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FORMULAE FOR BUCKLING LOAD BEARING CAPACITY OF GLASS STRUCTURE ELEMENTS

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Development of contemporary building industry and related search for new aesthetical and functional solutions of monumental buildings in the centers of large cities resulted in the interest in glass as a structural material. Attractiveness of glass as a building material may be derived from the fact, that it combines transparency and aesthetical look with other functional features. Application of glass results in modern look of building facades, improves the indoor comfort without limiting the availability of natural daylight. Wide implementation of the new high performance float flat glass manufacturing technology, in conjunction with increasing expectations of the construction industry relating to new glass functions, has led to significant developments in glass structures theory, cf. [1, 3, 4, 5, 9, 10]. Many years of scientific research conducted in European Union countries have been crowned with a report CEN/TC 250 N 1050 [2], compiled as a part of the work of European Committee for Standardization on the second edition of Eurocodes - an extension of the first edition by, among others, the recommendations for the above mentioned design of glass structures, in particular modern procedures for the design of glass building structures. The procedures proposed in the pre-code [2] are not widely known in Poland, and their implementation in the design codes should be verified at the country level. This task is undertaken in this paper.

Keywords**:** *glass, capacity, strength, elastic buckling, lateral-torsional buckling*

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1. FORMULAE FOR BEARING CAPACITY IN SIMPLE STATES

1.1. MODELING ASSUMPTIONS

It is assumed in specialist bibliography cf. [6, 9, 10], as well as in the European pre-code CEN/TC 250 N 1060 [2] that building components made of structural glass exhibit the physical behavior of linear perfectly elastic material cf. Fig. 1b. Such assumption is substantiated by the results of laboratory experiments, conducted during certification of glass products in the quality laboratories of glass manufacturers.

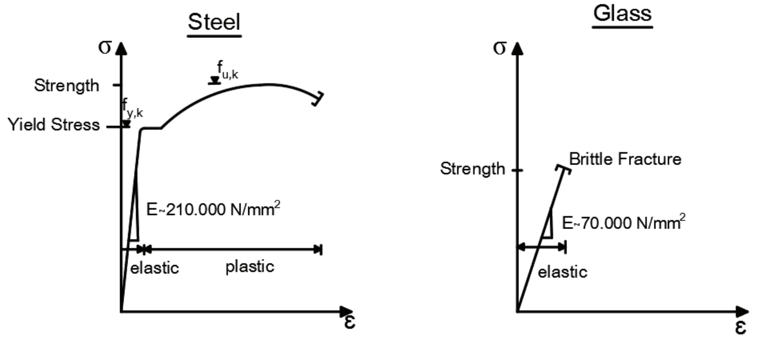

Fig. 1. Physical model for the low carbon steel and glass according to pre-code [2]

A clear difference exists between the strength of a glass bar subjected to axial tension and strength of the same bar subjected to pure bending, especially the bending strength is higher than strength in tension. Glass strength in tension is not a constant value, characteristic for this material, but depends on many factors, such as: surface finish of glass component, its size, history of loads, residual stresses and working conditions.

A comparison of σ-ε graphs for low carbon steel and glass, cf. Fig. 1, shows important model differences between both materials (elastic-plastic and linearly elastic material models). The yield limit $f_{y,k}$ and short term breaking strength $f_{u,k}$ of a steel sample subjected to axial tension are unique values, which may be specified in statistical analyses. In the case of glass samples, certified in Poland at the level of sorting minima, the characteristic strength f_k is a contractual value. In the contemporary domestic commercial offer glass products for building industry are based mainly on the float glass technology (glass cast on liquid tin). With this technology annealed glass plates may be made, exhibiting the characteristic strength according to the pre-code [2], verified in the four point bending test: $f_k = f_{g,k} = 45$ MPa; heat strengthened glass HSG exhibiting the strength $f_k = f_{b,k} = 70$ MPa and thermally toughened glass TTG exhibiting the strength $f_k = f_{b,k} = 120$ MPa. In addition it should be noted, that the compressive strength $f_{c,k}$ is much higher that the tensile strength $f_{b,k}$ and on average reaches $f_{c,k} = 500$ MPa. Formulation of dimensioning building structures by the load and bearing capacity coefficients according to code PN-EN 1990 defines the characteristic value of material strength as a lower quantile at the level of probability ω = 5%, thus for a large sample the following formula holds:

(1.1) $$f_k = \bar{f}_{b,t}(1-1.64v_{fb}) = \frac{\bar{f}_{b,t}}{\gamma_m},$$

where:

$\bar{f}_{b,t}$ – average strength in a bending test with respect to the fiber under tension, v_{fb} –coefficient of variation (a measure of dispersion of the results around the average value).

