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Performance analysis of an eight channel demultiplexer using a 2D-photonic crystal quasi square ring resonator

V. Kannaiyan [a,*], R. Savarimuthu [a,*], S.K. Dhamodharan [b]

[a] Department of Electronics and Communication Engineering, Mount Zion College of Engineering and Technology, Pudukkottai, Tamil Nadu 622507, India
[b] Department of Electronics and Communication Engineering, National Institute of Technology, Tiruchirappalli, Tamil Nadu 622201, India

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ABSTRACT

Recent years, the design of photonic crystal (PC) based optical devices is receiving keen interest in research and scientific community. In this paper, two dimensional (2D) PC based eight channel demultiplexer is proposed and designed and the functional characteristics of demultiplexer namely resonant wavelength, transmission efficiency, quality factor, spectral width, channel spacing and crosstalk are investigated. The demultiplexer is designed to drop the wavelength centred at 1537.6 nm, 1538.5 nm, 1539.4 nm, 1540.4 nm, 1541.2 nm, 1541.9 nm, 1542.6 nm and 1543.1 nm. The proposed demultiplexer is primarily composed of bus waveguide, drop waveguide and quasi square ring resonator. The quasi square ring resonator and square ring micro cavity (inner rods) are playing a vital role for a desired channel selection. The operating range of the devices is identified through a photonic band gap (PBG) which is obtained using a plane wave expansion (PWE) method. The functional characteristics of the proposed demultiplexer are attained using a 2D finite difference time domain (FDTD) method. The proposed device offers low crosstalk and high transmission efficiency with ultra-compact size, hence, it is highly desirable for DWDM applications.

1. Introduction

Recent years, photonic crystal (PC) is an emerging technology for developing ultra-compact optical devices through periodic nanostructures. PCs provide low dispersion loss, high speed and small in size without diminishing the performance of the device. The PC regulates motion of photons through a photonic band gap (PBG). The PBG is the heart of the PC. The range of wavelength where the density of states or propagation of light inside the structure becomes zero is termed as PBG. A PBG [1] in a PC could control and establish beams of light, in the same way semiconductors control electric currents. Hence the PBG act as an optical insulator (OI) to a certain wavelength range. The defects introducing in a PC are employed to break up the PBG. This causes to pass the light signal through in it. Based on the nature of the defects, the localized mode is controlled in the PCs. The defects [1] are categorized as line, point and surface defects. The design of devices is made by utilizing defects in a PC structure. The defect is created by changing/removing the structural parameters such as radius, refractive index and lattice constant. If any one of the above mentioned parameters is modified or removed in a single rod, then it is termed as a point defect. The line defects are derived by removing or altering a row/column of rods. The surface defect is created by changing structural parameters of a rod in a surface of a PC lattice. The line and point defects are used to realize the PC based optical devices for photonic integrated circuits (PIC).

Typically, PCs are divided as one dimensional PC (1DPC), two dimensional PC (2DPC) and three dimensional PC (3DPC). The 2DPC has a complete generation of PBG than 1DPC, and less memory is required to perform simulation and high speed of operation is compared with a 3DPC. Hence, the 2DPC is preferred for device design. The 2DPC structure is classified as periodic array of air pores (triangular lattice) and dielectric rods in air medium (square lattice). In a triangular lattice [2], air pores depth and shape are strongly introducing the radiation losses. Hence, a unique design of air pores in a triangular lattice is very difficult. Alternatively, the 2DPC square lattice provides a simple structure, low propagation loss, accurate calculation of PBG, easy confinement of mode while signal travelling inside the structure. Hence, the proposed structure is designed using a 2DPC square lattice structure.

2DPC is suitable to design almost all kinds of optical devices and useful to integrate the photonic devices in a single chip. The photonic devices namely LED [3], sensors [4], filters [5–8], multi-

* Corresponding authors.
E-mail addresses: venkatachalamece@gmail.com (V. Kannaiyan), mail2robinson@gmail.com (R. Savarimuthu).

