translated into scalar outputs describing the degree of damage to the building. Adopting this approach made it possible to separate damage resulting from additional effects coming from mining-induced ground deformations and the natural wear and tear of masonry structure. By using statistical analysis an exponential function for the condition of building damage and the function of natural wear and tear were developed. Both phenomena were subject to studies as a function of time regarding the technical age of building structure. The results obtained were used to develop a model for the course of technical wear of traditionally constructed buildings used within mining areas.

In the course of natural wear and tear buildings located in mining areas are additionally exposed to forced ground deformations. The increase of internal forces in structure components induced by those effects results in creating an additional stress factor and damage. The hairline cracks and cracks of building structure components take place when the intensity value of mining effects becomes higher than the component stress resistance and repeated effects result in the decrease of structure rigidity. The observations of building behaviour in mining areas show that the intensity of mining activity and the multiplicity of its effect play a substantial role in the course of technical wear of buildings. The studies show that the level of damage resulting from mining effects adds up to natural wear and tear of the building and impairs the global technical condition as compared to similar buildings used outside mining areas.

Keywords: fuzzy inference, diagnosing the damage condition, "if-then" rules

W artykule przedstawiono wyniki badań odnoszących się do oceny przebiegu w czasie zużycia technicznego budynków murowanych położonych w obszarach oddziaływania górniczych deformacji podłoża. Wykorzystując zasady logiki rozmytej (FIS) i reguły „if-then", reprezentatywne etykiety językowe opisujące faktyczne uszkodzenia zarejestrowane w elementach konstrukcji, zamieniono na wyjścia

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skalarne opisujące stopień uszkodzenia budynku. Dzięki takiemu postępowaniu wydzielono uszkodzenia wynikające z dodatkowych oddziaływań pochodzących od deformacji gruntu wskutek robót górniczych oraz naturalne zużycie konstrukcji murowej. Z wykorzystaniem analizy statystycznej opracowano wykładniczą funkcję stanu uszkodzenia budynku oraz funkcję zużycia naturalnego. Oba zjawiska przebadano w funkcji czasu odnoszącego się do wieku technicznego konstrukcji budynku. Na podstawie uzyskanych wyników badań opracowano model przebiegu zużycia technicznego budynków o konstrukcji tradycyjnej użytkowanych w granicach terenów górniczych.

W przebiegu procesu zużycia naturalnego, budynki położone na terenach górniczych narażone są dodatkowo na oddziaływanie wymuszonych deformacji podłoża. Wywołany tym oddziaływaniem wzrost sił wewnętrznych w elementach konstrukcji, powoduje pojawienie się dodatkowego czynnika naprężeniowego i powstanie uszkodzeń. Zarysowanie lub spękanie elementu konstrukcji budynku następuje, kiedy wartości oddziaływania górniczego przewyższają poziom możliwych do przejęcia przez element naprężeń, a wielokrotne oddziaływania powodują spadek sztywności konstrukcji. Z obserwacji zachowania się budynków na terenach górniczych wynika, że intensywność eksploatacji górniczej i krotność jej oddziaływania odgrywają zasadniczą rolę w przebiegu zużycia technicznego budynków. Wykazano w badaniach, że stan uszkodzenia pochodzący od oddziaływań górniczych sumuje się z naturalnym zużyciem budynku i powoduje pogorszenie się globalnego stanu technicznego w stosunku do budynku podobnego użytkowanego poza terenami górniczymi. Wraz z upływem czasu i wzrostem wieku technicznego w budynkach występuje szybszy wzrost ich globalnego zużycia technicznego. Wzrost ten wynosi od ok. 20% dla budynku o wieku technicznym 20 lat do ok. 50%, dla budynków o wieku technicznym 120 lat. Wzrost ten interpretować należy, jako udział szkód górniczych w globalnej ocenie zużycia budynków na terenach górniczych.

Słowa kluczowe: konstrukcje murowe, zużycie techniczne, diagnostyka stanu uszkodzenia, wnioskowanie rozmyte, reguły „if-then”

1. Introduction

Structures during their normal use are subject to ageing processes that affect their technical condition. Therefore, the technical condition is a feature changing in time, related directly to progressive reduction in the strength properties of their components. The process of structure technical condition change in time is called technical wear and frequently referred to as natural wear. However, in mining areas the structures are subject to higher loads resulting from forced structure ground deformation. Therefore, in the case of such structures the technical wear is the sum of natural ageing processes and the wear caused by mechanical damage resulting from mining effects. The aim of the studies is to develop a methodology of determining the share of mining effects and the processes of natural structure ageing in the assessment of global technical wear of structures – masonry buildings. Determining the level of structure technical wear is important to define the amount of compensation for the so-called “mining damage”. However, in practice the amount of compensation is calculated based on a specific building reinstatement value reduced by the degree of its natural wear.

