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EXPERIMENTAL INVESTIGATIONS OF THE HYDROELASTICITY OF A LIQUID-FILLED TANK

The article presents the results of experimental investigations of interactions between a deformable structure and a liquid. The investigations were performed on two prismatic tanks with elastically deformable top walls. During the investigations, different levels of tank filling with liquid were examined.

The investigation of this phenomenon has direct reference to frequently recorded real events, such as collision of a tanker with another ship or a harbour berth, rapid braking of a road or rail tanker, etc. Recognition of this phenomenon is based on simultaneous measurements of the following parameters:

1. excited accelerations of the tank-liquid system,
2. elastic accelerations of top walls of the tank,
3. hydrodynamic pressures on the deformable top walls.

Notation

a, b, 1 – tank dimensions
D – flexural stiffness of the plate
E – Young Modulus
f_1, f_2 – pressure fluctation frequencies
g – gravitation
h – tank filling height
k – spring constants
m_p – mass of the front plate
$m_{cz,p}$ – mass of the acceleration sensor
$m_{cz,c}$ – mass of the pressure sensor
m_w – mass of water in the tank
m_o – mass of the casing

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t – time
T_1, T_2 – fluctuation periods
W – horizontal deflection of the tank
γ – numerical coefficient depending on tank dimensions: a, h, 1
ν – Poisson ratio
λ_1, λ_2 – matrix eigenvalues
ΔS – deformation of springs

1. Introduction

The article aims at determining the interaction of water with an elastically deformable wall in a limited-size tank having the shape of a rectangular prism. To preserve uniqueness and make analytical simplifications possible, the tank used in the experiment had four non-deformable lengthwise walls (parallel to the longitudinal axis of the tank), and two top walls (perpendicular to the longitudinal axis) which can undergo elastic deformations. The tank, filled with water to a variable level, was exposed to acceleration acting along the longitudinal axis.

The research rig had the form of a pendulum on which the tank dynamics was examined. Two tanks of the same length but different cross sections were alternatively used. Each tank was equipped with two different top walls. Different levels of filling were used, including: empty and full tank, and three intermediate stages.

For each case of the measuring system – defined by the type of tank, level of filling, and type of deformable wall – five different levels of horizontal acceleration acting along the longitudinal axis of the tank were used.

2. Model investigations

The examined phenomenon refers to possible practical events, such as collision of a tanker with a harbour wharf, another ship, iceberg, or a drilling platform, as well as rapid braking of a cistern or tank car, etc.

The object of examination will be the tank exposed to hydrodynamic strokes acting on its walls, in the direction perpendicular to the acceleration vector. The top walls are not, in fact, infinitely rigid and reveal ability to deform elastically, which makes them vibrate as a consequence of the action of the starting impulse. Certain volume of the liquid takes part in the vibration, as the added mass, together with the vibrating wall. In case of partial filling, an additional effect is the motion of the liquid with respect to the tank. All this interaction determines the distribution of hydrodynamic pressures acting on the elastic tank walls. The phenomenon takes its course at the presence of mutual couplings of characteristics. Its identification requires simultaneous measurements of the following quantities:

- forced accelerations of the tank-liquid system,
- accelerations of the deformable top walls of the tank,
- hydrodynamic pressures acting on the deformable tank walls.

The following methodology of measurements was adopted. According to Fig. 1 the measuring system consists, in general, of the tank treated as the pendulum mass, and elastic constraints (springs) with elasticity k.

One end of each spring is fixed (point S) to the theoretically infinitely rigid construction, while the other end (point C) is connected via a flexible and, by definition, inextensible connector (steel cord) with the tank (point D). When the pendulum B is in a static position, the length of the connector and that of the line segment CD are the same, and the points C and D are on the same level. The initial conditions of the measurement are given by the deflected position A (Fig. 1) of the tank, defined by the horizontal displacement "W". At this position the connector hangs down between the deflected tank and the end C of the springs. The tank moves from its starting position A towards position B according to the principles of pendulum motions. When the system reaches point B (Fig. 1) the connector takes a horizontal position and further motion towards point B' takes place at definite action of elastic forces generated by the deformed constraints (stroke projected onto the strings).

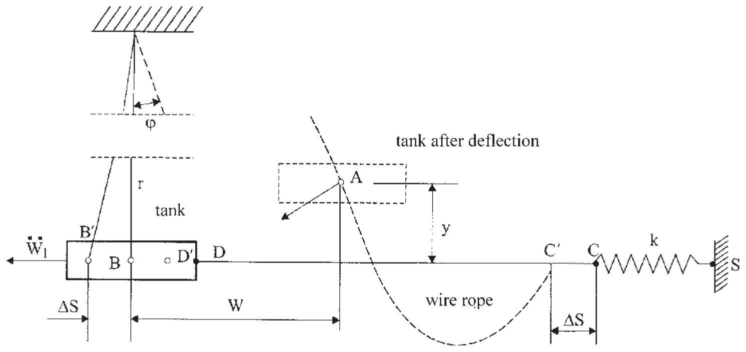

Fig. 1. Sketch of the measuring system suspended on ropes

The position B' is determined by the maximum spring deflection ΔS, the range of which results from the conversion of kinetic energy of the tank at point B into the work done by elastic forces of the deformed constraints on the way ΔS. The return movement from point B' to the position B is the consequence of the conversion of potential energy stored in the sprigs into the kinetic energy of the system. During the return motion from point *B* to point *A* (Fig. 1) the connector separates from the springs and the further motion of the tank takes place according to the principles of pendulum motion. The time interval in which the system is exposed to rapid accelerations resulting from spring braking is connected with the tank motion along the way $B \rightarrow B' \rightarrow B$. The course of the phenomenon in this particular time interval is the main object of investigations.

Treating the measuring system as an ideal pendulum and assuming that the elasticities of the constraints are linear (deflections proportional to forces), the following elementary relations can be written to describe geometric, kinematic, and dynamic parameters of the system:

– vertical elevation (ordinate) "y" of the system in the initial position *A* with respect to the lowest position *B* (Fig. 1):

$$y = r - \sqrt{r^2 - W^2} \tag{1}$$

– velocity $\mathrm{v_B}$ in the lowest position *B* after starting from point *A*:

$$\mathrm{v_B} = \sqrt{2gy} \tag{2}$$

where: g is the gravitational acceleration,

– relation between the elasticity coefficient k of the constraint, elevation y (deflection W), elastic deflection ΔS of the constraint, and mass m of the system:

$$k(\Delta\, S)^2 = 2mgy \tag{3}$$

– frequency ω_s, of system vibration in horizontal direction under the action of springs:

$$\omega_s = \frac{2\pi}{T_s} = \sqrt{\frac{g}{r} + \frac{k}{m}} \tag{4}$$

– horizontal force P in elastic constraints:

$$P = \sqrt{km}\ \mathrm{v}_B \sin \omega_s t \tag{5}$$

where t is time,

– maximum force P_{max}:

$$P_{\max} = \sqrt{km}\ \mathrm{v}_B = k\Delta S \tag{6}$$

– horizontal accelerations $\ddot{W}_1$ of the system

$$\ddot{W}_1 = -\mathrm{v}_B \omega_s \sin \omega_s t \tag{7}$$

– maximum accelerations $\ddot{W}_{1\max}$:

$$\ddot{W}_{1\max} = -\mathrm{v}_B\ \omega_s \tag{8}$$

It results from the abovenamed relations that for the assumed system parameters (mass m, length r) the expected accelerations are obtained by selecting the horizontal deflection W and springs with relevant elasticity coefficients k. The system of braking transmission from the springs via the connector fixed in a controlled way to the rear end of the tank eliminates additional (undesired) rotations of the system in the horizontal plane.

