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FUZZY LOGIC ENHANCEMENT OF MATERIAL STRAIN HARDENING DATA OBTAINED IN THE HEYER'S TEST

Fuzzy logic determination of the material hardening parameters based on the Heyer's method was applied in this research. As the fuzzy input variables, the length of two measuring bases and the maximum force registered in the Heyer's test were used. Firstly, the numerical experiment (the simulation of the fuzzification of the input data) with the assumed disturbance of input variables was performed. Next, on the basis of experimental investigations (eleven samples made from the same material), the membership functions associated with the input data were created. After that, the fuzzy analysis was examined. Fuzzy material hardening constants obtained by means of the α-level optimization and the extension principle methods were compared. Discrete values of the hardening data are found in the defuzzification process, by application of the mass center method.

Keywords: fuzzy logic, plasticity, material hardening parameters, Heyer's method

1. Introduction

In various materials forming processes, many parameters are usually imprecise or uncertain. Load magnitudes, material data and friction coefficient might change from one product to another. Moreover, properties of the final products might vary – even if produced from the same material. The experimental determination of the influence of the variation of the parameters on the final results is often problematic and time-consuming. It has led to the development of non-deterministic computer methods in mechanical engineering [1].

Soft computing-based methods accept the imprecision and uncertainty of material parameters. One of the most popular approaches is the fuzzy sets theory. Fuzzy logic enables to find the system response to the external factors assuming that the input data are fuzzed. Fuzzy theory is especially welcomed in problems where the information about the process is uncertain or partially certain, as well as, when the knowledge about the mathematical model is unknown [2]. One of the most important advantages of the fuzzy logic analysis is a possibility of finding the most reliable solution (crisp value) which cannot be ensured in classical experimental investigations.

Fuzzy logic is mostly applied in control processes. Microwave ovens or washing machines are equipped with fuzzy logic controllers. ABS systems are also exemplary application. Apart from these, the fuzzy theory is also applied in academic research. This method is being tested in many fields of research, e.g. in the powder metallurgy, composites forming, and in theory of plasticity [3-6]. In [7] the response of the Bodner-Partom material to external loads with the use of fuzzy set theory was investigated. In [8] fuzzy logic was used for determination of the best arrangement of the composite materials. The influence of the input parameters (mixing time, rotation speed, fibers fraction and lengths) on the tensile strength and destruction of composite materials was examined. The fuzzy set theory was also used [9] for diagnosis of casting defects. These and others not mentioned here examples confirm the usefulness of the fuzzy logic in different practical applications.

In this research the Hollomon strain hardening equation (1) is considered

$$\sigma = C\varphi^n \tag{1}$$

where σ and φ are true stress and true strain, respectively, and C and n are material data of the research interest. It should be mentioned that the relation (1) has some limitations. For strain less than 0.2% the material response is elastic and the Hooke's law should be applied instead. The Hollomon equation does not consider the saturation of the yield stress and provides unrealistic response for a large strain, therefore. It does not consider the Bauschinger effect and cannot be applied to yielding caused by the reverse load. Despite these limitations, the Hollomon model is quite often used by researches and engineers in plasticity theory.

A correct determination of the aforementioned parameters has a great significance in material processing and forming and/or in numerical calculations. In this research the application of the fuzzy logic for the determination of material strain hardening parameters is presented. Two fuzzy methods: α-optimization and the extension principle are described and compared, including

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their advantages and disadvantages. As the fuzzy input variables, the lengths of two measured bases from Heyer's test and the maximum force registered were used. The fuzzy logic analysis provides not only the information about the variation of n and C parameters, but also allows finding the level of acceptance of these parameters. In order to select the most reliable values of considered parameters, the defuzzification of the output membership functions is made with the use of the mass center method.

