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Vibration control of a rotating shaft by passive mass-spring-disc dynamic vibration absorber

Shaft is a machine element which is used to transmit rotary motion or torque. During transmission of motion, however, the machine shaft doesn't always rotate with a constant angular velocity. Because of unstable current or due to sudden acceleration and deceleration, the machine shaft will rotate at a variable angular velocity. It is this rotary motion that generates the moment of inertial force, causing the machine shaft to have torsional deformation. However, due to the elasticity of the material, the shaft produces torsional vibration. Therefore, the main objective of this paper is to determine the optimal parameters of dynamic vibration absorber to eliminate torsional vibration of the rotating shaft that varies with time. The new results in this paper are summarized as follows: Firstly, the author determines the optimal parameters by using the minimum quadratic torque method. Secondly, the maximization of equivalent viscous resistance method is used for determining the optimal parameters. Thirdly, the author gives the optimal parameters of dynamic vibration absorber based on the fixed-point method. In this paper, the optimum parameters are found in an explicit analytical solutions, helping the scientists to easily find the optimal parameters for eliminating torsional vibration of the rotating shaft.

Nomenclature

Abbreviations

CPVA	centrifugal pendulum vibration absorber
DVA	dynamic vibration absorber
FP	fixed-point
MEVR	maximization of equivalent viscous resistance

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MQT	minimum quadratic torque
TMD	tuned mass damper

Symbols

c_a	damping coefficient of damper
c_{eqv}	the equivalent resistance coefficient of DVA
D	the amplitude magnification factor of the rotating shaft
e_1	radial position of spring
e_2	radial position of damper
J_a	mass moment of inertia of DVA
J_r	mass moment of inertia of rotor
k_a	stiffness of spring of DVA
k_s	torsional stiffness of the rotating shaft
$\mathbf{L}$	the quadratic torque matrix
M	excitation torque
M_{eqv}	the equivalent resistance torque of DVA
m_a	mass of DVA
m_r	mass of rotor
n	number of spring-damper sets
$\mathbf{q}_1$	the response of the system

Greek symbols

α	tuning ratio
α_{opt}^{FP}	optimal tuning ratio of DVA by using the fixed-point method
α_{opt}^{MEVR}	optimal tuning ratio of DVA by using the maximization of equivalent viscous resistance method
α_{opt}^{MQT}	optimal tuning ratio of DVA by using the minimum quadratic torque method
β	frequency ratio
γ	ratio between radial position of spring and radius of gyration of rotor
ε	the angular acceleration of the rotating shaft
η	ratio between radius of gyration of passive disk and rotor
θ	torsional vibration of the rotating shaft
θ_0	initial condition of the torsional vibration angle
$\hat{\theta}$	complex form of torsional vibration of the rotating shaft
$\lvert\hat{\theta}\rvert$	amplitude of torsional vibration of the rotating shaft
λ	ratio between radial position of damper and radius of gyration of rotor
μ	ratio between mass of DVA and mass of the rotating shaft
ξ	damping ratio
ξ_{opt}^{FP}	optimal damping ratio of DVA by using the fixed-point method
ξ_{opt}^{MEVR}	optimal damping ratio of DVA by using the maximization of equivalent viscous resistance method

ξ_{opt}^{MQT}	optimal damping ratio of DVA by using the minimum quadratic torque method
ρ_a	radius of gyration of DVA
ρ_r	radius of gyration of rotor
φ	angular displacement of the rotating shaft
φ_a	relative torsional angle between passive disk and rotor
φ_r	angular displacement of rotor
ω	excitation torque frequency
ω_a	natural frequency of DVA
Ω_s	natural frequency of the rotating shaft

1. Introduction

This paper describes how to eliminate torsional vibration of rotating shafts. As we know, the shaft is one of the most important parts of a machine. The shaft is used to transmit torque and rotation from one part to another part of the machine. The characteristic movement of the shaft is rotary motion. The torsional vibration of the rotating system is mainly due to the uneven transmission of torque between the rotating parts of the machine. Excessive torsional vibration in the machine system leads to noise or fatigue destruction. So, they must be stopped or controlled immediately to ensure the reliability of the system. Passive vibration control has been applied frequently due to its simplicity and the effect which is acceptable, therefore, it has been studied by many scientists, as in references [1–23].

Centrifugal pendulum vibration absorbers (CPVA) are used for the reduction of torsional vibrations in rotating and reciprocating machines. They consist of the masses mounted on a rotor in such a way that they can freely move in accordance with the specified paths related to rotating systems. The movement of the masses is used to counteract the torque, thus reducing torsional vibration of the rotating parts [4–14]. The CPVA with different designs was introduced for the use in different conditions of the system. Author of [4] proposed the CPVA for the use in aircraft engines with variable speed conditions, in which the weight of the centrifugal mass was designed so that the recovery force changed with speed. Author of [5] introduced a CPVA including a compact design pendulum with rollers applicable for aircraft engines. Until early 1980, most CPVAs designs used circular profiles. Later, various non-circular path types were considered for the design of the CPVA, such as cycloidal path [6], tautochronic curve [7] and epicycloidal path [8, 9], etc. In 2002 the dynamic behavior of multiple CPVAs was studied in [10]. In 2006, authors of [11] studied an investigation of the dynamic response and performance of the CPVA. In 2014, authors of [12] presented a general approach to the design of tautochronic pendulum vibration absorbers. These results provided a basis for the design and analysis of tautochronic bifilar and non-bifilar vibration absorbers. In 2015, Ref. [13] studied aims to highlight the vibration absorbing capabilities of CVPAs, emphasizing the configurations with cycloidal and epicycloidal paths.

In 2016, Ref. [14] studied ways to reduce vibration for a rigid rotor with moving components simultaneously tilting, rotating and translating using the CPVA. To the best knowledge of the authors, there has been no study that identifies the optimum parameters of the CPVA applicable for eliminating torsional vibration of the rotating systems.