It is worth mentioning that the real measuring system differs to some extent from the idealised pendulum: the mass of the system is not concentrated at one point, elasticity is revealed not only by the springs, etc. That was why, from the methodological point of view, the above-presented measuring system underwent dynamic identification in the following aspect:

– detecting free vibrations of the deformable tank walls with the installed instruments and determining the effect of the vibrations of constructional elements of the system on those walls. This detection was done by applying a classical impulse (stroke) to a freely suspended tank at rest, empty or filled with water to an arbitrary level. The detection referring to the empty tank can be treated as dynamic identification of the measuring system as the mechanical system, and makes the background for similar identification of the tank filled with liquid – in the conditions of contact of the deformable wall with the liquid (harmonized, added mass of water),

– detecting dynamic behaviour of the measuring elements when the empty tank is braked by the springs (additional elasticities accompanying those represented by springs, vibrations of all system components, etc.). This detection, in turn, can be treated as the identification background for the basic measurements of interaction between the liquid and the deformable walls of the tank exposed to rapid accelerations (spring braking).

The above-described methodology of research was taken into account when deciding on constructional solutions of the research rig and the course of realisation of the basic part of the investigations.

3. Research rig

A general scheme of the research rig is given in Fig. 2

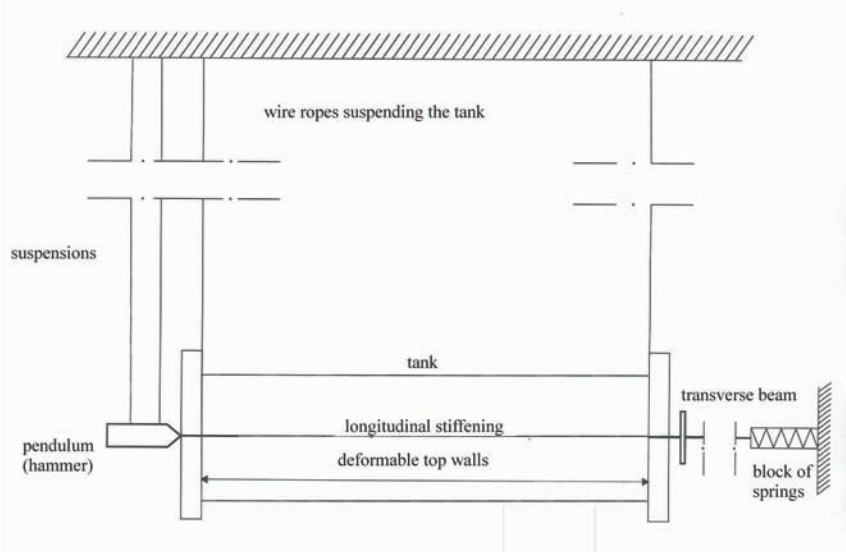

Fig. 2. General scheme of the research rig

The longitudinal section of basic components of the rig with the installed measuring instruments and attributed registration channels (paths) are shown in Fig. 3.

In general, the research rig consisted of the following three groups of components:
– tank with the suspension system and the installed instrumentation,
– spring braking system (generation of accelerations),
– pendulum-hammer.

The rigid construction of the thick-wall tank had the form of a cylindrical segment of rectangular cross-section with two caps (front and rear, Fig. 2 and 3). These elements were made of plastic (epoxide resin, glass reinforcement). The caps were connected with the tank using flanges. Thin-wall steel plates, playing the role of deformable walls, were mounted between the main part of the tank and the caps. Precisely flat surfaces of the flanges secured tightness and good support for the fixed plates around the entire perimeter. A space designed for filling with water was between the rigid tank walls and the deformable end plates. An accelerometer and 2 pressure sensors were installed at the central point of each plate (Fig. 3). Playing the role of the base for the mounted plates, the caps also protected the sensors installed on them. Other roles played by the caps were the following :
– the rear cap was the base for the accelerometer that measured the acceleration of the entire system (tank with water),
– the front cap took over (like an anvil) the impact generated by the pendulum-hammer to initiate the measured free vibrations in the system.

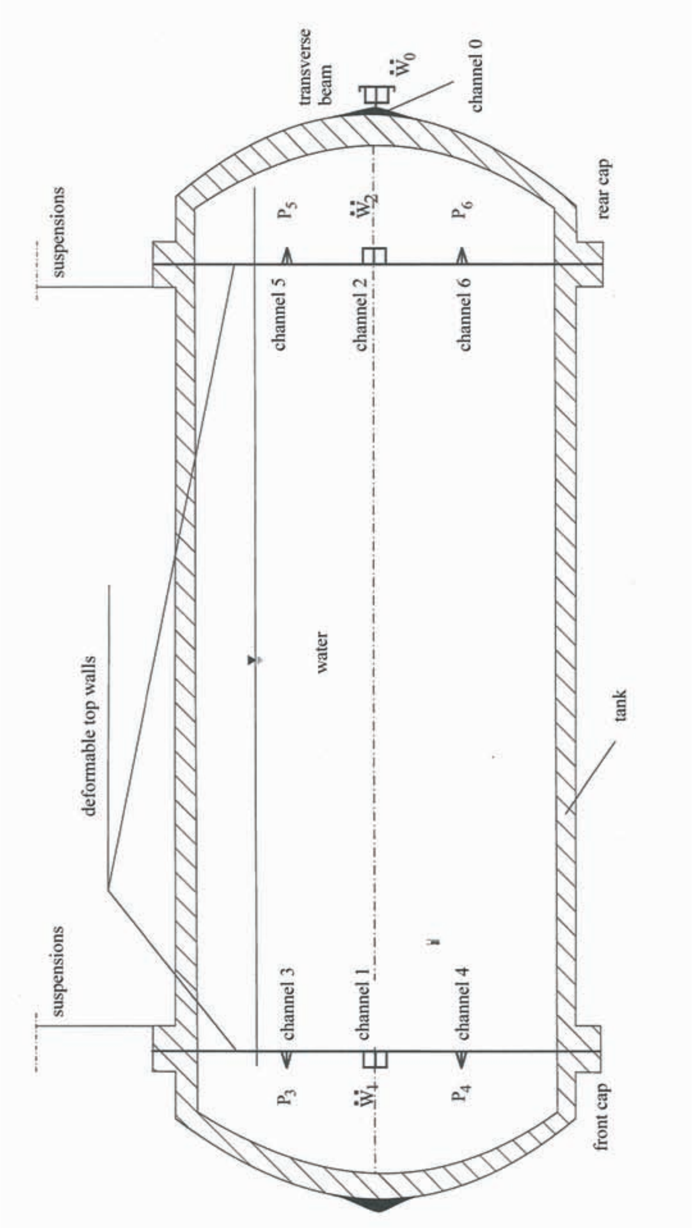

Fig. 3. Longitudinal section of the tank with measuring equipment components, and acceleration and pressure registration channels

The vertical suspension system of the tank consisted of 4 hangers fixed to tank flanges and 4 flexible steel cords (Fig. 2 and 3) of 6.8 m in length and 2 mm in diameter. The ends of the cords haved the form of threaded rods for adjusting purposes. The above suspension lengths are were not identical with the calculated length of an idealised pendulum. The measured periods of free oscillations of the tank were equal to 5.16 ÷ 5.17 s, which corresponds to the calculated (virtual) pendulum length 6.62 ÷ 6.65 m (6.65 was assumed).