2. Fuzzy logic application in engineering problems

The fuzzy theory introduced by Zadeh in 1965 is used for modeling of complex systems for which mathematical model or/and data are unknown [10-11]. Fuzzy set is an expansion of the classical sets [12] for which membership values are zeros or ones. In the fuzzy sets theory values (of membership functions) change in range between 0 and 1. As above, fuzzy set A might be defined as follows (Eq. 2):

$$A = \{x, \mu_A(x) : x \in X\} \tag{2}$$

where: X is input space, x are elements of X input space, $\mu_A(x)$ is a membership function of x in A fuzzy set.

In materials forming, the X input space may represent selected properties of material. Each x element of this space ($x \in X$) presents the specific material property.

In Eq. (2), fuzzy input variables are described by means of their membership functions $\mu(x)$. They determine the level of truth that a variable has a specific value [13]. Membership functions also present the scattering of input parameters and the acceptance level of dispersion [4]. They might be created on the basis of measurements or researcher's experience. Triangular, trapezoidal, and pseudo-Gaussian membership functions are usually used in the fuzzy logic [14]. The pseudo-Gaussian distribution with finite support is adequate for the most problems but in practice linear distributions are commonly applied [10] because their satisfy the partition of unity condition [13].

In numerical calculations, the fuzzy set theory is aimed to the mapping of fuzzy input variables into the result space (Fig. 1). It is realized by means of analysis algorithm which includes the mathematical model. As the result, output variables characterized by their membership functions are obtained. Two methods: α-optimization and the extension principle are considered here.

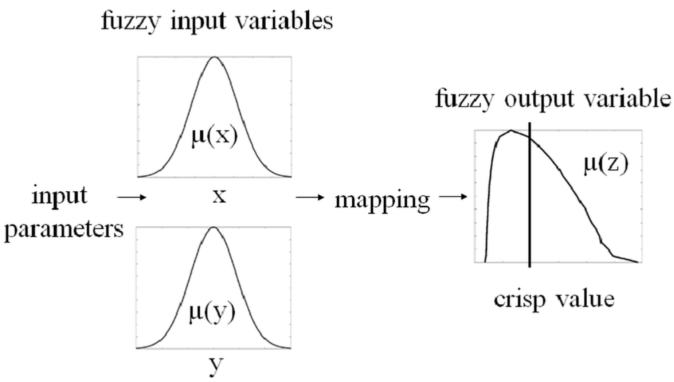

Fig. 1. The membership functions associated with input and output variables

The extension principle presents the mapping of input set X on Z fuzzy set with the use of mapping function $z = f(x, y)$. The membership function $\mu(z)$ of Z fuzzy set is calculated with the use of sup-min operator according to the following formula (Eq. 3) [7]:

$$\mu(z) = \sup\min\left[\mu(x), \mu(y)\right], \exists z = f(x, y) \tag{3}$$

where: sup min are mathematical operators, $\mu(x)$ and $\mu(y)$ are membership functions of x and y input variables, z is an output variable, $\mu(z)$ is a membership function of z output variable.

The extension principle is very sensitive to a number of combinations of x and y elements of fuzzy input data. The application of the *sup* operator in the case of floating-point numbers can be done for certain ranges of z i.e. $z \pm \Delta z$, where Δz is a small number. Thus, the range of z variation should be divided into many subdomains, each of the length of $2\Delta z$. Small amount of subdomains provides a jagged solution. Better results might be achieved for an increased number of subdomains. According to [10], acceptable smoothing of z membership function can be obtained if the number of subdomains >100. It is important that the smooth envelope of $\mu(z)$ might be obtained only for dense search of the fuzzy input data. In the case of sophisticated mapping function it could be very time-consuming, and therefore the extension principle may became not effective. The detailed information about the extension principle method is included in [15].

Better results with the limited number of computations of the mapping function might be achieved by the use of α-optimization method. This concept relies on the discretization of support and the selection of sufficiently high number of α-levels. The subspace assigned to α_K level $\in [0; 1]$ is determined by extreme $x_{\alpha kl}$ and $x_{\alpha kr}$ values (Fig. 2). For particular α-level of fuzzy input variables, the minimum $z_{\alpha kl}$ and maximum $z_{\alpha kr}$ values are searched [7, 10]. Extreme z values for all α-levels define the shape of output membership function $\mu(z)$.

