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ANALYSIS OF THE DISTRIBUTION INFLUENCE OF THE DENSITY OF COST-FORMING FACTORS ON RESULTS OF THE LCCA CALCULATIONS

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The paper evaluates the relationship between the selection of the probability density function and the construction price, and the price of the building's life cycle, in relation to the deterministic cost estimate in terms of the minimum, mean, and maximum. The deterministic cost estimates were made based on the minimum, mean, and maximum prices: labor rates, indirect costs, profit, and the cost of equipment and materials. The net construction prices received were given different probability density distributions based on the minimum, mean, and maximum values. Twelve kinds of probability distributions were used: triangular, normal, lognormal, beta pert, gamma, beta, exponential, Laplace, Cauchy, Gumbel, Rayleigh, and uniform. The results of calculations with the event probability from 5 to 95% were subjected to the statistical comparative analysis. The dependencies between the results of calculations were determined, for which different probability density distributions of price factors were assumed. A certain price level was assigned to specific distributions in 6 groups based on the t-test. It was shown that each of the distributions analyzed is suitable for use, however, it has consequences in the form of a final result. The lowest final price is obtained using the gamma distribution, the highest is obtained by the beta distribution, beta pert, normal, and uniform.

Keywords: LCCA, statistical calculations, probability density function

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1. INTRODUCTION

The life cycle cost analysis (LCCA) is an analytical tool for assessing the value of alternative investments. It covers the total cost of ownership, operation, and maintenance for a given building during a specific life span. The papers [6, 9, 10,11, 12, 13] presents LCCA analyses that account for risk and apply fuzzy logic. The calculation requirements are defined in the ISO 15686-5 standard "Buildings and constructed assets – Service life planning – Part 5: Life-cycle costing" [7]. The standard recommends the LCCA calculations for several alternatives to the same construction activity. The analysis should be carried out in accordance with the 5.2 point of the ISO 15686-5 for the period in which the building will maintain the function planned, in accordance with the investor's requirements. If the planned operation time of the facility is longer than 100 years, the calculation should be carried out for a period of 100 years. The shorter the period for which calculations are made, the less accurate and representative the result are, e.g., the future repairs or the recycling of building materials at the end of the building's life cycle are omitted. These costs can be a very heavy financial burden. According to the authors, the LCCA calculations for the period of durability of building's structural elements seem to be advisable. The 8.3 section of the ISO 15686-5 obliges to perform calculations taking into account statistical methods such as the Monte Carlo analysis. The results of the calculations should be given with a probability of 10%, 50%, and 90%. The statistical calculations are possible to perform when the probability density distributions of cost estimates prices are established. The probability density functions of prices in scientific papers are adopted in various ways. The discount rate has a normal distribution in all studies, while the distributions of other costs are different. The prices for the implementation and maintenance of pedestrian routes in [3] were determined by the triangular distribution, the prices of components, energy, and maintenance in [4] were determined by the normal and Uni distribution. In the LCCA [1], loglogistic, lognormal, and Weibull distributions were adopted – distributions were selected using the χ^2 test according to [2]. In [5, 8], the lognormal distributions were adopted to select the correct distribution using a one-way regression analysis. In [14], to calculate the LCCA of a highway, the beta distribution of the probability density of price-forming factors was used. In the paper, 12 probability density distributions were adopted for calculations: triangular, normal, lognormal, beta pert, gamma, beta, exponential, Laplace, Cauchy, Gumbel, Rayleigh, and uniform. With regard to cost estimates for investment construction activities, it was assumed to create 3 cost estimates based on minimum, mean, and maximum values of price-forming factors. For 3 values it is not possible to analyze the correctness of probability density distributions with the χ^2 test. The probabilistic approach provides more realistic information on the uncertainty of results and enables a more useful analysis of the potential benefits of design options. The probabilistic method described in this paper is based on the analysis of uncertainty and sensitivity using the Monte Carlo (MC) method. The aim of the paper is to determine what consequences cause the assumed to the calculations, the probability density distribution of the cost of building construction. The performed analysis of the results obtained may facilitate the selection of the probability density distribution in specific conditions by people, who conduct the LCCA analyzes.