The system used for generating the acceleration by rapid braking of the moving tank treated as the pendulum consisted of (Figs. 2 and 3):

– a rigid resistance beam with a set of elements for fixing springs (adjusting screw, crossbars, etc.),
– two longitudinal bars and a transverse beam, mounted on the tank,
– flexible connector (steel cord 3.5 mm) with articulated ends, to transmit forces of spring braking onto the tank via the transverse beam and longitudinal bars.

The scheme of transmission of spring braking forces from the flexible connector to the transverse beam and then via the longitudinal bars to the front flange takes into account the need to elimination of tensile stresses in the tank and to avoid its unsealing during the braking.

When complementing the above description it is worth repeating that two tanks of different dimensions were the object of investigations. The inner dimensions of these tanks were the following:

– large tank: cross section 20 × 40 cm, length 105 cm,
– small tank: cross section 10 × 30 cm, length 105 cm.

Each tank was equipped with two types of deformable end walls:

– made of steel plates of 0,8 mm in thickness,
– made of phosphor-bronze plates of 1,0 mm in thickness.

These plates revealed different elasticities, due to different materials used and plate thickness.

Summing up the description of the research rig, it we should be stressed that the rig was designed and machined in the way securing clear differentiation of the frequencies representing:

– changes in motion of the tank as a free pendulum,
– vibrations of the tank-springs system during rapid braking,
– free vibrations of deformable tank walls at different levels of its filling with water.

Without this differentiation, clear decomposition of the recorded resultant vibrations, of high complexity, into individual components describing the examined phenomenon would be extremely difficult, or even impossible.

4. Description of investigations

In each of four examined cases, five levels of tank filling with water were used, which were (Fig. 4): $h = 0$ (empty), $h = b$ (full),
$h = b - 5$ mm, $h = 0.75b$ and $h = 0.5b$.

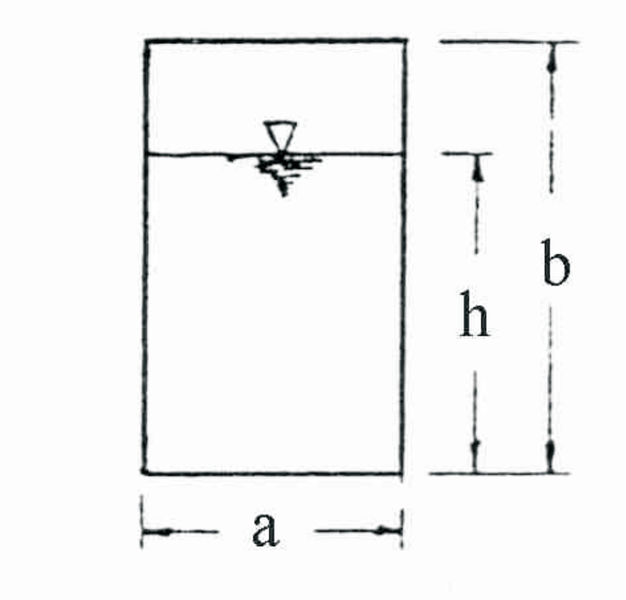

Fig. 4. Symbols used for defining the filling status in the tank cross-section

The size of the tank, type of the end plate, and the filling status define the measuring system and are characteristic discriminants of each individual series of measurements. The range of investigations covered 20 measurement series. The measuring system attributed to each series was the object of two-fold investigation procedure:
– the pendulum-hammer rapidly struck the front cap of the freely suspended tank; the stroke was directed along the longitudinal axis of the tank,
– the system was first started to the oscillating motion from the deflected static position (Fig. 1 deflection W) and then was exposed to rapid acceleration after braking the tank by the springs. The braking took place after the time corresponding to one fourth of the pendulum period counted from the start.

The strokes done by the pendulum-hammer aimed at recognising free vibrations in the measuring system. This recognition, done on the empty tank, determined dynamic characteristics of the mechanical system and created the background for determining dynamic interaction of the water with the vibrating deformable walls of the tank at rest. At the same time, the strokes done to the tank filled with water provided the dynamic background for analysing the interaction of the tank water with the deformable walls in rapid acceleration (spring braking) conditions. The same pendulum-hammer was used in all investigations. Its weight was 49,5 N. The range of the striking impulse projected to the tank was defined by the starting static deflection "u" of the pendulum-hammer with respect to the tank at rest. As a rule, two deflections of $u = 20$ cm and $u = 30$ cm were used.

During the basic investigations of the interaction of the water with the deformable walls in the conditions of rapid tank braking, a principle was adopted to conduct 5 experiments within the framework of each of the above-mentioned 20 measurement series. A discriminant of the experiment is the regime of horizontal accelerations applied to the tank during its braking. In each series the regime covered 5 different tank accelerations ranging from about 1 g to about 5 g (g – gravitational acceleration). The limitations in the applied accelerations mainly result from measuring ranges of the used accelerometers; beyond 10 g indications of the accelerometers can be more and more nonlinear, although the value for which the sensor is expected to be damaged is as big as 1000 g. A principle was preserved for the measured accelerations, being the superposition of tank acceleration and free vibrations, not to exceed $10g$. The assumed acceleration was obtained by selecting a set of braking springs and the value of initial static tank deflection (Fig. 1). Different sets of springs (up to 5 pieces) were used. The elasticity coefficients"k", experimentally determined for each spring, were assumed constant for linear elongations within the range of elastic deformations of the spring. The initial starting deflection "W" of the tank was applied using a thin steel cord fixed to the tank and equipped with a stopping slider. The role of the slider was to measure precisely the required distance "W" on the cord and to connect the cord to the resistance beam in order to keep the tank in the deflected position. Releasing the slider from the catch was the beginning of tank motion as the pendulum.

To sum up, one measurement series consisted of 6 experiments: one stroke with the pendulum-hammer and 5 tank braking experiments at different accelerations. 20 measurement series collected 120 experiments, in total.

The measured quantities were recorded using the numerical a digital technique. A simplified block diagram of the measuring and recording system is given in Fig. 5. The acceleration of the tank and its two deformable walls was measured using accelerometers and transmission paths produced by Endevco, USA, and used in outer space technology. The hydraulic pressure measurements on the deformable walls (2 points per wall) were done using sensors and transmission paths produced by Peltron, a Polish company, which made use of Swiss subassemblies. Before starting the measurements, the pressure sensors with their paths (channels) were tested in overpressure and underpressure conditions. The recorded signals were transmitted to the PC/AT computer equipped with a special measuring card of a maximum total sampling frequency (from all channels) equal to 8000 Hz.

5. Measurement results

In order to obtain experimental material of satisfying volume and accuracy, each of 120 experiments was repeater four times using the following configurations of recording paths with sampling frequencies in each path attributed to them:

- configuration A: simultaneous measurement of tank accelerations and front plate parameters (acceleration, pressure at 2 points), 4 paths in total, frequency – 2000 Hz per path,
- configuration B: simultaneous measurement of tank accelerations and rear plate parameters (acceleration, pressure at 2 points), 4 paths in total, frequency – 2000 Hz per path,
- configuration C: simultaneous measurement of all quantities, i.e. tank accelerations and 6 parameters on both end plates (2 acceleration paths and 4 pressure paths) 7 paths in total, frequency – 1000 Hz per path,
- configuration CP: repetition of the experiment with the recording configuration C.

