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A robust model free controller for a class of SISO nonaffine nonlinear systems: Application to an electropneumatic actuator

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This paper presents a robust model free controller (RMFC) for a class of uncertain continuous-time single-input single-output (SISO) minimum-phase nonaffine-in-control systems. Firstly, the existence of an unknown dynamic inversion controller that can achieve control objectives is demonstrated. Afterwards, a fast approximator is designed to estimate as best as possible this dynamic inversion controller. The proposed robust model free controller is an equivalent realization of the designed fast approximator. The perturbation theory and Tikhonov's theorem are used to analyze the stability of the overall closed-loop system. The performance of the developped controller are verified experimentally in the position control of a pneumatic actuator system.

Key words: dynamic inversion control, model free controller, nonaffine nonlinear systems, singular perturbation theory

1. Introduction

The design of model free control for nonlinear dynamic systems is becoming an increasingly important area of research in the automatic control field. Two main reasons can be cited for this interest: The first is the difficulty of implementing the most techniques stemming from model-based control theory and the second is the limitations of classical PID controllers especially for high-order

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processes and unstable systems. The real-time application of the model-based control schemes involves a good understanding of system dynamics and their operational environment. However, it is difficult to establish a good dynamic model of the conntrolled system and the knowledge of the different disturbances that influence its behavior is usually a difficult task. We can qualify as model free control any kind of control method which does not require the knowledge of the model of the controlled system for its implementation, but just general informations on the system are required, among other things class and order of the system, its inputs and its outputs.

Because of its importance in the industrial control system, the model-free control techniques have been studied in several research works in recent years and many control schemes have been proposed. Michel Fliess and his collaborators have introduced a new model free control scheme [1, 2] (so-called intelligent PID or i-PID) based on a local nonphysical model and the use of an online numerical differentiator. This method rests on an instantaneous identification, such that the mathematical model describing the dynamics of the system in a large operating range is replaced by a local model, valid on a very short time and updated step by step. The i-PID controllers have been successfully applied to various processes, such as shape memory alloys actuator [3], experimental greenhouses [4], glycemia of type-1 diabetes [5], quadrotor UAV [6], Servo Systems [7], autonomous vehicles [8] acute inflammation [9]. In the literature, model free control approaches are developed based on fuzzy logic such as in [7, 10] and other approaches are developed based on neural networks [11, 12]. In [13], the authors present two model-free sliding mode control schemes with an experimental validation in the position control of a twin rotor aerodynamic system. A model free control method is proposed in [14], this control law combines ideas from event-triggered control, optimal control and Q-learning theories. In [15], the authors suggest a model-free steepest-descent iterative learning controller with an experimental validation on real inverted elastic cantilever beam. A model-free optimal control scheme is proposed in [16] for a class of systems with multiple delays in state, control and output vectors based on adaptive dynamic programming which combines a similar Q-learning method with a value iteration algorithm. In [17], a model-free adaptive fractional order PID control is presented for a stable linear time-varying systems based on the numerical optimization of a frequency-domain criterion. Other model-free control schemes can be found in [18–21].

In this research work, a robust model free control strategy is developed for a class of uncertain continuous SISO minimum-phase nonaffine-in-control systems based on the singular perturbation theory and Tikhonov's theorem [22–25]. The idea of this approach is based on the estimation of an ideal dynamic inversion control by choosing a suitable estimator with fast dynamics. Firstly, the implicit function theory [25] will be used to prove the existence of such an ideal dynamic inversion controller. Then, we design a fast approximator to estimate as best as possible this ideal dynamic inversion controller. Finally, we will show that the designed fast approximator admits a robust model free controller realization. Within this scheme, the singular perturbation theory and Tikhonov's theorem are used to study the stability of the closed-loop system, the tracking performances and to show that when the dynamic of control law is chosen to be sufficiently fast, the control signal approximates with effectiveness the ideal dynamic inversion controller. The ability and the performances of the proposed robust model free control are examined experimentally in the position control of a pneumatic actuator system.

The main advantages of the proposed model free control scheme and the contributions of the current work are listed as follows:

1. The control law is designed based directly on a general class of SISO nonaffine nonlinear systems with internal dynamics, under certain acceptable assumptions. The proposed RMFC does not rely on a physical model of the controlled system, assuming only the knowledge of the order and relative degree. The stability of the overall closed-loop system is studied using the perturbation theory and Tikhonov's theorem.

2. The developped model free controller is an equivalent realization of the designed fast approximator of an unknown dynamic inversion controller that can achieve control objectives.

3. The proposed model free controller is an efficient and simple solution for practitioners seeking a general method to control minimum phase, nonaffine-in-control systems and it is really easy to implement.

The rest of the paper is organized as follows: The problem statement is introduced in section 2. The design of the proposed RMFC and the stability analysis of the overall closed-loop system are given in section 3. Section 4 discusses the real-time application of the presented control scheme and shows the experimental results that demonstrate its performance. Finally, we present the conclusion of this work in section 5.

2. Problem statement

The considered class of systems to be studied in this paper is an n-th order SISO nonaffine-in-control system represented in the following normal form

$$\begin{cases} \dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \quad \ldots, \quad \dot{x}_{r-1} = x_r\,, \\ \dot{x}_r = f(x, z, u), \\ \dot{z} = Q(x, z, u), \\ y = x_1\,, \end{cases} \tag{1}$$

where $x(0) = x_0$ and $z(0) = z_0$ for all $(x, z, u) \in D_x \times D_z \times D_u$ with $D_x \subset \mathbb{R}^r$, $D_z \subset \mathbb{R}^{n-r}$ and $D_u \subset \mathbb{R}$ are domains containing their respective origins. The overall state vector of the system (1) is $[x^T,\ z^T]^T$ where $x = \left[y, y^{(1)}, \ldots, y^{(r-1)}\right]^T \in \mathbb{R}^r$ and y is the system output, u is the control input, r is the relative degree of the system, and $f : D_x \times D_z \times D_u \to \mathbb{R}$, $Q : D_x \times D_z \times D_u \to \mathbb{R}^{n-r}$ are continuously differentiable functions of their arguments.

The input-output form of (1) can be rewritten as the following

$$\begin{cases} y^{(r)} = f(x, z, u), & x(0) = x_0\,, \\ \dot{z} = Q(x, z, u), & z(0) = z_0\,. \end{cases} \tag{2}$$

The problem is to design a controller law $u(t)$ such that the output $y(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded. Throughout this paper we make the following assumptions regarding the system (1) and the desired trajectory.

