BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 68, No. 2, 2020
DOI: 10.24425/bpasts.2020.133114

SPECIAL SECTION

# Performance metrics in OFDM wireless networks supporting quasi-random traffic

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**Abstract.** We consider the downlink of an orthogonal frequency division multiplexing (OFDM) based cell that accommodates calls from different service-classes with different resource requirements. We assume that calls arrive in the cell according to a quasi-random process, i.e., calls are generated by a finite number of sources. To calculate the most important performance metrics in this OFDM-based cell, i.e., congestion probabilities and resource utilization, we model it as a multirate loss model, show that the steady-state probabilities have a product form solution (PFS) and propose recursive formulas which reduce the complexity of the calculations. In addition, we study the bandwidth reservation (BR) policy which can be used in order to reserve subcarriers in favor of calls with high subcarrier requirements. The existence of the BR policy destroys the PFS of the steady-state probabilities. However, it is shown that there are recursive formulas for the determination of the various performance measures. The accuracy of the proposed formulas is verified via simulation and found to be satisfactory.

**Key words:** OFDM, congestion, quasi-random, reservation, loss model, recursive.

## 1. Introduction

Teletraffic modelling is an inseparable part of the information and communication technology infrastructure. Whatever changes new networking technologies may bring, the essential task of teletraffic models is to determine and evaluate the main quality of service (QoS) parameters such as call blocking probabilities (CBP) and network resources utilization. This task is complex in contemporary networks, not only due to the growth of network traffic, but also due to the high diversity of traffic streams [1]. The latter requires the development of call/packet-level loss/queueing models based on the input traffic stream (e.g., [2–7]). Such models assist in network optimization and dimensioning procedures and can also be used as an input to computational intelligent techniques such as fuzzy analytical hierarchy process techniques (e.g., [8, 9]). In this paper, we concentrate on call-level teletraffic loss models.

The simplest call arrival process, adopted in teletraffic modelling, is the Poisson process since it leads to analytically tractable formulas for the determination of the various performance measures (e.g., CBP). In the Poisson process, calls originate from an infinite number of traffic sources. Thus, the Poisson process cannot capture the case of calls generated via a finite number of sources. The latter can be described by the quasi-random arrival process which is smoother than the Poisson process. For possible applications of the quasi-random process in loss systems, an interested reader may resort to [10–15].

We consider the case of quasi-random traffic and study the downlink of an orthogonal frequency division multiplexing (OFDM) based cell that accommodates calls from different service-classes with different QoS requirements. The springboard for the analysis of this system are models from [16–19], in which the Poisson call arrival is considered. More specifically, in [16], Paik and Suh (P-S) consider the downlink of an OFDM-based cell that accommodates Poisson arriving calls generated by multiservice classes. The system is described via a multirate loss model, i.e., new calls are blocked and lost if their required resources are not available. This implies that the resource sharing policy used in the P-S model is the classical complete sharing (CS) policy. It is characterized as complete, since the only restriction in call admission is the "complete" system capacity.

Contrary to [17] and [18], where the acceptance of a new call in the cell depends only on the availability of subcarriers, in the P-S model both the subcarriers and power are modelled as system resources and participate in call admission. The P-S model is noteworthy since power is a limited resource in OFDM wireless networks and should be taken into account in call admission. In addition, the steady-state probabilities in the P-S model are described using a product form solution (PFS). The latter is important in teletraffic modelling, because it usually results in computationally efficient formulas for the determination of performance measures. However, the calculation of CBP and

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Manuscript submitted 2019-07-22, initially accepted for publication 2019-10-06, published in April 2020

resource utilization in the P-S model is based on highly complexed formulas which are not attractive for network planning and dimensioning procedures. To solve this problem, a recursive formula is proposed in [19] for determining the occupancy distribution. This formula substantially reduces the complexity of the calculations of the P-S model.