Using the theory of fuzzy sets and the membership function (Zadeh, 1965; Łachwa, 2001) is currently one of the basic tools for building diagnostic procedures in many fields of science. The results of studies presented in medicine, economics and technical science (Jankowski & Janiak, 2014; Ying He Y. & Rui Xiu, 2018) indicate large opportunities for the effective use of this approach in searching for diagnostic conclusions. The opportunity to use many items of information regarding factors that affect the course of this phenomenon makes the cognitive procedures more real, also if one of the factors includes expert’s opinion (Nazemi et al., 2012). This is also a theory used to develop the models for assessing mining activity impact on buildings (Firek, 2017) and engineering structures (Rusek & Kocot, 2017; Malinowska et al., 2016).

2. Methodology of the research

2.1. Study group

The study covered in total 260 low-rise residential buildings made of masonry and with wall-based bearing system, built from about 1900 to 1980. Average technical age of the building was 55 years (±27 years). The statistical unit was a single-family building with a surface area of approx. 150 m^2. About 73% of the buildings were ones erected at a fixed level, without basement or with basement covering its whole horizontal projection. The buildings were made of bricks and/or stone (wall footings) and with brickwork lintels over window and door openings. The ceilings of buildings in this group were mainly made as brickwork sectional or concrete on steel joists (supporting beams), at basement floors and timber joists at overground floors.

In the course of gathering data on buildings the information on the frequency of refurbishments and the level of maintenance in the buildings was also collected. The majority of buildings (57% examined) was found to show a good level of maintenance. 32% of buildings were clearly found to have no routine repairs and maintenance, while average maintenance level was found in just 10% of cases examined. Based on the visual assessment of buildings' masonry wall structure condition an expert assessment of natural wear was performed. 63% of buildings examined were found to show an average level of natural wear. The share of buildings with technical wear assessed to be low or high was similar in population and amounted to: 16% and 20%, respectively. 63% of buildings examined were found to show a medium level of natural wear of walls. The total share of buildings with low and medium natural wear amounted to 79%, which correlates with the number of houses showing a good and medium level of maintenance. No routine maintenance was found in 32% of buildings, and in 20% of buildings it translated into high natural wear of walls.

The effect of mining activity on building structure was expressed by the value of index describing the horizontal ground displacement caused by the response of overburden to underground mining activity in the rock mass. The level of effect was determined based on the mining forecast and land surveys of field observation lines. In the group of study-included buildings the value of ground horizontal displacement index ranged from 0.2 mm/m to 9 mm/m.

2.2. Data types, units of quantitative and qualitative variables

The general population includes the group of buildings, where the statistical unit is defined by quantitative measurable explanatory variables expressed by numerical values, i.e. "building technical age" and "horizontal ground deformation" as well as qualitative variables, unmeasurable and expressed in nominal and order categories: "Projection shape" (symmetric, asymmetric, irregular or with an extension), "building type" (detached, medium-density, high-density development", "foundation level" (fixed, variable), "basement type" (at a fixed level, at a variable level, with full, partial or without basement), "degree of wear of walls" (low, medium, high), "maintenance level" (good, average, none), "mining area category" (I, II, III, IV).

The dependent explanatory variable is the damage condition described for individual structure components "*Su* damage level" directly resulting from the effects of foundation soil deformation.

2.3. Damage condition function

The damage condition of buildings was identified depending on the mechanical damage observed resulting from exceeding the ultimate tensile strength of the structure materials and its finishing components. The cracks were recorded by: describing the crack width and locating them within structural components: load-bearing walls, lintels, window and door openings and ceilings (floors). Recording was performed by distinguishing the occurrence of damage in deep basements (marked with P) and overground floors (marked with N) (Tab. 1). The cracks in structure components were classified by measuring their maximum width (marked with "*a*"). There were two sets of cracks: less and more than 1 mm wide. Each structure component was given a classification grade 1, 2, 3 or 4 (Tab. 2). Depending on the description used, 6 variables in total were obtained to detail the condition of structure damage for each of the buildings under test. After analysing the data, the assessment of damage was expressed in the form of variable referred to as "damage level" marked as "*Su*".