Assumption 1 *For the system (1), the function $f_u(x, z, u) = \partial f/\partial u$ is nonzero and bounded as $0 < \delta_0 < |\partial f/\partial u| < \delta_1$ for all $(x, z, u) \in D_x \times D_z \times D_u$ with δ_0 and δ_1 are some positive constants. This implies that only knowledge required of the system is the sign of the control effectiveness, sgn $(\partial f/\partial u) \in \{-1, +1\}$.*

Assumption 2 *The desired trajectory $y_d(t)$, is a known bounded function assumed to be r-times differentiable with bounded known derivatives.*

Let us define the tracking error as $e_1(t) = y_d(t) - y(t)$, the desired state vector by $x_d = \left[y_d, y_d^{(1)}, \ldots, y_d^{(r-1)}\right]^T \in \mathbb{R}^r$, the error vector as $e = x - x_d = \left[e_1, e_1^{(1)}, \ldots, e_1^{(r-1)}\right]^T \in \mathbb{R}^r$ and the filtered tracking error σ as follows

$$\sigma(t) = \left(\frac{d}{dt} + \lambda\right)^{r-1} e_1(t), \quad \lambda > 0. \tag{3}$$

Using (3), $\sigma(t) = 0$ represents a linear differential equation whose solution implies that the tracking error $e_1(t)$ converges to zero with a time constant $(r-1)/\lambda$. In addition, the derivatives of $e_1(t)$ up to the $r-1$ order also converges to zero [26]. Thus, the control objective becomes the design of a controller to keep $\sigma(t)$ at zero, therefore, the original stabilizing problem of the (r)-dimensional vector e, is reduced to that of keeping the scalar $\sigma(t)$ at zero. Moreover, bounds on $\sigma(t)$ can be directly translated into bounds on the tracking error. Specifically, if we have $|\sigma(t)| \leqslant \Phi$ where Φ is a positive constant, we can conclude that [26]: $\left|e_1^{(i)}(t)\right| \leqslant 2^i \lambda^{(i-r+1)}\Phi, i = 0, \ldots, r-1$. These bounds can be reduced by increasing the design parameter λ.

The time derivative of the filtered error (3) can be rewritten as

$$\dot{\sigma} = y_d^{(r)}(t) + \beta_{r-1} e_1^{(r-1)} + \cdots + \beta_1 \dot{e}_1 - f(x, z, u), \tag{4}$$

with $\beta_i = \dfrac{(r-1)!}{(r-i)!(i-1)!}\lambda^{r-i}$, $i = 1, \ldots, r-1$.

From (2) and (4), we obtain the following dynamics

$$\begin{cases} \dot{\sigma} = y_d^{(r)}(t) + \beta_{r-1} e_1^{(r-1)} + \cdots + \beta_1 \dot{e}_1 - f(x, z, u), & \sigma(0) = \sigma_0\,, \\ \dot{z} = Q(x, z, u), & z(0) = z_0\,. \end{cases} \tag{5}$$

Let us denote $f_e(t, e, z, u) = y_d^{(r)}(t) + \beta_{r-1} e_1^{(r-1)} + \cdots + \beta_1 \dot{e}_1 - f(e + x_r, z, u)$, then (5) can be rewritten as

$$\begin{cases} \dot{\sigma} = f_e(t, e, z, u), & \sigma(0) = \sigma_0\,, \\ \dot{z} = Q(e + x_r, z, u)\,, & z(0) = z_0\,. \end{cases} \tag{6}$$

Based on implicit theorem [25, 27], since $\partial f/\partial u \neq 0$, there exists an ideal dynamic inversion control $u^\star$ for (6) which can be found by solving the following equation with respect to u

$$f_e(t, e, z, u) = -K\sigma - K_0 \tanh\left(\frac{\sigma}{\varepsilon_0}\right). \tag{7}$$

where $K > 0$ and $K_0 > 0$, ε_0 is a small positive constant and $\tanh(\cdot)$ is the hyperbolic tangent function, which is a globally Lipschitz function. Consequently, with the ideal dynamic inversion control $u^\star$ and in light of (7), the dynamics (6) becomes

$$\begin{cases} \dot{\sigma} = -K\sigma - K_0 \tanh\left(\dfrac{\sigma}{\varepsilon_0}\right), & \sigma(0) = \sigma_0\,, \\ \dot{z} = Q(e + x_r, z, u^\star)\,, & z(0) = z_0\,. \end{cases} \tag{8}$$

From the first equation in (8), it results that $\sigma(t) \to 0$ as $t \to \infty$ and, therefore, all elements of the error vector e converge to zero as $t \to \infty$.

It is worth to point out that the implicit function theory only guarantees the existence of an ideal dynamic inversion controller for the system (1). Moreover, since the control input u does not appear linearly in (1) and the function $f(x, z, u)$ is unknown, the equation (7) cannot be solved explicitly for u. In fact, even if the non-affine non-linear function $f(x, z, u)$ is known exactly, it is not easy to find a solution of (7). In the next section, we design a fast approximator to estimate as best as possible the exact dynamic inversion controller solution of (7). Then, we show that this fast approximator admits a robust model free controller realization.

3. Robust Model Free Controller design

Based both on the approximate dynamic inversion control scheme and singular perturbation theory, we will design a stable and robust model free controller for the general class of nonaffine-in-control systems (1). In this scheme, the singular perturbation theory and Tikhonov's theorem will be used to show that when the controller dynamic is chosen to be sufficiently fast, the control signal approximates effectively the ideal dynamic inversion controller.

3.1. Fast Approximation of the Dynamic Inversion Control

The exact dynamic inversion solution of (7) will be approximated via the fast dynamics given by

$$\epsilon\dot{u} = -\alpha\left(f_e(t,e,z,u) + K\sigma + K_0\tanh\left(\frac{\sigma}{\varepsilon_0}\right)\right), \tag{9}$$

where ϵ is a positive controller design parameter and $\alpha = \operatorname{sgn}\left(\frac{\partial f}{\partial u}\right)$. Note that the parameter ϵ must be chosen sufficiently small to achieve closed-loop stability and obtain the best approximation for the exact dynamic inversion solution of (7). We denote $F(t,\sigma,z,u) = f_e(t,e,z,u) + K\sigma + K_0\tanh\left(\frac{\sigma}{\varepsilon_0}\right)$, then we obtain the system Σ_ϵ as follows

$$\Sigma_\epsilon : \begin{cases} \dot{\sigma} = F(t,\sigma,z,u) - K\sigma - K_0\tanh\left(\frac{\sigma}{\varepsilon_0}\right), & \sigma(0) = \sigma_0, \\ \dot{z} = Q\left(e + x_r, z, u\right), & z(0) = z_0, \\ \epsilon\dot{u} = -\alpha F(t,\sigma,z,u), & u(0) = u_0\,. \end{cases} \tag{10}$$

The stability analysis of the overall dynamics (10), or the perturbed system Σ_ϵ, using the singular perturbation theory and Tikhonov's theorem will be the topic of the next subsection.

3.2. Stability analysis of the overall closed-loop system

As we have explained previously, to achieve a good approximation of the ideal dynamic inversion control, the positive controller design parameter ϵ must be chosen small enough, which makes the dynamics of the controller fast with respect to the dynamics of σ and z. The coexistence of fast and slow dynamics in the overall dynamics (10) allows us to use the Tikhonov's theorem to analyze the tracking performance and stability in closed loop system. The Tikhonov's theorem was introduced in 1952 on systems of differential equations containing small parameters in the derivative and represents the main theorem of the singular perturbation theory [25].