The above recording procedure has made it possible to use full potential of the measuring instruments, at the same time taking into account the requirements resulting from the expected frequencies of changes of the measured quantities. As a result of 120 experiments, each repeated four times, over 470 data files were obtained (some CP configurations were omitted) which were then stored on disquettes. The data collected in the files describe physical properties of the measured quantities (acceleration in m/s^2, pressure in kPa). For easy identification of the measuring system type and its dynamics and recording configuration, the following notation of the data file names was used (the 8-character name limit):

- M or D – tank: small M (10 × 30 cm) or large D (20 × 40 cm),
- S or F – end walls: steel S or phosphor-bronze F,
- U or H – stroke: by pendulum-hammer U or spring braking H,
- P or W – tank: empty P or filled with water W,
- numbers from 1 to 21 correspond to the following data set: tank filling "h", initial tank deflection "W" or initial pendulum-hammer deflection "u", aggregate elasticity "k" of the set of braking springs. The tank size and the parameters "h", "W" and "k" are in some relation with the maximum accelerations $\ddot{W}_{1\max}$ recorded in the spring braking process,
- A or B or C – recording configuration,
- P – repeated experiment and recording (configuration C).

Sample names of data files:

- MFUW2A,
- DSHW19CP.

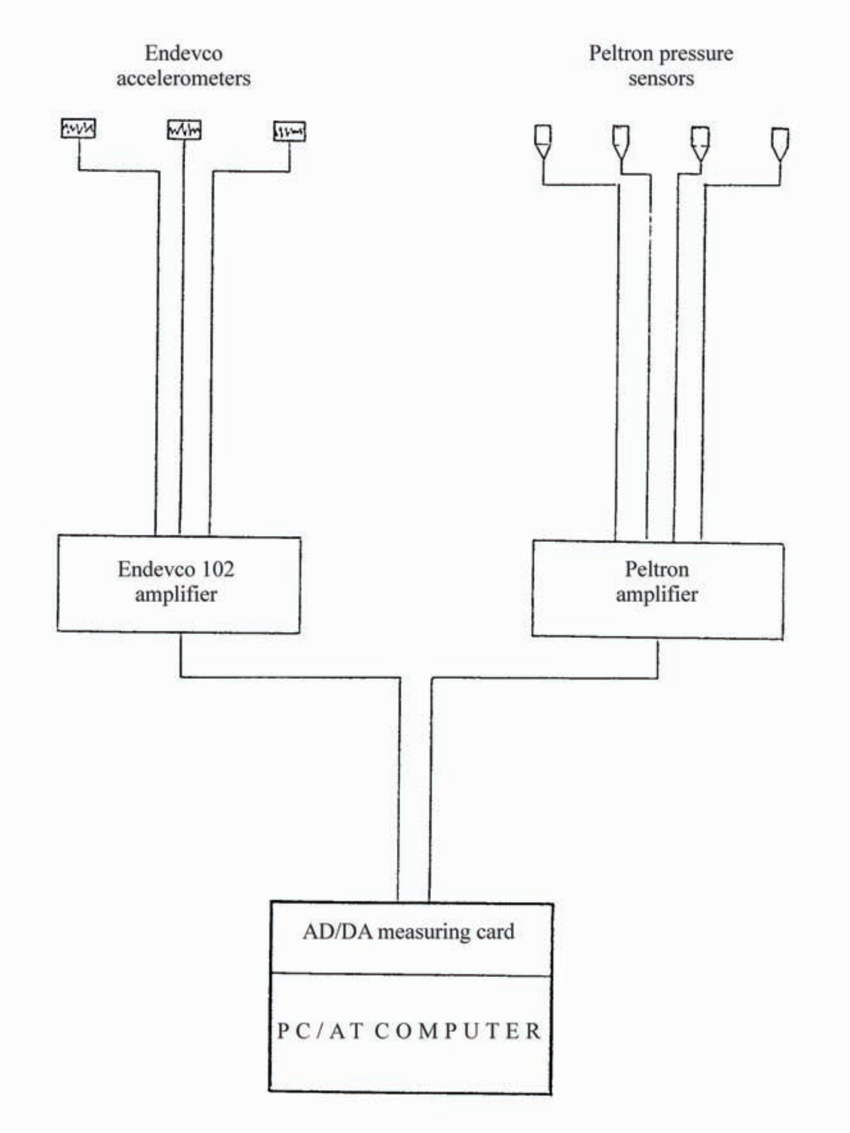

Fig. 5. Simplified block diagram of the measuring and recording equipment

Sample time-histories of the measured quantities are shown in Fig. 6. They represent the recording configuration C, i.e. simultaneous recording of all 7 quantities during one experiment. From top to bottom, the presented curves describe:

– channel 0 – accelerations of the tank construction filled with water,
– channel 1 – accelerations of the front tank plate,
– channel 2 – accelerations of the rear tank plate,
– channel 3 – pressures recorded by the upper sensor on the front plate,

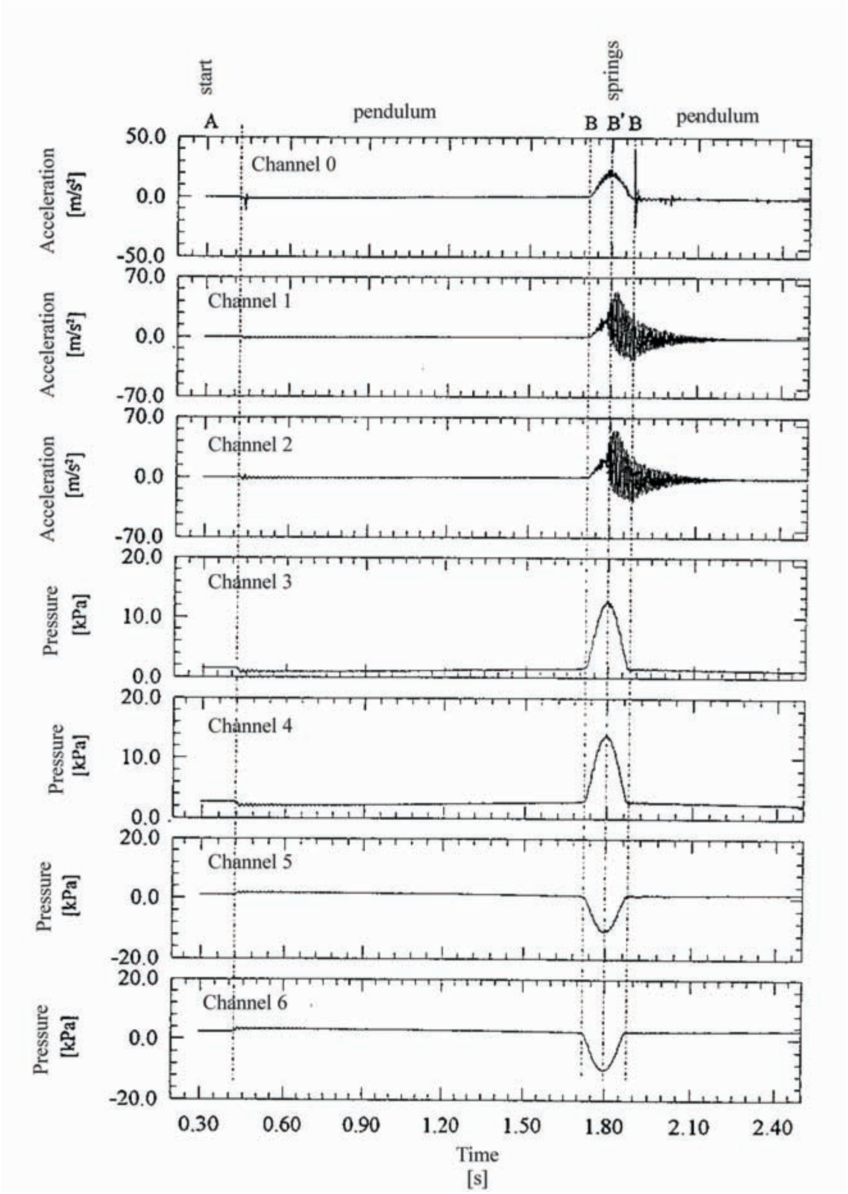

Fig. 6. Sample time-histories of the measured quantities (tank filled with water, file MSHW3C)

– channel 4 – pressures recorded by the lower sensor on the front plate,
– channel 5 – pressures recorded by the upper sensor on the rear plate,
– channel 6 – pressures recorded by the lower sensor on the rear plate.