In this paper, firstly we extend the models of [16, 19] by incorporating the bandwidth reservation (BR) policy (P-S/BR model). The BR policy allows the reservation of subcarriers in order to favor calls with high subcarrier requirements. In that sense, and contrary to the CS policy which is unfavorable to service-classes of high subcarrier requirements, the BR policy can provide a certain QoS to calls of certain service-classes [20–24]. Due to the existence of the BR policy, the steady-state probabilities in the P-S/BR model do not have a PFS. However, we show that recursive formulas do exist for the determination of the main performance measures. Secondly, we extend the P-S model by examining the quasi-random call arrival process. We name the proposed model quasi-random P-S model (qr-P-S model). Thirdly, we show that the steady-state probabilities in the proposed qr-P-S model can be analytically described via a PFS. Fourthly, we propose recursive formulas for the determination of time congestion (TC) and call congestion (CC) probabilities as well as resource utilization, which are attractive for network planning and dimensioning procedures. TC probabilities for calls of a particular service-class can be determined by the proportion of time during which the system has no available resources for this service-class. CC probabilities for calls of a particular service-class can be determined by the proportion of arriving calls that find no available resources in the system. Note that TC and CC probabilities coincide in the case of the Poisson process. Finally, we further extend the qr-P-S model by considering the BR policy (qr-P-S/BR model). Due to the BR policy, the steady-state probabilities in the qr-P-S/BR model do not have a PFS. However, we show that recursive formulas do exist for the determination of performance measures. The accuracy of the proposed formulas in all models (P-S/BR, qr-P-S and qr-P-S/BR) is verified via simulation and found to be quite satisfactory.

This paper is organized as follows. In Section 2, we review the P-S model and present the formulas for the CBP determination and resource utilization. In Section 3, we propose the P-S/BR model and the corresponding recursive formulas. In Section 4, we propose the qr-P-S model, show the PFS and present recursive formulas for the determination of the various performance measures. In Section 5, we propose the qr-P-S/BR model. In Section 6, we compare the analytical with simulation results for the P-S, the P-S/BR and the qr-P-S models. The comparison verifies the accuracy of the proposed formulas. We conclude in Section 7.

## 2. Review of the P-S loss model

To describe the P-S model, consider the downlink of an OFDM-based cell that has $M$ subcarriers and let $R$, $P$ and $B$ be the average data rate per subcarrier, the available power in the cell and the system's bandwidth, respectively. We assume that the entire range of channel gains or signal to noise ratios per unit power is partitioned into $K$ consecutive and non-overlapping intervals and denote as $\gamma_k$, $k = 1,\dots,K$ the average channel gain of the $k$th interval. By further assuming $L$ subcarrier requirements and $K$ average channel gains, the cell accommodates $KL$ service-classes. A new call of service-class $(k,l)$ ($k = 1,\dots,K$ and $l = 1,\dots,L$) requires $b_l$ subcarriers in order to be accepted in the cell. This means that each call has a data rate requirement $b_l R$. In addition, it has an average channel gain $\gamma_k$. If these subcarriers are not available, then call blocking occurs. Otherwise, the call remains in the cell for a generally distributed service time with mean $\mu^{-1}$. To calculate the power $p_k$ required to achieve the data rate $R$ of a subcarrier assigned to a call whose average channel gain is $\gamma_k$, we use the Shannon theorem: $R = (B/M)\log_2(1+\gamma_k p_k)$.

Assuming that service-class $(k,l)$ calls follow a Poisson process with rate $\lambda_{kl}$ and that $n_{kl}$ is the number of in-service calls of service-class $(k,l)$, then the system can be described as a multirate loss model whose steady-state probabilities $\pi(\boldsymbol{n})$ have the following PFS [16]

$$\pi(\boldsymbol{n}) = G^{-1}\left(\prod_{k=1}^{K}\prod_{l=1}^{L} a_{kl}^{n_{kl}} \Big/ n_{kl}!\right), \tag{1}$$

where $\boldsymbol{n} = (n_{11},\dots,n_{k1},\dots,n_{K1},\dots,n_{1L},\dots,n_{kL},\dots,n_{KL})$, $G$ is the normalization constant, $G = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}}\left(\prod_{k=1}^{K}\prod_{l=1}^{L} p_{kl}^{n_{kl}} \big/ n_{kl}!\right)$, $\boldsymbol{\Omega}$ is the state space of the system denoted as

$$\boldsymbol{\Omega} = \left\{\boldsymbol{n} : 0 \le \sum_{k=1}^{K}\sum_{l=1}^{L} n_{kl} b_l \le M,\ 0 \le \sum_{k=1}^{K}\sum_{l=1}^{L} p_k n_{kl} b_l \le P\right\}$$

and $a_{kl} = \lambda_{kl}/\mu$ is the offered traffic-load (in Erlang) of service-class $(k,l)$ calls.