Consider the fast dynamics in (10), that of the controller: $\epsilon \dot{u} = -\alpha F(t, \sigma, z, u)$, set $\epsilon = 0$ and solve the resulting algebraic equation

$$F(t, \sigma, z, u) = 0 \tag{11}$$

with respect to u. Let $u = h(t, \sigma, z)$ be an isolated root of (11). In accordance with the singular perturbations theory, the reduced system Σ_0 is obtained by setting $\epsilon = 0$ and $u = h(t, \sigma, z)$ in the first two equations of the system Σ_ϵ in (10), which leads to

$$\Sigma_0 : \begin{cases} \dot{\sigma} = -K\sigma - K_0 \tanh\left(\dfrac{\sigma}{\varepsilon_0}\right), & \sigma(0) = \sigma_0 , \\ \dot{z} = Q\left(e + x_r, z, h(t, \sigma, z)\right), & z(0) = z_0 . \end{cases} \tag{12}$$

Now, let us proceed to the change of variables $\vartheta = u - h(t, \sigma, z)$ and $\tau = \dfrac{t}{\varepsilon}$. The resulting system in the ϑ coordinates is called the boundary layer system Σ_b, such that

$$\Sigma_b : \frac{\mathrm{d}\vartheta}{\mathrm{d}\tau} = -\alpha F\left(t, \sigma, z, \vartheta + h(t, \sigma, z)\right), \quad \vartheta(0) = u_0 - h\left(0, \sigma_0, z_0\right). \tag{13}$$

Before proceeding, the following assumption on the system Σ_ϵ in (10) is made in order to establish stability.

Assumption 3 *Consider the system Σ_ϵ and let $u = h(t, \sigma, z)$ be an isolated root of (11). For all $[t, \sigma, z, u - h(t, \sigma, z), \varepsilon] \in [0, \infty) \times D_{\sigma,z} \times D_v \times [0, \epsilon_0]$ with $D_{\sigma,z} \subset \mathbb{R}^n$ and $D_v \subset \mathbb{R}^m$ are domains containing their respective origins, we suppose that the following conditions are met:*

A. On any compact subset of $D_{\sigma,z} \times D_v$, the functions f, Q and their first partial derivatives with respect to (x, z, u) are continuous and bounded, the two functions $h(t, \sigma, z)$ and $\dfrac{\partial f}{\partial u}(x, z, u)$ have bounded first partial derivatives with respect to their arguments, and $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial z}, \dfrac{\partial Q}{\partial x}, \dfrac{\partial Q}{\partial z}$ as functions of $(x(t), z, h(t, \sigma, z))$ are Lipschitz in σ and z uniformly in t.

B. The origin is an exponentially stable equilibrium of the system

$$\dot{z} = Q\left(x_r(t), z, h(t, 0, z)\right).$$

The map $(\sigma, z) \mapsto Q\left(x(t), z, h(t, \sigma, z)\right)$ is continuously differentiable and Lipschitz in (σ, z) uniformly in t.

C. $(t, \sigma, z, v) \mapsto \left|\dfrac{\partial f}{\partial u}(x(t), z, v + h(t, \sigma, z))\right|$ *is bounded below by some positive nomber for all* $(t, \sigma, z) \in [0, \infty) \times D_{\sigma,z}$.

At this stage, it is helpful to point out the equivalence between the various variables and functions evoked in Tikhonov's theorem [25, Theorem 11.2, pp. 439–440] and those of the system Σ_ϵ in (10). This identification of variables and functions simplifies the application of Tikhonov's theorem for the system Σ_ϵ and check the satisfaction of its three assumptions for this system. First of all, we identify x, z, y, and $h(t, x)$ in Theorem 11.2 [25], denoted here by x_s, z_s, y_s, and $h_s(t, x_s)$ repectively for distinction, with quantities of the system Σ_ϵ by $x_s \sim \left[\sigma^T, z^T\right]^T$, $z_s \sim u$, $y_s \sim \vartheta$ and $h_s(t, x_s) \sim h(t, \sigma, z)$. We identify also the functions f and g of Theorem 11.2 in [25], denoted here by f_s and g_s, with those of the system Σ_ϵ as the following

$$f_s \sim \left[F(t, \sigma, z, u) - K\sigma - K_0 \tanh\left(\frac{\sigma}{\varepsilon_0}\right), \quad Q\left(\bar{e} + x_r, z, h(t, \sigma, z)\right)\right]^T, \qquad (14)$$
$$g_s \sim -\alpha F(t, \sigma, z, u).$$

The following remark will help us to analyze and discuss the stability of the system Σ_ϵ:

Remark 1 *For the functions introduced in (14), it is clear that to obtain* f_s *and* g_s *continuous and bounded for any compact subset of* $D_{x_s} \times D_{y_s}$ *requires that* f*,* Q*,* x_d *and* $y_d^{(r)}$ *be continuous and bounded for any compact subset of* $D_{\sigma \times z} \times D_v$*. Moreover, the condition on the continuity and boundedness of the first partial derivatives of* f_s *and* g_s *with respect to* (x_s, z_s, ε) *is fulfilled when the first partial derivatives* f *and* Q *with respect to* (x, z, u) *be continuous and bounded. The condition that the first partial derivative of* g_s *with respect to* t*, corresponds in the system (10) to the first partial derivative of* $-\alpha F(t, \sigma, z, u)$ *with respect to* t*, requires that* $\dfrac{\partial f}{\partial x}$ *and* $y_d^{(r+1)}$ *to be continuous and bounded. Similarly, to have the first partial derivatives of* $\dfrac{\partial g_s(t, x_s, z_s, 0)}{\partial z_s}$ *with respect to its arguments be bounded requires the boundedness of the first partial derivatives of* $\dfrac{\partial f(x, z, u)}{\partial u}$ *with respect to its arguments and the boundedness of* $y_d^{(r)}$*. The requirement on the first partial derivatives of* $h_s(t, x_s)$ *with respect to its arguments to be bounded is satisfied if* $h(t, \sigma, z)$ *fulfills the same condition.*

Remark 2 *The first part of Assumption 3, i.e. Assumption 3.A, summarizes in fact the conventional regularity assumptions on the system dynamics. More precisely, this assumption is required to guarantee the existence and uniqueness of* *solutions of differential equations involved in the control design. Assumption 3.B is introduced to guarantee that the controlled system (1) is minimum phase and, then, the dynamic inversion problem is well defined. Assumption 3.C represents the controllability assumption for the system (1), the control effectiveness of this later is assumed to be bounded away from zero.*

Now, we can give the following theorem about the stability and performance of the system Σ_ϵ in (10):