2. Calculation Method

The proposed method of calculation consists of the following stages:

Stage 1 – preparation of three cost estimates (minimum, mean, and maximum) of the same construction project; as a result of the calculations, the minimum, mean, and maximum prices were obtained.

Stage 2 – selection of types of the probability density distributions for three cost estimates; all available distributions in the Risky Project Professional software were selected for calculation: triangular, normal, lognormal, beta pert, gamma, beta, exponential, Laplace, Cauchy, Gumbel, Rayleigh, and uniform.

Stage 3 – calculating the prices of construction works for various probability values of completing the activities in the price given by the probability distributions; probabilities 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95% were used for calculations.

Stage 4 – graphical evaluation of the results obtained using the STATISTICA software, statistical calculations of the correlation coefficients and values of the t-test differences coefficients.

Stage 5 – analysis of the calculation results.

3. Calculation of Value of Construction Cost

The calculations were carried out in a deterministic and probabilistic approach.

3.1. Deterministic calculations

The cost estimate was based on the superstructure project of a multi-family building. The construction project was divided into 30 processes. The Norma Pro software was used for the cost estimation. The differentiation of cost estimates consisted of the subsequent adoption of minimum, mean, and maximum cost estimate and material, equipment, and labor prices. Indirect costs were equal to 40, 66, and 80%, materials purchase costs were equal to 1, 6, and 13%, profit – 5, 11, and 20%, respectively. The results of net calculations are presented in Table 1.

Table 1. List of construction costs divided into processes; minimum, mean, and maximum values.

No	Costing items	Minimum price [$]	Mean price [$]	Maximum price [$]
1	Demolition of cover	16 484.95	23 268.88	31 794.02
2	Removal of the structure	63 233.77	92 342.89	131 142.30
3	Demolition of masonry structures	68 069.07	96 039.84	131 283.39
4	Exports and disposal of rubble	93 482.72	101 688.60	164 064.05
5	Masonry work	45 382.11	95 837.50	124 246.77
6	Assembly of steel structures	467 846.87	567 578.99	681 453.89
7	Demolition of ceilings	110 399.43	154 333.77	210 177.28
8	Densely ribbed ceilings 24 cm thick	329 157.93	381 181.80	546 230.62
9	Densely ribbed ceilings 29 cm thick	16 927.26	19 602.63	28 090.42
10	Thick ribbed ceilings 26 cm thick	11 272.22	13 053.81	18 706.00
11	Preparation of reinforcement	7 975.24	10 273.82	11 972.47
12	Replenishment of walls and bricklaying of chimneys	24 731.31	52 227.34	67 709.17
13	Lightweight roof cladding	50 733.28	75 123.44	112 981.72
14	Roof slope formwork	37 945.08	47 680.89	60 903.08
15	Ventilation mat	10 907.58	17 808.00	23 073.73
16	Roof covering with galvanized sheet metal for rabbet	94 320.21	120 605.46	153 979.19
17	Roof deflectors	5 782.01	7 993.60	10 933.80
18	Roof gutters	9 993.84	13 547.15	19 232.89
19	Drain pipes	5 729.14	7 951.78	10 870.51
20	Roof accessories - snow fencing	39 857.76	48 320.07	71 465.93
21	Thermal insulation and plasterboard linings	193 356.43	256 024.40	368 080.28
22	Underfloor thermal insulation	429 266.00	461 022.26	872 148.97
23	Underfloor moisture insulation	59 536.25	78 306.10	112 776.53
24	Cement screeds	170 620.73	228 189.49	309 879.68
25	Sealing slurry insulation	2 567.49	14 991.55	16 395.48
26	Gres tiles cladding	145 436.72	226 150.17	366 780.59
27	Wall sockets	66 732.48	89 428.57	123 025.29
28	Ceiling linings with plasterboards	60 731.50	82 655.90	111 287.12
29	Painting	6 102.69	8 769.24	12 899.12
30	Acoustic ceilings	56 029.13	69 935.34	123 867.17
31	SUM	2 700 611.20	3 461 933.28	5 027 451.46

As a result of the calculations carried out, the deterministic net price of the superstructure was obtained: a minimum equal to 2 700 611.20 USD, a mean – 3 461 933.28 USD, and a maximum – 5 027 451.46 USD.

