archives
of thermodynamics
Vol. **41**(2020), No. 1, 219–243
DOI: 10.24425/ather.2020.132956

Wave propagation in 2D between two elastic media in contact in theory of generalized two-temperature thermoelasticity with gravity effect

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Abstract The theory of generalized two-temperature thermoelasticity is used to solve the boundary value problems between two elastic media with two different types of temprature under the influence of gravity.The classical dynamical coupled theory and Lord-Şhulman theory are used to obtain the general solution of the governing equations and investigate the effect of surface waves in an isotropic elastic medium subjected to gravity field. The harmonic vibrations method is used to obtain the displacement components, stress tensor and temperature distribution in the considerd physical domain with comparison with the two theories. The obtained analytic solution of the problem is applied for special cases for which the effect of two temperatures is studied. The conductive and dynamical temperatures as well as stress and strain components are shown graphically for a suitable material. Some comparisons are also introduced in the absence and in the presence of gravity, and two-temperature parameter. The differences in the obtained results between the two theories are considered.

Keywords: Generalized thermoelasticity; Surface waves; Two temperature; Lord-Şhulman theory; Influence of gravity

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Nomenclature

a	–	two temperature parameter
Ce	–	specific heat at constant strain
e	–	cubical dilatation
e_{ij}	–	components of the strain tensor
$\overrightarrow{E}$	–	electric intensity
e	–	dilatation
$\overrightarrow{F}$	–	Lorentz's body forces vector
$\overrightarrow{h}$	–	perturbed magnetic field over the constant primary magnetic field vector
$\overrightarrow{H_0}$	–	primary magnetic field vector
$\overrightarrow{J}$	–	electric current density
K	–	thermal conductivity
P	–	initial stress
T	–	absolute temperature
T_0	–	temperature of the medium in its natural state assumed to be $\left\|(T - T_0)/T_0\right\| < 1$
t	–	time
u, v	–	two components of the displacement
u_x, u_y, u_z	–	components of the displacement vector
$u_{i,j}$	–	differentiation of the displacement tensor
x, y	–	Cartesian coordinates

Greek symbols

α_t	–	coefficient of linear thermal expansion
δ_{ij}	–	Kronecker delta function
ε_0	–	electric permittivity
h	–	initial stress parameter
$\theta = T - T_0$	–	thermodynamical temperature
λ, μ	–	Lame's parameters
γ	–	the volume thermal expansion
μ_0	–	magnetic permeability
Π, Ψ	–	two scalar functions
ρ	–	density of the medium
σ_{ij}	–	stress tensor
τ_0	–	thermal relaxation time
$\phi = \phi_0 - T$	–	conductive temperature
ω	–	the complex time constant

1 Introduction

The study of influence of surface waves in microstructure materials such as metals, polymers, composites, solids, rocks and concrete are very important for earthquake engineering and seismologists . As the earth is made up of different layers, the dynamical problems of propagation of surface waves in homogeneous and non-homogeneous elastic and thermoplastic media are of considerable importance. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves, especially when elastic vibrations are characterized by high frequencies and small wavelengths, the influence of the microstructure material becomes significant.

The classical coupled theory in thermoelasticity problems proposed by Biot with the introduction of the strain-rate term in the Fourier heat conduction equation leads to a parabolic type of the heat conduction equation, called the diffusion equation [1]. This theory predicts the finite propagation speed for elastic waves but an infinite speed for thermal disturbances, and this result is not accepted physically. To overcome such absurdity, the generalized thermoelasticity theories have been propounded by Lord and Şhulman [2]. Green and Lindsay have advocated the existence of a finite thermal wave speed in solids [3]. The generalized thermoelasticity theories have been developed by introducing one or two relaxation times in the thermoelastic process, either by modifying Fourier′s heat conduction equation or by correcting the energy equation and Neuman-Duhamel relation [4,5]. Because of its application to geophysical problems and certain topics in optics and acoustics, the Maxwell electromagnetic field with the motion of deformable solids is often considered by many investigators. A comprehensive review of earlier contributions to the subject can be found in the study by Puri [6]. Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Nemat-Nasser who studied the propagation of plane waves in a solid under the influence of an electromagnetic field [7]. Roy Choudhuri extended these results to rotating media [8]. Ezzat *et al.* have studied the problem of generalized magneto-thermoelastic waves in two dimensions by considering a thermal shock in a perfectly conducting half-space [9,10].

Most studies of the classical or generalized thermoelastic problems used the displacement potential function approach. However, Bahar and Hetnarski outlined several disadvantages of the potential function approach based on the fact that the boundary and initial conditions of the problem are not related directly to the potential function, as it has no physical meaning explicitly [11,12]. More stringent assumptions must be made on the behaviour of the potential functions than on the actual physical quantities. It was found that many integral representations of physical quantities are convergent in the classical sense while their potential function representations only converge in the mean. To get rid of these difficulties, Bahar and Hetnarski [13] introduced the state space formulation in thermoelastic problems. This state space approach has been further developed by Sherief to include the effect of heat sources [14]. Sherief and Anwar surveyed a two-dimensional generalized thermoelasticity in an infinitely long cylinder [15]. Youssef and El-bary put forward an analysis for a generalized thermoelastic infinite layer problem under three theories using the state space approach [16]. State space formulation to the vibration of a gold nanobeam in femtosecond scale was done by Elsibai and Youssef [17].

The theory of heat conduction in a deformable body, formulated by Chen and Gurtin [18] and Chen *et al.* [19,20] depends on two different temperatures, the conductive temperature and the thermo dynamical temperature. Chen *et al.* have suggested that the difference between these two temperatures is proportional to heat supply [21]. In absence of heat supply, these two temperatures are identical for a time independent situation. However, for time dependent cases, particularly for problems related to wave propagation, the two temperatures are in general different, regardless of heat supply. The two-temperature thermoelasticity theory has gained much attention of the researchers in the recent years. The existence, structural stability, convergence and spatial behaviour of two temperature thermoelasticity has been described by Quintanilla [22]. Youssef *et al.* have developed various solutions of problems with a new model of generalized thermoelasticity that depends on two temperatures: T and ϕ [23–29].