A dynamic vibration absorber (DVA), or tuned mass damper (TMD), is a well-known device used to eliminate vibration. The absorber consists of a moving mass attached to the main structure through springs and dampers. The description of use of the DVA, or the TMD, as an additional tool for eliminating torsional vibration of the rotating systems is very limited in the literature. Until 2017, Ref. [21] used the fixed-point method to give optimal parameters of the DVA in the form of a passive mass-spring-disc for the rotating shafts. When designing absorbers to eliminate vibration for the main system, one can apply diverse shapes of the absorbers, depending on the type of structure to be installed. Therefore, in [22] the author studied how to eliminate torsional vibration of rotating shafts with a symmetric TMD in the form of mass-spring-pendulum by using the principle of minimum kinetic energy. The studies in references [21, 22] only considered the shaft rotating at a constant angular velocity. In reality, however, many rotating systems do not always rotate with constant angular velocities. Because of unstable current or due to sudden acceleration and deceleration, the angular velocity of rotating shafts can vary with time. It is this variability in rotation of the rotating systems that generate the moment of inertial force, causing the rotating systems to have torsional deformation. But due to the elasticity of the material, the rotating systems produce torsional vibration. Therefore, the research results in references [4–14, 21–23] were developed to overcome the limitations of the rotating systems. In this paper, the author continues the work aimed at finding the optimal parameters of the DVA for the case of the rotating shaft that varies with time. The optimum parameters are found in an explicit analytical solutions, helping the scientists to easily find the optimal parameters when applicable for eliminating torsional vibration of the rotating shafts. The results presented in this paper enrich knowledge of optimal control and help to eliminate torsional vibration of the rotating systems.

2. Shaft modelling and vibration equations

Fig. 1 shows a rotating shaft which has an attached passive mass-spring-disc dynamic vibration absorber (DVA). Fig. 2 shows the passive mass-spring-disc dynamic vibration absorber [21]. The parameters of the rotating shaft with DVA are listed in the Nomenclature.

By using the second-order Lagrange equation, the motion equations of the system are written as follows [21]

$$\begin{aligned} &(J_r + J_a)\,\ddot{\varphi}_r + J_a\ddot{\varphi}_a + k_s\left(\varphi_r - \Omega_0 t\right) = M(t), \\ &J_a\ddot{\varphi}_r + J_a\ddot{\varphi}_a + nk_a e_1^2\varphi_a + nc_a e_2^2\dot{\varphi}_a = 0 \end{aligned} \tag{1}$$

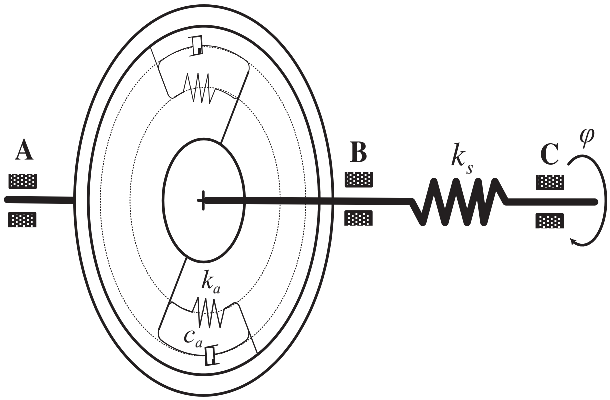

Fig. 1. The rotating shaft with DVA

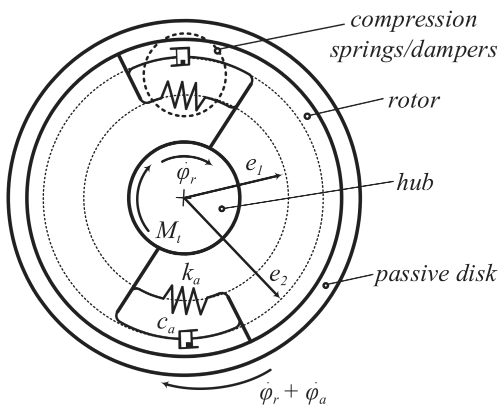

Fig. 2. Modeling of the DVA

where

$$J_r = m_r \rho_r^2, \qquad J_a = m_a \rho_a^2, \qquad \varphi = \Omega_0 t. \tag{2}$$

Because the rotating shaft varies with time: $\varphi = \varphi(t)$, we have

$$\varphi_r - \varphi(t) = \theta \quad \to \quad \varphi_r = \theta + \varphi(t), \quad \dot\varphi_r = \dot\theta + \dot\varphi(t), \quad \ddot\varphi_r = \ddot\theta + \ddot\varphi(t), \quad M(t) = 0. \tag{3}$$

By substituting equations (2), (3) into equations (1), the motion equations of the system are represented as

$$\left(m_r \rho_r^2 + m_a \rho_a^2\right)\ddot\theta + m_a \rho_a^2 \ddot\varphi_a + k_s \theta = -(m_r \rho_r^2 + m_a \rho_a^2)\ddot\varphi(t), \tag{4}$$

$$m_a \rho_a^2 \ddot\theta + m_a \rho_a^2 \ddot\varphi_a + n c_a e_2^2 \dot\varphi_a + n k_a e_1^2 \varphi_a = -m_a \rho_a^2 \ddot\varphi(t). \tag{5}$$

To write the equations in non-dimensional form, the following symbols are introduced

$$k_a = m_a\omega_a^2, \quad k_s = m_r\rho_r^2\Omega_s^2, \quad m_a = \mu m_r, \quad \rho_a = \eta\rho_r, \\ e_1 = \gamma\rho_r, \quad e_2 = \lambda\rho_r, \quad \omega_a = \alpha\Omega_s, \quad c_a = \xi m_a\omega_a. \tag{6}$$

The symbols in the expressions of equation (6) are described in Nomenclature.