Table 1. Mean strength and material coefficient for tempered glass according to [11]

Thickness of sample [mm]	Glass type: $f_{b,k}$					
	TTG: $f_{b,k}$ = 120 MPa			TTG and glazed: $f_{b,k}$ = 75 MPa		
	Number of samples	$\bar{f}_{b,t}$ [MPa]	$\gamma_{m,exp} = \frac{\bar{f}_{b,t}}{f_{b,k}}$	Number of samples	$\bar{f}_{b,t}$ [MPa]	$\gamma_{m,exp} = \frac{\bar{f}_{b,t}}{f_{b,k}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
3	37	180	1.500	72	148	1.973
4	447	189	1.575	190	144	1.920
5	186	176	1.467	10	101	-
6	169	174	1.450	10	111	-
8	77	189	1.575	-	-	-
10	46	172	1.433	-	-	-
12	22	164	1.367	-	-	-
15	40	185	1.542	-	-	-

It directly follows from the formula (1.1), that the quality of glass products is determined by the coefficient of variation v_{fb} , as it directly affects the material coefficient: $\gamma_m = 1/(1\text{-}1.64\, v_{fb})$. In spite of many modern glass and glass processing works operating in Poland, the published statistical strength analyses of domestic glass products are rather infrequent. Results of research conducted by the Institute of Glass, Ceramics, Building and Fire Resistant Materials, Glass Division in Cracow, summarized in [11] and listed in the Table 1 are available. Taking into account the numerical values listed in column (4), based on the condition (1.1) for $\gamma_m = \gamma_{m,exp}$ one gets an estimate of the variation coefficient for bending strength of thermally toughened glass: v_{fb} = 0.16÷0.22. Analogous calculations for glazed glass yield the following value v_{fb} = 0.30. The obtained results seem to indicate a large if not very large dispersion of analyzed glass samples strength around average value, thus their credibility is questionable. Bearing in mind the regional verification of the new edition of Eurocodes with respect to the design of glass structures and a significant share of domestic makers in the European market of building products made of glass, a complete certification of domestic products, confirming their quality is postulated. Therefore statistical verification of the strength variation coefficients v_{fb} for various types of glass is necessary.

It has been proved, that under certain conditions fragile materials, including glass, are destroyed at stress levels much lower than those indicated by the strength hypotheses. In order to define the failure criterion in the case of glass one should turn to fracture mechanics. The classical design methods, consisting in determining the stresses in the structure and comparing these stresses with glass strength reduced by the material coefficients are sufficient at the level of engineering calculations. The strength criterion is recommended in CEN/TC 250 N 1060 [2] both for verification of material effort as well as general stability of glass structures. In the second case, however, one should take into account an initial geometrical imperfection having the shape of a circular arc with the height e(x) according to Fig. 2. This model applied in the bearing capacity analysis of glass bars – compressed, but bent as well, raises objections in the context of linearly elastic material model, according to Fig. 1, which precludes elastic-plastic deformation of the structures.

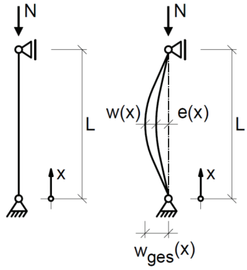

Fig. 2. Bar model with initial arched deflection assumed for bearing capacity analysis

1.2. BEARING CAPACITY FORMULAE FOR MONOLITHIC GLASS BARS

All the instability issues of glass structures in view of the CEN/TC 250 N 1060 [2] have been derived based on the model of a compressed bar with initial arched imperfection $e(x)$ according to Fig. 2, which, after application of the external load N is increased to reach the amount of $w(x)$. The arched imperfection induces bending moments $M(x)$ reaching the extreme value in the mid span of the bar, where the initial deflection is equal to $e_o = e(L/2)$. The equation of bending, taking into account the bending function: $M(x) = N[e(x) + w(x)]$, results in a differential formula combining the function of bending moment with the function of small deflections for the bar:

$$EI_y \frac{d^2 w(x)}{dx^2} + N \cdot w(x) = -N \cdot e(x)\,. \tag{1.2}$$

Assuming the initial bending function to agree with buckling form of a perfect bar:

$$e(x) = e_o \sin \frac{\pi x}{L}, \tag{1.3}$$

one obtains a complete solution for the formula (1.2) in the following form:

$$w(x) = A \cdot \sin kx + B \cdot \cos kx + \frac{k^2 e_o \sin \frac{\pi x}{L}}{\left(\frac{\pi}{L}\right)^2 - k^2}, \tag{1.4}$$