The time-histories shown in Fig. 6 refer to the scheme of the measuring system shown in Fig. 1 (tank positions *A*, *B* and *B*′). In all diagrams the beginning instant of the tank motion as the pendulum starting from the deflected static position A is visible. The beginning of the motion is accompanied by the presence of accelerations, especially noticeable on the rigid tank casing (channel 0). When the tank, starting from position A, reached position B, it crashes into the springs and is rapidly braked during the time $B \rightarrow B'$. Position B' corresponds to the maximum deflection of the springs and the extreme values of dynamic parameters. The line segment $B' \rightarrow B$ represents the time when the tank is pushed back by the springs. In the return motion, starting from the time when it reaches the position B for the second time, the tank looses contact with the springs and becomes the pendulum again. The dynamic interactions observed in the system are again of free vibration nature. In the presented time-histories especially visible are free vibrations of the deformable tank walls in contact with water (channels 1 and 2).

In Fig. 6, the line segment AB corresponds to one fourth of the period in the tank motion, while the line segment $B' \rightarrow B \rightarrow B$ represents the tank motion controlled by the springs. The diagrams clearly show the differences in time intervals of pendulum-like tank motion, oscillatory spring-controlled tank motion, and free vibrations of the deformable walls in contact with water. This clear differentiation of particular intervals allows easy decomposition of the recorded time-histories into individual components. It is also noteworthy (Fig. 6) that in conditions of rapid system braking the appearance of overpressures on the front wall (channels 3 and 4) is accompanied by the appearance of corresponding underpressures on the rear wall (channels 5 and 6).

6. Selected results of investigations

The range of the performed laboratory tests covers a number of dynamic system configurations and variable parameters, including: tank size (small or large tank), parameters of braking springs, level of tank filling with water (tank empty, partially filled or fully filled) and the range of tank deflection from the equilibrium position. Additionally, the tests were performed in which the tank at rest was struck with a hammer.

The designed research rig made it possible to separate four phases (stages) of the course of the phenomenon. This has its reflection in the obtained records of investigation results, which were also divided into four stages. Stage 1 includes the time interval from the release of the deflected pendulum to tank coupling with the braking spring. In this phase the pendulum motion and plate vibrations with respect to the casing are observed. In phase 2 the entire system is exposed to rapid braking. Of high importance in the description of this phase of motion are the forces in the braking springs. The springs used in the experiments were selected in such a way that the free vibration frequencies of the relative motion of the plate were much higher than those of the braking system. This provided opportunities for separating (decomposing) the effect of particular components on the dynamics of the entire system. This decomposition was done applying the Kalman filtration to the recorded vibration time-histories. The braking springs were released in the intermediate phase 3. In phase 4, the vibrations of the tank suspended on flexible connectors, without contact with the braking springs, were recorded. This phase includes the damping of plate vibration as a clear dominating form. For the tank partially filled with water, the appearance of frequencies connected with undulation of the liquid free surface can be expected.

To illustrate the recorded measurement data, below presented are those obtained for the dynamic system configuration labelled as DSHW3A (large tank braked by springs and fully filled with water). To identify the free vibration frequency of the front plate, the test consisting in striking the empty tank with the hammer was performed. The obtained record of plate acceleration is given in Fig. 7, along with its energy spectrum.

To identify the parameters of the elastic plate, numerical test calculations of its vibrations were performed using FEM. The free vibration energy spectrum and vibration modes were calculated. The first five free vibration frequencies for this plate are the following:

$$\begin{array}{ll} f_1 = 71,4Hz, & f_4 = 280,4Hz \\ f_2 = 81,3Hz, & f_5 = 312,7Hz \\ f_3 = 108,1Hz & \end{array} \tag{9}$$

For the vibrations of a plate being in contact with water this frequency spectrum will be moved down. The basic part of the experiment was performed for the tank fully filled with water. The quantities of interest which were recorded in the experiment included the accelerations of the centre of the elastic plate, and pressure changes at two selected points on the plate. A record of these quantities is shown in Fig. 8.

The recorded time changes of the abovenamed quantities reveal that the examined problem involves system vibrations of many freedom degrees of freedom. In order to separate individual quantities, the time-histories shown in Fig. 8 were filtered, after which the components corresponding to particular system vibration frequencies were separated. Selected results of the decomposition are shown in Figs. 9 and 10, in the form of vibration components for the selected time interval (stage 2 and stage 4).

It is noteworthy that the amplitude range for these two components is the same for all frequencies, which does not correspond directly with their effect on plate displacements. During the acceleration recording process,

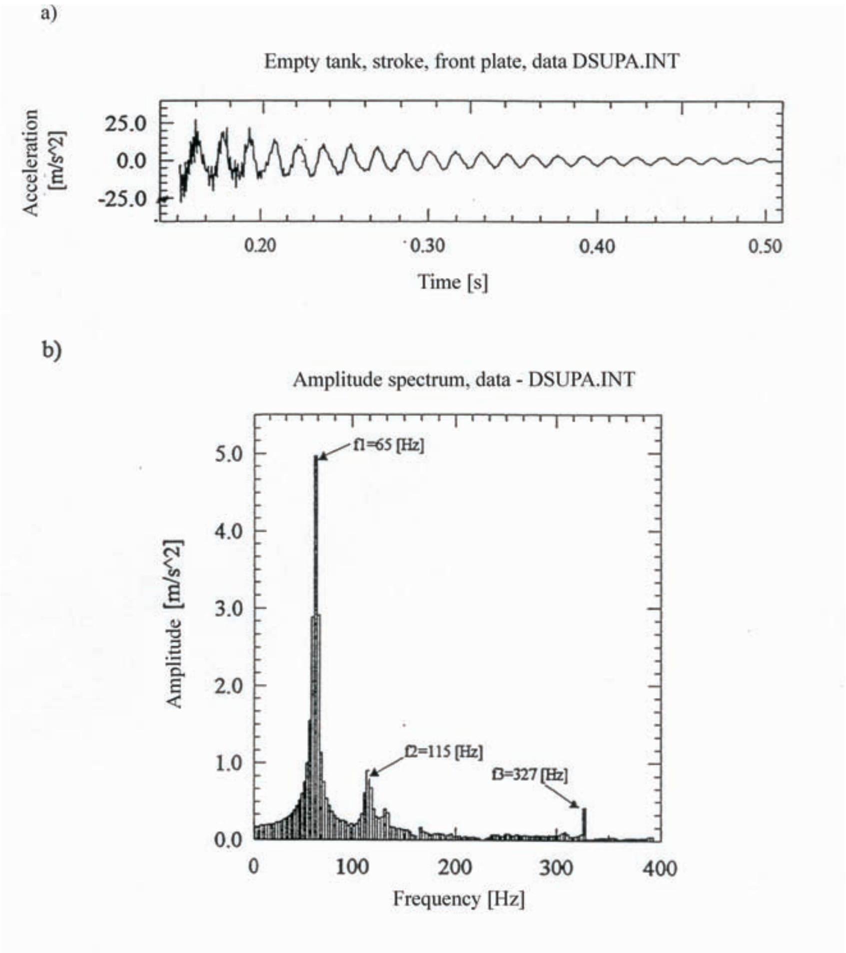

Fig. 7. Records of plate centre acceleration (a) and amplitude spectrum of its vibrations (b)

double-frequency displacement components are amplified four times (acceleration increases with the square of frequency). The recorded pressure time-history (Fig. 8) is much smoother that the acceleration curve. Its decomposition is shown in Fig. 11.