TABLE 1

Scheme of registration of cracks in the construction of buildings

No.	The crack locating within structural components	Floor	Variables
1	on the surfaces of load-bearing walls	N	x_{1N}
		P	x_{1P}
2	on the surfaces of window and/or door lintels	N	x_{2N}
		P	x_{2P}
3	on the surfaces of ceilings	N	x_{3N}
		P	x_{3P}:

TABLE 2

Classification scheme of variables

The cracks with a width greater or equal to 1 mm occurring in building ($a \geq 1$ mm)	Classification grade
if there are no cracks;	1
if cracks cover up to approx. 5% of the component surface area;	2
if cracks at most cover up to approx. 50% of the component surface area;	3
if cracks at most cover up to approx. 70% of the component surface area;	4

By using the "if-then" rule based on fuzzy inference system (FIS) (Zadeh, 1965), corresponding language labels describing actual damage recorded in structure components were translated into scalar outputs describing the degree of damage to the building (Tab. 3).

Fuzzy inference system (FIS) defines non-linear mapping of input data to a scalar output by using fuzzy rules. The process of mapping includes the membership function of input/output, FL operators, fuzzy if-then rules, output set aggregation and the final classification. Classifying the *Su* damage level requires a numerical representation of linguistic data describing the difference between individual damage levels. In practice, damage to buildings is of various intensity, course and range, assigning the same damage level to buildings does not mean that the number of hairline cracks and their course within the structure are identical. In the studies described, the

TABLE 3

Classification of the degree of damage on the basis of the rules "if-then"

Damage level	Language labels describing damage		Assigned numerical value
Su – M "low"	if structural components are free from damage and damage in building finishing components is single,	then the building damage level is low;	$Su = 0.15$
Su – S "medium"	if damage to structural components with a crack width higher than 1 mm covers the surface area of approx. 5% of all structural components and damage to finishing components has a width lower than 1 mm,	then the building damage level is medium;	$Su = 0.25$
Su – D "high"	if damage to structural components with a crack width higher than 1 mm covers the surface area of higher than 5%, but less than 50% of all structural components and damage to finishing components has a width higher than 1 mm	then the building damage level is high;	$Su = 0.50$
Su – BD "very high"	if damage to structural components with a crack width higher than 1 mm covers the surface area of higher than 50%, but less than 70% of all structural components and damage to finishing components has a width higher than 1 mm	then the building damage level is very high;	$Su = 0.70$

assessment in the form of percentage assigned is a fuzzy assessment, while the 5%, 50% and 70% values should be treated as mean values of certain intervals (Łachwa, 2001, Piegat & Olchowy, 2009).

For the needs of interpretation the damage level (*Su*) should be treated as a random variable with a distribution replaceable by the continuous probability distribution, expressed in the form of an exponential density function (Papoulis, 1972; Plucińska & Pluciński, 2000) (Fig. 1a):

$$f_{s_u}(Su) = ke^{-\lambda s_u}; \quad \text{for} \quad s_u \geq 0 \tag{1}$$

Where l and *k* are constant parameters of the density function. The *k* constant can be determined from the dependency showing that the integral of function $f_{Su}(Su)$, within interval from 0 to ∞ must be equal to one:

$$\int_0^\infty ke^{-\lambda s_u} ds_u = \frac{-k}{\lambda} e^{-\lambda s_u} = \frac{k}{\lambda} = 1; \quad k = \lambda \tag{2}$$

and

$$f_{Su}(Su) = \lambda e^{-\lambda s_u}; \quad \text{for} \quad s_u \geq 0 \tag{3}$$