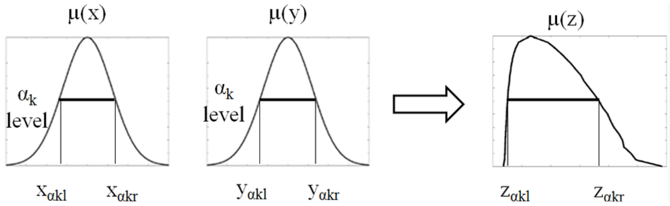

Fig. 2. The α-optimization method in fuzzy analysis

The α-optimization method might be used only if the mapping operator is continuous and the fuzzy result space is convex. This concept gives better results when compared to an extension principle and requires less calculations. In comparison to the extension principle, α-optimization method enables to obtain a definitely smoother shape of the output membership function.

In extension principle, such a smooth line might be achieved only for very dense searching of the input fuzzy variables [10]. For this reason, the α-optimization method is the main numerical tool used in presented research. The extension principle is applied here additionally in order to check the consistency and convexity of the results.

In defuzzification step, fuzzy output variables are transformed back into discrete value in order to search for the most reliable solution. The defuzzification of z variable might be obtained by the use of the mass center method. This concept is based on searching for the center of space below the membership function plot according to the following formula (Eq. 4) [13]:

$$z_0 = \frac{\int z \cdot \mu(z) dz}{\int \mu(z) dz} \tag{4}$$

where: z_0 is the most reliable value of z variable, $\mu(z)$ is the membership function of z.

3. Numerical simulation

At the beginning, the fuzzy logic analysis is made for the single exemplary specimen (Fig. 3). In numerical test as the fuzzy input variables the lengths of two measuring bases l_B and l_C and the maximum force P_{max} are considered. This allows a true strain and true stress calculations in sections B and C (true stress calculation assumes satisfaction of the incompressibility condition for plastic deformation). The output fuzzy results are: strain hardening index n and strength coefficient C (Fig. 4).

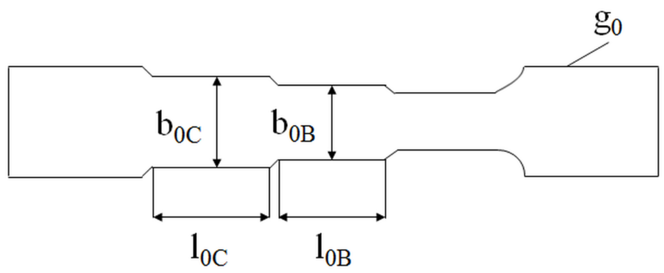

Fig. 3. Dimensions of considered sample

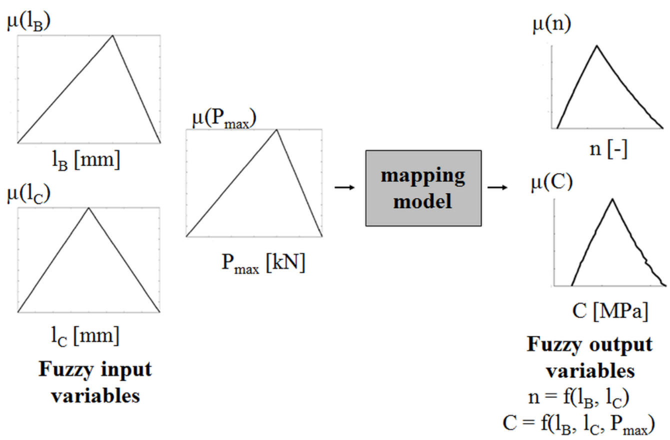

Fig. 4. The mapping of fuzzy input variables into a result space

The range of l_B, l_C and P_{max} variations can be assumed from the experience gained in the previously made Hayer's tests. The following ranges of l_B, l_C and P_{max} are assumed: [33.7; 34] mm, [32; 32.4] mm and [4.5; 4.8] kN, respectively. On the basis of the aforementioned values, triangle membership functions were built (Fig. 5). The membership functions maximums are reached for: $l_B = 33.9$ mm, $l_C = 32.2$ mm and $P_{max} = 4.7$ kN.