Theorem 1 *Consider the system Σ_ϵ in (10), the reduced system Σ_0 given by (12) and the boundary layer system Σ_b introduced in (13), and let $u = h(t,\sigma,z)$ be an isolated root of (11). We suppose that Assumptions 1–3 are satisfied for all $[t,\sigma,z,u-h(t,\sigma,z),\varepsilon] \in [0,\infty)\times D_{\sigma,z}\times D_v\times[0,\epsilon_0]$ for some domains $D_{\sigma,z} \subset \mathbb{R}^n$ and $D_v \subset \mathbb{R}^m$ containing their respective origins. Moreover, let Ω_b be a compact subset of R_v, where $R_v \subset D_v$ denotes the region of attraction of the autonomous system*

$$\frac{\mathrm{d}v}{\mathrm{d}\tau} = -\alpha F\left(0,\sigma_0,z_0,v+h\left(0,\sigma_0,z_0\right)\right).$$

Then, for each compact subset $\Omega_{z,\sigma} \subset D_{z,\sigma}$, there exists a positive constant $\epsilon_\star$ and a $T > 0$ such that for all $t \geqslant 0$, $(\sigma_0,z_0) \in \Omega_{z,\sigma}$, $u_0 - h\left(0,\sigma_0,z_0\right) \in \Omega_v$ and $0 < \epsilon < \epsilon_\star$, the system Σ_ϵ has a unique solution $\sigma(t,\epsilon)$, $z(t,\epsilon)$, $u(t,\epsilon)$ on $[0,\infty)$ and

$$\begin{aligned}
\sigma(t,\epsilon) - \bar{\sigma}(t) &= o(\epsilon),\\
z(t,\epsilon) - z_r(t) &= o(\epsilon),\\
u(t,\epsilon) - h\left(t,\sigma,z_r(t)\right) &= o(\epsilon)
\end{aligned}$$

hold uniformly for $t \in [0,\infty)$, where $\bar{\sigma}(t)$ and z_r denote the solution of the reduced system Σ_0. Furthermore, there exists $T < \infty$ such that the output tracking error converges to a small neighborhood of the origin.

Proof. For the purpose of proving the previous theorem, we follow a procedure similar to the proofs of Theorem 2 in [28]. Therefore, we need to verify that the three assumptions of Tikhonov's theorem [25, Theorem 11.2, pp. 439–440] are satisfied for the system Σ_ϵ. Firstly, in the light of remark 1 and taking into account Assumptions 1, 2 and 3.A, it is clair that the first assumption of Tikhonov's theorem holds.

In the following, we will demonstrate that the second assumption of Tikhonov's theorem is fulfilled. Thus, we need to show that the origine of the reduced system Σ_0 in (12) is exponentially stable. To this end, we follow a procedure similar to the proofs of Lemma 4.7 in [25, pp. 180] and Theorem 4 in [29].

For the reduced system Σ_0, we have $\sigma = 0$ is an exponentially stable equilibrium point, and its solution $\sigma(t)$ fulfills for all $t \geqslant t_0$

$$|\sigma(t)| \leqslant k_\sigma |\sigma(t_0)| \exp\left(-\lambda_\sigma (t - t_0)\right), \tag{15}$$

with $t_0 \geqslant 0$ is the initial time, k_σ and λ_σ are some positive constants.

On the other hand, Assumption 3.B, i.e. the unforced system $\dot{z} = Q(x_r(t), z, h(t, 0, z))$ has 0 as en exponentially stable equilibrium point, implies that the system

$$\dot{z} = Q(e + x_r, z, h(t, \sigma, z))$$

with σ as input, is input-to-state exponentially stable, see the proof of Lemma 4.6 in [25, pp. 176], and its solution $z(t)$ satisfies for all $t \geqslant \kappa \geqslant t_0$

$$\|z(t)\| \leqslant k_z \|z(\kappa)\| \exp\left(-\lambda_z(t - \kappa)\right) + \sup_{\kappa \leqslant \zeta \leqslant t} c_z \|\sigma(\zeta)\|, \tag{16}$$

where k_z, λ_z and c_z are some positive constants. Furthermore, using (15) and (16), we obtain

for all $t \geqslant t_0 \geqslant 0$

$$\|x_{\sigma z}(t)\| \leqslant k_{\sigma z} \|x_{\sigma z}(t_0)\| \exp\left(-\lambda_{\sigma z}(t - t_0)\right), \tag{17}$$

where $x_{\sigma z} = \left[\sigma^T, z^T\right]^T$ is the composite state, $\lambda_{\sigma z} = \dfrac{1}{2}\min\{\lambda_\sigma, \lambda_z\}$ and $k_{\sigma z} = (1 + c_z) k_\sigma + c_z k_\sigma k_z + k_z^2$ (see details in Appendix). So, we conclude that $x_{\sigma z} = (0, 0)$ is an exponentially stable equilibrium point of the reduced system Σ_0.

From a converse Lyaounov theorem, there exists a Lyapunov function $V\colon [0, \infty) \times D_{\sigma,z} \to [0, \infty)$ that satisfies

$$c_1 \|x_{\sigma z}\|^2 \leqslant V(t, x_{\sigma z}) \leqslant c_2 \|x_{\sigma z}\|^2,$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_{\sigma z}} \widehat{F}(t, x_{\sigma z}) \leqslant -c_3 \|x_{\sigma z}\|^2,$$

where

$$\widetilde{F}(t, x_{\sigma z}) = \begin{bmatrix} -K\sigma - K_0 \tanh\left(\dfrac{\sigma}{\varepsilon_0}\right) \\ Q(\overline{e} + x_r, z, h(t, \sigma, z)) \end{bmatrix} \in \mathbb{R}^3$$

which implies that the Lyapunov function condition mentioned in the second assumption of Tikhonov's theorem is fulfilled with $W_1(r) = c_1 \|r\|^2$, $W_2(r) = c_2 \|r\|^2$, and $W_3(r) = c_3 \|r\|^2$. Moreover, by choosing c sufficiently small, the set $\left\{ x_{\sigma z} \in D_{\sigma,z} \middle| W_1\left(x_{\sigma z}\right) = c_1 \left\|x_{\sigma z}\right\|^2 \leqslant c \right\}$ can be made compact. Hence, from the previous analysis, satisfaction of assumption 3.B implies satisfaction of the second assumption of Tikhonov's theorem [25, Theorem 11.2, pp. 439–440].

Now, we interest to prove satisfaction of the third assumption of Thikhonov's theorem, i.e. the origin of $\frac{\mathrm{d}v}{\mathrm{d}\tau} = -\alpha F\left(t, \sigma, z, v + h(t, \sigma, z)\right)$ is exponentially stable. We have $u = h(t, \sigma, z)$ is the solution of $F(t, \sigma, z, u) = 0$, this implies that $v = 0$ is an equilibrium point of the boundary layer system (13). Let us denote $\widetilde{g}(\tau, v) = -\alpha F\left(\epsilon\tau, \sigma(\epsilon\tau), z(\epsilon\tau), v + h\left(\epsilon\tau, \sigma(\epsilon\tau), z(\epsilon\tau)\right)\right)$, with $\sigma(\epsilon\tau)$ and $z(\epsilon\tau)$ viewed as the exogenous time-varying signals, then the linear system corresponds to the boundary layer system $\frac{\mathrm{d}v}{\mathrm{d}\tau} = \widetilde{g}(\tau, v)$ can be written as $\frac{\mathrm{d}\widetilde{v}}{\mathrm{d}\tau} = A(\tau)\widetilde{v}$, such that

$$A(\tau) = \frac{\partial\widetilde{g}}{\partial v}(\tau, v)\bigg|_{v=0} = -\left|\frac{\partial f}{\partial u}\left(x(\epsilon\tau), z(\epsilon\tau), v + h\left(\epsilon\tau, \sigma(\epsilon\tau), z(\epsilon\tau)\right)\right)\right|.$$

From the preceding, and using Assumption 3.A, we conclude that, for all $(\tau, x, z) \in [0, \infty) \times D_x \times D_z$, the origin is an exponentially stable equilibrium point of the linear system, which translates directly, based on the theorem 4.13 in [25], to exponential stability of the origin of the boundary layer system (13).