where in the formula (1.4) the following denotations hold: $k^2 = N/EI_y$, while A and B – integration constants, which for the boundary conditions depicted in Fig. 2 assume the value of A = B = 0. For the critical force $N_{cr} = \pi^2 EI_y/L^2$ the condition (1.4) yields the following formula for the deflected axis of the bar:

$$w(x) = \frac{k^2 e_o \sin \frac{\pi x}{L}}{\left(\frac{\pi}{L}\right)^2 - k^2} = \frac{e_o \sin \frac{\pi x}{L}}{\frac{1}{k^2}\left[\left(\frac{\pi}{L}\right)^2 - 1\right]} = \frac{e(x)}{\frac{N}{N_{cr}} - 1}\,. \tag{1.5}$$

For a bar with initial imperfection Perry proposed the following strength criterion:

$$\sigma = \frac{N}{A} + \frac{N \cdot z_{max}}{I_y} \frac{e_o}{1 - \frac{N}{N_{cr}}} \leq f_{y,k}. \tag{1.6}$$

In the contemporary works cf. [3, 4, 6], dedicated to the bearing capacity analysis of glass bars, the strength criterion for monolithic glass is assumed directly from the formula (1.6), while for the laminated glass it is assumed with modification of geometrical properties (effective characteristics). Since the resistance against tension of glass bars is much lower than the resistance against compression $f_{u,t} \neq |f_{u,c}|$, the condition (1.6) should be verified in the tensioned and compressed fibers separately:

$$\sigma_t = -\frac{N}{A} + \frac{N \cdot z_{max}}{I_y} \frac{e_o}{1 - \frac{N}{N_{cr}}} \leq f_{u,t} = 70\,\text{MPa (for HSG) or } 120\,\text{MPa (for TTG)}, \tag{1.7}$$

$$\sigma_c = -\frac{N}{A} - \frac{N \cdot z_{max}}{I_y} \frac{e_o}{1 - \frac{N}{N_{cr}}} \leq f_{u,c} = -500\,\text{MPa}. \tag{1.8}$$

In the bearing capacity limit state, for $\sigma_t = f_{u,t}$ and $N = N_u$ after reformulation one may write:

$$\frac{\sigma_t}{f_{u,t}} = -\frac{N_u}{A \cdot f_{u,t}} + \frac{N_u \cdot A}{W_y \cdot A \cdot f_{u,t}} \frac{e_o}{1 - \frac{\bar{\lambda}_t^2 \cdot N_u}{A \cdot f_{u,t}}} = 1, \tag{1.9}$$

where:

$W_y = \frac{I_y}{z_{max}}$ – strength index, $\bar{\lambda}_t = \sqrt{\frac{A \cdot f_{u,t}}{N_{cr}}}$ – relative slenderness of the bar related to the tensile strength.

Denoting the dimensionless bearing capacity of the bar (buckling coefficient) as $\chi_t = N_u/A \cdot f_{u,t}$ and substituting $\eta = e_o \cdot A/W_y$ one obtains, after rewriting, the following relationship:

$$-\chi_t + \frac{\eta \cdot \chi_t}{1 - \bar{\lambda}_t^2 \chi_t} = 1 \rightarrow \chi_t. \tag{1.10}$$

The above equation is solved by the function:

(1.11) $$\chi_t = \frac{1}{\phi_t + \sqrt{\phi_t^2 + \bar{\lambda}_t^2}} \quad \text{for} \quad \phi_t = 0.5\left(-1 + \alpha \cdot \bar{\lambda}_t + \bar{\lambda}_t^2\right).$$

The equation (1.8) may be used to derive analogous buckling coefficient with respect to the compressed edge of the bar cross section:

(1.12) $$\chi_c = \frac{1}{\phi_c + \sqrt{\phi_c^2 - n_f \bar{\lambda}_t^2}} \quad \text{for} \quad \phi_c = 0.5\left(1 + \alpha \cdot \bar{\lambda}_t + n_f \cdot \bar{\lambda}_t^2\right) \text{ and } n_f = \left|\frac{f_{u,c}}{f_{u,t}}\right|.$$

The parameters of functions (1.11) and (1.12) have been analyzed in several works, but the pre-code [2] turns to the proposal of K. Langosch [6], according to which the imperfection parameter α having the following form:

(1.13) $$\alpha = \sqrt{3}\,\frac{e_o}{L}\sqrt{\frac{\pi^2 E}{f_{u,t}}},$$

correctly depicts the results of known laboratory experiments for heat strengthened glass HSG with assumed initial displacement e_o = L/400 (yields α = 0.430) and e_o = L/300 for thermally toughened glass TTG (yields α = 0.329), cf. Fig. 3, curves under the Euler's hyperboloid.
Buckling coefficient function based on the one assumed by Eurocode 3 for steel columns is a solution of the equation (1.10) alternative to (1.11), and recommended in [2] for columns made of both monolithic glass types:

(1.14) $$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}_t^2}} \quad \text{for} \quad \phi = 0{,}5\left(1 + \alpha(\bar{\lambda}_t - \bar{\lambda}_{t,0}) + \bar{\lambda}_t^2\right),$$

where:

$\bar{\lambda}_t = \frac{L_{cr}}{i \cdot \lambda_1}$, $\lambda_1 = 99.3\sqrt{\frac{70}{f_{u,t}}}$ – relative and comparative slenderness, respectively.

The function (1.14) is depicted in Fig. 3 (the bottom continuous line) for unified imperfection parameters specified in the pre-code [2], and having the values of $\alpha = 0.43$ and $\bar{\lambda}_{t,0} = 0.89$.

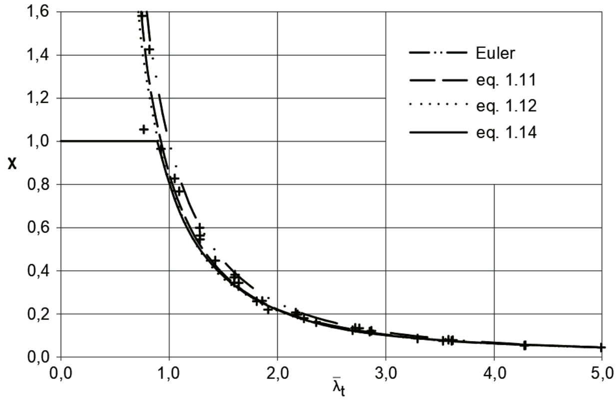

Fig. 3. Buckling curves, from the top: according to Euler, formulae (1.11) and (1.14). Source [2].

The dimensionless bearing capacity formula for beam-ribs susceptible to lateral-torsional buckling subjected to bending and made of monolithic glass has not been specified in CEN/TC 250 N 1060 [2], only the following bearing capacity condition has been formulated:

(1.15)
$$\frac{M_{\mathrm{Ed}}}{\chi(\bar{\lambda}_{\mathrm{LT}})\cdot M_{\mathrm{el,d}}} \leq 1{,}0 \ \text{ for } \bar{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{el}} \cdot f_{\mathrm{u,t}}}{M_{\mathrm{cr}}}}.$$

where:

$W_{el} = b \cdot t^2/6$ – strength index of a glass pane having the width b and thickness t, M_{cr} – critical moment at lateral-torsional buckling of a bar subjected to bending and having the same cross section as above.

1.3. BEARING CAPACITY FORMULAE FOR BARS MADE OF LAMINATED GLASS

In the case of bars made of laminated glass two or three ply panes are used, having the dimensions indicated in Fig. 4. The bonding layer is characterized by high susceptibility to rheological phenomena, which, at the stage of engineering analyses may be accounted for by reduced geometrical characteristics. The formulae for bearing capacity of columns made of laminated glass may be derived according to CEN/TC 250 N 1060 [2] from the model of a bar burdened with initial arched imperfection, but with real geometrical properties superseded by effective characteristics

(1.16)
$$\sigma_t = -\frac{N}{\Sigma A_i} + \frac{N\cdot}{W_{\mathrm{i,eff}}}\frac{e_o}{1-\frac{N}{N_{\mathrm{cr,eff}}}} \le f_{\mathrm{u,t}} = 70\,\mathrm{MPa}\ (\text{for HSG})\ \text{or}\ 120\,\mathrm{MPa}\ (\text{for TTG}),$$

(1.17)
$$\sigma_t = -\frac{N}{\Sigma A_i} - \frac{N\cdot}{W_{\mathrm{i,eff}}}\frac{e_o}{1-\frac{N}{N_{\mathrm{cr,eff}}}} \le f_{\mathrm{u,c}} = -500\,\mathrm{MPa}.$$

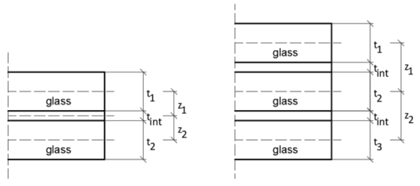

Fig. 4. Denotation of dimensions in laminated glass panes

where the effective strength index of double laminated glass for the i-th pane is:

(1.18)
$$W_{\mathrm{i,eff}} = \frac{1}{\left[\pm\left(\frac{m}{b\cdot t_{\mathrm{i}}} \pm \frac{t_i}{2(I_1+I_2)}\cdot[1-(z_1+z_2)\cdot m]\right)\right]},$$

for

(1.19)
$$m = \frac{G_{\mathrm{int}}b}{t_{\mathrm{int}}}\cdot\frac{z_1+z_2}{E(I_1+I_2)}\cdot\frac{1}{\left(\frac{\pi}{L}\right)^2+\alpha_o^2}, \qquad \alpha_o^2 = \frac{G_{\mathrm{int}}b}{E\cdot t_{\mathrm{int}}}\left(\frac{(z_1+z_2)^2}{I_1+I_2}+\frac{1}{A_1}+\frac{1}{A_2}\right).$$

The effective strength index of triple laminated glass in document [2]. Assuming the above ancillary expressions (t_{int}, G_{int} – thickness and modulus of elasticity in shear of the bonding layer), the effective critical force may be expressed as:

(1.20)
$$N_{\mathrm{cr,eff}} = \frac{\pi^2 E\cdot I_{\mathrm{eff}}}{L^2} = \frac{\pi^2 E}{L^2}\cdot\frac{\Sigma I_{\mathrm{i}}}{1-\psi\cdot m},$$

where:

I_i – moments of inertia of single panes (I_1 or I_2), $\psi = z_1 + z_2$.

The formula for dimensionless bearing capacity of a bar made of laminated glass may be derived based on conditions (1.16) and (1.17):

(1.21) $$\chi_{\text{t,eff}} = \frac{1}{\phi_{\text{t,eff}} + \sqrt{\phi^2_{\text{t,eff}} + \overline{\lambda}^2_{\text{t,eff}}}} \quad \text{for} \quad \phi_{\text{t,eff}} = 0.5\left[-1 + \alpha(\overline{\lambda}_{\text{t,eff}} - \overline{\lambda}_{\text{t,0}}) + \overline{\lambda}^2_{\text{t,eff}}\right],$$

(1.22) $$\chi_{\text{c,eff}} = \frac{1}{\phi_{\text{c,eff}} + \sqrt{\phi^2_{\text{c,eff}} - n_f \cdot \overline{\lambda}^2_{\text{t,eff}}}} \quad \text{for} \quad \phi_{\text{c,eff}} = 0.5\left[1 + \alpha(\overline{\lambda}_{\text{t,eff}} - \overline{\lambda}_{\text{t,0}}) + \overline{\lambda}^2_{\text{t,eff}}\right],$$

The bearing capacity criterion may then be expressed as:

(1.23) $$\frac{N_{\text{Ed}}}{\chi_{\text{eff}} \Sigma A_i f_{\text{u,t}} / \gamma_{\text{M}}} \leq 1{,}0 \quad \text{for} \quad \overline{\lambda}_{\text{t,eff}} = \sqrt{\frac{\Sigma A_i \cdot f_{\text{u,t}}}{N_{\text{cr,eff}}}}\,.$$

2. MEMBERS IN BENDING AND AXIAL COMPRESSION

The formulae for bearing capacity of glass components subjected to complex state of eccentric bending have been restricted in CEN/TC 250 N 1060 [2] to the case of eccentricities introduced during assembly in structures made of monolithic glass. Columns made of heat strengthened glass HSG may be burdened with increased initial deflection having the maximum value of e_o = L/400 + e_p ≈ 1/170, while the components made of thermally toughened glass TTG may have the maximum initial deflection increased to e_o = L/300 + e_p ≈ 1/130.

The increased initial deflections are to be accounted for in the formula (1.13) and introduced into the dimensionless formulae for bearing capacity of columns χ_t according to (1.11) and χ_c according to (1.12). The formula for buckling coefficient (1.14) with imperfection parameters listed in the pre-code [2], having the values of α = 1.0 and $\overline{\lambda}_{\text{t,0}} = 0.20$ may be treated as alternative solution.

The bearing capacity problems of columns subjected to eccentric compression or compression and bending have been treated in the pre-code [2] marginally, redirecting the interested reader to specialist literature, though indicating the concept proposed by M. Feldman and K. Langosch cf. [7]. Cited authors derive the appropriate formulae for bearing capacity from the generalized problem of a compressed column burdened with initial arched imperfection $e(x)$ compounded by bending induced deflections $f_M(x)$. The strength conditions in the most stressed cross section then take the following form:

(2.1) $$\sigma_t = -\frac{N}{A} + \frac{N \cdot e_o + M_Q^I}{W_y} \frac{1}{1 - \frac{N}{N_{cr}}} \le f_{u,t} = 70\ \text{MPa (for HSG) or } 120\ \text{MPa (for TTG)},$$