The n and C strain hardening parameters [16] can be calculated from the formulas (Eq. 5).

$$n = \frac{\ln \frac{b_{0C}}{b_{0B}} + \varphi_B - \varphi_C}{\ln \varphi_B - \ln \varphi_C} \qquad C = \frac{P_{max}}{b_{0C} \cdot g_0 \cdot \frac{l_{0C}}{l_C} \cdot \varphi_C{}^n} \tag{5}$$

where φ_B and φ_C are true strains in B and C segments of specimen, P_{max} is the maximum force registered in the tensile test.

Eq. (5) constitutes the mapping model of the considered problem. In the numerical simulation the following specimen dimensions are assumed: $g_0 = 1$ mm, $b_{0B} = 16$ mm, $b_{0C} = 17$ mm, $l_{0B} = 30$ mm, $l_{0C} = 30$ mm, where: g_0 – thickness of specimen, b_{0B} and b_{0C} – widths of B and C segments of specimen, l_{0B} and l_{0C} – initial lengths of B and C segments of specimen, l_B and l_C – lengths of B and C segments of specimen after the tensile test.

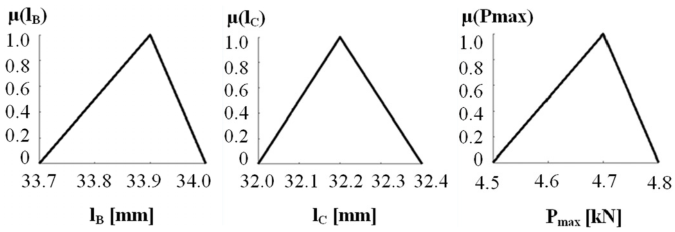

Fig. 5. Assumed membership functions for fuzzy input variables

The fuzzy results for n and C obtained by the use of α-optimization and extension principle methods are presented in Fig. 6. The maximum membership functions (value 1) are obtained for $n = 0.205$ and $C = 510.9$ MPa. The black envelope represents the solution obtained for α-optimization method, small blue 'x' characters represent the solution of the extension principle without the *sup* operator.

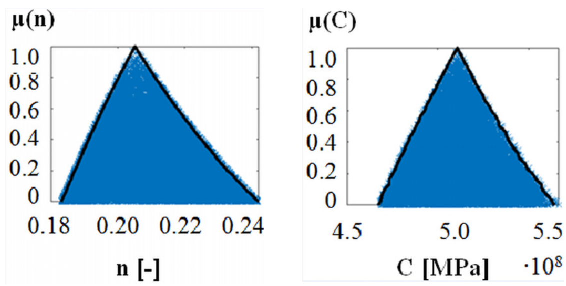

Fig. 6. Fuzzy results obtained by means of α-optimization method

The last step of presented numerical experiment is the search for the most reliable n and C values. The defuzzification step provides the mapping of fuzzy output variables into discrete values (Fig. 7). In this research, the mass center method was applied. For demonstrative fuzzy analysis, the mass centers of n and C membership functions are 0.208 and 512.26 MPa, respectively. These are the most reliable values of n and C parameters for fuzzy logic analysis of the Heyer's method.

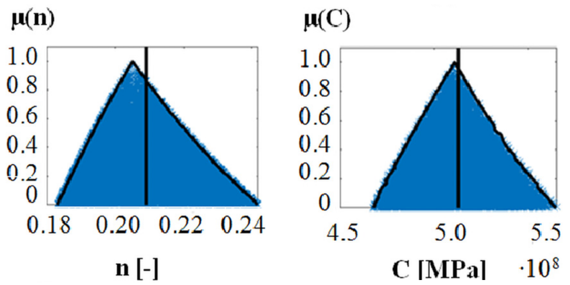

Fig. 7. Crisp values of n and C parameters obtained with the use of mass center method

4. Description of experimental research

Appropriate determination of strain material hardening data (Eq. 1) requires an investigation of several samples. In this research eleven samples made from DC04 low-carbon steel were used as shown in Fig. 8. The dimensions of specimens used in a tensile tests are shown in Fig. 8.