Finally, from the previous analysis, we conclude that all assumptions of Tikhonov's theorem are fulfilled, and so it follows that for each compact set $\Omega_{\sigma,z}$ given by

$$\Omega_{\sigma,z} \subset \left\{ x_{\sigma z} \in D_{\sigma,z} \middle| W_2\left(x_{\sigma z}\right) = c_2 \left\|x_{\sigma z}\right\|^2 \leqslant \rho c,\ 0 < \rho < 1 \right\}$$

with c is selected above, there exists a positive constant $\epsilon^\star$ such that for all $t > 0$, $(\sigma_0, z_0) \in \Omega_{z,\sigma}$, $u_0 - h\left(0, \sigma_0, z_0\right) \in \Omega_v$, and $\epsilon \in (0, \epsilon^\star)$, the system (10) has a unique solution $\sigma(t, \epsilon)$, $z(t, \epsilon)$, $u(t, \epsilon)$ on $[0, \infty)$, such that $\sigma(t, \epsilon) - \bar{\sigma}(t) = o(\epsilon)$, $z(t, \epsilon) - z_r(t) = o(\epsilon)$, $u(t, \epsilon) - h\left(t, \sigma, z_r(t)\right) = o(\epsilon)$, hold uniformly for $t \in [0, \infty)$, where $\bar{\sigma}(t)$ is the solution of exponentially stable system $\dot{\sigma} = -K\sigma - K_0 \tanh\left(\frac{\sigma}{\varepsilon_0}\right)$, $\sigma(0) = \sigma_0$. Clearly that for any $\epsilon > 0$, there exists $T < \infty$ such that the solution $\bar{\sigma}(t)$, the solution of the exponentially stable system $\dot{\sigma} = -K\sigma - K_0 \tanh\left(\frac{\sigma}{\varepsilon_0}\right)$, $\sigma(0) = \sigma_0$, fulfils $\bar{\sigma}(t) \leqslant \epsilon$ for all $t \geqslant T$. In addition, it follows that there exists a small bound $\Phi(\epsilon)$ such that $\sigma(t, \epsilon) \leqslant \Phi(\epsilon)$ for $t \geqslant T$. Hence, the tracking errors converge to residual sets as: $\left|e^{(i)}(t)\right| \leqslant 2^i \lambda^{(i-r+1)}\Phi(\epsilon)$, $i = 0, \ldots, r-1$. This completes the proof. □

3.3. Equivalent Robust Model Free Controller

Corollary 1 *For every approximate dynamic inversion controller (7), there exists a model free controller realization in the following form*

$$u = -\frac{\alpha}{\epsilon}\left(\sigma(t) + \int_0^t \left(K\sigma(\tau) + K_0 \tanh\left(\frac{\sigma(\tau)}{\varepsilon_0}\right)\right) \mathrm{d}\tau + \sigma(0)\right) + u_0, \tag{18}$$

where $u_0 = u(0)$.

Proof. Substituting the equation (7) into (9), we obtain

$$\epsilon\dot{u} = -\alpha\left(\dot{\sigma} + K\sigma + K_0 \tanh\left(\frac{\sigma}{\varepsilon_0}\right)\right). \tag{19}$$

Integrating both sides of (19) and dividing by $\epsilon >$, we get (18). □

Remark 3 *The proposed model free controller (18) may exhibit a peaking phenomenon, in which the control input peaks to an extremely large value during the transient stage. This undesirable problem can be eliminated by saturating the control input u outside a compact region of interest in order to create a buffer that protects the system from peaking [25, 30, 31].*

4. Experimental validation of the proposed controller

4.1. Electropneumatic actuator description

The robust model free controller presented in this paper was examined through real-time experimental study on position control for a pneumatic system [32, 33]. This latter is composed of two antagonist actuators controlled by two servodistributors, as displayed in Fig. 1. The actuator considered by the control is named main actuator, it contains two chambers denoted P (as "positive" – the left hand side chamber) and N (as "negative" – the right hand side chamber). The second actuator is named perturbation actuator, it is mechanically connected to the main actuator and used to produce an external force acting as an unknown perturbation. For the main actuator, two three-way servodistributors modulate the air mass flow rates q_m entering in the chambers P and N. The pneumatic jack horizontally moves a load carriage of mass M.

The complete model of this electropneumatic experimental set-up (as detailed in [32, 33]) is derived based on three physical laws [33]: the mass flow rate under a restriction, the pressure behavior in a chamber with variable volume and the

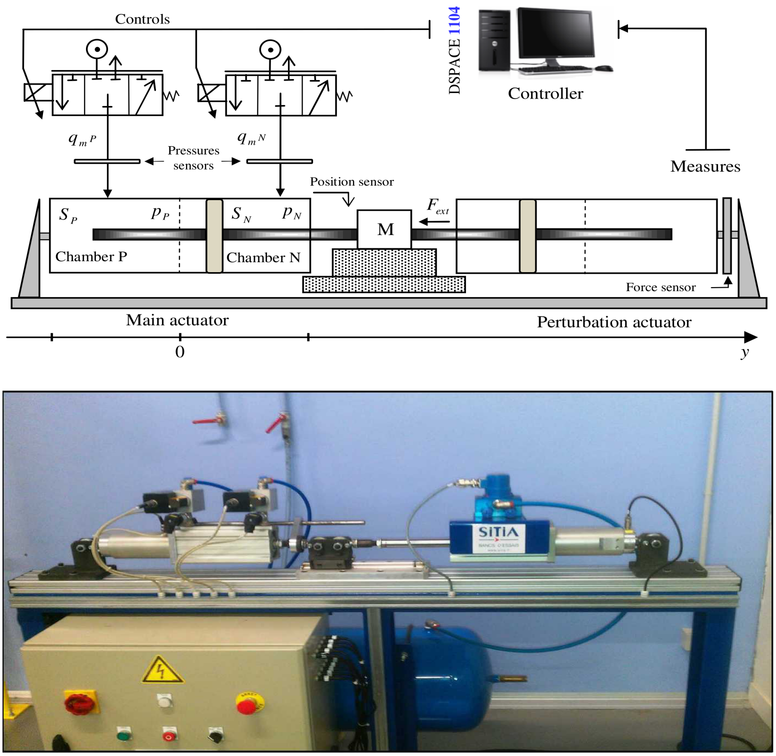

Figure 1: Description scheme of the electropneumatic system (top) and photo of the used pneumatic experimental set-up located at LS2N (bottom).