(2.2) $$\sigma_c = -\frac{N}{A} - \frac{N \cdot e_o + M_Q^I}{W_y} \frac{1}{1 - \frac{N}{N_{cr}}} \le f_{u,c} = -500\ \text{MPa}.$$

Transforming the formulae (2.1) and (2.2) to dimensionless bearing capacity, one obtains the formulae for generalized instability coefficients:

(2.3) $$\chi_t(\mu_t) = \frac{1 - \mu_t}{\phi_t + \sqrt{\phi_t^2 + \bar{\lambda}_t^2 (1 - \mu_t)}} \quad \text{for } \phi_t = 0{,}5\left(-1 + \alpha(\bar{\lambda}_t - \bar{\lambda}_o) + \bar{\lambda}_t^2\right),$$

(2.4) $$\chi_c(\mu_t) = \frac{n_f - \mu_t}{\phi_c + \sqrt{\phi_c^2 - \bar{\lambda}_t^2 (n_f - \mu_t)}} \quad \text{for } \phi_c = 0{,}5\left(1 + \alpha(\bar{\lambda}_t - \bar{\lambda}_o) + n_f \cdot \bar{\lambda}_t^2\right),$$

in which the imperfection parameters are equal to α = 0.430 for HSG glass, α = 0.329 for TTG glass and $\bar{\lambda}_o = 0$ for both curves, according to [6]. The parameter μ_t has the following form in formulae (2.3) and (2.4):

(2.5) $$\mu_t = \frac{M_Q^I}{M_{el}} = \frac{M_Q^I}{W_y \cdot f_{u,t}},$$

where M_Q^I – extreme bending moment determined according to the 1st order theory, the remaining denotations used in (2.3) and (2.4) are identical to those assumed in chapter 1. The ultimate bearing capacity condition for columns subjected to eccentric compression is thus expressed by the following formula:

$$\text{(2.6)} \qquad \frac{N}{\chi_t(\mu_t)\cdot N_{el}} = \frac{N}{N_{el}} \frac{\phi_t + \sqrt{\phi_t^2 + \left(1 - \dfrac{M_Q^I}{M_{el}}\right)\bar{\lambda}_t^2}}{1 - \dfrac{M_Q^I}{M_{el}}} = \frac{\chi_t}{\chi_t(\mu_t)\cdot \chi_t} \cdot \frac{N}{N_{el}} = 1,$$

may be linearized to the following form:

$$\text{(2.7)} \qquad \frac{\chi_t(\mu_t)}{\chi_t} + \mu_t = \frac{N}{\chi_t \cdot N_{el}} + \frac{M_Q^I}{M_{el}} = 1.$$

.

a) b)

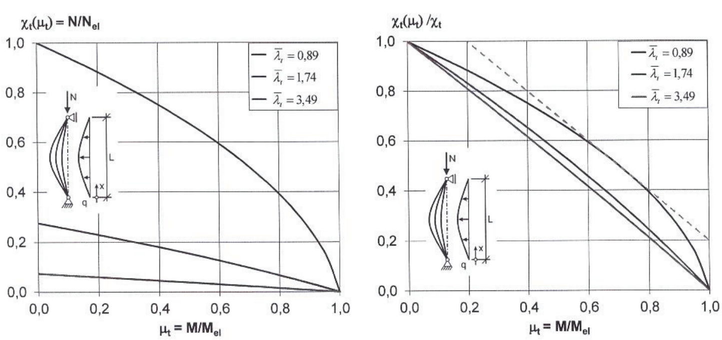

Fig. 5. Interaction curves in the dimensionless coordinates: a) ($N/N_{el} – M/M_{el}$), b) ($N/\chi_t{\cdot}N_{el} – M/M_{el}$).
Source: K. Langosch [6]

The denotation $\chi_t = \chi_t\,(\mu_t = 0)$ has been assumed in the formulae (2.6) and (2.7), i.e. it is a buckling coefficient for a compressed column according to (2.3). The graphs of interaction curves according to (2.6) and [6] for selected relative slendernesses are depicted in Fig. 5. Fig. 5/a depicts the solution in (N/N_{el} – M/M_{el}) coordinates, while Fig. 5/b depicts the solution in ($N/\chi_t{\cdot}N_{el}$ – M/M_{el}) coordinates.