The cumulative distribution function of the (*Su*) variable then adopts the form of the following formula (4): (Fig. 1b):

$$f_{Su}(Su) = 1 - e^{-\lambda s_u}; \quad \text{for} \quad s_u \geq 0 \tag{4}$$

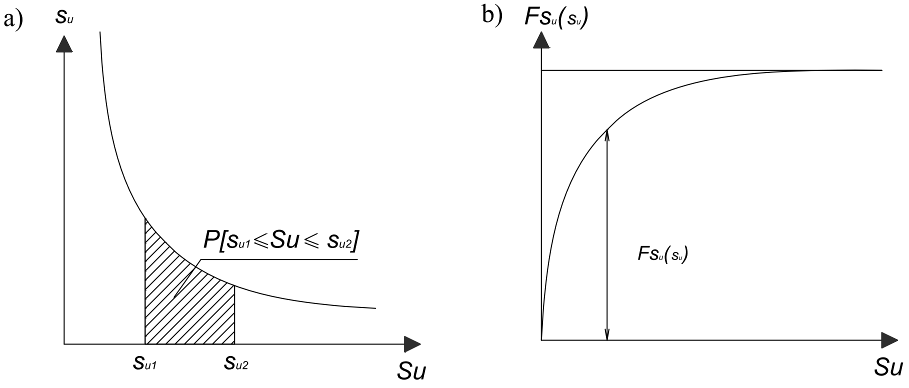

Fig. 1. The exponential function of damage condition
(a) density function, (b) cumulative distribution function *Su*

The assessment value of damage level for each building is the probability of an event that the random variable (*Su*) adopts the value lower or equal to the argument of the function according to (5):

$$F_{Su}(s_u) = P[Su \leq s_u] \tag{5}$$

During studies four damage levels were assumed, which in practice, meant that each building was assigned to one of them at various actual conditions possible. The damage level of a building was determined by using membership function (6) (Łachwa, 2001), which value lies within the range {0.1} and the damage level was determined by using the "if-then" rule, *low*, *medium*, *high* or *very high* and, subsequently, the maximum assessment within the range, 0.15, 0.25, 0.50 and 0.70, respectively, should be assigned.

$$\begin{array}{|l|} \hline F_{Su}(Su-M) \\ F_{Su}(Su-S) \\ F_{Su}(Su-D) \\ F_{Su}(Su-BD) \\ \hline \end{array} \Longrightarrow P[Su \leq s_u] \tag{6}$$

The preparation stage of damage condition description for statistical analysis consisted in assigning *Su* function value to each building by using a subjective assessment determined by taking into account the damage found in individual structure components by adopting "if-then" rule classifications.

The assessment of natural wear of building walls used an *in situ* examination of construction material, general condition of wall component structure, corrosion of brickwork in the form of hairline cracks, stratifications, crumbled parts, damp walls, plaster/render condition, including the hairline cracks not induced by mechanical factors. The natural wear of walls was classified based on an expert assessment by taking into account the general rule for assessing the condition of walls (Borusiewicz, 1975) and the following rules were formulated:

Rule 1: If the condition of wall component structure is good and there is no signs of binding material erosion as well as there are no cracks in plaster/render, then the natural wear is low.

Rule 2: If there is a structure disturbance in single wall components, brickwork corrosion is local, while there are fine cracks in render/plaster, local cavities or plaster/render loosening, then the natural wear is medium.

Rule 3: If there is a structure disturbance in many wall components, the decay or corrosion of brickwork occurs in larger surface area of walls, brick binding material erosion is visible, there are numerous cracks in plaster/render and/or major loss and plaster/render loosening, then the natural wear is high.

The approach presented is part of uncertainty analyses developed recently also as Ferson's *p*-boxes and Neumaier's clouds (Destercker et al., 2008).

2.4. Schematic of statistical analysis

Because of the fact that the variables under analysis apart from technical age are qualitative rather than not quantitative, it indicated to use nonparametric statistics and the analysis of variances (ANOVA) in data analyses (Volk, 1973). The studies used the following schematic of statistical analysis:

Step 1: The analysis of general group using the basic statistics and the so-called simple cross-sections. Analysing in a selected sample of variable explanations, i.e. determining mean values, determining distributions and variable histograms, correlation coefficients and studying the co-occurrence of variables.

Step 2: Assuming that mean *Su* values in the individual classifications of explanatory variables are equal and making the null hypothesis on the equality of variances at the significance level of $p < 0.05$.

Step 3: Calculating the variance for individual explanatory variables, assigning $F_{obl.}$ statistics.

Step 4: Comparing the calculated statistics $F_{obl.}$ to tables ($F_{p,dfA,dfR}$).

Step 5: Testing null hypothesis (H_0) and alternative hypothesis (H_1), if $F_{obl.} \geq F_{p,dfA,dfR}$ null hypothesis on the equality of variances is rejected and an alternative hypothesis is adopted for testing.