Since the pressure sensors were installed at one fourth of the plate height, filtrating the pressure record does not have to lead to the components identical to the acceleration components. Some free vibration modes of the plate have nodes located close to the pressure sensor, while other vibration modes have nodes close to the position of the acceleration sensor.

To identify the model used in the experimental investigations, below presented are numerical values of the calculation model described as the simplified dynamic model with two freedom degrees. Numerical parameters of the model used in the experimental investigations are the following:

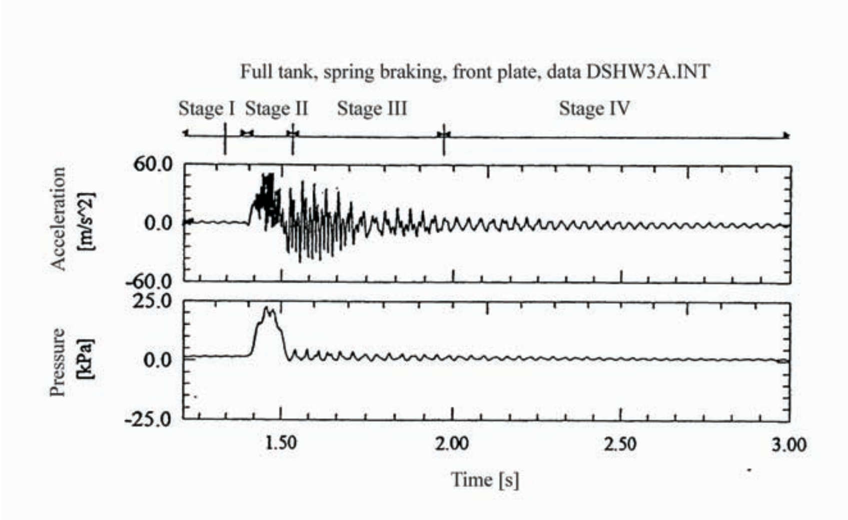

Fig. 8. Records of elastic plate acceleration and water pressure at plate centre

$$\begin{gathered} m_p = 0,504kg, \quad m_{cz.c} = 0,132kg, \quad m_{cz.p} = 0,067kg \\ m_w = 84,0kg, \quad m_0 = 62,13kg, \quad \gamma = 0,13806 \\ D = \frac{E\delta^3}{12(1-\nu^2)} = 9,61172Nm \end{gathered} \tag{10}$$

The virtual spring constant of the plate is equal to:

$$k_0 = \frac{D}{0,00722a^a} = 33,281N/m \tag{11}$$

The same constant determined from the plate deformation potential energy is equal to:

$$k_0 = 43,156N/m \tag{12}$$

The virtual pendulum constants is:

$$\begin{gathered} --\text{for empty tank}: \quad k_1 = 89,411N/m \\ --\text{for full tank}: \quad k_1 = 207,13N/m \end{gathered} \tag{13}$$

Since the acceleration and pressure sensors are fixed to the model plate via flanges, the bending stiffness of the plate is larger than that determined using formula (10), therefore from now on the constant (11) will be used.

Firstly, let us consider the first phase of motion and determine vibration frequencies for the system working without contact with the spring k_1.

A) Empty tank

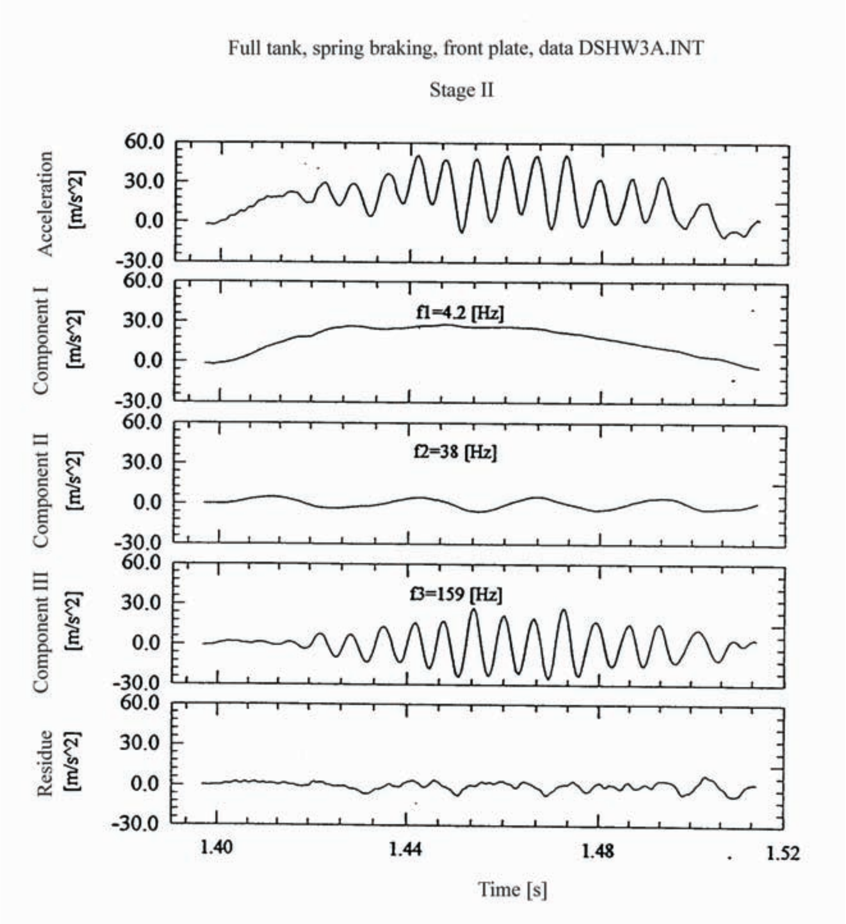

Fig. 9. Decomposition of accelerations into components corresponding to system free vibration frequencies in motion phase II

The total mass of the system is equal to $m_\Sigma = 63,80$ kg. The virtual spring constants are equal to:

$$\begin{aligned} k_0^* &= 4k_0 = 172,625 N/m \\ k_1^* &= 2k_1 = 178,822 N/m \end{aligned} \tag{14}$$

The mass matrix elements are:

$$\begin{aligned} m_{11} &= 0,8155 kg \\ m_{12} &= 1,3000 kg \\ m_{22} &= 127,600 kg \end{aligned} \tag{15}$$

The calculated results are the following:

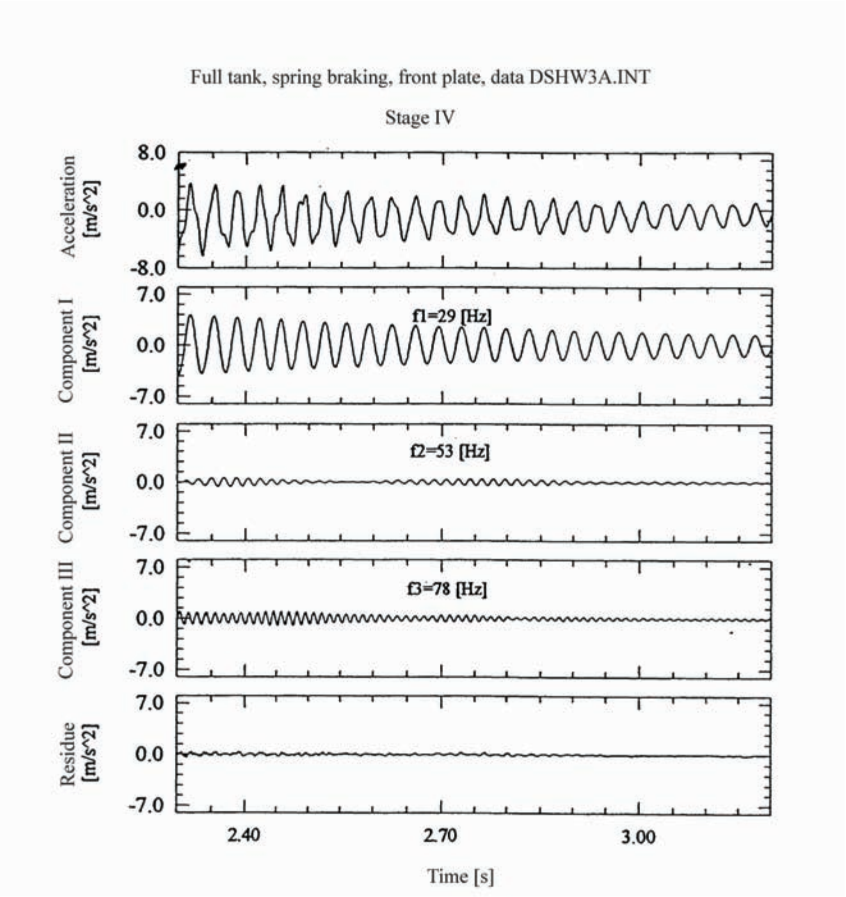

Fig. 10. Decomposition of accelerations into components corresponding to system free vibration frequencies in motion phase IV

$$\begin{aligned} \lambda_1 &= 0,71356 \quad \lambda_2 = 4,6474 \cdot 10^{-6} \\ f_1 &= 0,18841Hz \quad f_2 = 73,8269Hz \\ T_1 &= 5,30755s \quad T_2 = 0,01355s \end{aligned} \tag{16}$$

B) Tank filled with water

The virtual spring constant is equal to:

$$k_1^* = 2k_1 = 414,26229N/m \tag{17}$$

The mass matrix elements are:

$$m_{11} = 12,412kg \quad m_{12} = 43,300kg \quad m_{22} = 295,600kg \tag{18}$$

The calculated results are the following:

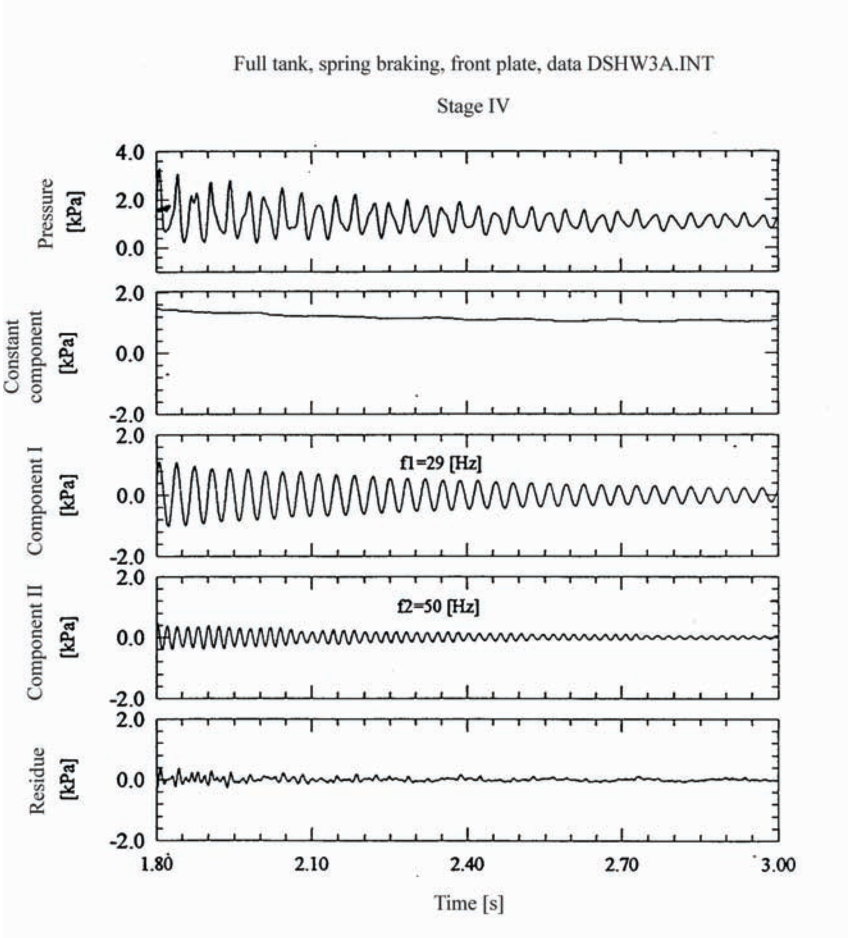

Fig. 11. Decomposition of the pressure record at the plate centre

$$\begin{gathered} \lambda_1 = 0,71359 \quad \lambda_2 = 3,51619 \cdot 10^{-5} \\ f_1 = 0,18841Hz \quad f_2 = 26,84009Hz \\ T_1 = 5,30769s \quad T_2 = 0,03726s \end{gathered} \tag{19}$$

Next let us consider the second phase of motion, during which the tank comes into contact with the spring k_1 (see Fig. 6).

A) Empty tank

The virtual spring constants are equal to (large tank –DSHW3A experiment):

$$\begin{aligned} k_1^* &= 400,819N/m \\ k_0^* &= 172,624N/m \end{aligned} \tag{20}$$

The mass matrix elements are given by formulas (15). The calculated results are the following:

$$\begin{array}{ll} \lambda_1 = 3,18426 \cdot 10^{-4} & \lambda_2 = 4,64626 \cdot 10^{-6} \\ f_1 = 8,91899Hz & f_2 = 73,83601Hz \\ T_1 = 0,11212s & T_2 = 0,01354s \end{array} \tag{21}$$

B) Tank filled with water

The virtual spring constant is equal to:

$$k_1^* = 401,054N/m \tag{22}$$

The mass matrix elements are given by formulas (18). The calculated results are the following :

$$\begin{array}{ll} \lambda_1 = 7,75544 \cdot 10^{-4} & \lambda_2 = 3,34187 \cdot 10^{-5} \\ f_1 = 5,71501Hz & f_2 = 27,53123Hz \\ T_1 = 0,17498s & T_2 = 0,036322s \end{array} \tag{23}$$

Knowing initial tank deflection (DSHW3A) u = 0,85 m, let us calculate the system velocity when the first phase of motion ends (the lowest position). For this purpose we can treat the pendulum as the system with one freedom degree. The calculated horizontal component of the tank casing velocity is equal to:

$$\mathrm{v}_1 = -1,00625m/s \tag{24}$$

It results from the theoretical analysis that the amplitudes of the plate surface centre acceleration components are of an order of $5g$, i.e. five times as large as the gravitational acceleration.

For the tank partially filled with water, the nature of the phenomenon is different due to the presence of the liquid free surface. To illustrate this case, Fig. 12 shows the acceleration time-history recorded when the tank filled with water up to the half of its height was braked.