3.2. Probabilistic calculations

The data obtained (minimum, mean, and maximum prices) from the deterministic calculations were used for the probabilistic calculations in the Risky Project Professional software. The different distributions of probability density were given to the same data. The following distributions were used: triangular, normal, lognormal, beta pert, gamma, beta, exponential, Laplace, Cauchy, Gumbel, Rayleigh, and uniform. Table 2 presents the results of calculations: histograms and cumulative distributions for each of the distributions analyzed. The construction costs were then calculated for different probability density distributions with respect to the percent probability of the construction completion. For the calculations, probabilities from 5 to 95% with gradation at 5% were assumed. The calculation results are summarized in Table 3.

Table 2. Histograms and cumulative distributions for the same output data using different probability density functions.

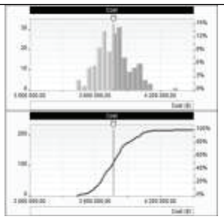
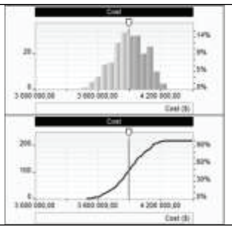
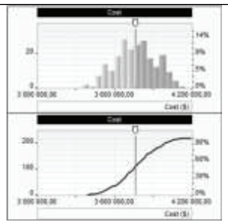
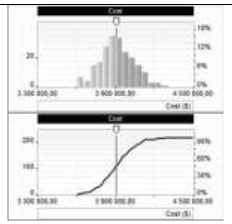
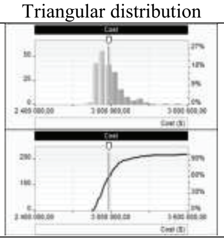
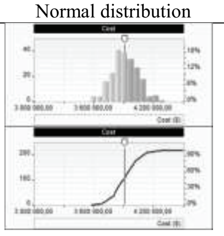
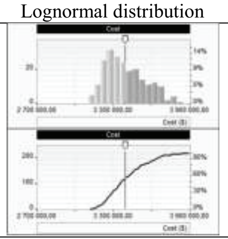
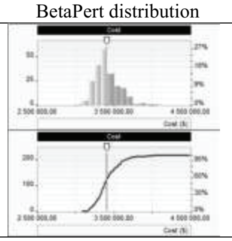
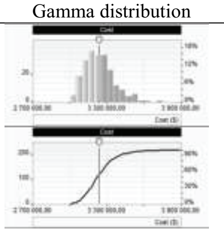
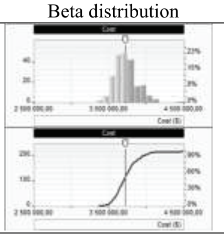
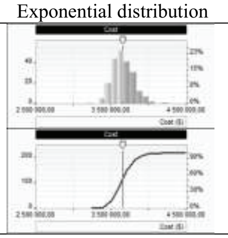
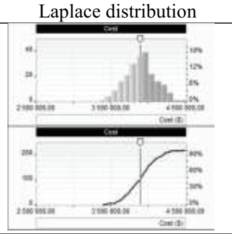
			
Triangular distribution	Normal distribution	Lognormal distribution	BetaPert distribution
			
Gamma distribution	Beta distribution	Exponential distribution	Laplace distribution
			
Cauchy distribution	Gumbel distribution	Rayleigh distribution	Uniform distribution

Table 3. A comparison of the percentage probability of construction prices for different probability density distributions.