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plexers [9], demultiplexers [10–24], etc. are already reported. In the literature, the demultiplexer is designed using square lattice and triangular lattice. Rostami et al. [10] designed a four channel DWDM demultiplexer using a T shaped resonant cavity and the average transmission efficiency of 49% was reported. The seven channel demultiplexer was designed through a triangular lattice where the channels were dropped by having a unique dropping path with a different radius of holes at the border [11]. The transmission efficiency started from 24% to 56% from channel 1 to channel 8. In Alipour Banaei et al. [12,13], four X shaped ring resonators with different rod radius in each resonator were utilized to select the required channel. Further, the scatterer rod was placed in each corner of the X shaped ring resonator. A T-shaped structure with four dropping paths that have different dimension of resonant cavity to select the wavelength was suggested [15]. The transmission efficiency of all the channels was very low. Three quasi square ring resonators with a unique refractive index that are arranged in a cascaded manner to drop the required wavelength [16] were reported. Hamed Alipour et al. [17] designed a demultiplexer using quasi square ring resonators, here, the resonant wavelength is selected through an inner rod with a different lattice constant. Djavid et al. [18] and Rakhshani et al. [19] designed and investigated the characteristics of the demultiplexer through ring resonators with a different refractive index and obtained 82%, 90% of output efficiency. Elliptical resonator and rectangular ring resonator based demultiplexer were designed to select different channels [20].

Mehdizadeh et al. [21] designed the eight channel demultiplexer using different dimensions of resonant cavities in a square lattice. Three quasi square ring resonators with a distinctive inner rod radius are used to select three respective channel wavelengths [22]. Two channels demultiplexer based on photonic crystal ring resonator was designed by Ghorbanpour et al. [23]. The functional characteristics of the reported demultiplexer are listed in Table 3. From the literature survey, it is clearly noticed that the reported demultiplexers were designed by using different shapes of ring resonators and defects. The required channels are dropped by altering the refractive index value, radius and lattice constant of the structure. Though, there all several attempts that are made. The transmission efficiency and crosstalk are not reasonably good for real time applications. In order to mitigate the above reported issues, the quasi square ring resonators based eight channel demultiplexer is proposed and designed.

In this paper, a quasi-square ring resonator is used to construct an eight channel demultiplexer where the odd number of channels and even number of channels are grouped separately in order to minimize the crosstalk. The functional characteristics of the demulitplexer such as resonant wavelength, Q factor, spectral width, channel spacing and crosstalk are investigated. The propagation modes and output response of the proposed device are obtained through a plane wave expansion (PWE) method [25] and a finite difference time domain (FDTD) method [26], respectively.

The rest of this paper is organized as follows. The design of eight the channel demultiplexer is discussed in Section 2. In Section 3, the simulated results and its significance are discussed. Finally, the work is concluded in Section 4.

2. Design of an eight channel demultiplexer

Fig. 1 shows the band diagram of the proposed structure which is obtained using a PWE method. It gives the propagation modes and PBG of the designed demultiplexer. The structural parameters such as lattice constant (a) of 560 nm, radius (r) of 0.1 μm and refractive index (n) of rod 3.24 are considered to design the proposed demultiplexer. It has a TE and TM PBG. The normalized frequency of TE PBG is from 0.31744a/λ to 0.44767a/λ and its corresponding

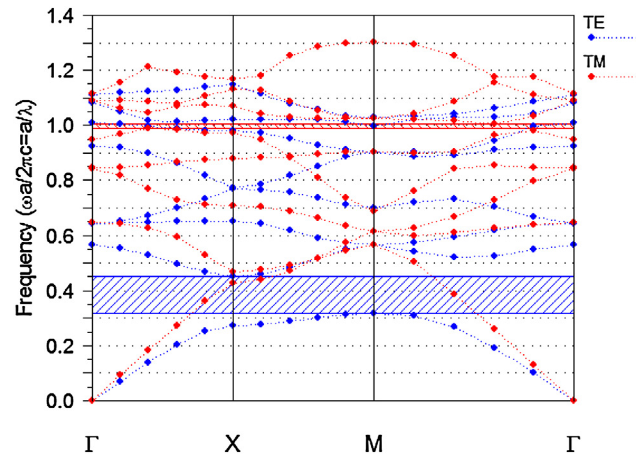

Fig. 1. Band diagram of 2DPC before introducing the defects.