Hypothesis H_0: For each variable classification (test parameter), the average building damage level (*Su*) does not show a significant statistical difference, $\mu_1 = \mu_2$;

Conclusion: There is an equality of variances at level $p < 0.05$, adopting null hypothesis.

Hypothesis H_1: For individual variable classification (test parameter), the average building damage level (*Su*) shows a significant statistical difference, while the change of its value correlates positively with the increase of damage level value (*Su*), $\mu_1 \neq \mu_2$;

Conclusion: There is no ground to announce the equality of variances. Rejecting the null hypothesis, analysing an alternative hypothesis according to *Spearman's* rank correlation coefficient (r_s) analysis.

Step 6: Creating the model of multiple regression considering explanatory variables (test parameters), the alternative hypothesis is adopted for.

Step 7: Determining model parameters: the coefficient of determination R^2 and *Beta* regression coefficients at significance level $p < 0.05$, creating *Su* scatter diagrams against to test parameters (explanatory variables).

Step 8: Analysis of *Spearman's* rank correlation coefficients (r_s) and *gamma* coefficient for factors (explanatory variables).

Step 9: Determining *Su* linear dependencies on mining effect described by the value of horizontal ground deformation (ε).

The statistical analyses performed adopted $p < 0.05$ as the acceptable error limit level. The result obtained below this value was assessed as statically significant, while below the $p < 0.001$ value, as highly significant.

3. Research results

3.1. Variance analysis

According to the adopted schematic of statistical analysis a variance analysis was performed to separate building features that justify most the level of (*Su*) damage under assessment. In addition, the analysis covered the impact of time and mining effect level expressed by horizontal ground deformation on this condition. Then, a regression model was built for the explanatory variables of (*Su*) damage level. Based on calculated statistical value $F_{obl.}$ and the assessment of significance level for the obtained result (p), a set of variables (test parameters), for which the variance analysis showed statistically significant differences between mean (*Su*) for individual variable classifications, was obtained, Once the variance analysis and the verification of null hypothesis H_0 and alternative H_1 were completed, the following was adopted for further analysis: „Horizontal ground deformation" ($F_{obl.} = 4.573$; $F_p = 2.21$), „mining area category" ($F_{obl.} = 5.809$; $F_p = 3.00$), „basement type" ($F_{obl.} = 4.154$; $F_p = 2.08$), „shape of horizontal projection" ($F_{obl.} = 3.063$; $F_p = 3.00$), „development type" ($F_{obl.} = 5.419$; $F_p = 3.00$),"building technical age t": ($F_{obl.} = 2.844$; $F_p = 2.20$), "natural wear of load-bearing walls" marked *Zn* ($F_{obl.} = 26.56$; $F_p = 3.00$). The significance level for features was within the p range: 0.0005÷0.0034. For factors, for which the variance analysis did not show any significant differences between mean values, the statistical analysis of the data was completed.

3.2. Analysis of multiple regression, stepwise

As a result of regression, a model was obtained that by using the assessment of the coefficient of determination R^2, in approx. 39% of cases considered, explains the reason for damage in masonry structures unprotected against mining effects. As a result of multiple regression because of high correlation coefficient values ($\beta : 0.442 \div 0.004$) and the model significance level obtained ($p < 0.00$) made it possible to perform a multiple stepwise regression according to the sequence taking into account the significance of data under analysis. In the created model the best correlated explanatory variable is "building technical age t", then the value of index describing the "horizontal ground deformation", "natural wear of walls *Zn*" and the "basement type". The correlation of those variables with (*Su*) was confirmed by the high level of statistical significance. All variables, except for "basement type" achieved $p < 0.001$, assessed in analysis as highly significant results. The "development type" variable because of exceeding the significance level ($p < 0.07$) was excluded from the model explaining the (*Su*) damage level ($p < 0.05$).

3.3. Spearman's rank correlation coefficient test

Another stage of studies included performing the *Spearman's* rank and *gamma* correlation tests. For features under analysis the following results were obtained: *Spearman's coefficient*

$r_s : 0.131 \div 0.300$ and $gamma : 0.240 \div 0.375$, significance level ($p < 0.05$). *Spearman's r_s coefficient* and *gamma* coefficient should be interpreted as a measure of correlation between variables. The best correlated variables with "damage level *Su*" appear to be the variables in the following sequence: „Building technical age *t*", "ground horizontal deformation *ε*", "natural wear of walls *Zn*", "development type", "basement type" and "foundation level". A compliance of *Spearman's coefficient* and *gamma* coefficient was obtained in the study. At the same time, the *gamma* coefficient in all cases provided higher values that the *Spearman's* coefficient. It should be emphasised that the results of rank correlation coefficient test correspond to models obtained as a result of the independent multiple regression analysis performed.