In the classical theory of elasticity, the gravity effect is generally neglected. The gravity effect in the problem of wave propagations in solids, particularly on an elastic globe, was first studied by Bromwich [30]. Love considered the effect of gravity and showed that the velocity of Rayleigh waves increased to a significant extent due to the gravitational field for large wave lengths [31]. De and Sengupta studied the gravity effect on surface waves, on the propagation of waves in an elastic layer and Lamb's problem on a plane [32–34]. Sengupta *et al.* studied the influence of gravity on the propagation of waves in a thermoelastic layer [35–38]. Many authers developed the generalized thermo-micro-stretch elastic medium in two dimensions with Rayleigh waves in a magnetoelastic medium and studied the photo-thermal-elastic interaction for two-temperature problems with gravitational effect and initial stress for different theories and other fileds [39–55].

In the present investigation, we shall formulate the two-dimensional generalized thermoelasticity problem with two-temperature theory for two elastic isotropic semi-infinite solid media perfectly in contact. The governing equations under the influence of gravity of an infinite space weakened by a finite linear opening are solved for the considered variables. The normal mode method is used to obtain the exact expressions for the considered variables. The distributions of the considered variables have been discussed and represented graphically using some particular cases.

2 Mathematical formulation of the problem

Let $\coprod_1$and $\coprod_2$ be two elastic isotropic semi-infinite solid media perfectly welded in contact to prevent any relative motion or sliding before and after the disturbances and that the continuity of displacement, stress etc. is held across the common boundary surface. Further the mechanical properties of $\coprod_1$ are different from those of $\coprod_2$. These media extend to an infinite great distance from the origin and are separated by a horizontal plane boundary. Let $\coprod_2$ be above $\coprod_1$ and Oxyz be a set of orthogonal Cartesian coordinates such that O be any point of the plane boundary and Oz – points vertically downward to the medium $\coprod_1$. Consider the plane waves travelling in Ox direction such that the disturbance is largely confined to the neighborhood of the boundary at any instant. Also assume that all particles parallel to x-axis have equal displacements. These two assumptions confirm the neuter of the considered waves, consequently the physical fields are independent of z variable.
The heat conduction equation takes the form [23]

$$K\varphi_{,ii} = (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})(\rho C_E T + \gamma T_0 u_{i,j})\,. \tag{1}$$

The constitutive equation takes the form

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma T\delta_{ij}\,. \tag{2}$$

The equation of motion without body force takes the form

$$\rho\ddot{u}_i = \sigma_{ij,j}\ , \quad (i,j = 1,2,3), \tag{3}$$

where 'over-dots' denote the second derivative with respect to time.

The relation between the heat conduction and the dynamical heat takes the form

$$\varphi - T = a\varphi_{,ii}\ , \tag{4}$$

where $a > 0$ is two-temperature parameter.

Now, we will suppose elastic and homogenous half-space $x \geq 0$ which obeys Eqs. (1)–(4) and is initially quiescent where all the state functions depend only on the dimensions x, y, and the time t.

The displacement components for two dimensionl medium have the form

$$u_x = u(x, y, t), \quad u_y = \mathrm{v}(x, y, t) \quad \text{and} \quad u_z = 0\,. \tag{5}$$

The strain component takes the form

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})\,. \tag{6}$$

The heat conduction equation takes the form

$$K(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}) = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\rho C_E T + \gamma T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{7}$$

The constitutive equations take the form:

$$\sigma_{xx} = (2\mu + \lambda)\frac{\partial u}{\partial\, x} + \lambda\,\frac{\partial v}{\partial\, y} - \gamma\; T\,, \tag{8}$$

$$\sigma_{yy} = (2\mu + \lambda)\frac{\partial v}{\partial\, y} + \lambda\,\frac{\partial u}{\partial\, x} - \gamma\, T\,, \tag{9}$$

$$\sigma_{xy} = \mu(\frac{\partial u}{\partial\, y} + \frac{\partial v}{\partial\, x})\,. \tag{10}$$

For a two dimensional problem (xy-plane) all quantities depend only on space coordinates x, y, and time t too. The field equations and constitutive relations in generalized linear thermoelasticity with the influence of gravity and without body forces and heat sources are:

$$\rho\left(\frac{\partial^2 u}{\partial t^2} - g\frac{\partial v}{\partial x}\right) = \mu\nabla^2 u + (\mu + \lambda)\frac{\partial e}{\partial x} - \gamma\frac{\partial \mathrm{T}}{\partial x}\,, \tag{11}$$

$$\rho\left(\frac{\partial^2 v}{\partial t^2} + g\frac{\partial u}{\partial x}\right) = \mu\nabla^2 v + (\mu + \lambda)\frac{\partial e}{\partial y} - \gamma\frac{\partial \mathrm{T}}{\partial y}\,. \tag{12}$$

The relation between the heat conduction and dynamical heat takes the form

$$\varphi - T = a\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right). \tag{13}$$

For simplicity, we will use the following non-dimensional variables:

$$\begin{aligned} &(x', y',\ u', v') = c_0\eta(x, y,\ u, v), \quad (t',\ \tau_0', \upsilon_0') = c_0^2\eta(t,\ \tau_0, \upsilon_0), \\ &(\theta',\ \varphi') = \frac{(T,\varphi) - T_0}{T_0}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{2\mu + \lambda}, \quad g' = \frac{g}{C_0\eta}, \end{aligned} \tag{14}$$

where

$$\eta = \frac{\rho C_E}{K}, \quad C_2^2 = \frac{\mu}{\rho}, \quad C_0^2 = \frac{2\mu + \lambda}{\rho}.$$

Hence, we have (dropping primes for convenience)

$$\nabla^2\varphi - \left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t} - \varepsilon\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} = 0, \tag{15}$$

$$\varphi - \theta = \beta\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right), \tag{16}$$

and the equations of motion take the form:

$$\frac{\partial^2 u}{\partial t^2} = a_1^*\nabla^2 u + a_2\frac{\partial e}{\partial x} - a_0\frac{\partial\theta}{\partial x} + a_3\frac{\partial v}{\partial x}, \tag{17}$$

$$\frac{\partial^2 v}{\partial t^2} = a_1^*\nabla^2 v + a_2\frac{\partial e}{\partial y} - a_0\frac{\partial\theta}{\partial y} - a_3\frac{\partial u}{\partial x}, \tag{18}$$

where

$$\varepsilon = \frac{\gamma}{\rho C_E}, \quad \beta = a\eta^2 c_0^2, \quad a_1^* = \frac{\mu}{\rho C_0^2}, \quad a_2 = \frac{\mu + \lambda}{\rho C_0^2}, \quad a_0 = \frac{\gamma T_0}{\rho C_0^2}, \quad a_3 = \frac{g}{C_0^2}.$$