By substituting the expressions of equation (6) into equations (4), (5) the latter can be rewritten in the matrix form:

$$\begin{bmatrix} 1+\mu\eta^2 & \mu\eta^2 \\ \mu\eta^2 & \mu\eta^2 \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{\varphi}_a \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & n\xi\alpha\Omega_s\mu\lambda^2 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \dot{\varphi}_a \end{bmatrix} \\ + \begin{bmatrix} \Omega_s^2 & 0 \\ 0 & n\alpha^2\Omega_s^2\mu\gamma^2 \end{bmatrix}\begin{bmatrix} \theta \\ \varphi_a \end{bmatrix} = \begin{Bmatrix} -1-\mu\eta^2 \\ -\mu\eta^2 \end{Bmatrix}\ddot{\varphi}(t). \tag{7}$$

3. Determination of optimal parameters by using the minimum quadratic torque method (MQT)

The minimum quadratic torque method is applied when the angular acceleration of the rotating shaft, $\varepsilon(t)$, is assumed to have a form of a white noise of spectral density S_ε. This method, which bases on the calculation of quadratic torque, has been presented in [1]. From the vibration equation (7), the equation of state is constructed:

$$\dot{\mathbf{q}}_1(t) = \mathbf{A}\,\mathbf{q}_1(t) + \mathbf{N}_\varepsilon\varepsilon(t), \tag{8}$$

where $\mathbf{q}_1(t)$, $\varepsilon(t)$, respectively, are the response of the system and the angular acceleration of the rotating shaft, they are defined as follows

$$\mathbf{q}_1(t) = \begin{Bmatrix}\theta & \varphi_a & \dot{\theta} & \dot{\varphi}_a\end{Bmatrix}^T, \tag{9}$$

$$\varepsilon(t) = \ddot{\varphi}(t). \tag{10}$$

Using equations (7)–(10), the matrices $\mathbf{A}$ and $\mathbf{N}_\varepsilon$ are determined, respectively

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}, \tag{11}$$

$$\mathbf{N}_\varepsilon = \begin{bmatrix}0 & 0 & -1 & 0\end{bmatrix}^T \tag{12}$$

in which

$$\begin{aligned}
&A_{11} = 0, \quad A_{21} = 0, \quad A_{31} = -\Omega_s^2, \quad A_{41} = \Omega_s^2, \\
&A_{12} = 0, \quad A_{22} = 0, \quad A_{32} = n\alpha^2\Omega_s^2\mu\gamma^2, \quad A_{42} = -\frac{\left(1+\mu\eta^2\right)n\alpha^2\Omega_s^2\gamma^2}{\eta^2}, \\
&A_{13} = 1, \quad A_{23} = 0, \quad A_{33} = 0, \quad A_{43} = 0, \\
&A_{14} = 0, \quad A_{24} = 1, \quad A_{34} = n\xi\alpha\Omega_s\mu\lambda^2, \quad A_{44} = -\frac{\left(1+\mu\eta^2\right)n\alpha^2\Omega_s^2\lambda^2}{\eta^2}.
\end{aligned} \tag{13}$$

According to [1], the quadratic torque matrix, **L**, is a solution of Lyapunov equation in equation (14).

$$\mathbf{A}\,\mathbf{L} + \mathbf{L}\,\mathbf{A}^T + S_\varepsilon \mathbf{N}_\varepsilon \mathbf{N}_\varepsilon^T = \mathbf{0}. \tag{14}$$

By solving the system of equations (11)–(14), the matrix **L** is determined as

$$\mathbf{L} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \tag{15}$$

where

$$L_{11} = \frac{1}{2}\frac{\left[\begin{array}{c}\gamma^4\alpha^4n^2\left(1+\mu\eta^2\right)^4 + \left(n\xi^2\lambda^4\left(\mu\eta^2+1\right)\right. \\ \left.+\eta^2\gamma^2\left(\mu\eta^2-2\right)\right)\left(1+\mu\eta^2\right)^2 n\alpha^2 + \eta^2\end{array}\right]S_\varepsilon}{n\xi\alpha\Omega_s^3\mu\lambda^2\eta^4}, \tag{16}$$

$$L_{32} = -\frac{1}{2}\frac{\left(1+\mu\eta^2\right)^2 S_\varepsilon}{\eta^2\Omega_s^2\mu}, \tag{17}$$

$$L_{33} = \frac{1}{2}\frac{\left[\begin{array}{c}\gamma^4n^2\left(1+\mu\eta^2\right)^3\alpha^4 + \left(\mu\eta^2+1\right)\left(n\xi^2\lambda^4\left(\mu\eta^2+1\right)\right. \\ \left.-2\eta^2\gamma^2\right)n\alpha^2 + \eta^4\end{array}\right]S_\varepsilon}{n\xi\alpha\Omega_S\mu\lambda^2\eta^4}, \tag{18}$$

$$L_{34} = \frac{1}{2}\frac{\left[\gamma^2 n\left(1+\mu\eta^2\right)^2\alpha^2 - \eta^2\right]S_\varepsilon}{n\xi\alpha\Omega_S\mu\lambda^2\eta^2}. \tag{19}$$

In the quadratic torque matrix **L**, the response of the main system is L_{11}. The smaller the L_{11} is, the faster the torsional vibration of the system turns off. So, the minimum conditions are expressed as

$$\left.\frac{\partial L_{11}}{\partial \alpha}\right|_{\alpha_{opt}^{MQT}=\alpha} = 0, \tag{20}$$

$$\left.\frac{\partial L_{11}}{\partial \xi}\right|_{\xi_{opt}^{MQT}=\xi} = 0. \tag{21}$$

By solving the system of equations (16), (20), (21), the optimal parameters of the DVA, α_{opt}^{MQT} and ξ_{opt}^{MQT}, can be obtained as

$$\alpha_{opt}^{MQT} = \frac{1}{2}\frac{\eta\sqrt{2n\left(2-\mu\eta^2\right)}}{\gamma n\left(1+\mu\eta^2\right)}, \tag{22}$$

$$\xi_{opt}^{MQT} = \frac{\eta^3\sqrt{4\mu\gamma+3\mu^2\eta^2-\mu^3\eta^4}}{\left(1+\mu\eta^2\right)\lambda^2\sqrt{2n\eta\left(2-\mu\eta^2\right)}}. \tag{23}$$