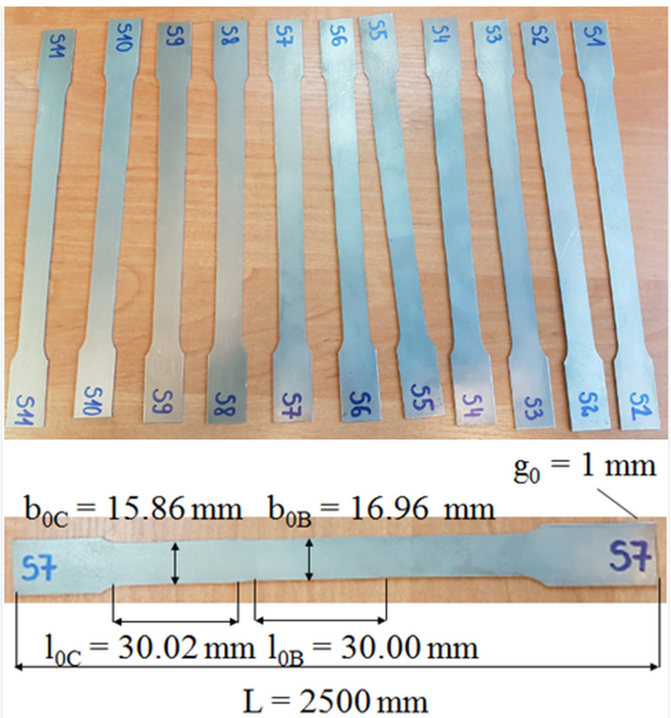

Fig. 8. Specimens used in tensile test

The laboratory research was carried out as follows. Two measuring bases with the length of 30 mm were marked in B and C zones. After that, the lengths of l_{0B} and l_{0C} segments were measured by Nikon MM-800 measuring microscope (Nikon, Japan) (Fig. 9) with 1 μm measurement precision. The widths of b_{0B} and b_{0C} segments were determined with the use of digital caliper. The thickness g was measured by thickness gauge.

Fig. 9. Measurement of l_{0B} and l_{0C} lengths by NIKON MM-800 microscope

The tensile tests were carried out at room temperature on ZWICK Z030 testing machine (Fig. 10) according to PN-EN ISO 6892-1:2016-09 [17]. For all samples, the maximum load was registered. The tensile tests were carried out until the sample s are broken (Fig. 11). The lengths of l_B and l_C segments after tensile test were measured on a microscope.

Fig. 10. ZWICK Z030 testing machine used in the laboratory research

Fig. 11. Tensile test; appearance of "neck" (a), cracked sample (b)

The following formulas were applied in calculations [16]:

- true strain in B (φ_B) and C (φ_C) segments of specimen (Eq. 6)

$$\varphi_B = \ln \frac{l_B}{l_{0B}} \qquad \varphi_C = \ln \frac{l_C}{l_{0C}} \tag{6}$$

- true stress in B (σ_B) and C (σ_C) segments of sample (Eq. 7)

$$\sigma_{PB} = \frac{P_{\max}}{S_B} \qquad \sigma_{PC} = \frac{P_{\max}}{S_C} \tag{7}$$

- actual cross-sections of B (S_B) and C (S_C) segments (Eq. 8)

$$S_B = g_0 \cdot b_{0B} \cdot \frac{l_{0B}}{l_B} \qquad S_C = g_0 \cdot b_{0C} \cdot \frac{l_{0C}}{l_C} \tag{8}$$

Strain hardening index n and strength coefficient C were calculated from Eq. (5).

5. Fuzzy analysis of experimental data

The results of tensile tests carried out under laboratory conditions are presented in Table 1. They were later on entered into the fuzzy analysis. Basic statistical parameters as: mean average (M), standard deviation (SD) and coefficient of variation (V) are also included in Table 1.