Assuming the two scalar potential functions $\Pi(x, y, t)$ and $\psi(x, y, t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial\Pi}{\partial x} + \frac{\partial\psi}{\partial y}, \qquad \mathrm{v} = \frac{\partial\Pi}{\partial \mathrm{y}} - \frac{\partial\psi}{\partial \mathrm{x}}, \tag{19}$$

by using (19) and (14) in Eqs. (17) and (18), we obtain

$$\left[\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\Pi - a_3\frac{\partial\psi}{\partial x} - a_0\theta = 0, \tag{20}$$

$$\left(\nabla^2 - a_1\frac{\partial^2}{\partial t^2}\right)\psi + a_4\frac{\partial \Pi}{\partial x} = 0\,, \tag{21}$$

where $a_4 = \frac{a_3}{a_1^*}$, $a_1 = \frac{1}{a_1^*}$, $a_1^* + a_2 = 1$. Also Eq. (15) takes the form

$$\nabla^2\varphi - \left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t} - \varepsilon\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2\Pi = 0 \tag{15*}$$

and similar relations are held in $\coprod_2$ with $\rho, \lambda, \mu, \alpha, \gamma$ replaced by $\rho^+, \lambda^+, \mu^+, \alpha^+$, and γ^+.

3 Soltuion of the problem

To solve Eqs. (20), (21), (15*) and (16) assume the following:

$$[\Pi, \psi, \varphi, \theta, \sigma_{ij}](x, y, t) = [\Pi^*(y), \psi^*(y), \varphi^*(y), \theta^*(y), \sigma_{ij}^*(y)]\,\exp i\omega(x - ct)\,, \tag{22}$$

where $u^*(x), \varphi^*(x), \theta^*(x)$, and $\sigma_{ij}^*(x)$ are the amplitude of the field quantities. Using Eqs. (22), (20), (21), (15)*, and (16) we get a set differential equations for medium $\coprod_1$ as follows:

$$[D^2 - A_1]\,\Pi^* - a_3^*\psi^* - A_2\theta^* = 0\,, \tag{23}$$

$$\left(D^2 - A_4\right)\psi^* + a_4^*\Pi^* = 0\,, \tag{24}$$

$$[D^2 - A_3]\varphi^* = -\beta^*\theta^*\,, \tag{25}$$

$$(D^2 - \omega^2)\varphi^* - A\theta^* - B(D^2 - \omega^2)\Pi^* = 0\,, \tag{26}$$

where $A = ic\omega(ic\omega\tau_0 - 1)$, $B = \varepsilon A$, $A_1 = (1 - c^2)\omega^2$, $A_2 = a_0$, $A_3 = (\beta\omega^2 + 1)/\beta$, $\beta^* = \frac{1}{\beta}$, $A_4 = \omega^2(1 - a_1 c^2)$, $a_3^* = ia_3\omega$, $a_4^* = ia_4\omega$, $D = \frac{d}{dy}$.
Eliminating $\theta^*(x)$, $\Pi^*(x)$, $\psi^*(x)$ and $\varphi^*(x)$ between Eqs. (23)–(26), we obtain the partial differential equation satisfied by $\Pi^*(x)$

$$[D^6 - ED^4 + FD^2 - G]\,\Pi^*(x) = 0\,, \tag{27}$$

where $A_5 = A_7 + A_2 B$, $A_6 = (\beta^*\omega^2 + AA_3)$, $A_7 = (\beta^* + A)$. Since

$$E = [(A_1 + A_4)A_7 + A_6 + A_2 B(A_4 + A_3 + \omega^2]/A_5\,, \tag{28}$$

$$F = [(A_1 A_4 + a_3^* a_4^*)A_7 + A_2 B(A_3 A_4 + \omega^2(A_3 + A_4)) + A_6(A_1 + A_4)]/A_5\,, \tag{29}$$

$$G = [(A_1A_4 + a_3^*a_4^*)A_6 + A_2A_3A_4B\omega^2]/A_5 \;, \tag{30}$$

in a similar manner, we get

$$[D^6 - ED^4 + FD^2 - G]\;(\theta^*, \varphi^*, \psi^*)(x) = 0 \;. \tag{31}$$

The above equation can be factorized

$$\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)\;\left(D^2 - k_3^2\right)\Pi^*(x) = 0 \;, \tag{32}$$

where, k_n^2 $(n = 1\,, 2, 3)$ are the roots of the following characteristic equation:

$$k^6 - Ek^4 + Fk^2 - G = 0 \;. \tag{33}$$

The solution of Eq. (32) which is bounded as $y \to \infty$, is given by

$$\Pi^*(y) = \sum_{n=1}^{3} M_n\; \exp(-k_n y) \;. \tag{34}$$

Similarly

$$\theta^*(y) = \sum_{n=1}^{3} M_n'\; \exp(-k_n y) \;, \tag{35}$$

$$\psi^*(y) = \sum_{n=1}^{3} M_n''\; \exp(-k_n y) \;, \tag{36}$$

$$\varphi^*(y) = \sum_{n=1}^{3} M_n'''\; \exp(-k_n y) \,, \tag{37}$$

since,

$$u^*(y) = \mathrm{i}\omega\Pi^* + \mathrm{D}\psi^* \;, \tag{38}$$

$$v^*(y) = \mathrm{D}\Pi^* - \mathrm{i}\omega\,\psi^* \;, \tag{39}$$

$$\mathrm{e}^*(y) = \mathrm{i}\omega u^* + D\mathrm{v}^* \;. \tag{40}$$

Equations (38) and (39) can be rewritten in order to obtain the amplitude of the displacement components u and v, which are bounded as $x \to \infty$

$$u^*(y) = i\omega \sum_{n=1}^{3} M_n\; \exp(-k_n y) - \sum_{n=1}^{3} k_n M_n''\; \exp(-k_n y) \;, \tag{41}$$

$$v^*(y) = \sum_{n=1}^{3} -M_n k_n \exp(-k_n y) - i\omega \sum_{n=1}^{3} M_n'' \exp(-k_n y) \;, \tag{42}$$

where M_n, M'_n, M''_n , and M'''_n are some parameters depending on β, c, and ω.