4. Determination of optimal parameters by using the maximization of equivalent viscous resistance method (MEVR)

The objective of this method is to maximize the equivalent resistance coefficient of the system [2]. The greater the equivalent resistance coefficient of the system, the faster the decay of vibration of the system. Firstly, it is necessary to determine the equivalent resistance torque of the DVA acting on the main system. Using equations (4), (5), (10) gives

$$m_r\rho_r^2\ddot{\theta} + k_s\theta = nk_a e_1^2\varphi_a + nc_a e_2^2\dot{\varphi}_a - m_r\rho_r^2\varepsilon(t). \tag{24}$$

From (24), the equivalent resistance torque of the DVA acting on the main system is obtained as

$$M_{eqv} = nk_a e_1^2\varphi_a + nc_a e_2^2\dot{\varphi}_a\,. \tag{25}$$

According to [2], the equivalent resistance coefficient of the DVA acting on the main system is determined as follows

$$c_{eqv} = -\frac{\left\langle M_{eqv}\dot{\theta}\right\rangle}{\left\langle\dot{\theta}^2\right\rangle} = -\frac{nk_a e_1^2\left\langle\varphi_a\dot{\theta}\right\rangle + nc_a e_2^2\left\langle\dot{\varphi}_a\dot{\theta}\right\rangle}{\left\langle\dot{\theta}^2\right\rangle}. \tag{26}$$

In this method, the angular acceleration of the rotating shaft, $\varepsilon(t)$, is assumed to have a form of white noise with spectral density S_ε. Therefore the average values of equation (26) are the components of the matrix $\mathbf{L}$ in equation (14). In this case, we have

$$c_{eqv} = -\frac{nk_a e_1^2 L_{32} + nc_a e_2^2 L_{34}}{L_{33}}. \tag{27}$$

By substituting equations (6), (17)–(19) into equation (27), the equivalent resistance coefficient can be determined as

$$c_{eqv} = \frac{-\rho_r^2 m_a \eta^4 n \xi \alpha \Omega_s \lambda^2}{\gamma^4 n^2 \left(\gamma^2\mu+1\right)^3 \alpha^4 + \left(\gamma^2\mu+1\right) n \left(\lambda^4 \xi^2 \left(\gamma^2\mu+1\right) n - 2\eta^2\gamma^2\right) \alpha^2 + \eta^4}. \tag{28}$$

The conditions for the equivalent resistance coefficient of the DVA, c_{eqv}, yield maximum value as follows

$$\left.\frac{\partial c_{eqv}}{\partial \alpha}\right|_{\alpha_{opt}^{MEVR}=\alpha} = 0, \tag{29}$$

$$\left.\frac{\partial c_{eqv}}{\partial \xi}\right|_{\xi_{opt}^{MEVR}=\xi} = 0. \tag{30}$$

By solving the system of equations (28)–(30), the optimal parameters of the DVA, α_{opt}^{MEVR}, ξ_{opt}^{MEVR}, are determined as

$$\alpha_{opt}^{MEVR} = \frac{\eta}{\gamma\left(1+\mu\eta^2\right)\sqrt{n}}, \tag{31}$$

$$\xi_{opt}^{MEVR} = \frac{\gamma\eta^2\sqrt{\mu}}{\lambda^2\sqrt{n\left(1+\mu\eta^2\right)}}. \tag{32}$$

5. Determination of optimal parameters by using the fixed-point method (FP)

In this method, one considers the rotating shaft whose angular velocity varies with time. The angular acceleration is presented in the harmonic form:

$$\ddot{\varphi}(t) = \varphi_0 e^{i\Omega t}. \tag{33}$$

By solving the system of equations (7), (33), the torsional angle of the rotating shaft, $\theta(t)$, can be obtained as

$$\theta(t) = \hat{\theta} e^{i\Omega t} \tag{34}$$

where

$$\hat{\theta} = \frac{\left[\begin{array}{l} \xi\alpha\beta\lambda^2 n\left(\mu\eta^2+1\right) i \\ +\left(\alpha^2\eta^2\gamma^2\mu n + \alpha^2\gamma^2 n - \beta^2\eta^2\right) \end{array}\right] \varphi_o\Big/\Omega_s^2}{\left[\begin{array}{l} \alpha\beta\xi\lambda^2 n\left(\mu\beta^2\eta^2+\beta^2-1\right) i \\ +\left(\alpha^2\beta^2\eta^2\gamma^2\mu n + \alpha^2\beta^2\gamma^2 n - \beta^4\eta^2 - \alpha^2\gamma^2 n + \beta^2\eta^2\right) \end{array}\right]}. \tag{35}$$

The amplitude of torsional vibration of the rotating shaft is

$$\left|\hat{\theta}\right| = \sqrt{\frac{\left[\xi\alpha\beta\lambda^2 n\left(\mu\eta^2+1\right)\right]^2 + \left[\alpha^2\eta^2\gamma^2\mu n+\alpha^2\gamma^2 n-\beta^2\eta^2\right]^2\left[\varphi_0/\Omega_s^2\right]^2}{\left[\alpha\beta\xi\lambda^2 n\left(\mu\beta^2\eta^2+\beta^2-1\right)\right]^2 + \left[\beta^2\left(\alpha^2\eta^2\gamma^2\mu n+\alpha^2\gamma^2 n-\beta^2\eta^2+\eta^2\right)-\alpha^2\gamma^2 n\right]^2}}\,. \tag{36}$$

Using equation (36), we determine the amplitude magnification factor of the rotating shaft as follows

$$D = \sqrt{\frac{\left[\xi\alpha\beta\lambda^2 n\left(\mu\eta^2+1\right)\right]^2 + \left[\alpha^2\eta^2\gamma^2\mu n+\alpha^2\gamma^2 n-\beta^2\eta^2\right]^2}{\left[\alpha\beta\xi\lambda^2 n\left(\mu\beta^2\eta^2+\beta^2-1\right)\right]^2 + \left[\beta^2(\alpha^2\eta^2\gamma^2\mu n+\alpha^2\gamma^2 n-\beta^2\eta^2+\eta^2)-\alpha^2\gamma^2 n\right]^2}}\,. \tag{37}$$

Fig. 3 presents the graphs of the amplitude magnification factor D versus the frequency ratio β corresponding to some different values of the DVA's damping ratio ξ.