On the basis of the tensile test results, the fuzzification of l_B, l_C and $P_{\max}$ was done. For this purpose, the aforementioned values were lumped together in groups (Fig. 12). After that, membership functions for l_B, l_C and $P_{\max}$ were developed. Triangular membership functions are appropriate for l_B and $P_{\max}$. The trapezoidal membership function is adequate for l_C variable.

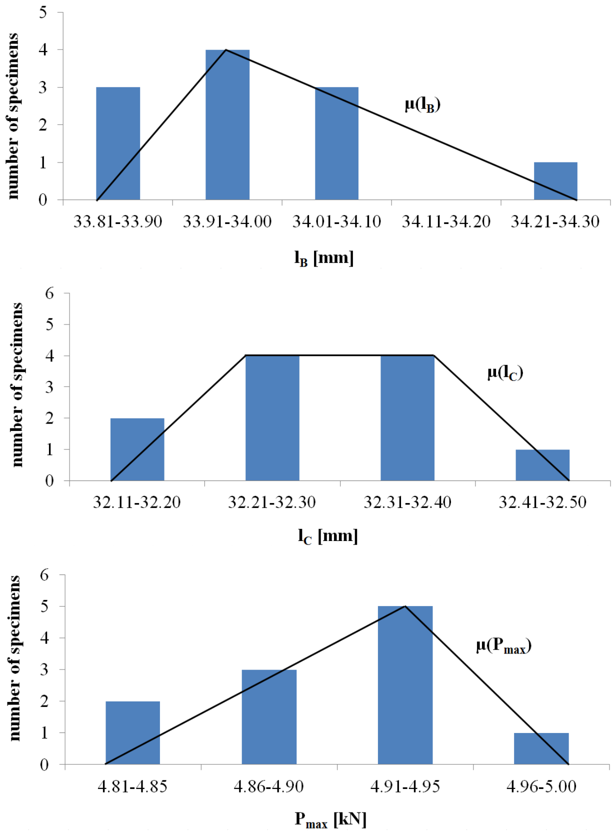

Fig. 12. Histograms of l_B, l_C and $P_{\max}$ measurements and assumed membership functions

TABLE 1

Results of a tensile test for eleven steel samples (S1-S11)

Symbol	g_0 [mm]	b_{0B} [mm]	b_{0C} [mm]	l_{0B} [mm]	l_{0C} [mm]	$P_{\max}$ [kN]	l_B [mm]	l_C [mm]	φ_B [-]	φ_C [-]
S1	1.00	15.86	16.96	30.00	30.02	4.90	34.00	32.29	0.125	0.073
S2						4.84	33.87	32.25	0.121	0.072
S3						4.97	33.94	32.22	0.123	0.070
S4						4.93	33.99	32.40	0.125	0.076
S5						4.90	33.97	32.31	0.124	0.074
S6						4.85	33.81	32.14	0.120	0.068
S7						4.90	33.88	32.20	0.122	0.070
S8						4.94	34.09	32.45	0.128	0.078
S9						4.95	34.03	32.31	0.126	0.073
S10						4.95	34.27	32.38	0.133	0.076
S11						4.92	34.06	32.29	0.127	0.070
M						4.91	33.99	32.29	0.124	0.073
SD						0.05	0.12	0.09	0.004	0.003
V [%]						0.92	0.37	0.29	2.93	3.93

One can see in Fig. 10 that membership functions are adjusted for the following ranges of l_B, l_C and P_{max} variations: [33.81; 34.30] mm, [32.11; 32.50] mm and [4.81; 5.00] kN, respectively.

The n and C fuzzy results obtained with the use of extension principle and α-optimization are shown in Fig. 13. A random dense search for all fuzzy input variables is made.