Substituting Eqs. (34)–(36) into Eqs. (23)–(26), we have

$$M'_n = H_{1n} M_n\,, \quad n = 1,\, 2, 3\,, \tag{43}$$

$$M''_n = H_{2n} M_n\,, \quad n = 1,\, 2, 3\,, \tag{44}$$

where

$$H_{1n} = -\frac{B(k_n^2 - A_3)(k_n^2 - \omega^2)}{A_7 k_n^2 - A_6}\,, \quad n = 1,\, 2, 3, \tag{45}$$

$$H_{2n} = -\frac{a_4^*}{(k_n^2 - A_4)}\,, \quad n = 1,\, 2, 3., \tag{46}$$

$$H_{3n} = \frac{-\beta^* H_{1n}}{k_n^2 - A_3}\,, \quad n = 1,\, 2, 3\,. \tag{47}$$

Thus, we have

$$\theta^*(y) = \sum_{n=1}^{3} H_{1n}\, M_n\, \exp(-k_n y)\,, \tag{48}$$

$$\psi^*(y) = \sum_{n=1}^{3} H_{2n} M_n \exp(-k_n y)\,, \tag{49}$$

$$\varphi^*(y) = \sum_{n=1}^{3} H_{3n} M_n\, \exp(-k_n y)\,. \tag{50}$$

Substitution of Eqs. (14), (22), (40), and (41) into Eqs. (8)–(10), we get

$$\sigma_{xx}^* = \sum_{n=1}^{3} h_n\, M_n\, \exp(-k_n y)\,, \tag{51}$$

$$\sigma_{yy}^* = \sum_{n=1}^{3} h'_n\, M_n\, \exp(-k_n y)\,, \tag{52}$$

$$\sigma_{xy}^* = \sum_{n=1}^{3} h''_n\, M_n\, \exp(-k_n y)\,, \tag{53}$$

$$u^*(\mathrm{y}) = \sum_{n=1}^{3} (i\omega - k_n H_{2n}) M_n\, \exp(-k_n y)\,, \tag{54}$$

$$v^*(\mathrm{y}) = -\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n \exp(-k_n y)\,, \tag{55}$$

where

$$h_n = \left[i\omega(i\omega - k_n H_{2n}) + \frac{\lambda k_n(k_n + i\omega H_{2n})}{2\mu+\lambda} - \frac{\gamma T_0 H_{1n}}{2\mu+\lambda}\right], \tag{56}$$

$$h_n' = \left[k_n(k_n + i\omega H_{2n}) + \frac{i\lambda\omega(i\omega - k_n H_{2n})}{2\mu+\lambda} - \frac{\gamma T_0 H_{1n}}{2\mu+\lambda}\right], \tag{57}$$

$$h_n'' = -\frac{\mu[k_n(i\omega - k_n H_{2n}) + i\omega(k_n + i\omega H_{2n}]}{2\mu+\lambda}\,. \tag{58}$$

It should be noted that Eqs. (22)–(58) are also satisfied for medium $\coprod_2$, therefore the plus sign is used for the physical quantities of medium $\coprod_2$.

4 Applications

We consider the following set of boundary conditions:
I The displacement components are continuous:

$$(u,v)|_{\coprod_1} = (u,v)|_{\coprod_2}\,, \quad \text{at} \quad x = 0\,. \tag{59}$$

Using Eqs. (19), (54), and (55) we get:

$$\left[\sum_{n=1}^{3}(i\omega - k_n H_{2n})M_n\right]\coprod_1 = \left[\sum_{n=1}^{3}(i\omega - k_n H_{2n})M_n\right]\coprod_2, \tag{60}$$

$$\left[\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n\right]\coprod_1 = \left[\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n\right]\coprod_2\,. \tag{61}$$

II The stress components σ_{xy}, σ_{xx}, and σ_{yy} are continuous:

$$[\sigma_{xy}, \sigma_{xx}, \sigma_{yy}]\coprod_1 = [\sigma_{xy}, \sigma_{xx}, \sigma_{yy}]\coprod_2\,, \quad \text{at} \quad x = 0\,. \tag{62}$$

Substituting from Eqs. (22), (51)–(53) we obtain:

$$\left[\sum_{n=1}^{3} h_n''\, M_n\right]\coprod_1 = \left[\sum_{n=1}^{3} h_n''\, M_n\right]\coprod_2, \tag{63}$$

$$\left[\sum_{n=1}^{3} h_n\, M_n\right]\coprod_1 = \left[\sum_{n=1}^{3} h_n\, M_n\right]\coprod_2, \tag{64}$$

$$\left[\sum_{n=1}^{3} h_n'\, M_n\right]\coprod_1 = \left[\sum_{n=1}^{3} h_n'\, M_n\right]\coprod_2\,. \tag{65}$$

5 Thermal shock problem

To investigate the possibility of thermal shock problem in anisotropic elastic media, we replace medium $\coprod_2$ by a vacuum, in the proceeding problem. Since the boundary $x = 0$ is adjacent to vacuum, it is free from surface traction. In this section we determine the parameters, M_n $(n = 1, 2, 3)$.

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The constants M_1, M_2, and M_3 must be chosen such that the boundary conditions on the surface at $x = 0$ take the form:

1) The surface of the half-space is subjected to thermal shock (thermal boundary conditions)

$$\theta(0, y, t) = f(0, y,\ t)\ . \tag{66}$$

2) The surface of the half-space is traction free (mechanical boundary condition):

$$\sigma_{yy}(0, y,\ t) = \ 0\ , \tag{67}$$

$$\sigma_{xy}(0, y, t) = 0\ . \tag{68}$$

Applying the boundary conditions we obtain the following equations:

$$\sum_{n=1}^{3} H_{1n}\, M_n(b, \beta^*, \omega)\ = f^*\ , \tag{69}$$

$$\sum_{n=1}^{3} h_n\, M_n(b, \beta, \omega)\ = 0\ , \tag{70}$$

$$\sum_{n=1}^{3} h''_n\, M_n(b, \beta, \omega)\ = 0\ , \tag{71}$$

where

$$f(x, y,\ t) = f^*(y) \exp i\omega(x - ct)\ . \tag{72}$$

Invoking the boundary conditions at the surface, $x = 0$ of the plate, we obtain a homogeneous system of algebraic equations. Applying the inverse matrix method, we get three constants M_{j}, $j = 1, 2, 3$ and therefore, we obtain the expressions of displacements, temperature distribution and all other physical quantities of the plate.