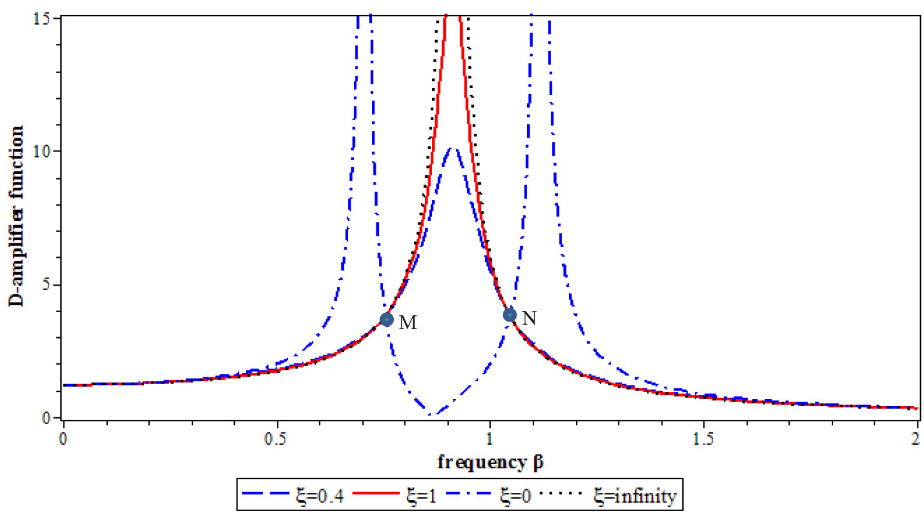

Fig. 3. Graphs of the amplitude magnification factor versus the frequency ratio β

Fig. 3 shows that the curve $D = f(\beta)$ always passes through two points M, N which are fixed and independent of the viscous resistance coefficient ξ. So, it is possible to apply the fixed-point method to find the optimal parameters to reduce harmful vibrations of the rotating shaft. According to [3], the peak of the smallest amplitude of vibration can be achieved by choosing such coefficients of the two points M, N that have equal values and reach the maximum value at that point. We have

$$\frac{\partial D}{\partial \xi} = 0, \tag{38}$$

$$D_M = D_N \,. \tag{39}$$

By solving the system of equations (37)–(39), the optimal parameters, α_{opt}^{FP} and β, are identified as

$$\alpha_{opt}^{FP} = \alpha = \frac{\eta\sqrt{2}\,(2-\mu\eta^2)}{2\gamma\sqrt{n}\,(\mu\eta^2+1)}\,, \tag{40}$$

$$\beta_1^2 = \frac{2+\eta\sqrt{2\mu}}{2\,(\mu\eta^2+1)}\,, \tag{41}$$

$$\beta_2^2 = \frac{2-\eta\sqrt{2\mu}}{2\,(\mu\eta^2+1)}\,. \tag{42}$$

Then, the optimum absorber damping can be identified from the following condition

$$\frac{\partial D}{\partial \beta} = 0. \tag{43}$$

By solving the system of equations (37), (40)–(43), the DVA's damping ratios can be determined as

$$\xi_1^2 = \frac{\mu\left(1+\mu\eta^2\right)^2\left(\sqrt{2}\eta\sqrt{\mu}+6\right)\gamma^2\eta^4}{2n\lambda^4\,(2+5\mu\eta^2+3\mu^2\eta^4-\mu^3\eta^6-\mu^4\eta^8)} \tag{44}$$

and

$$\xi_2^2 = \frac{\mu\gamma^2\eta^4\left(\mu\eta^2+1\right)\left(6+\eta\sqrt{2\mu}\right)}{2n\lambda^4\,(2+3\mu\eta^2-\mu^3\eta^6)}\,. \tag{45}$$

The optimal value of ξ is found as

$$\xi_{opt}^{FP} = \xi_{opt} = \sqrt{\frac{\xi_1^2+\xi_2^2}{2}}\,. \tag{46}$$

By substituting equations (44) and (45) into equation (46), the optimal parameter of DVA, ξ_{opt}^{FP}, is determined as

$$\xi_{opt}^{FP} = \frac{\gamma\eta^2}{2\lambda^2}\sqrt{\frac{2\mu(\mu\eta^2+2)\left(6-2\eta\sqrt{2\mu}-\mu\eta^2\right)}{n(\mu\eta^2-2)\left(\eta\sqrt{2\mu}-2\right)}}\,. \tag{47}$$

From equations (22), (23), we obtain the optimal parameters of the DVA by using the minimum quadratic torque method. Based on the maximization of equivalent viscous resistance method, the optimal parameters of the DVA are given in equations (31), (32). The fixed-point method is used for finding the optimal parameters of the DVA, which are defined in equations (40) and (47). The optimal expressions (equations (22), (23), (31), (32) and (40), (47)) for DVA are used to eliminate the torsional vibration of the rotating shaft that varies with time. These optimal expressions are different from the optimal expressions that have been found in reference [21]. From this fact it follows that the optimal parameters determined to eliminate torsional vibration of the rotating shafts are different for shafts rotating with constant and variable velocity. Therefore, when applying the optimal parameters to eliminate torsional vibration of the rotating shaft we have to consider if the shaft rotates at a constant or variable angular velocity to use the proper optimal expressions for determining the optimal parameters in order to increase the effect of eliminating torsional vibration.

6. Numerical simulation

To evaluate the reliability of the optimal parameters that are found in this paper, the author uses Maple software to simulate the vibration of the rotating shaft with the DVA when it is optimally designed.

The author performs calculations to eliminate the torsional vibration of the rotating shaft with the input data taken from [21] which are summarized in Table 1.

Table 1.

The input data of the rotating shaft and the DVA

Parameter	m_r	ρ_r	k_s	m_a	ρ_a	e_1	e_2	N
Value	5 kg	0.1 m	10^4 Nm/rad	0.05 kg	0.1 m	0.06 m	0.08 m	5

Based on the parameters in Table 1 and the expressions of equation (6), the non-dimensional parameters of the system are determined and summarized in Table 2.

Table 2.