Both resulting membership functions are nonlinear. They present quite a similar shape. It is worth highlighted that for highly non-symmetrical membership function associated with P_{max} fuzzy variable, shapes of output membership functions will be clearly different. It can be seen in Fig. 13, that the variation of n and C parameters is [0.194; 0.261] and [514.10; 615.30] MPa, respectively. For n = [0.213; 0.236] and C = [549.1; 576.3 MPa], the membership functions receive maximum value. Results obtained for both tested methods are similar, but the α-optimization envelope is much smoother.

The final step of the fuzzy analysis is the conversion of the fuzzy output sets into crisp values. This way the discrete values of n and C parameters are found. There are several alternative defuzzification algorithms, such as: height method, centroid method or level rank method [7,13]. In this research the algorithm based on searching for the mass center of output membership functions was applied. The discrete values of n and C are 0.226 and 563.04 MPa, respectively (see Fig. 13). These are the most reliable strain hardening parameters for the tested steel samples.

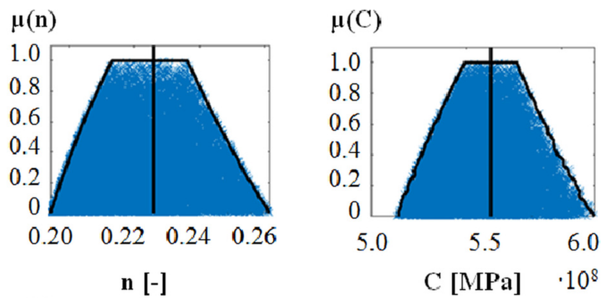

Fig. 13. The n and C fuzzy results and their discrete values

For the classical discrete analysis, mean values of n and C of investigated eleven samples are 0.221 and 557.04 MPa, respectively. The comparison of hardening curves for n and C values obtained both by means of fuzzy logic, as well as, with the use of discrete analysis is presented in Fig. 14.

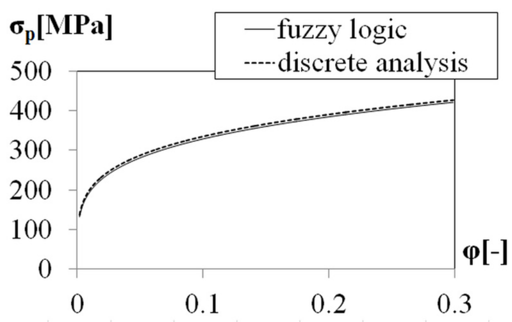

Fig. 14. The hardening curves for n and C values obtained by means of fuzzy logic and classical analysis

It can be observed that the fuzzy analysis gives little overestimated results when compared to typical experimental research. The differences are caused by the fact that the fuzzy analysis unlike classical one, includes the characteristic of the mapping model (Fig. 15). Moreover, fuzzy logic includes the statistics of both input and output parameters. In Fig. 15a there is presented a special case of the fuzzy analysis which is equivalent to discrete approaches. All input parameters are taken into consideration with the same weights (rectangular membership functions), statistic is made only for output variables, and the influence of the mapping model on results is neglected.