Rayleigh waves

To investigate the occurrence of Rayleigh waves in anisotropic fibre-reinforced elastic media, we replace medium $\coprod_2$ by a vacuum in the proceeding problem. So the stress boundary condition in this case may be expressed as $\sigma_{xy} = \sigma_{xz} = \sigma_{yy} = 0$.

Stoneley waves

It is the generalized form of Rayleigh waves in which we assume that the waves are propagated along the common boundary of two semi-infinite media $\coprod_1$and $\coprod_2$. Therefore Eqs. (63)–(65) determine the wave velocity equation for Stoneley waves in a solid elastic media under the influence of gravity.

Clearly from Eqs. (63)–(65) , the wave velocity of the Stoneley waves depends upon the parameters of the two-temperature material medium, gravity and densities of both media. Thus, the wave velocity in Eqs. (63)–(65) for Stoneley waves under the present circumstances depends on the particular value of ω and creates a dispersion of a general wave form.

6 Numerical results and discussions

In this section we consider the copper material as a numerical example to illustrate the analytical procedure presented earlier. The results of this example depict the variation of temperature, displacement and stress fields in the context of two theories, all computations are carried out at time $t = 0.1$.

The physical constants for copper material are:
$\lambda = 7.59 \times 10^9\,\mathrm{N/m^2}$, $\mu = 3.86 \times 10^{10}$ kg/ms^2, $C_E = 383.1$ J/(kgK),
$\alpha = -1.28 \times 10^9$ N/m^2, $\rho = 7800$ kg/m^2, $K = 386$ N/Ks, $\tau_0 = 0.02$,
$f^* = 1, \alpha_t = 1.78 \times 10^{-5}$ N/m^2, $a = 1$, $T_0 = 293\,\mathrm{K}$, $\omega = \omega_0 + i\xi$,
$\omega_0 = 2$, $\xi = 1$, $\eta = 8886.73$ m/s^2, $\varepsilon = 0.0168$.
The used numerical technique concentrated on the study of distribution of the real part of thermal temperature θ and ϕ, the displacement components (u, v), and the stress $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ distribution for the problem. The field quantities, temperature, displacement components and stress components depend not only on space x and time t but also on the thermal relaxation time τ_0. Here, all variables are taken in non dimensional forms.

Figure 1 shows the comparisom between classical dynamical (CD) and Lord-Shulman LS models by considering variation of thermodynamical heat θ, displacement u, normal stress σ_{yy} and shear stresses σ_{xy} with respect to x. It can be noticed that θ and u take large values for the CD theory comparing with its corresponding values for the LS theory for various values of x, while σ_{yy} takes a large value for CD at $x \in [0, 0.01)$ comparing with the LS model while its behavior is inversed at $x \in [0.01, 0.06]$. Also, it is seen that σ_{xy} takes small values for the CD theory at $x \in [0, 0.05)$ comparing with the LS model but assumes an inverse behavior at $x \in [0.05, 0.06]$.

Figure 2 shows the gravity effect on thermodynamical heat θ, displacement u, shear stresses σ_{xy} and normal stress σ_{xx} with respect to x. It can be noticed that the thermodynamical heat θ starts from unity at the interface surface (i.e., $x = 0$) and tends to zero at $x \to \infty$, while the displacement component u, the shear stresses σ_{xy} and the normal stress σ_{xx} start from zero at the interface surface and tend to unity when x tends to infinity. It is also noted that, the gravity has a positive effect on the thermodynamical heat θ but a negative effect on σ_{xx}. Also, it is shown that gravity g affects increasingly the displacement component u at $x \in [0, 0.003)$, but decreasingly at $x \in [0.003, 0.01]$ and vise-versa with respect to σ_{xy}.

Figure 3 shows influence of β on thermodynamical heat θ, displacement u, normal stress σ_{xx} and shear stresses σ_{xy} with respect to x. It noted that θ, u and σ_{xx} increase with an increasing of $\beta = \{0, 0.1\}$ but σ_{xy} decreases.

Three-dimensional grafs for heat distribution ϕ, stress σ_{xx} and σ_{yy} with respect to x and y axis are presented in Figs. 4–6. All these quantities increase and decrease periodically tending to zero as x tends to infinity. Also, with an increasing y all of them decrease.

Figure 7 shows the effect of variation of gravity and phase velocity on the secular function. It clearly decreases and increases tending to zero as g and c tend to infinity.

Figure 8 exhibits the effect of variation of gravity and phase velocity on Stoneley waves. It is clear that the Stoneley wave velocity decreases and increases periodically tending to unity as c tends to infinity, but decreases and increases with an increasing gravity field.

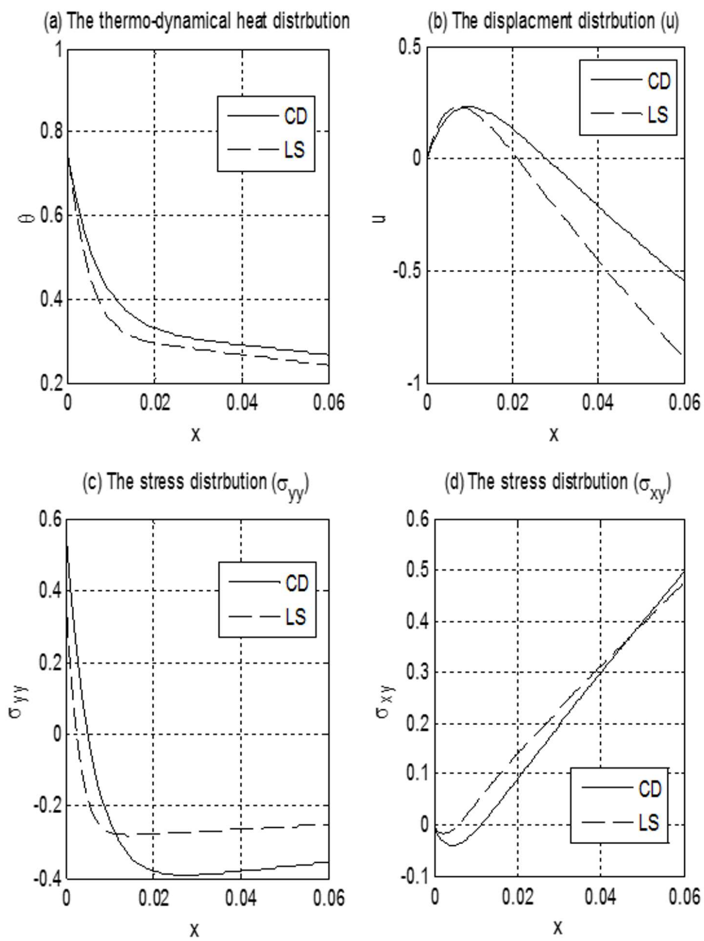

Figure 1: Comparisom between CD and LS models on variation of thermodynamical heat θ, displacement u, normal stress σ_{yy} and shear stresses σ_{xy} with respect to x.