Percentile probability	GAMA	CAUCHY	EXPONENTIAL	LAPLACE	RAYLEIGH	LOG NORMAL
[%]	[$]	[$]	[$]	[$]	[$]	[$]
5	2854103	2995929	3153611	3178187	3429602	3501468
10	2862382	3020263	3200368	3224935	3470918	3538258
15	2866682	3039187	3223556	3255068	3512341	3579767
20	2879413	3060906	3240589	3281552	3536980	3631481
25	2889379	3075058	3260916	3307997	3553294	3657562
30	2896372	3091688	3278187	3324624	3573718	3673495
35	2906217	3100574	3295262	3337452	3592524	3695142
40	2912210	3126960	3326648	3350535	3617174	3714156
45	2921955	3137993	3334279	3371656	3626914	3744673
50	2932418	3151415	3350148	3380589	3647541	3760913
55	2938646	3161627	3375316	3398813	3662790	3779238
60	2957384	3179132	3402193	3415391	3683027	3806447
65	2970828	3179132	3431846	3437371	3699022	3831024
70	2979638	3195815	3451504	3459828	3717615	3843713
75	2995796	3219043	3489767	3497757	3747909	3882069
80	3007227	3231522	3529155	3537569	3774779	3905303
85	3032840	3282320	3572936	3582935	3799471	3952575
90	3095089	3323271	3619744	3604134	3849073	3981867
95	3183623	3412826	3664145	3690564	3912647	4025113
Percentile probability	TRIAN GULAR	GUMBEL	UNIFORM	NORMAL	BETA	BETA PERT
[%]	[$]	[$]	[$]	[$]	[$]	[$]
5	3534982	3508496	3569165	3610190	3698856	3698856
10	3586856	3547258	3640567	3666565	3738230	3738230
15	3615475	3580429	3678849	3715363	3781274	3781274
20	3647045	3601439	3713807	3749747	3809459	3809459
25	3665597	3626558	3754803	3787363	3822147	3822147
30	3675590	3637286	3786496	3806482	3836510	3836510
35	3698768	3651037	3804577	3833712	3853257	3853257
40	3721005	3678532	3839147	3848815	3869499	3869499
45	3749020	3692800	3884972	3867790	3888570	3888570
50	3765708	3708152	3904570	3885609	3915133	3915133

55	3781792	3727865	3928117	3906938	3928407	3928407
60	3800116	3748543	3946034	3926531	3944705	3944705
65	3819844	3770867	3973992	3947990	3958621	3958621
70	3833286	3786552	3995871	3975788	3976382	3976382
75	3853020	3805833	4028809	4004764	4002059	4002059
80	3884134	3835825	4066877	4031533	4016619	4016619
85	3938508	3880868	4106559	4078172	4044698	4044698
90	3986283	3938317	4173023	4098998	4085114	4085114
95	4026906	4002965	4228044	4117310	4123311	4123311

4. ANALYSIS OF THE CALCULATION RESULTS

The results of the probabilistic calculations, summarized in Table 3, and the minimum, mean, and maximum prices from the deterministic cost estimates are presented in graphical form in Figure 1.

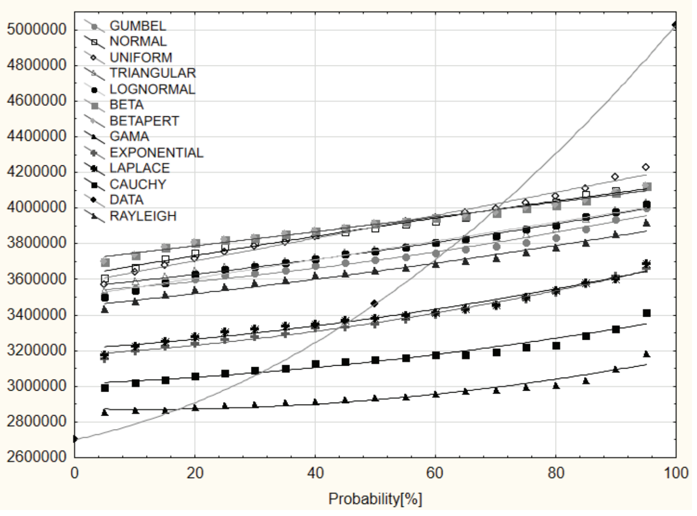


Fig. 1. Graphical presentation of probabilistic and deterministic calculations.

By analyzing the graph in Figure 1, it can be seen that all lines of the probabilistic values (probability density distributions) are strongly linearly correlated with each other. The calculated Poisson correlation coefficients reach values from 0.94 to 1.00. Tab. 4.

Table 4. A summary of the calculations results of the Poisson correlation coefficients for all probability density functions analyzed.