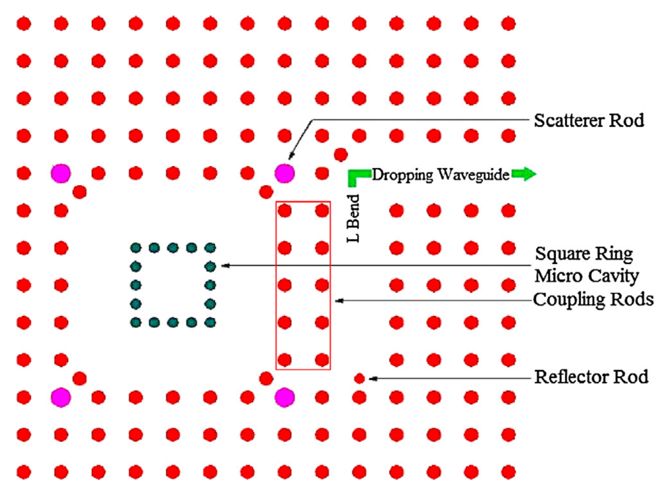

Fig. 2. Schematic representation of single QSRR.

wavelength range is covering from 1251 nm to 1764 nm. The normalized frequency of second TM PBG is spanning from 0.98895a/λ to 1.0052a/λ whose corresponding wavelength range is ranging from 557.1 nm to 566.2 nm. As the wavelength range of the first TE PBG is covering the desired range for DWDM applications, author considered the first TE PBG for further investigation.

Fig. 2 represents the sectional view of single a quasi square ring resonator (QSRR). It consists of QSRR, square ring inner rod, coupling rods, L bend waveguide, scatterer rods and reflector rod.

The QSRR is designed by removing 16 pillars in total in a square shape of X–Z direction and one pillar is inserted at each corner of the square ring to make QSRR. The square ring micro cavity consists of 16 pillars in X and Z direction. Two columns of pillars are positioned left and right side of QSRR which are called as coupling rods. The dropping waveguide is formed by removing pillars (single row and column) in an L shape. The scatterer rod is located at each corner of QSRR and the reflector rod is positioned at the L bend dropping waveguide.

Fig. 3(a) shows three dimensional (3D) views of a single QSRR and its size of about 16 μm^2 and Fig. 3(b) represents a 3D view of a square ring micro cavity whose overall size is of about 2.56 μm^2. The number of rods in the inner ring and outer ring of the QSRR is of 16 rods and 24 rods, respectively.

The proposed QSRR based eight channel demultiplexer is shown in Fig. 4. The proposed demultiplexer is devised by having dielectric pillars of 1392 where 29 rows of rods in X direction and 48 rows of rods in Z direction are positioned. The overall size of the structure is

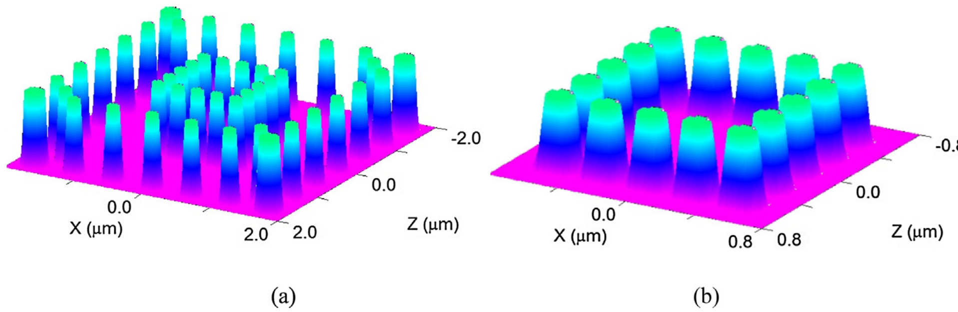

Fig. 3. 3D view of (a) single QSRR and (b) square ring cavity.