The effect of variation of gravity and phase velocity on the attenuation coefficient is illustrated in Fig. 9. The attenuation coefficient starts to increase from its minimum value and tends to zero as g tends to infinity, but increases with an increasing gravity and phase velocity.

Figure 10 shows the effect of variation of gravity and phase velocity on the secular function for Rayleigh waves. It is shown that it decreases and increases periodically tends to unity as c tends to infinity but decreases and increases with an increasing of gravity.

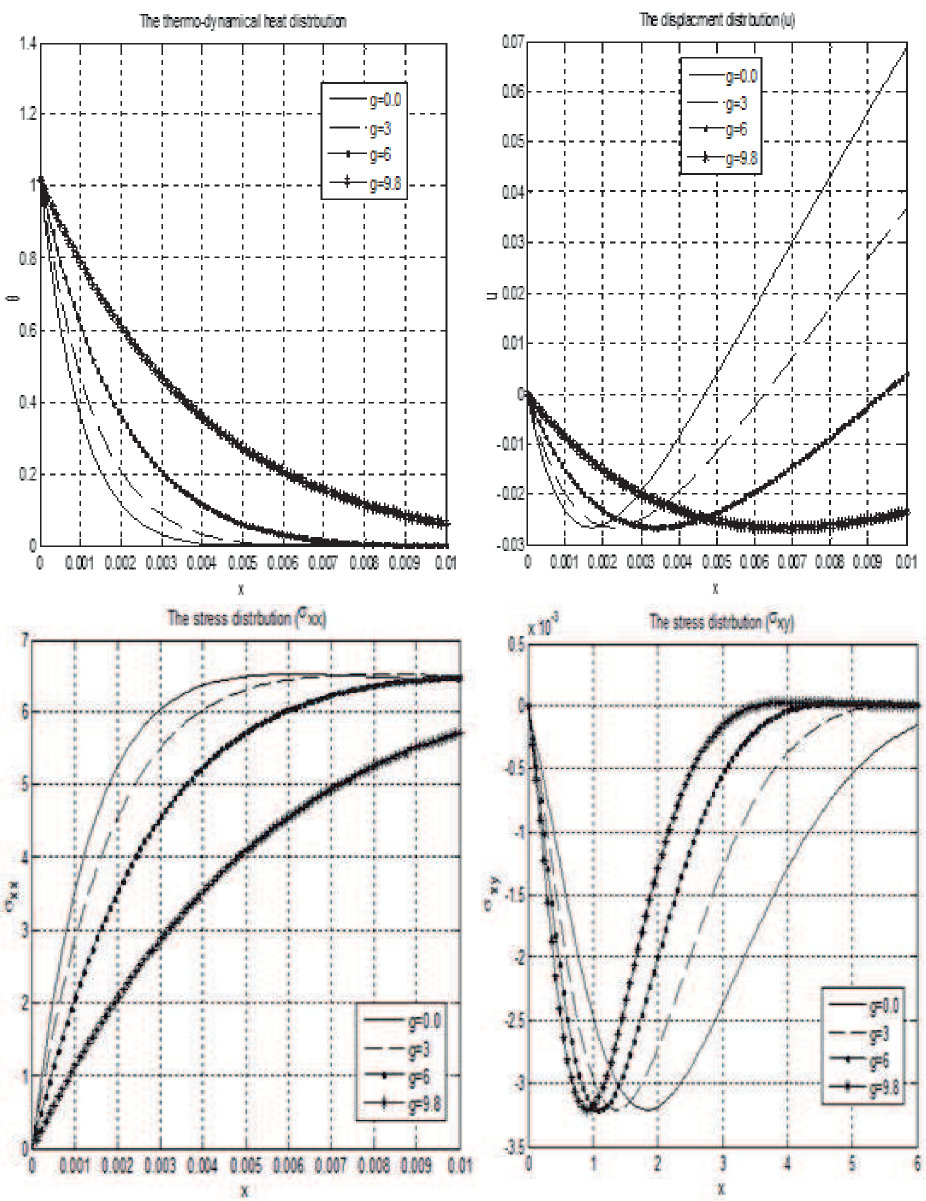

Figure 2: Variation of gravity on thermodynamical heat θ, displacement u, shear stresses σ_{xy} and normal stress σ_{xx} with respect to x.