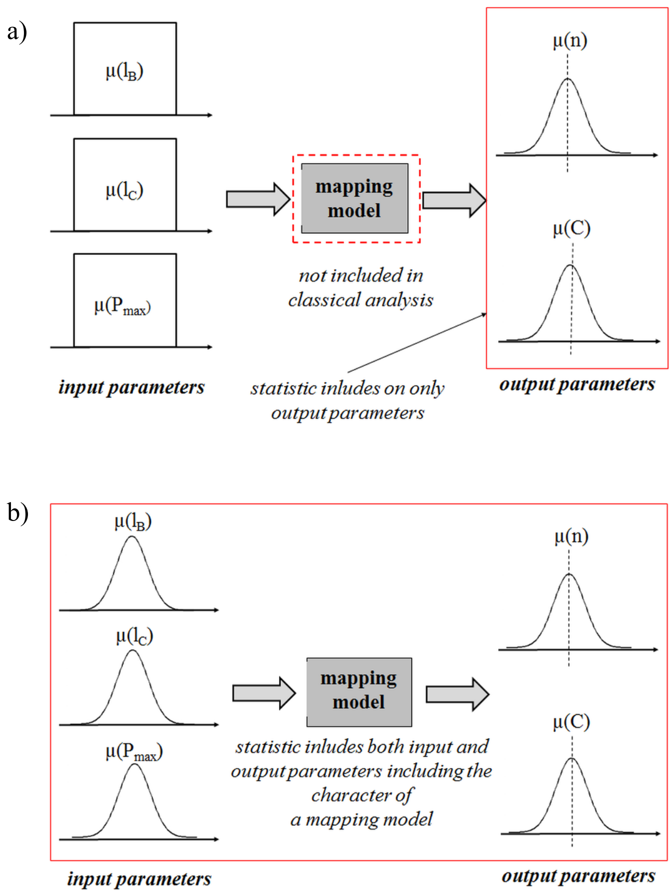

Fig. 15. Classical analysis (a) vs. fuzzy analysis (b)

6. Summary and conclusions

The fuzzy set theory is an efficient tool for solving engineering problems for which mathematical model or data are uncertain to some extent. Fuzzy logic investigates the influence of selected parameters on the model response assuming that input parameters are fuzzed. The fuzzy set theory also enables to find the most reliable (crisp) value of fuzzy output parameters.

In this paper, the fuzzy analysis was applied in calculations of strain hardening n and C parameters resulting from the Heyer's method. As the fuzzy variables, the lengths of two measuring bases and the maximum force are selected. The comparison of results obtained by means of classical statistics and results obtained by application of the fuzzy set theory are presented.

From the performed research the following remarks can be concluded:

1. The fuzzy analysis might be applied for a single sample in the limiting case. In the presented article, the numerical simulation was made for exemplary specimen in order to investigate in general the influence of input variables on the final results. It is very important in cases for which the number of measurements is limited by: the cost, access to specimen, short-period of time, etc. In these cases the fuzzy logic allows to obtain the most reliable solution (crisp value) in an objective way. The correct definition of the membership functions required in the fuzzy analysis can be based on the experts knowledge.
2. The shape of the input membership functions affects the shape of output membership function but does not influence significantly on the crisp value (the most reliable solution).
3. The shape of both resulting n and C membership functions slightly differs. The small differences are caused by the fact that C parameter depends on the length of two l_B and l_C measuring bases and the maximum force registered during the tensile test. The n strain hardening index depends only on l_B and l_C values. Because the distribution of the maximum force was symmetrical, the shapes of n and C membership functions are similar. Considerable differences will be observed if the distribution of $P_{\max}$ will be non-symmetric.
4. The discrete values of n and C membership functions obtained in fuzzy analysis were 0.226 and 563.04 MPa, respectively. Average n and C values obtained from the experimental research are 0.221 and 557.04 MPa, accordingly. The differences are caused by the fact that the fuzzy set theory considers the characteristic of the mapping model. If the number of measurements registered in the experimental investigation is sufficient, the solution based on statistics as well as the fuzzy logic results are similar – see Fig. 14.
5. As the fuzzy set theory might be used as enhancement of the experimental data analysis, it can provide the reference solution for other numerical methods.
6. The application of the fuzzy logic in engineering problems can reduce the cost and time of the experimental research.

In this paper, fuzzy logic was implemented in relatively simple tensile test. The Hollomon equation used here as the mapping model provides explicit formula for stress vs. strain. For more advanced models based on the yield condition associated with the flow rule, the numerical integration is required which complicates the mapping model and its numerical implementation. The obtained results have shown the usefulness and the potential of this method in mechanical engineering, as well as, in plasticity theory. The convergence of the statistical and the fuzzy sets analyses (sufficient number of measurements was available) has proven the reliability of the proposed approach.

Further research will be focused on the application of the fuzzy set theory for adjusting various elastic-plastic material models to the experimental stress-strain curves obtained in cyclic tension-compression tests. In future, the proposed approach will be extended on models including more than two material parameters (Ramberg-Osgood, Voice isotropic hardening, Frederick-Armstrong or Chaboche non-linear kinematic hardening) which can give more realistic description of the material response.

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