of 434.16 μm^2. The demultiplexer primarily consists of single linear or bus waveguide, eight QSRR with square ring shape of inner rods and L bend dropping waveguides. The bus waveguide is common for all the channels, however, the separated QSRR and L bend waveguides are employed to drop a specific channel. The bus waveguide is designed by removing 44 rods as line defects from the centre of the bottom of the demultiplexer towards to *Z* direction. Eight QSSRs are positioned around the bus waveguide where each QSSR is responsible for particular channel selection. The square ring shaped inner rods with 72 nm are placed inside the QSSR. The radius of an inner rod is kept 72 nm as constant for all the channels. The refractive index of inner rods of the QSRR is varied from 3.16, 3.26, 3.36, 3.46, 3.56, 3.66, 3.76 and 3.86 for channel 1 ($\lambda 1$) to channel 8 ($\lambda 8$). The selected channels are coupled to an L bend dropping waveguide through the two columns of coupling rods and the reflector rod is located at the end of the L bend waveguide with a 50 nm spacing. The reflector rod size is varied from one channel to other by 1 nm i.e., 70 nm for channel 1 ($\lambda 1$) and 77 nm for channel 8 ($\lambda 8$). The rod reflects the selected channel properly and sends to its respective output ports. The scatterer rod is positioned at each corner of the QSSR (pink colour) that is used to reduce the counter propagation loss. The radius of the scatterer rod is of 145 nm for all the channels.

The size of ring resonator remains same for all channels however, refractive index of each square ring cavity (inner rods) and reflector rod radius is different. The channel selection is primarily carried out by having the QSRR with a different refractive index of