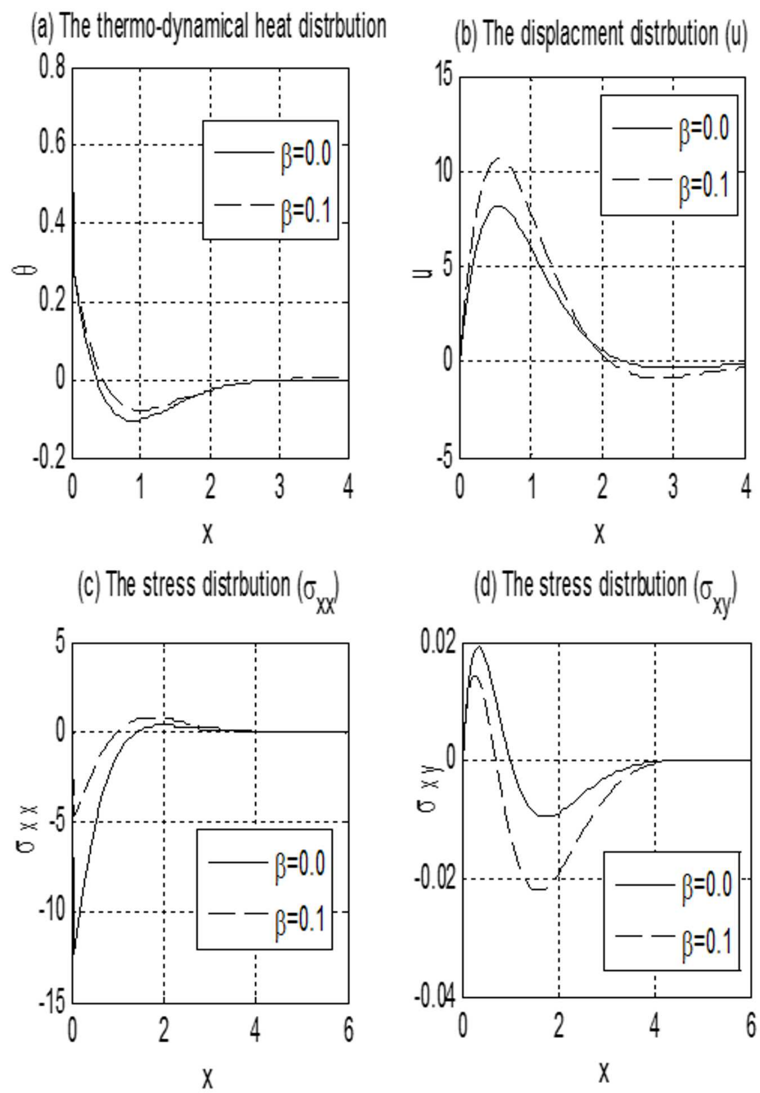

Figure 3: Variation of β on thermodynamical heat θ, displacement u, normal stress σ_{xx} and shear stresses σ_{xy} with respect to x.

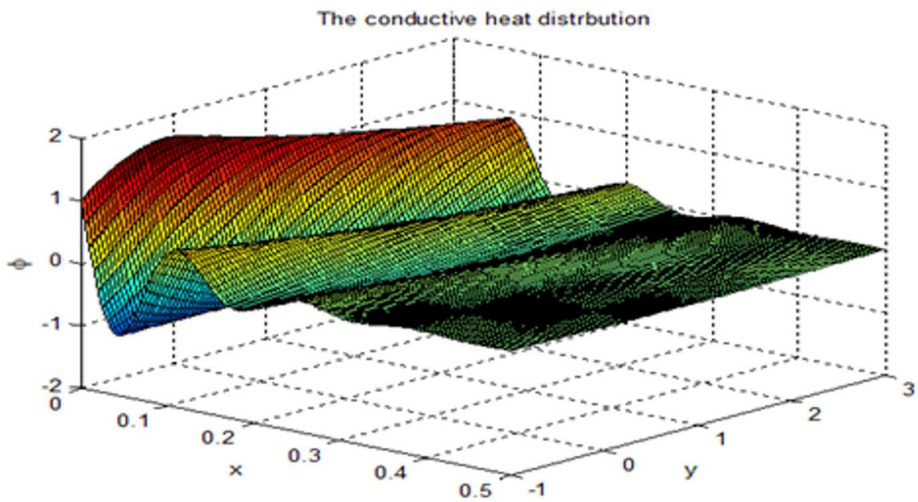

Figure 4: 3D variation of conductive heat distribution ϕ with the distance (x,y).

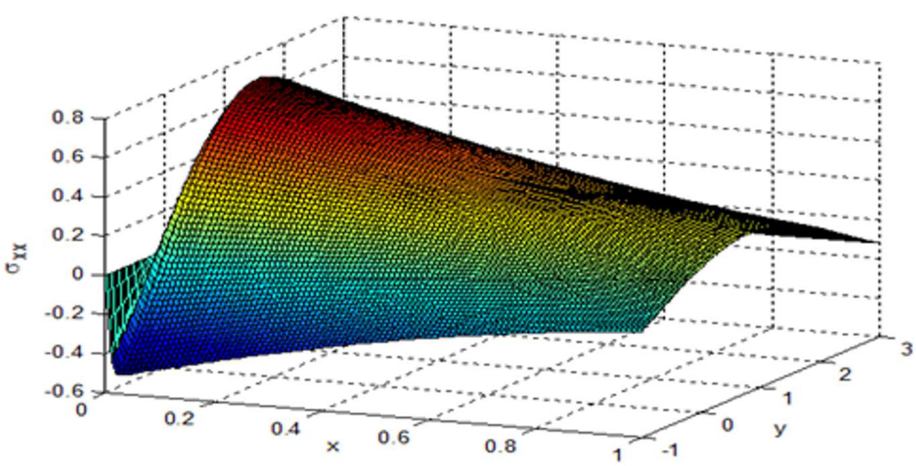

Figure 5: 3D variation of normal stress σ_{xx} (x,y) with the distance (x, y).

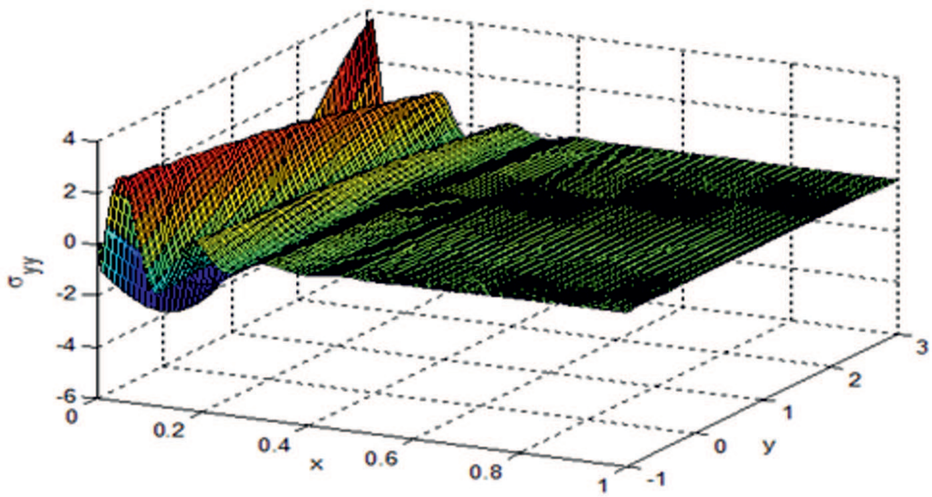

Figure 6: 3D variation of shear stress σ_{xy} with the distance (x, y).