The figure clearly shows the presence of the second stroke, after time $\cong 1.50s$ from the beginning of braking. Calculating the undulation frequency for this tank with the walls at rest returns a set of wave numbers:

$$k_r = r\frac{\pi}{l}, \quad r = 1, 2, ... \tag{25}$$

For $r = 1$ we get the lowest undulation frequency, which corresponds to the interval:

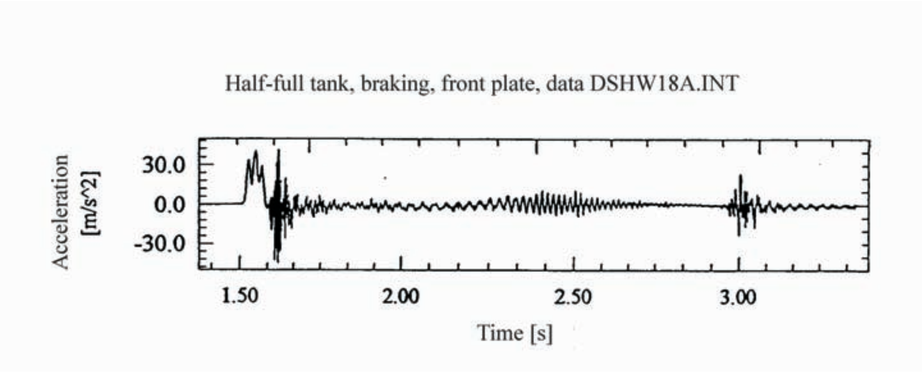

Fig. 12. Braking the tank partially filled with water

$$T_1 = 1.5842s \tag{26}$$

This last result means that the stroke at time $t = 3s$ (see Fig. 12) is caused by the vibrations of the water free surface.

The above-presented approximate solution of the dynamics of a tank filled with water allows an order of magnitude of basic dynamic quantities to be determined. The amplitudes of plate acceleration and pressures acting on it change depending on the stiffness of the springs that brake the tank. Comparing the calculated results with those obtained in the experiment reveals that the basic free vibration frequencies of the plate differ by less than 10%. The accuracy of the calculation model can be increased by replacing the physical system by that with a bigger number of freedom degrees.

7. Conclusions resulting from the experimental investigations

1. Two tanks were used in the experiment. They had the same length of 105 cm, but different cross sections areas: 10 × 30 cm and 20 × 40 cm (outer dimensions). Each tank was equipped with two sets of end walls, treated as deformable. The first set was made of steel plates of 0.8 mm in thickness, while the second set was made of phosphor-bronze plates of 1.0 mm in thickness. Assuming that "b" is the height of the cross section and "h" is the water filling depth, filling of each tank was described by the following 5 states: $h = 0$ (empty), $h = 0.5b$, $h = b$-0.5 cm and $h = b$ (fully filled). The combinations of the above named system parameters (tank size, type of deformable walls, filling level) composed 20 measurement series. Each series included 6 experiments consisting of:
   - dynamic identification of the system (free vibrations of elements) done by applying a sudden impulse to the tank casing (stroke with a so so-called pendulum-hammer, Figs. 2 and 3),

   - applying 5 different accelerations, of an order between $1g$ and $5g$, to the system (g – gravitational acceleration).
2. The dynamics of the interaction was examined by measuring accelerations and pressures acting on the deformable end walls. On each wall (plate), the pressure was measured at two points distant by $0.25b$ from the lower and upper edge, while the acceleration was measured at the central point of the plate. Numerical Digital recording was carried our using 7 measurement paths (channels). Moreover, due to the nature of the phenomenon and high frequencies recorded in the system, each experiment was repeated four times using the following recording configurations, and recording frequencies attributed to them:
   A – tank accelerations and front plate data, 4 measuring channels (paths), 2000 Hz per path,
   B – tank accelerations and rear plate data, 4 paths, 2000 Hz per path,
   C – measurements done in all 7 paths, 1000 Hz per path,
   CP – repetition of configuration C.
   As a result of the measurements performed in such a wide range (different tanks, deformable walls, filling levels, accelerations, free vibrations – strokes, recording configurations) about 480 data files were collected and recorded on disquettes, thus composing voluminous material that provided opportunities for comprehensive analysis of dynamic interaction of the water with the deformable walls in the tank.
3. A preliminary analysis of the recorded measurement data allows the following qualitative conclusions to be formulated:
   - during the appearance of accelerations (braking) in the system, underpressure is recorded on the rear wall,
   - when the tank is fully filled, the interaction of the liquid with the deformable wall reveals the nature of a model dynamic phenomenon,
   - when the tank is nearly full (an order of 98% of full filling) the interaction phenomena have a course without radical changes with respect to that recorded for the full tank,
   - for partial tank filling (75% and less) the course of the phenomenon becomes more complicated, to the extent that forces taking into account nonlinear interactions.
4. Stage 4 of the system motion with spring braking reveals the same qualitative nature as the system motion caused by a stroke done by a hammer, excluding a short time interval directly after the stroke. In the first, short time interval, of highest significance are wave phenomena combined with the appearance of extremely high vibration components. After this time, the system motion can be described using the linear theory of vibration, which was done in the above -presented approximate analysis. The pre-

liminary analysis of phase 2 (spring braking) reveals that the approximate description within the framework of the linear theory of vibration does not secure a complete description of the behaviour of the here examined dynamic system.

5. The constructed experimental models are not ideal. A real physical system is that of an infinite number of freedom degrees of freedom. When working out experimental models, a general principle is in force for the free vibration frequencies of particular model components to be distant as far as possible from each other. In the reported experiment this condition was fulfilled to a satisfactory extent. Free vibration frequencies of the elastic plates are beyond the range of those of the pendulum and braking springs. The collected experimental material provides opportunities for verification of the adopted calculation models. On the other hand, this material reflects the physical situation in which additional interactions are observed (the effect of flexible connectors of the pendulum, the effect of damping on the system dynamics). For the tank partially filled with water, an additional phenomenon, neglected in the present analysis, takes place which is the excitation of displacements of the free surface and mass of liquid in the tank. These displacements of the mass of water will generate extra load to the end walls, which can be a decisive factor in some ranges of the phenomenon. Filtrating the recorded data allows the effect of particular system components (plate, casing) on the dynamics of the entire system to be identified. The goal of the experimental investigations has been reached which, among other applications, provides opportunities for verification of other, more precise calculation models, which can be worked out to describe the phenomena observed in the performed experiments.

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Badania eksperymentalne hydrosprężystości zbiornika wypełnionego cieczą

Streszczenie

W artykule przedstawiono wyniki badań eksperymentalnych dotyczących współoddziaływania konstrukcji odkształcalnej z cieczą. Badaniaprzeprowadzono na dwóch zbiornikach prostopadłościennych ze sprężyście odkształcalnymi ścianami szczytowymi. Stosowano różne stopnie napełnienia zbiorników cieczą.

Badanie tego zjawiska ma odniesienie do często spotykanych zdarzeń praktycznych jak zderzenie zbiornikowca z innym statkiem lub nabrzeżem portowym. Gwałtowne hamowanie cystern samochodowych lub kolejowych itp. Rozpoznanie zjawiska doprowadziło do jednoczesnych pomiarów następujących wielkości:

1. Wymuszanych przyspieszeń układu zbiornik-ciecz
2. Przyspieszeń sprężystych ścian szczytowych zbiornika
3. Ciśnień hydrodynamicznych odkształcalnych ścian szczytowych