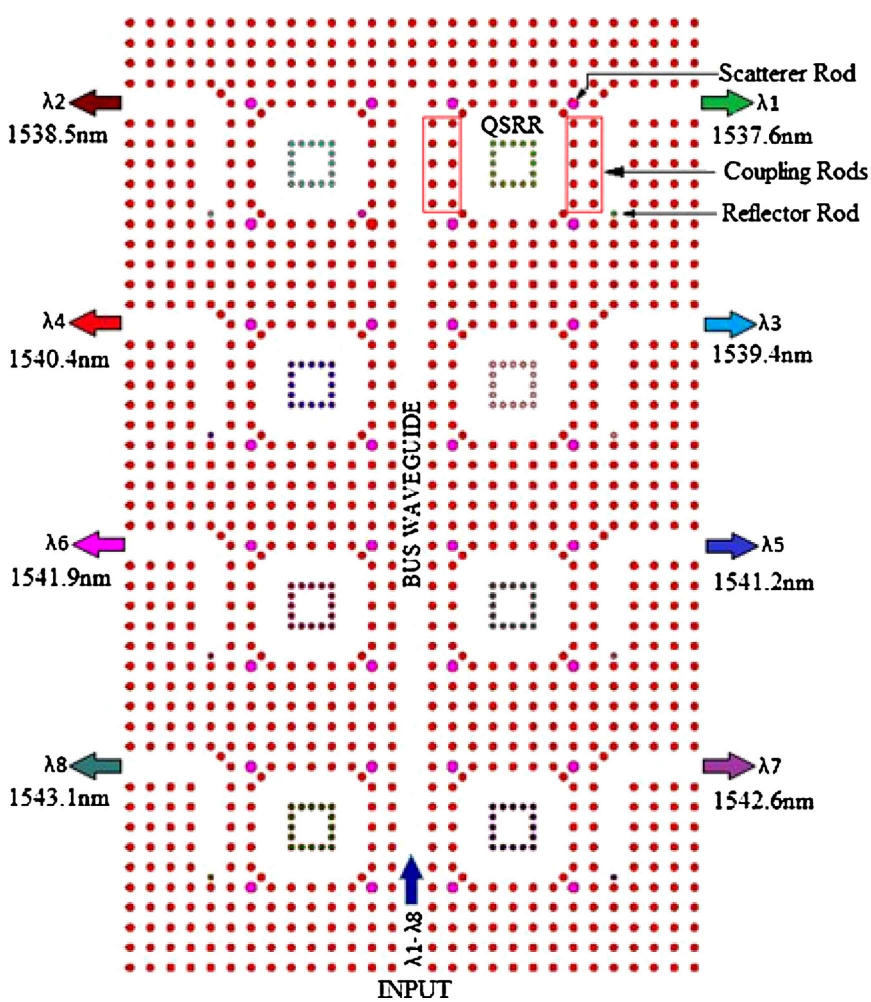

Fig. 4. Schematic representation of proposed 2DPC QSRR based eight channel demultiplexer.

inner rod and a different size of a reflector rod. The output ports are grouped into odd ports and even ports in order to minimize the crosstalk and enhance the transmission efficiency.

3. Simulation results and discussion

The Gaussian light signal is launched at the bottom source end of the structure. The light signal is entered into the bus waveguide and propagates inside in the QSRR (at resonance condition) through coupling rods. At off resonance the signal is travelled to the bus waveguide and reflected to the source, however, at on resonance, the signal is coupled into the respective QSRR and entered into the L bend dropping waveguides which in turn output port. The QSSR and variation in refractive index of the inner core are selecting the respective channel which turn it is dropped in the L bend dropping waveguide through a reflector rod.

Fig. 5(a) depicts a single channel output spectrum for channel 3 and its resonant wavelength and transmission efficiency is $\lambda 3 = 1539.4$ nm, 97%. Fig. 5(b) shows the output spectra of the proposed QSRR based eight channel demultiplexer. The 2D FDTD method is used to obtain the normalized output spectra of the proposed eight channel demultiplexer.

The perfectly matched layer (PML) as absorbing boundary condition is incorporated to minimize the reflection from the launching field. The width of PML is of 500 nm. The following solution is used to compute the time steps (Δt) required for simulation in order to provide the stability of the numerical solution. The time step (Δt) is calculated as follows.

$$\Delta t \leq \frac{1}{c\sqrt{(1/\Delta x^2) + (1/\Delta z^2)}} \tag{1}$$

where c is the velocity of light in free space, Δx and Δz are the space steps along x, z directions. The 2D FDTD could follow this rule to manipulate required time steps and to perform channel selection. The required time step is of about $\Delta t = 0.0248$ of 22 MB memory size for the proposed QSRR eight channel demultiplexer. The resonant wavelengths of the demultiplexer are of $\lambda 1 = 1537.6$ nm, $\lambda 2 = 1538.5$ nm, $\lambda 3 = 1539.4$ nm, $\lambda 4 = 1540.4$ nm, $\lambda 5 = 1541.2$ nm, $\lambda 6 = 1541.9$ nm, $\lambda 7 = 1542.6$ nm and $\lambda 8 = 1543.1$ nm. From Fig. 5(b), it is noticed that resonant wavelength, spectral width, channel spacing, transmission efficiency and Q factor of the channel 3 ($\lambda 3$) are of 1539.4 nm, 0.8 nm, 1 nm, 97%, and 1924, respectively. The output spectra of the proposed eight channel demultiplexer in dB scale are shown in Fig. 5(c). It is obtained by taking logarithmic value of normalized transmission value which is calculated from Fig. 5(b). The impact of Fig. 5(b) and (c) is remaining the same. However, Fig. 5(c) is giving crosstalk value of the demultiplexer clearly.