The non-dimensional parameters of the system

Parameter	μ	η	γ	λ
Value	0.01	1.0	0.6	0.8

By applying equations (6), (22), (23), (31), (32), (40), (47) and parameters from Table 2, the optimal parameters of the DVA are defined in Table 3.

Based on the parameters from Table 1 and Table 3, and using Maple software, we simulate the torsional vibration of the rotating shaft, as shown in Figs 4–12.

Table 3.

The optimal parameters of the DVA

The minimum quadratic torque method				
Parameter	α_{opt}^{MQT}	ξ_{opt}^{MQT}	k_a	c_a
Value	0.736	0.054	5418.86 N/m	0.89 Ns/m
The maximization of equivalent viscous resistance method				
Parameter	α_{opt}^{MEVR}	ξ_{opt}^{MEVR}	k_a	c_a
Value	0.738	0.042	5446.09 N/m	0.688 Ns/m
The fixed-point method				
Parameter	α_{opt}^{FP}	ξ_{opt}^{FP}	k_a	c_a
Value	0.736	0.042	5418.86 N/m	0.692 Ns/m

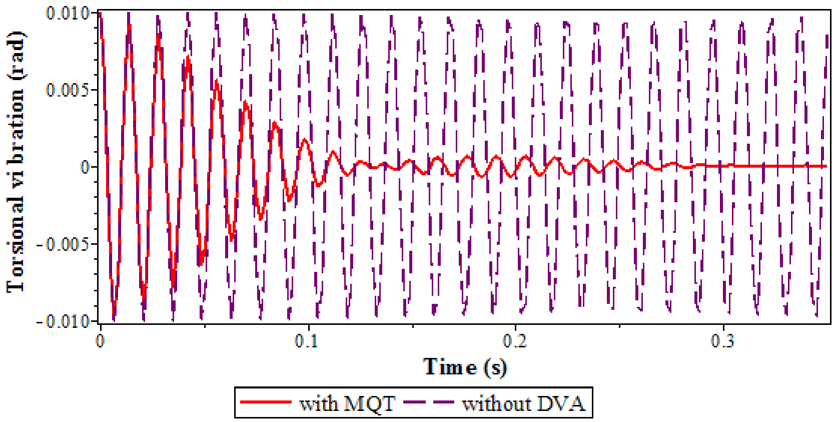

Fig. 4. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) of the minimum quadratic torque method

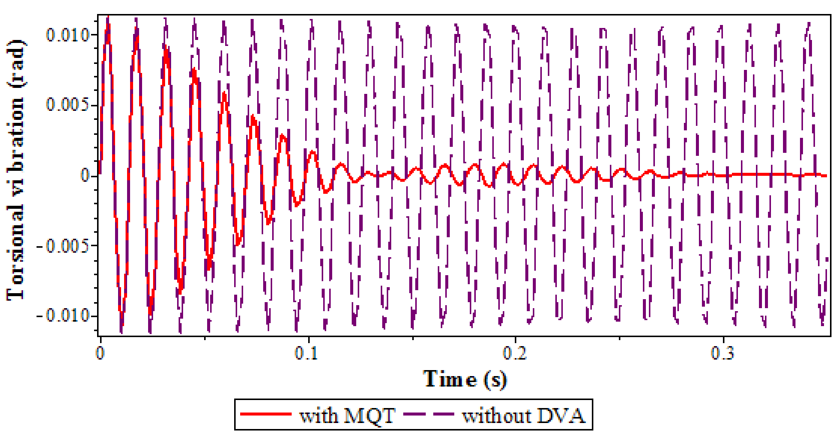

Fig. 5. The vibration of the rotating shaft with $\dot{\theta}_0 = 5.0$ (rad/s) of the minimum quadratic torque method

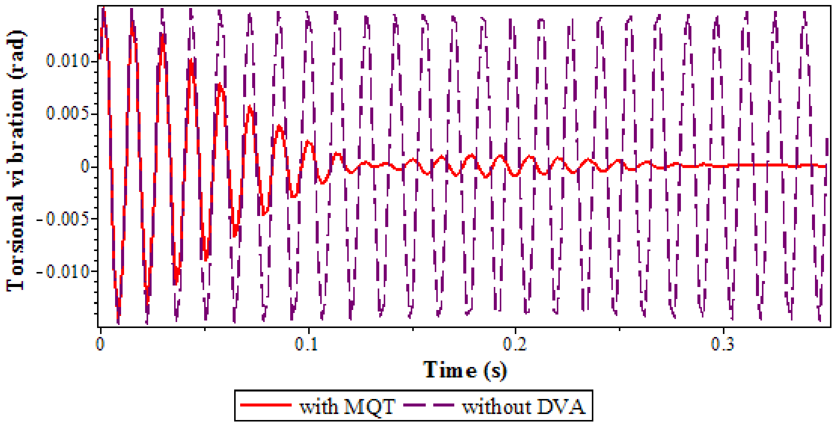

Fig. 6. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) and $\dot{\theta}_0 = 5.0$ (rad/s) of the minimum quadratic torque method

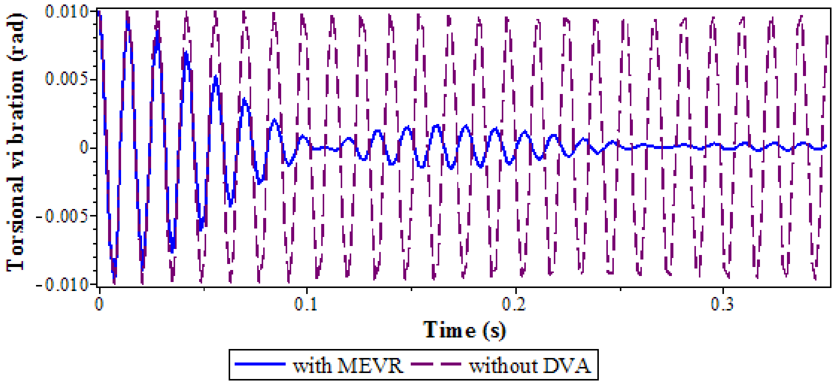

Fig. 7. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) of the maximization of equivalent viscous resistance method