Correlations	Normal	Beta	Betapert	Gama	Cauchy	Exponential	Laplace	Rayleigh	Triangular	Lognormal	Gumbel	Uniform
Gama	0.92	0.94	0.94	1.00	0.99	0.97	0.97	0.96	0.96	0.94	0.97	0.94
Cauchy	0.97	0.98	0.98	0.99	1.00	0.99	0.99	0.99	0.99	0.98	1.00	0.98
Exponential	0.98	0.99	0.99	0.97	0.99	1.00	1.00	0.99	0.99	0.99	1.00	0.99
Laplace	0.98	0.99	0.99	0.97	0.99	1.00	1.00	1.00	0.99	0.99	1.00	0.99
Rayleigh	0.99	1.00	1.00	0.96	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
Triangular	0.99	1.00	1.00	0.96	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00
Lognormal	1.00	1.00	1.00	0.94	0.98	0.99	0.99	1.00	1.00	1.00	0.99	1.00
Gumbel	0.98	0.99	0.99	0.97	1.00	1.00	1.00	1.00	1.00	0.99	1.00	0.99
Uniform	1.00	1.00	1.00	0.94	0.98	0.99	0.99	1.00	1.00	1.00	0.99	1.00
Beta	1.00	1.00	1.00	0.94	0.98	0.99	0.99	1.00	1.00	1.00	0.99	1.00
Betapert	1.00	1.00	1.00	0.94	0.98	0.99	0.99	1.00	1.00	1.00	0.99	1.00

This means that none of the probability density functions is an invalid function. The decision to accept a specific function will result in a selection of the solution with a specific price range: extra low, very low, low, medium, high, and very high. Giving probabilistic values causes relaxation of the deterministic extreme values. In order to determine the statistical significance of differences between groups, the t-test was made assuming a 5% chance of making a mistake at evaluation (p = 0.05) and the number of freedom equal to 9. The number of degrees of freedom is the number of independent observation results minus the number of relations that combine these results with each other. In the case analyzed, the number of relations combining the results is 3 and they are the minimum, mean, and maximum prices from the deterministic cost estimates. The number of independent results equals 12 – the number of the probability density functions. The critical value of the t-test for the probability of 0.05 and 9 degrees of freedom is equal to 0.7027. If the absolute value of the t-test will have a value above 0.7027, it will mean that the hypothesis of equality of variables can not be accepted – the variables will be different. Table 5 presents the results of the significance test of the t-test.

Table 5. The values of the t coefficient of the t-test for the probability density functions.

Correlations	Gama	Cauchy	Exponential	Laplace	Rayleigh	Triangular	Lognormal	Gumbel	Uniform	Normal	Beta	Betapert
Gama	**0.00**	-6.58	-11.04	-12.29	-19.88	-22.41	-20.71	-21.44	-20.54	-24.31	-24.31	-29.11
Cauchy	6.58	**0.00**	-5.36	-6.20	-12.96	-15.54	-14.47	-14.54	-15.30	-17.71	-17.71	-20.86
Exponential	11.04	5.36	**0.00**	**-0.50**	-6.15	-8.55	-8.05	-7.62	-9.67	-10.78	-10.78	-12.43
Laplace	12.29	6.20	**0.50**	**0.00**	-5.87	-8.36	-7.84	-7.40	-9.51	-10.67	-10.67	-12.43
Rayleigh	19.88	12.96	6.15	5.87	**0.00**	-2.69	-2.45	-1.65	-4.76	-5.28	-5.28	-6.48
Triangular	22.41	15.54	8.55	8.36	2.69	**0.00**	**0.09**	**1.04**	-2.48	-2.64	-2.64	-3.51
Lognormal	20.71	14.47	8.05	7.84	2.45	**-0.09**	**0.00**	**0.90**	-2.46	-2.61	-2.61	-3.40
Gumbel	21.44	14.54	7.62	7.40	1.65	**-1.04**	**-0.90**	**0.00**	-3.37	-3.67	-3.67	-4.66
Uniform	20.54	15.30	9.67	9.51	4.76	2.48	2.46	3.37	**0.00**	**0.16**	**0.16**	**-0.29**
Normal	24.31	17.71	10.78	10.67	5.28	2.64	2.61	3.67	**-0.16**	**0.00**	**0.00**	**-0.54**
Beta	29.11	20.86	12.43	12.43	6.48	3.51	3.40	4.66	**0.29**	**0.54**	**0.54**	**0.00**
Betapert	29.11	20.86	12.43	12.43	6.48	3.51	3.40	4.66	**0.29**	**0.54**	**0.54**	**0.00**

The calculations show that the probability density functions analyzed form 6 groups. No statistically significant differences in the following groups can be specified:

Group 1 – extramely low price – the gamma function,

Group 2 – low price – the Cauchy function,

Group 3 – medium price – the exponential and Laplace functions,

Group 4 – medium high price – the Rayleigh function,

Group 5 – high price – the triangular, lognormal, and Gumbel functions,

Group 6 – very high price – uniform, normal, beta, and beta pert functions.