3. SUMMARY

The bearing capacity formulae juxtaposed in this paper according to the pre-code [2] for monolithic or laminated glass bars subjected to compression or bending raise doubts in terms of modeling assumptions listed in chapter 1.1, results of research documented in analytical works [3, 5, 7, 9],

[10], as well as experimental verification [8]. The bearing capacity of monolithic glass columns has been verified experimentally in 2004 in the dissertation of A. Lubile [8]. The later theoretical analyses, for instance [6, 7], as well as the pre-code CEN/TC 250 N 1060 [2] refer to the results of these experiments. Fig. 3 depicts the scanned results of experiments performed by A. Lubile, as cited in [6], and denoted with crosses. The results of these experiments do not justify the application of Ayrton-Perry formula according to the formulas (1.11) or (1.14) to interpolate the experimental results. In addition, the analysis of curves (1.11) indicates, that the bearing capacity of glass columns does not depend on the glass type, as the differences between the bearing capacity curves obtained for HSG and TTG glass are negligible. According to the author of this paper, this is the result of incorrectly assumed model of the structure used to analyze buckling of glass columns. The Ayrton-Perry curves yield correct results for structures made of elastic-plastic material, like steel exhibiting clear yield limit. The continuous curve according to (1.11) or (1.14) with inflection point for average slendernesses correctly describes the bearing capacity of steel columns, but for glass ones loses cognitive value. The linear elastic material model depicted in Fig. 1 is appropriate for glass structures. The glass is a linearly elastic material until brittle failure. It does not exhibit plastic behavior and thus local stress concentrations may not be relieved by redistribution of internal forces, as is the case in case of steels exhibiting definite yield limit. The bearing capacity of columns made of such material is quite well explained by the buckling theorem due to Euler. The dimensionless Euler curve depicted in Fig. 3 does not depend on the glass type and is expressed by the following formula:

$$\chi_{\text{perf}} = \frac{1}{\bar{\lambda}^2}, \text{ where relative slenderness: } \bar{\lambda} = \sqrt{\frac{A \cdot f_{\text{u,t}}}{N_{\text{cr}}}}. \tag{3.1}$$

An analysis of the experimental results by A. Lubile [8], indicated with crosses in Fig. 3 shows, that the theoretical curve according to (3.1) yields values slightly overestimated with respect to the experiment. The glass structures in bearing capacity and stability limit states are characterized by large deformations, which in turn affect the results of statical and strength analyses. Thus when formulating the Euler problem for glass columns, one should account for the exact curvature of bar axis and solve the problem using appropriate computer programs. At the level of engineering analyses, with small modeling errors, one may alternatively use the simple solutions of reliability theory. In particular when dimensioning glass columns by the method of load and bearing capacity according to code PN-EN 1990, one should reduce the critical force of flexural buckling N_{cr}

applying the modeling error coefficient γ_{Rd}, i.e. in order to obtain satisfactory agreement between experimental results depicted in Fig. 3 and the theory one may modify the slenderness formula (3.1) to take the following shape:

$$\bar{\lambda} = \sqrt{\frac{A \cdot f_{\mathrm{u,t}}}{N_{\mathrm{cr}} / \gamma_{\mathrm{Rd}}}} = \sqrt{\gamma_{\mathrm{Rd}}} \cdot \bar{\lambda}_{\mathrm{t}}, \tag{3.2}$$

The buckling coefficient according to (3.1) may then be expressed as:

$$\chi_{\mathrm{t}} = \frac{1/\gamma_{\mathrm{Rd}}}{\bar{\lambda}_{\mathrm{t}}^2} = \frac{\eta}{\bar{\lambda}_{\mathrm{t}}^2} \; . \tag{3.3}$$

The modeling error coefficient γ_{Rd} in formula (3.3) may be determined by matching the values of function (3.3) and function (1.14) for the relative slenderness $\bar{\lambda}_t = 1{,}0$, this approach yields the value γ_{Rd} = 1.242. Fig. 6 depicts the buckling curves of glass columns according to the recommendations of pre-code [2] and formula (3.3) for η = 1/1.242 = 0.805.

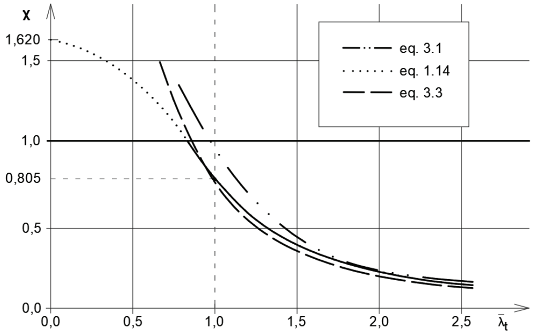

Fig. 6. Buckling curves for glass columns according to (1.14) and formula (3.3)