The electric field distribution of the proposed demulitplexer is shown in Fig. 6. At 1541.2 nm ($\lambda 5$), the signal from the bus waveguide is coupled into QSRR and reached into the output waveguide. Alternatively, the signal is not coupled into other ring resonator owing to off resonance.

The maximum performance parameters of the proposed eight channel demultiplexer namely transmission efficiency (η), channel spacing, spectral width and Q factor are of 98%, 1 nm, 0.9 nm and 2205, respectively. The refractive index of square ring cavity, resonant wavelength, spectral width, channel spacing, transmission efficiency and Q factor of the proposed eight channel demulitplexer are listed in Table 1. The arrived value of Q factor is highly sufficient for real time applications. From Table 1, it is noticed that, the observed functional parameters of eight channel demultiplexer is sufficient for DWDM applications.

Fig. 7 shows the impact of a resonant wavelength shift while varying the refractive index of the square ring cavity. The refractive index of inner rods in the square ring resonant cavity is changed from 3.16 to 3.86 with steps 0.10 from channel 1 ($\lambda 1$) to channel 8 ($\lambda 8$). It is investigated that there is about 0.7 nm of a resonant wavelength shift is noticed, for every increasing 0.10 value of refractive index of the square ring cavity. It means that dielectric constant of each square ring cavity is directly proportional to resonant wave-