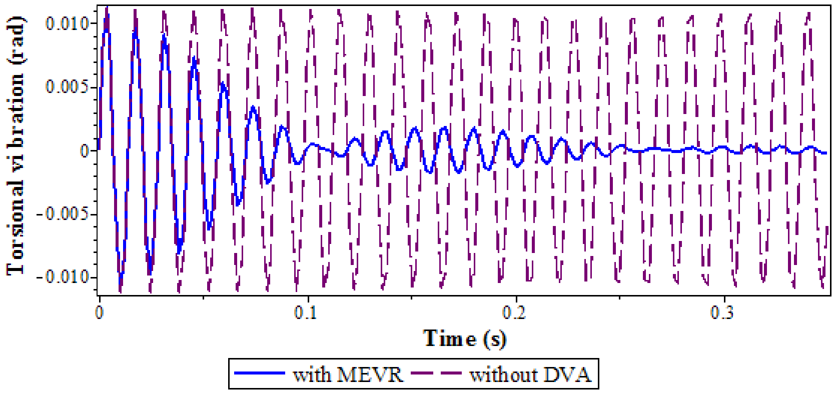

Fig. 8. The vibration of the rotating shaft with $\dot{\theta}_0 = 5.0$ (rad/s) of the maximization of equivalent viscous resistance method

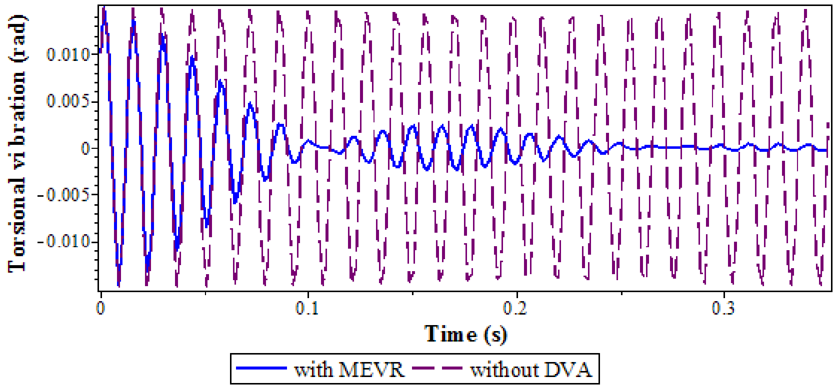

Fig. 9. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) and $\dot{\theta}_0 = 5.0$ (rad/s) of the maximization of equivalent viscous resistance method

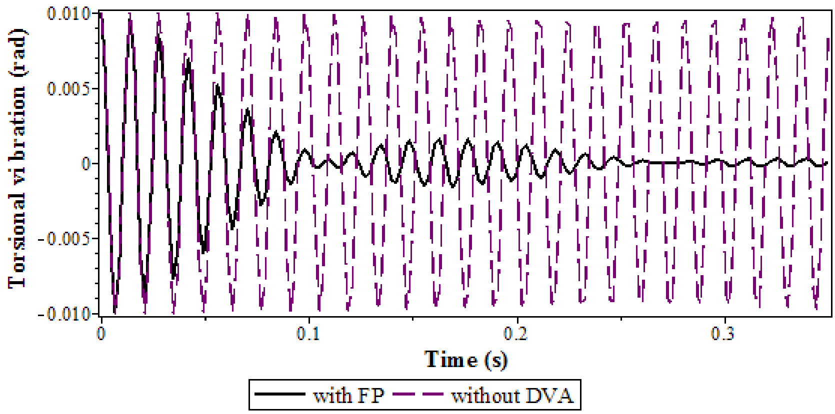

Fig. 10. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) of the fixed-point method

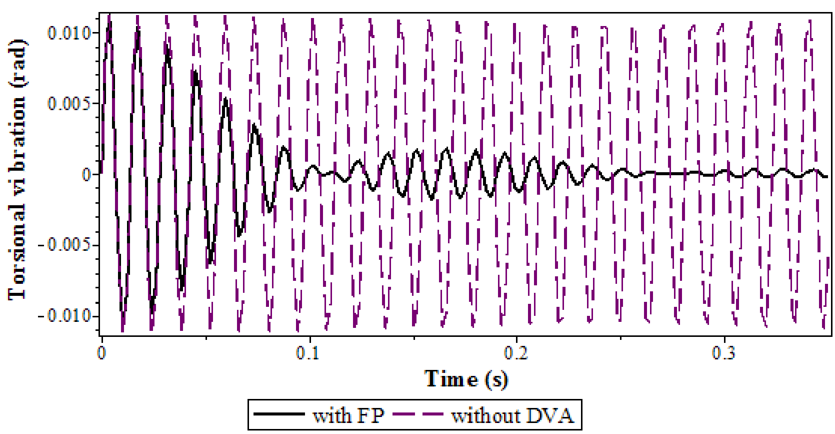

Fig. 11. The vibration of the rotating shaft with $\dot{\theta}_0 = 5.0$ (rad/s) of the fixed-point method

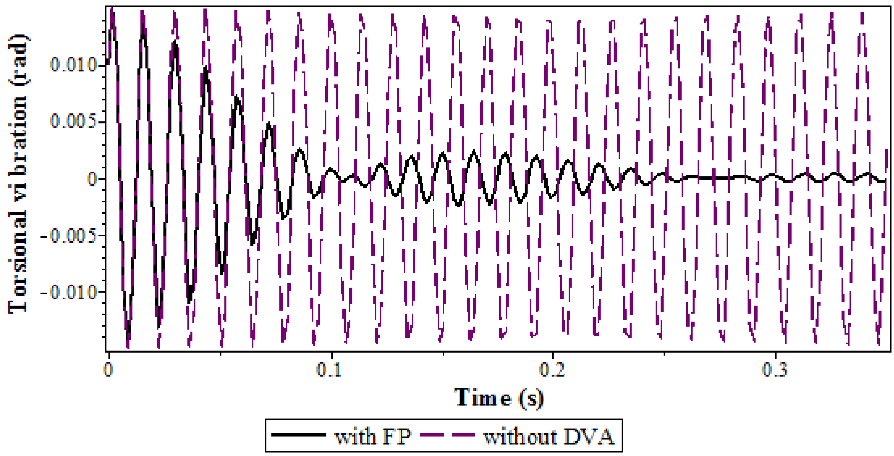

Fig. 12. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) and $\dot{\theta}_0 = 5.0$ (rad/s) of the fixed-point method

7. Comparison of the effect of three methods

In order to compare the effect of eliminating torsional vibration when applying the optimal parameters found by the three methods, the author presents simulation results of torsional vibrations of the rotating shaft in the same graph, as shown in Figs. 13–15.