The sequential chart of the raw data in Figure 2 illustrates the separation of 6 groups of the probability density functions (based on the t-test).

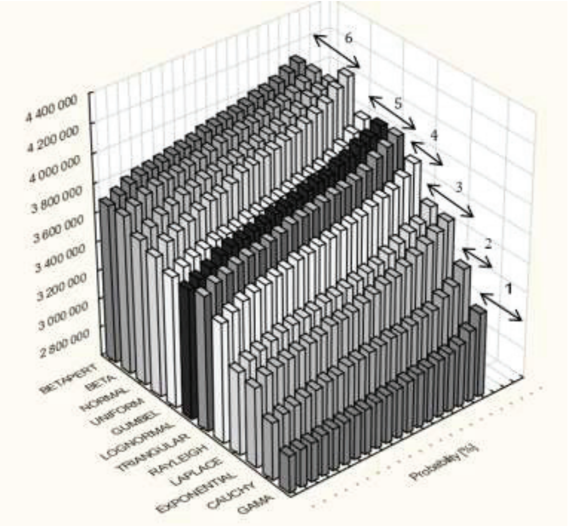


Fig. 2. The sequential chart of the raw data

5. SUMMARY AND CONCLUSIONS

When carrying out a cost analysis in accordance with the ISO 15686-5 standard “Buildings and constructed assets – Service life planning- Part 5: Life-cycle costing”, a specific probability density function should be adopted for the calculations. The paper shows the relationship between the selection of the probability density function and the analyst's intention as to the proposed construction price and the price of the building's life cycle, in relation to the deterministic cost estimate in terms of the minimum, mean, and maximum. Specific price levels have been assigned to specific distributions in 6 groups. It was shown that each of the distributions analyzed is acceptable, but it has consequences in the form of a final result. The lowest final price is obtained using the gamma distribution, the highest – the beta, beta pert, normal, and uniform distributions. The results were confirmed by the t-test.

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LIST OF FIGURES AND TABLES:

ANALIZA WPŁYWU ROZKŁADÓW GĘSTOŚCI PRAWDOPODOBIEŃSTWA CZYNNIKÓW CENOTWÓRCZYCH NA WYNIKI OBLICZEŃ LCCA

Słowa kluczowe: LCCA, obliczenia statystyczne, funkcja gęstości prawdopodobieństwa

STRESZCZENIE

W artykule wykazano związek pomiędzy wyborem funkcji gęstości prawdopodobieństwa a ceną budowy i ceną cyklu życia budynku, w odniesieniu do posiadanego deterministycznego kosztorysu w ujęciu minimalnym, średnim i maksymalnym. Wykonano kosztorysy deterministyczne realizacji inwestycji bazując na cenach minimalnych, średnich i maksymalnych : stawki robocizny, kosztów pośrednich, zysku oraz ceny pracy sprzętu i materiałów. Otrzymanym cenom netto budowy nadawano różne rozkłady gęstości prawdopodobieństwa bazując na wartości minimalnej, średniej i maksymalnej. Wykorzystano 12 rodzajów rozkładów prawdopodobieństwa: trójkątny, normalny, lognormalny, beta pert, gamma, beta, exponential, Laplaca, Cauchy, Gumbel, Rayleigh, uniform. Wyniki obliczeń przy prawdopodobieństwach zdarzenia od 5 do 95 % poddano statystycznej analizie porównawczej. Określono zależności pomiędzy wynikami obliczeń dla których przyjęto rożne rozkłady gęstości prawdopodobieństwa czynników cenotwórczych. Przypisano określonym rozkładom poziom wysokości cen w 6 grupach na podstawie przeprowadzonego testu t-Studenta. Wykazano, że każdy z analizowanych rozkładów nadaje się do stosowania, powoduje to jednak konsekwencje w postaci wyniku końcowego. Najniższą cenę końcową uzyskamy stosując rozkład Gama, najwyższą rozkład Beta, Betapert, Normalny i Uniform.

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