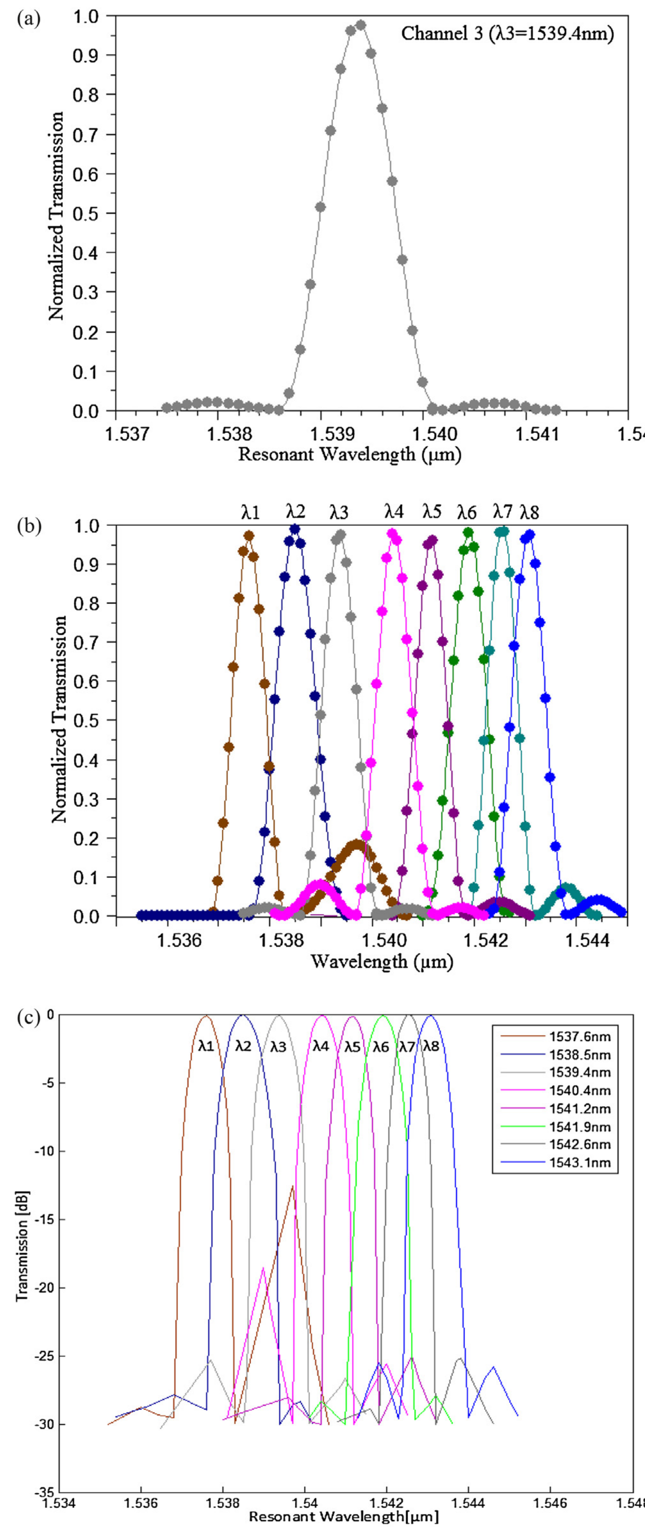

Fig. 5. Output spectral response of the proposed eight channel demultiplexer for (a) single channel ($\lambda 3 = 1539.4$ nm), (b) normal scale and (c) dB scale.

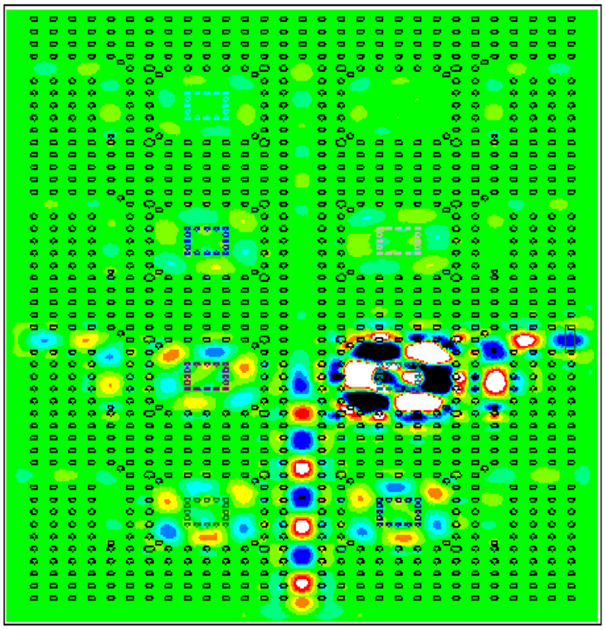

Fig. 6. Electric field distribution of proposed eight channel demultiplexer at λ5 = 1541.2 nm.

length of the channels. Hence, authors incorporated the unique refractive index value in the inner rod of the QSSR.

Generally, the quality of the demultiplexer is evaluated by crosstalk. If the demultiplexer offers lower crosstalk then the performance of the demultiplexer is better one. The crosstalk value of the proposed demulitplexer is listed in Table 2. From Table 2, it is clearly observed that the acceptable crosstalk among the output ports is noticed. The lower crosstalk value of the proposed demultiplexer is − of 28.9 dB at channel 5 (λ5) and channel 7 (λ7).

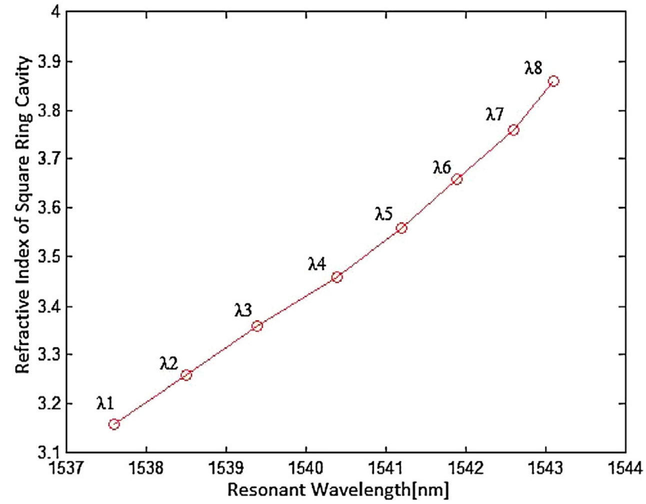

Fig. 7. Effect of resonant wavelength vs refractive index of square ring cavity.