A proposal to derive the formula for eccentric compression from the model of compressed bar with initial imperfection compounded by deflection due to bending according to the formulae (2.1) and (2.2) is as controversial as formulae in the case of compressed glass columns. The convex interaction curves *M-N* depicted in Fig. 5, appropriate for bars made of materials exhibiting elastic-plastic behavior, and not for glass structures are a result of such modeling. For glass structures, due to large deflections, the stiffening principle should be renounced. The solution of such problem yields concave (not convex) interaction boundaries, which in engineering calculations may be simplified to the following:

$$\left(\frac{N}{\chi_{\mathrm{t}} \cdot N_{\mathrm{el}}}\right)^{\xi} + \frac{M_{\mathrm{Q}}^{\mathrm{I}}}{M_{\mathrm{el}}} = 1, \tag{3.4}$$

where the buckling coefficient is assumed according to (3.4), and exponent $\xi \leq 1$ may be determined by model experiments. One should note, that the empirical basis necessary to verify the bearing capacity models for columns made of monolithic and laminated glass is insufficient and should be expanded or complemented by in-depth numerical modeling.

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LIST OF FIGURES AND TABLES:

Fig. 1. Physical model for the low carbon steel and glass according to pre-code [2]

Rys. 1. Model fizyczny stali niskowęglowej i szkła wg pre-normy [2]

Fig. 2. Bar model with initial arched deflection assumed for bearing capacity analysis

Rys. 2. Model pręta z początkowym wygięciem łukowym przyjęty w analizie nośności

Fig. 3. Buckling curves, from the top: according to Euler, formulae (1.11) and (1.14). Source [2]

Rys. 3. Krzywe wyboczenia od gory: wg Eulera, wzorów (1.11) i (1.14). Żródło [2]

Fig. 4. Denotation of dimensions in laminated glass panes

Rys. 4. Oznaczenia wymiarów szkła warstwowego

Fig. 5. Interaction curves in the dimensionless coordinates: a) $(N/N_{el} – M/M_{el})$, b) $(N/\chi_{f} \cdot N_{el} – M/M_{el})$. Source: K. Langosch [6]

Rys. 5. Krzywe interakcji we współrzędnych bezwymiarowych: a) $(N/N_{el} – M/M_{el})$, b) $(N/\chi_{f} \cdot N_{el} – M/M_{el})$. Źródło: K. Langosch [6]

Fig.6. Buckling curves for glass columns according to (1.14) and formula (3.3)

Rys. 6. Wykresy krzywych wyboczenia słupów wg wzoru (1.14) i formuły (3.3)

Tab. 1. Mean strength and material coefficient for tempered glass according to [11]

Tab. 1. Wytrzymałość średnia i współczynnik materiałowy szkła hartowanego wg pracy [11]

FORMUŁY NOŚNOŚCI WYBOCZENIOWEJ ELEMENTÓW KONSTRUKCJI SZKLANYCH

Słowa kluczowe: *szkło, nośność, wytrzymałość, wyboczenie sprężyste, zwichrzenie*

STRESZCZENIE. Rozwój nowoczesnego budownictwa i związane z tym poszukiwanie nowych rozwiązań w zakresie estetyki i funkcjonalności budynków reprezentacyjnych w centrach dużych miast, stało się przyczyną zainteresowania szkłem. Atrakcyjność szkła jako materiału budowlanego wynika z faktu, że łączy ono w sobie przeźroczystość i estetyczny wygląd z innymi cechami użytkowymi. Jego zastosowanie nadaje nowoczesny wygląd elewacjom budynków i polepsza komfort przebywania w pomieszczeniach, nie ograniczając przy tym naturalnego oświetlenia dziennego, Wdrożenie nowej, wysokowydajnej technologii produkcji szkła płaskiego float, w powiązaniu z rosnącymi wymaganiami budownictwa, dotyczącymi nowych funkcji szkła, doprowadziło do znacznego rozwoju teorii konstrukcji szklanych, por. prace [1, 3, 4, 5, 9, 10]. Wieloletnie badania naukowe prowadzone w krajach Unii Europejskiej zostały zwieńczone opracowanym dokumentem CEN/TC 250 N 1050 [2], zredagowanym w ramach prac Europejskiego Komitetu Normalizacyjnego nad drugą edycją Eurokodów. W wydaniu tym zaproponowano poszerzenie pierwszej edycji między innymi o rekomendacje w/z projektowania konstrukcji szklanych, a w szczególności o nowoczesne procedury w zakresie obliczania konstrukcji budowlanych szklanych. W Polsce zaproponowane w pre-normie [2] procedury nie są powszechnie znane, a ich implementacja do norm projektowania wymaga przeprowadzenia weryfikacji krajowej, co podejmuje niniejsza praca.

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