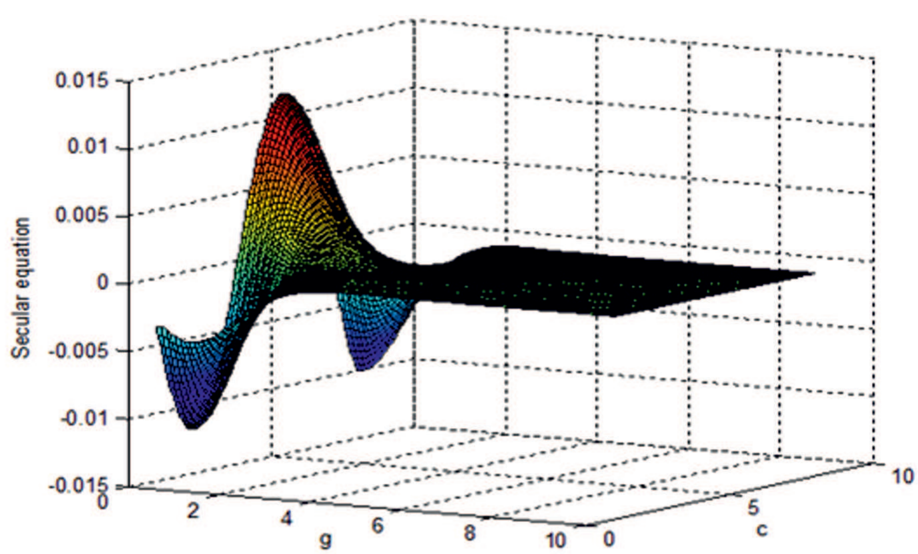

Figure 7: 3D variation of secular function with g and c.

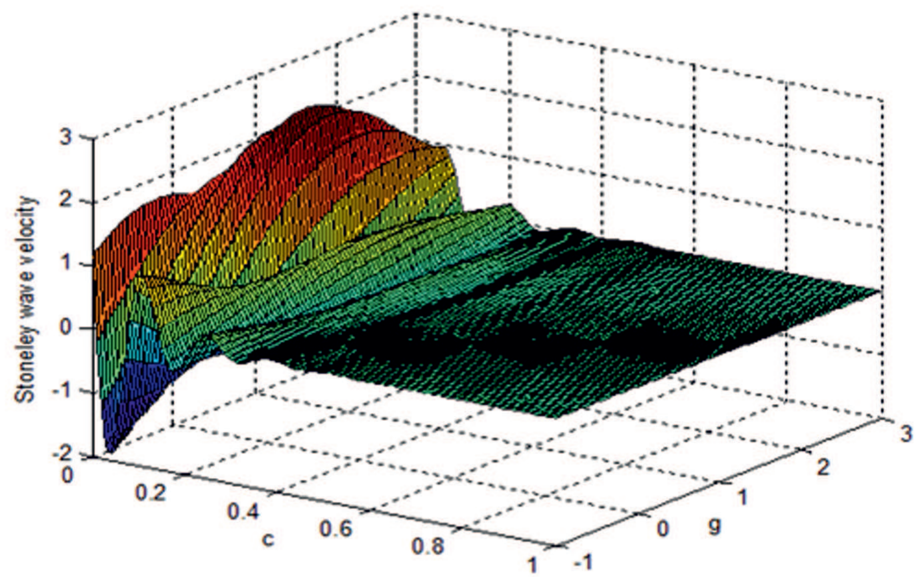

Figure 8: 3D variation of Stoneley wave velocity with g and c.

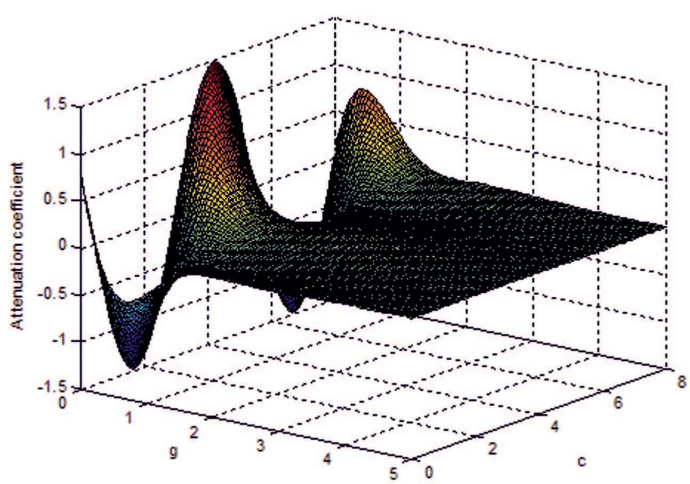

Figure 9: 3D variation of attenuation coefficient with g and c.

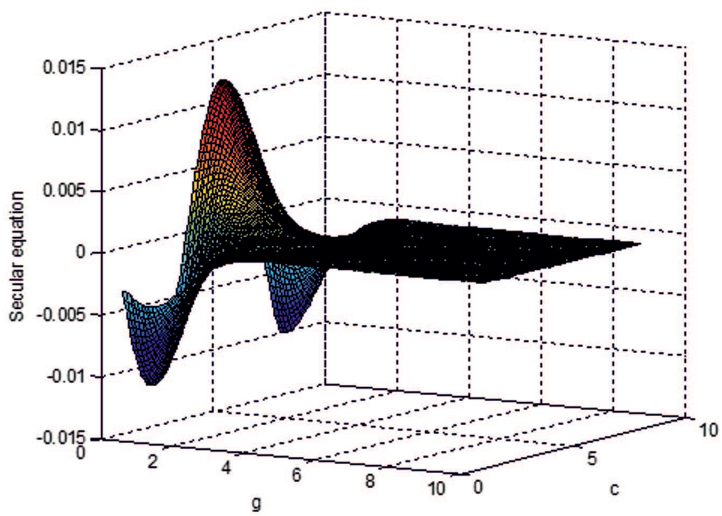

Figure 10: 3D Variation of the secular function for Rayleigh waves with varies values of g and c.

7 Conclusion

Based on the obtaind results, we concluded that:

1. The curves of physical quantities obtained from the CD theory in most figures have lower values in comparison with those of the LS theory due to the external effects parameters and the difference in the relaxation time.

2. The theory of two-temperatures generalized thermoelasticity gives a better description of behavior of the elastic body particles than the theory of one-temperature generalized thermoelasticity .

3. All physical quantities converge to zero as the non-dimensional distance x increases , therefore all functions are continuous and damped as the distance increses.

4. The deformation of the body depends on the nature of the applied force as well as on the type of boundary conditions and gravity.

5. The time parameter, relaxation time, gravity and phase velocity have a significant effect on all results obtained, including the displace-

ment, temperature, stresses, secular function, Rayleigh waves, Stoneley wave velocity and attenuation coefficient.

Received 10 September 2017, in revised form 21 February 2019

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