fundamental mechanical equation. The resulting dynamic model of the main actuator reads as a nonaffine nonlinear system [32, 33]

$$\begin{cases} \dot{y} = v, \\ \dot{v} = \dfrac{1}{M}\left(S\left(p_P - p_N\right) - b_v v - F_{\text{ext}}\right), \\ \dot{p}_P = \dfrac{\bar{k} r_0 T}{V_P(y)}\left(q_m\left(u, p_P\right) - \dfrac{S}{r_0 T} p_P v\right), \\ \dot{p}_N = \dfrac{\bar{k} r_0 T}{V_N(y)}\left(q_m\left(-u, p_N\right) + \dfrac{S}{r_0 T} p_N v\right), \end{cases} \tag{20}$$

where $V_P(y) = V_0 + S \times y$ and $V_N(y) = V_0 - S \times y$ with V_0 is the half-cylinder volume, S denotes the piston surface, y and v being the position and velocity of the actuator (the piston), p_P (reps. p_N) denote the pressure in P (resp. N) chamber, u the input voltage defined as $|u| \leqslant 10V$, $\bar{k}$ is the polytropic constant, T the chamber temperature, r_0 is the perfect gas constant, b_v is the viscous friction coefficient, and F_{ext} represents the external force generated by the "perturbation" actuator. In the mathematical model (20), the two first equations represent the pressure dynamics in the chambers whereas the motion of the actuator is governed by the two last equations.

To establish the state representation of (20), let us denote $x = [x_1,\ x_2,\ x_3]^T = [y,\ \dot{y},\ \ddot{y}]^T$ and $z = [z_1,\ z_2]^T = [p_P,\ p_N]^T$. From (20), the dynamic model of the controlled actuator reads as the following nonaffine nonlinear system

$$\begin{cases} \dot{x}_1 = x_2\,, \quad \dot{x}_2 = x_3\,, \\ \dot{x}_3 = f(x,z,u), \\ \dot{z} = Q(x,z,u), \\ y = x_1\,, \end{cases} \tag{21}$$

where the nonlinears functions $f(x,z,u)$ and $Q(x,z,u)$ in (21) are given as follows

$$\begin{aligned} f(x,z,u) = {} & \frac{S\bar{k}r_0T}{M}\left(\frac{q_m(u,z_1)}{V_P(x_1)} - \frac{q_m(-u,z_2)}{V_N(x_1)}\right) - \frac{\bar{k}S_2}{M}\left(\frac{z_1}{V_P(x_1)} + \frac{z_2}{V_N(x_1)}\right)x_2 \\ & - \frac{1}{M}\left(b_v x_2 + \dot{F}_{\text{ext}}\right), \end{aligned} \tag{22}$$

$$Q(x,z,u) = \left[\frac{\bar{k}r_0T}{V_P(x_1)}\left(q_m(u,z_1) - \frac{S}{r_0T}z_1x_2\right),\ \frac{\bar{k}r_0T}{V_N(x_1)}\left(q_m(-u,z_2) + \frac{S}{r_0T}z_2x_2\right)\right]^T. \tag{23}$$

Consequently, the input-output representation of (21) reads as

$$\begin{cases} y^{(3)} = f(x,z,u), \\ \dot{z} = Q(x,z,u) \end{cases} \tag{24}$$

which is in the general input-output form given by (2) with the relative degree $r = 3$ in this case.

Remark 4 *The mass flow rates $q_m\,(u,p_P)$ and $q_m\,(-u,p_N)$ play a key role in the dynamics of the controlled actuator. The knowledge of these quantities is not an easy task, in fact, their experimental identification is the main challenge of the dynamic model (21) and often needs to be done again when the servodistributors are changed. In literature, as a solution for the control design, the mass flow rates is experimentally identified by an affine model as follows [32, 34]: $q_m\,(u,p_P) = \varphi_P + \psi_P u$ and $q_m\,(-u,p_N) = \varphi_N - \psi_N u$ with $\varphi_j > 0$ and $\psi_j > 0,\ j \in (P,N)$, fifth-order polynomials with respect to p_j. It is important to point out that, within this control scheme, the experimental identification of these unknown quantities is not needed both in control design and implementation phases.*

Remark 5 *Since the mass flow rates $q_m(u, p_P)$ and $q_m(-u, p_N)$ are unknown quantities, it is not possible to check that the Assumption 1 is fulfilled for the system (21). For this reason, we just assume that $f_u(x, z, u)$ is strictly positive and bounded without proving it. Nevertheless, it is worthwhile to notice that using the experimental identification, it results that $f_u(x, z, u) = \frac{S\bar{k}r_0T}{M}\left(\frac{\psi_P}{V_P(y)} + \frac{\psi_N}{V_N(y)}\right) > 0$ and bounded, then the Assumption 1 holds with this choice.*

4.2. Experimental results and comparative study

To validate the robust model free controller introduced in the previous section, experimental tests were conducted on the experimental set-up located at LS2N's laboratory (Nantes, France), equipped with a dSpace DS1104 controller board on which the designed robust model free control law is implemented. For this experimental study, the control objective is to drive the piston position y to follow a desired reference $y_d(t) = 50\sin(0.2\pi t)\, mm$, in spite of the presence of perturbation forces. The implemented contol law was selected in the form of (18) and the filtered tracking error σ as $\sigma(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)^2 e_1(t)$ where $e_1(t) = y_d(t) - y(t)$.

The designed robust model free controller is implemented with sampling time set to $T_s = 0.001$ s and the design parameters are chosen as follows: $\lambda = 2$, $\epsilon = 0.01$, $K = 0.5$, $K_0 = 1$, $\varepsilon_0 = 0.1$ and $u_0 = 0$. The robustness of the proposed controller was tested by applying an external force produced by the perturbation actuator and it is illustrated in Fig. 2.

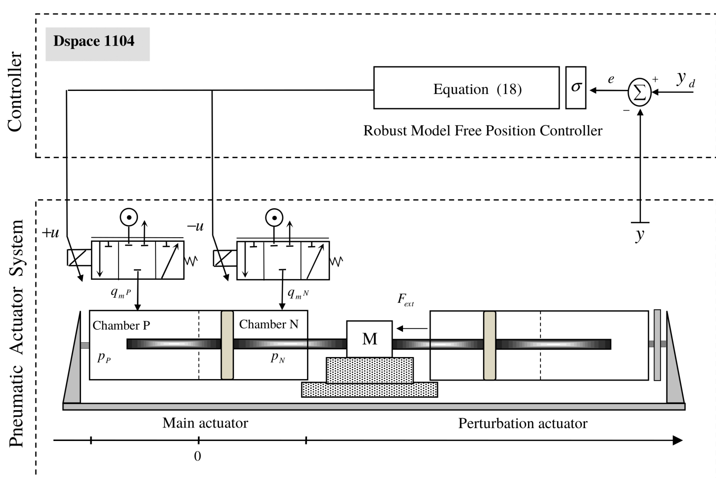

Figure 2: Proposed observer-based linear adaptive controller applied to the pneumatic actuator system.