3.4. The course of natural wear of walls in the building group under test

Based on the above-listed analyses it was found that the most significant impacts on the masonry structure condition in a mining area include "building technical age *t*", "horizontal ground deformation *ε*" and "natural wear of walls *Zn*". This conclusion constitutes the basis for modelling the exponential dependency of the "damage level *Su*" defined in the studies as a function of time i.e. as a function of building technical age *t*:

$$Su(t) = 0.199 + 0.0002 \cdot t + 1.9586 \cdot 10^{-5} \cdot t^2 \tag{6}$$

Based on the research data, statistical dependencies occurring between the recorded natural wear of walls *Zn* and the technical age of buildings as well as the wear of walls *Zn* and general maintenance level, were also identified. The classifications of natural wear assumed in the studies: "low", "medium", "high" were assigned the following grades, respectively: 0.15, 0.25, 0.50. Building where the natural wear of walls was assessed as *very high* were not classified in the studies. Based on *Spearman's rank correlation coefficient* (r_s) *test* result, positive correlations between technical age and the natural wear of walls were obtained, at the level of $r_s = 0.214$ ($gamma = 0.224$), and between natural wear and maintenance level, $r_s = 0.21$ ($gamma = 0.358$), at the test significance level $p < 0.05$. The above-mentioned allows us to conclude that there is a statistically significant dependency between: building technical age and the increase of recorded natural wear for walls in those buildings, as well as the recorded natural wear for walls in the buildings and their maintenance level, i.e. another level of maintenance assessment level (i.e. high, average, none) corresponds a higher level of natural wear.

Using the polynomial function a dependency between natural wear of walls *Zn* as a function of time expressing the "building technical age *t*" was obtained:

$$Zn(t) = 0.2338 + 0.0004 \cdot t + 5.668 \cdot 10^{-6} \cdot t^2 \tag{7}$$

4. Global model of building technical wear in mining areas

Regarding the statement that damage condition resulting from mining effects sums up with natural building wear, by using polynomial functions $Su(t)$ and $Zn(t)$, a graphical interpretation of the increased course of building wear in mining areas was proposed, according to Fig. 2.

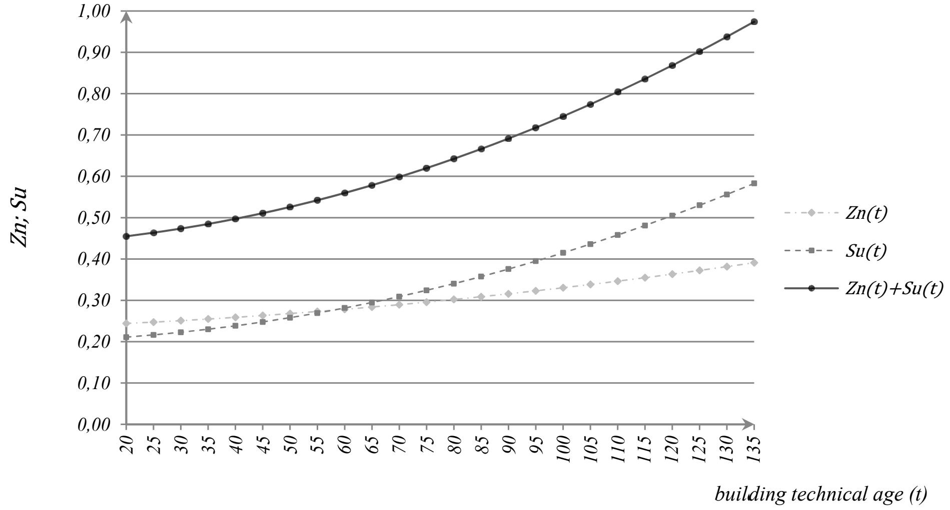

Fig. 2. The obtained model of building technical wear in mining areas

This is the so-called “a posteriori” model, developed by using an experimental study performed. Assuming the opportunity to superpose the values of wear of walls (*Zn*) and mining-induced damage level (*Su*) obtained from the studies, a curve was plotted to express the sum of $n(t)$ and $Su(t)$. functions.