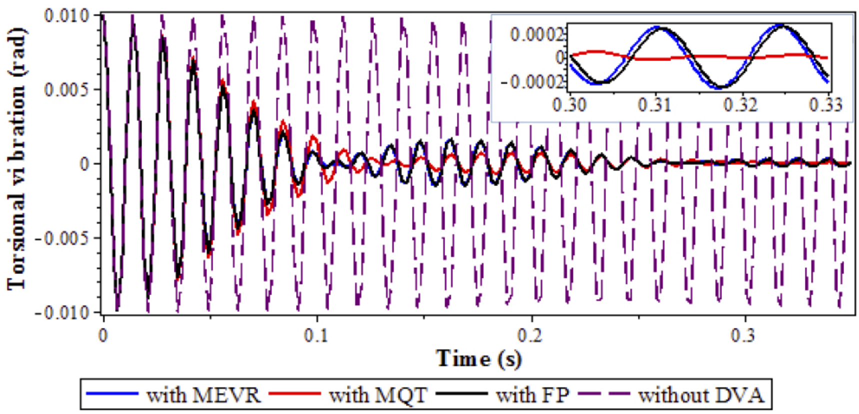

Fig. 13. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) of the three methods

Figs. 13–15 show the maximization of equivalent viscous resistance method and the fixed-point method, which give the same effect for reducing the torsional vibration. Within the first 0.12 s, the vibration amplitudes of the maximization of equivalent viscous resistance method and the fixed-point method are smaller than those of the minimum quadratic torque method. But, from 0.12 s onwards, the vibration amplitudes of the minimum quadratic torque method are smaller than

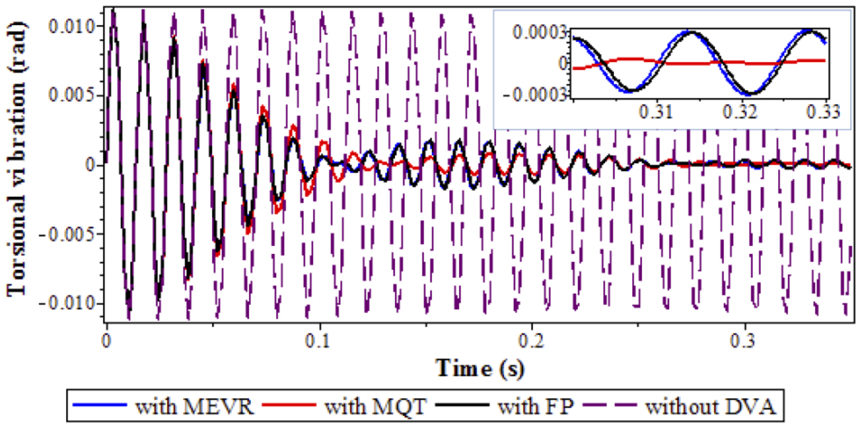

Fig. 14. The vibration of the rotating shaft with $\dot{\theta}_0 = 5.0$ (rad/s) of the three methods

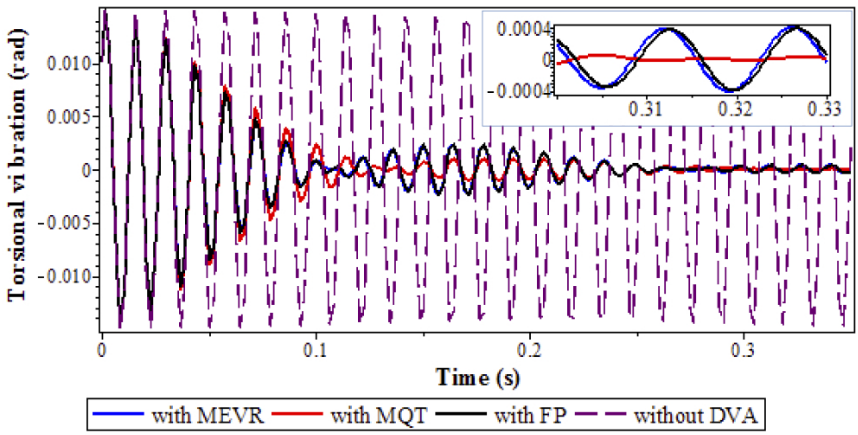

Fig. 15. The vibration of the rotating shaft with $\theta_0 = 0.01$ (rad) and $\dot{\theta}_0 = 5.0$ (rad/s) of the three methods

those of the maximization of equivalent viscous resistance method and the fixed-point method. Overall, however, among the three methods the minimum quadratic torque method has the greatest effect on eliminating torsional vibration.

8. Conclusions

This paper aims at determining the optimal parameters of the dynamic vibration absorber in order to eliminate torsional vibration of the rotating shaft that varies with time. The optimum parameters of the dynamic vibration absorber which are determined by using the minimum quadratic torque method are shown in equations (22) and (23). The optimal parameters of the dynamic vibration absorber that are chosen in order to maximize the equivalent resistance coefficient of the system are presented in equations (31) and (32). The optimum parameters of the dynamic vibration absorber given in equations (42) and (47) are obtained by using the fixed-point method. To evaluate the effect of eliminating torsional vibration, the author uses Maple software to simulate the vibration of the rotating shaft when it is optimally designed. Through vibration simulation, we find that the vibration amplitude of the rotating shaft is eliminated when the dynamic vibration absorber is installed. This confirms that the optimal parameters of the dynamic vibration absorber found in this paper are reliable and accurate.

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