The experiment results are shown in Figs 3–8. The plots of the piston position y and the reference trajectory y_r are reported in Fig. 3 and the resulting error curve $e = y_d - y$ is displayed in Fig. 5. It appears from these figures that the piston position tracks the reference trajectories satisfactorily. In addition, Fig. 4 indicates that the control input u applied to the main actuator is bounded in spite of the effects of the external force and the measurement noise, which confirms the good tracking performance and the robustness of the controller. The pressures p_P and p_N in the two chambers are bounded during the control process, as it is illustrated in Fig. 7 and 8.

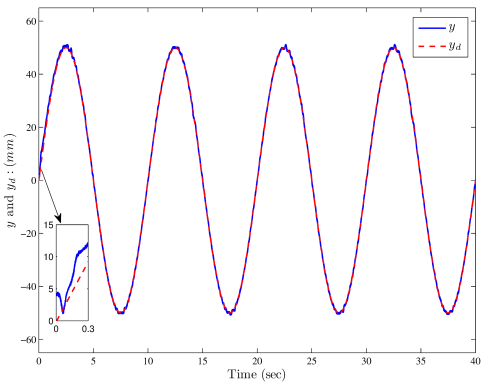

Figure 3: Actual position y (solid line) and its reference $y_d(t)$ (dotted line) (mm) versus time (sec).

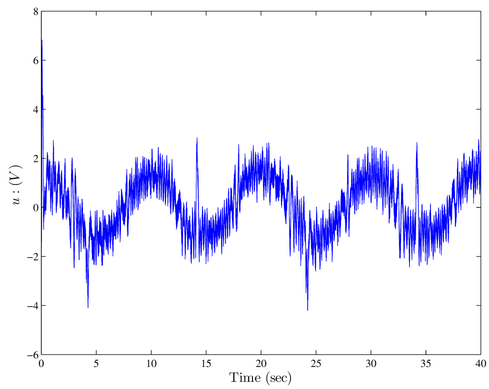

Figure 4: Control input $u(V)$ versus time (sec).

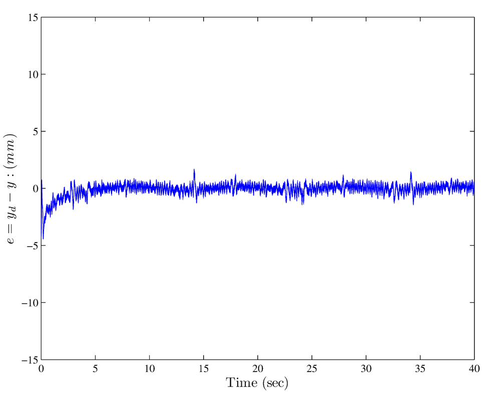

Figure 5: Tracking position error (mm) versus time (sec).

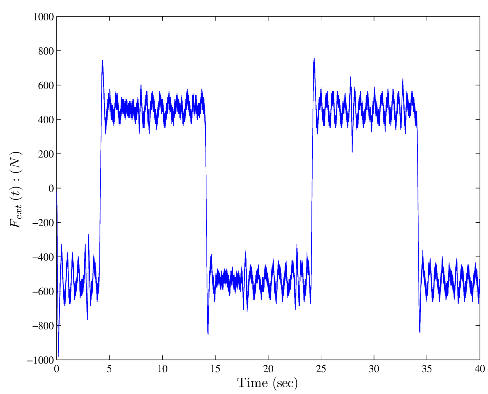

Figure 6: Perturbation force $F_{\text{ext}}(N)$ versus time (sec).

To further evaluate the performances of the proposed controller, an experimental comparative study of the robust model free controller against two conventional

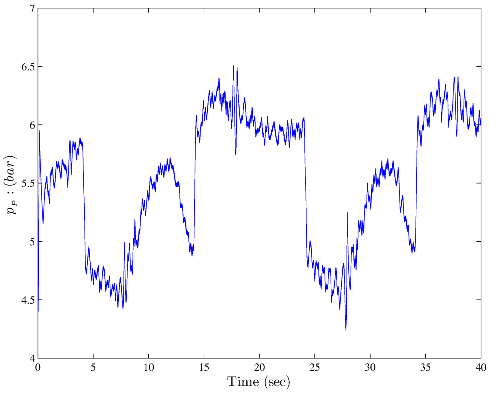

Figure 7: Pressure in chamber P (bar) versus time (sec).

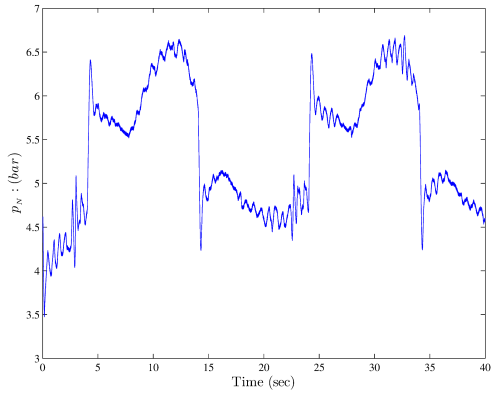

Figure 8: Pressure in chamber N (bar) versus time (sec).

controllers PID and PI has been conducted under the same conditions. We used the two PI and PID controllers that give the best performances in terms of desired trajectory tracking, their gains were obtained by trial and error method. We implemented the PID controller with the parameters $K_p = 250$, $K_I = 45$, $K_d = 20$ and the PI controller with $K_p = 250$, $K_I = 50$. The time evolution of the piston position with the PI controller is illustrated in Fig. 9 and that of the PID controller is shown in Fig. 10. The performances of the three implemented controllers are analyzed and compared based on the characteristics of the resulting position tracking error e in the time interval $t \in [5 \text{ s}, 35 \text{ s}]$. The average of the absolute value of the position tracking error e over the duration of the test, the range of e and its standard deviation are reported in Table 1. The obtained results in Table 1

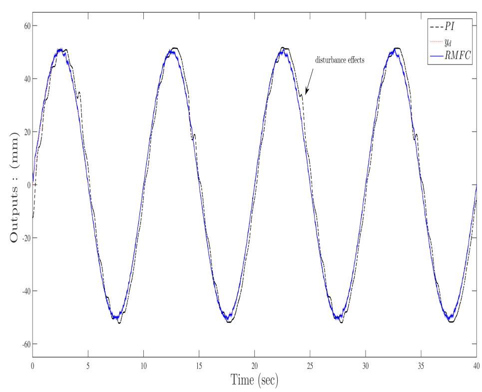

Figure 9: Comparative study with a PI controller.