If the so-called natural wear is a common phenomenon, and the $Zn(t)+Su(t)$ curve represents a global assessment of masonry building wear, then it can be assumed that $Su(t)$ represents the increase of mining-induced wear in those ildings. The study results show an increase of mining damage in buildings as time goes by. The $Su(t)$ value can be also interpreted as a share of mining damage in the global building wear assessment in mining areas.

5. Conclusions

The presented model of building wear in mining areas reflects well the actual course of phenomenon of increased building wear in mining areas both in a qualitative and quantitative way. The diagnostic procedure includes the descriptions of the course, intensity and hairline crack width being individual assessments provided by particular experts. In fact, the assessments, even if properly identified and providing a good description of actual condition bear all the hallmarks of the so-called “fuzzy” assessment (Łachwa, 2001). It also applies to the stage, where the actual conversion of damage information takes place i.e. structure damage condition data change into a measurable value of damage level. The opportunity to use the “if-then” rule and the membership function to classify the condition of buildings to a specified damage level makes it possible to use a deterministic variable to analyse the impact of damage on the global technical wear. This approach gives rise to specify the standards of procedures for: assessing the reasons for damage, assessing the level of usage risk and, at the final stage of “estimation”, the amount of mining-related damage compensation.

References

Borusiewicz W., 1971. *Konserwacja zabytków budownictwa murowanego*. Wydawnictwo Arkady, Warszawa.

Destercker S., Dudios D., Chojnacki E., 2008. *Unifying practical uncertainty Representations: I. Generalized P-Boxes*. International Journal of Approximate Reasoning **49** (3), 649-663, DOI:10.1016/j.ijar.2008.07.003.

Firek K., 2017. *Analysis of the influence of mining impacts on the intensity of damage to masonry building structures*. Journal Of Civil Engineering, Envi-ronment And Architecture, JCEEA, vol. XXXIV, from **64** (1/17) Rzeszów 2017.

Łachwa A., 2001. *Rozmyty świat zbiorów, liczb, relacji, faktów, reguł i decyzji.* Akademicka Oficyna Wydawnicza Exit. Warszawa, 2001.

Malinowska A., Hejmanowski R., Rusek J., 2016. *Estimation of the parameters affecting the water pipelines on the mining terrains with a use of an adaptive fuzzy system.* Arch. Min. Sci. **61**, 1, 183-197.

Nazemi Sh., Kazemi M., Okhravi A.H., 2012. *A conceptual model for recognition of improvement priorities: integrating performance gap and fuzzy groups.* Journal of Modeling in Engineering, January 2012, **9**, 27 (DOI:10.22075/jme.2017.1598).

Papoulis A., 1972. *Prawdopodobieństwo, zmienne losowe i procesy stochastyczne.* Wydawnictwa Naukowo Techniczne, Warszawa.

Piegat A., Olchowy M., 2009. *Czy istnieje optymalna forma eksperckiego modelu rozmytego?* Metody Informatyki Stosowanej **3** (20), 169-178.

Plucińska A., Pluciński E.: *Rachunek prawdopodobieństwa.* Statystyka matematyczna. Procesy stochastyczne. Wydawnictwa Naukowo-Techniczne. Warszawa 2000.

Rusek J., Kocot W., 2017. *Proposed assessment of dynamic resistance of the existing industrial portal frame building structures to the impact of mining tremors.* IOP Conference Series: Materials Science and Engineering **245** art. no. 032020, s. 1-10. Prague.

Zadeh L.A., 1965. *Fazzy Sets.* Information and Control **8** (3), 33-353.

Jankowski J., Janiak M., 2014. *Application of fuzzy inference models in the web recommending interface design.* Business Informatics **2** (32), 86-94.

Ying He Y., Rui Xiu, 2018. *Decision Model of Aero-Engine Design Integration Platform Implementation Solution Selection Based on Number Interval Number with Priority Order,* January 2018, MATEC Web of Conferences 228(2):03005, DOI: 10.1051/matecconf/201822803005 License.

Volk W., 1973. *Statystyka stosowana dla inżynierów.* Wydawnictwa Naukowo-Techniczne. Warszawa.