The functional parameter of the proposed demulitplexer is compared with the reported one which is listed in Table 3. From Table 3, it is noticed that the overall size, spectral width, channel spacing, transmission efficiency and Q factor of eight channels proposed

Table 1
Resonant wavelength, spectral width, channel spacing, transmission efficiency and Q factor of proposed eight channel demultiplexer.

Channel (λ)	Refractive index of square ring cavity	Resonant wavelengths (λo) (nm)	Spectral width (nm)	Channel spacing (nm)	Transmission efficiency (η) (%)	Q factor
λ1	3.16	1537.6	0.8	0.9	97	1922
λ2	3.26	1538.5	0.9	0.9	98	1709
λ3	3.36	1539.4	0.8	1.0	97	1924
λ4	3.46	1540.4	0.7	0.8	98	2200
λ5	3.56	1541.2	0.7	0.7	96	2202
λ6	3.66	1541.9	0.8	0.7	98	1927
λ7	3.76	1542.6	0.7	0.5	98	2204
λ8	3.86	1543.1	0.7	0.5	97	2205

Table 2
Crosstalk value of the proposed demultiplexer.

Crosstalk [dB]	λ1	λ2	λ3	λ4	λ5	λ6	λ7	λ8
λ1	–	−4.11	–	–	–	–	–	–
λ2	−4.15	–	−3.6	–	–	–	–	–
λ3	–	−3.7	–	−7.9	–	–	–	–
λ4	–	–	−7.9	–	−2.8	−20.4	–	–
λ5	–	–	–	−2.9	–	−3.0	−28.6	–
λ6	–	–	–	−20.8	−2.9	–	−2.3	−13.8
λ7	–	–	–	–	−28.9	−2.3	–	−1.5
λ8	–	–	–	–	–	−13.9	−1.5	–

Table 3
Comparison of functional characteristics of proposed demultiplexer with existed one.

Authors/year	Lattice structure	Dimension (μm^2)	Number of channels	Spectral width (nm)	Channel spacing (nm)	Transmission efficiency (%)	Quality (Q) factor	Crosstalk (dB)
Nikhil Deep Gupta et al. [15], 2014	Square	484	4	0.2	0.8	Less than 7%	7800	[a]
Hamed Alipour Banaei et al. [17], 2015	Square	[a]	4	0.5	3	95	2600	−19
Djavid et al. [18], 2008	Square	[a]	4	30	28	82	[a]	[a]
Farhad Mehdizadeh, et al. [21], 2015	Square	495	8	0.7	19.5	96	2200	−11.2
This work	Square	434.16	8	0.8	0.7	98	2037	−28.9

[a] Not discussed.

demultiplexer are better than the reported one. In Ref. [15], the Q factor was very high, however, the transmission efficiency was poor. The size was also very high. Alternatively, in Ref. [21], the Q factor is about 2200 with 96% of transmission efficiency. The higher level crosstalk value and larger in size was reported. Above all, the proposed demultiplexer provides better performance in all aspect, hence, it could be suitable for all optical photonic circuits.

4. Conclusion

The quasi square ring resonator based eight channel demultiplexer is proposed and designed using two dimensional photonic crystals. The functional characteristics of eight channel demultiplexer such as transmission efficiency, resonant wavelength, spectral width, channel spacing, crosstalk and Q factor are investigated using the 2D FDTD method. The inner rod of the quasi square ring resonator is having a unique refractive index which facilitates desired channel selection. The output ports are grouped into odd channels and even channels separately in order to reduce crosstalk performance of the demultiplexer. The average channel spacing and spectral width of channels are almost equal to 0.7 nm and the mean transmission efficiency is of about 98%. The maximum Q factor is obtained at channel 8 ($\lambda 8$) which is of about 2205. The overall size of the proposed device is of 434.16 μm^2. Hence, it is suitable photonic integrated circuit for DWDM applications.

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