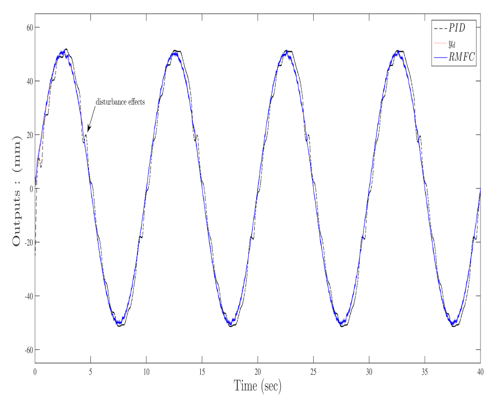

Figure 10: Comparative study with a PID controller.

and the Figs 9 and 10 show the superiority of the developped RMFC controller with respect to both controllers PID and PI, especially in terms of disturbance rejection and robustness against the effect of the external force produced by the disturbance actuator.

Table 1: Comparisons of the tracking error $e_1(t) = y_d(t) - y(t)$ for the desired trajectory $y_d(t) = 50\sin(0.2\pi t)$ mm.

Control method	Avg (abs(e_1))	Range of e_1	Standard deviation of e_1
Robust model free controller	2.60×10^{-4}	$(-14,\ 17) \times 10^{-4}$	3.38×10^{-4}
PI Controller	35×10^{-4}	$(-117,\ 64) \times 10^{-4}$	39×10^{-4}
PID Controller	30×10^{-4}	$(-75,\ 74) \times 10^{-3}$	35×10^{-4}

The presented experiment results in this subsection demonstrate the effectiveness and feasibility of the proposed robust model free controller.

5. Conclusion

A robust model free controller is proposed in this paper for a class of SISO minimum-phase nonaffine-in-control systems. The presented controller is derived from a fast approximator of an unknown dynamic inversion controller.Within this scheme, the singular perturbation theory and Tikhonov's theorem are used to analyze the stability in the closed-loop system. The ability and the performances of the presented robust model free controller have been experimentally examined in the position control of a pneumatic actuator system. The introduced control approach proposes an efficient and simple solution for practitioners seeking a general method to control minimum phase, nonaffine-in-control systems.

Appendix

In this appendix, we explain how to obtain the inequality (17) which shows that the origin of the reduced system Σ_0 is exponentially stable. By substitution $\kappa = \dfrac{(t+t_0)}{2}$ into (16), the solution $z(t)$ can be written as

$$\|z(t)\| \leqslant k_z \left\| z\left(\frac{t+t_0}{2}\right)\right\| \exp\left(-\frac{\lambda_z (t-t_0)}{2}\right) + \sup_{\frac{t+t_0}{2} \leqslant \zeta \leqslant t} c_z \left|\sigma(\zeta)\right| . \tag{25}$$

To estimate $\left\|z\left(\frac{t+t_0}{2}\right)\right\|$ in (25), we substitute $\kappa = t_0$ and replace t by $\frac{t+t_0}{2}$ in (16) to obtain

$$\left\|z\left(\frac{t+t_0}{2}\right)\right\| \leqslant k_z \left\|z\left(t_0\right)\right\| \exp\left(-\frac{\lambda_z\left(t-t_0\right)}{2}\right) + \sup_{t_0\leqslant\zeta\leqslant\frac{t+t_0}{2}} c_z\left|\sigma(\zeta)\right|. \tag{26}$$

From (15), we can deduce the following inequalities

$$\sup_{t_0\leqslant\zeta\leqslant\frac{t+t_0}{2}} c_z\left|\sigma(\zeta)\right| \leqslant c_z k_\sigma \left|\sigma\left(t_0\right)\right|, \tag{27}$$

$$\sup_{\frac{t+t_0}{2}\leqslant\zeta\leqslant t} c_z\left|\sigma(\zeta)\right| \leqslant c_z k_\sigma \left|\sigma\left(t_0\right)\right| \exp\left(-\frac{\lambda_\sigma\left(t-t_0\right)}{2}\right). \tag{28}$$

Now, let us define $x_{\sigma z} = \left[\sigma^T, z^T\right]^T$ as the composite state. Using (25), we can write

$$\begin{aligned}\left\|x_{\sigma z}(t)\right\| &\leqslant \left|\sigma(\zeta)\right| + \left\|z(t)\right\| \\ &\leqslant \left|\sigma(\zeta)\right| + k_z\left\|z\left(\frac{t+t_0}{2}\right)\right\|\exp\left(-\frac{\lambda_z\left(t-t_0\right)}{2}\right) + \sup_{\frac{t+t_0}{2}\leqslant\zeta\leqslant t} c_z\left|\sigma(\zeta)\right|.\end{aligned} \tag{29}$$

Substitution of (15) and (28) into (29), and using the fact that $\exp\left(-\left|a\right|\right) \leqslant \exp\left(-\frac{|a|}{2}\right)$ for all $a \in \mathbb{R}$, we obtain

$$\begin{aligned}\left\|x_{\sigma z}(t)\right\| \leqslant{}& \left(1+c_z\right) k_\sigma \left|\sigma\left(t_0\right)\right| \exp\left(-\frac{\lambda_\sigma\left(t-t_0\right)}{2}\right) \\ &+ k_z\left\|z\left(\frac{t+t_0}{2}\right)\right\| \exp\left(-\frac{\lambda_z\left(t-t_0\right)}{2}\right)\end{aligned} \tag{30}$$

using the inequality (26), we have

$$\begin{aligned}\left\|x_{\sigma z}(t)\right\| \leqslant{}& \left(1+c_z\right) k_\sigma \left|\sigma\left(t_0\right)\right| \exp\left(-\frac{\lambda_\sigma\left(t-t_0\right)}{2}\right) \\ &+ k_z \exp\left(-\frac{\lambda_z\left(t-t_0\right)}{2}\right)\left(\sup_{t_0\leqslant\zeta\leqslant\frac{t+t_0}{2}} c_z\left|\sigma(\zeta)\right| + k_z\left\|z\left(t_0\right)\right\|\exp\left(-\frac{\lambda_z\left(t-t_0\right)}{2}\right)\right)\end{aligned}$$

and from (27) we find the following inequality

$$\begin{aligned}\left\|x_{\sigma z}(t)\right\| \leqslant{}& \left(1+c_z\right) k_\sigma \left\|\sigma\left(t_0\right)\right\| \exp\left(-\frac{\lambda_\sigma\left(t-t_0\right)}{2}\right) \\ &+ c_z k_\sigma k_z \left|\sigma\left(t_0\right)\right| \exp\left(-\frac{\lambda_\sigma\left(t-t_0\right)}{2}\right) + k_z^2\left\|z\left(t_0\right)\right\| \exp\left(-\frac{\lambda_z\left(t-t_0\right)}{2}\right).\end{aligned}$$

Choose $\lambda_{\sigma z} = \frac{1}{2}\min\{\lambda_\sigma, \lambda_z\}$, and using the facts that $|\sigma(t_0)| \leqslant \|x_{\sigma z}(t_0)\|$ and $\|z(t_0)\| \leqslant \|x_{\sigma z}(t_0)\|$, we obtain finally (17).

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