To derive (1), it is required that the available power in the cell, $P$, and the power $p_k$ are integers. This is achieved by multiplying both $P$ and $p_k$ by a constant so as to have an equivalent representation of $\sum_{k=1}^{K}\sum_{l=1}^{L} p'_k n_{kl} b_l \le P'$ where $P'$ and $p'_k$ are integers. Thus, without loss of generality, it is assumed that $P$ and $p_k$ are integers.

According to [16], all performance metrics are based on the calculation of all $\pi(\boldsymbol{n})$ using (1). As an example, the CBP $B(k,l)$ of service-class $(k,l)$ calls is determined via

$$B(k,l) = 1 - G(P - p_k b_l, M - b_l)\big/G(\boldsymbol{\Omega}). \tag{2}$$

However, since the cardinality of $\boldsymbol{\Omega}$ grows as $(MP)^{KL}$, the applicability of (1) (and consequently of (2)) is limited to systems of moderate size and therefore is not recommended for network planning and dimensioning.

In [16], Paik and Suh propose the algorithms from [25] and [26] for the determination of $G(P - p_k b_l, M - b_l)$ (and consequently for the CBP calculation of $B(k,l)$) without providing explicit details. The algorithms of [25] and [26] are proposed in the literature for the CBP determination in circuit-switched networks (see e.g., [27, 28]). The algorithms from [25] are based on $z$-transforms and mean-value analysis. On the other hand, the algorithm from [26] is based on numerical inversion of generating functions which is a quite complex approach (see e.g., [29, 30]). Both algorithms: i) are applied to loss models whose steady-state probabilities have a PFS and ii) are less general than the Kaufman-Roberts (K-R) recursive formula ([31, 32]). The latter provides an efficient way for the CBP determination in a multirate loss system that accommodates Poisson traffic. Due to the effectiveness of the K-R formula, there is an extensive list of applications in PFS and non-PFS models (e.g., [33–40]).

To circumvent the complexity problem of (1), a recursive yet efficient formula that resembles the K-R formula is proposed in [19]. To present this formula, the following notation is necessary: $j_1 = \sum_{k=1}^{K}\sum_{l=1}^{L} n_{kl} b_l$ denotes the occupied subcarriers, i.e., $j_1 = 0,\ldots,M$ and $j_2 = \sum_{k=1}^{K}\sum_{l=1}^{L} p_k n_{kl} b_l$ denote the occupied power in the cell, i.e., $j_2 = 0,\ldots,P$. Also, let $q(\vec{j}) = q(j_1, j_2)$ be occupancy distribution, given by

$$q(\vec{j}) = q(j_1, j_2) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}} \pi(\boldsymbol{n}), \tag{3}$$

where $\boldsymbol{\Omega}_{\vec{j}}$ is the set of states in which the occupied subcarriers and the occupied power in the cell is $j_1$ and $j_2$, respectively.

The determination of all $q(j_1, j_2)$ is based on the following recursive formula [19]

$$q(j_1,j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \dfrac{1}{j_1}\displaystyle\sum_{k=1}^{K}\sum_{l=1}^{L} a_{kl} b_l q(j_1-b_l, j_2-p_k b_l) \\ \quad \text{for } j_1 = 1,\ldots,M \text{ and } j_2 = 1,\ldots,P \end{cases}. \tag{4}$$

The recursive form of (4) and its lower computational complexity, in the order of $O(MPKL)$, makes (4) attractive for network planning and dimensioning.

Having obtained the unnormalized values of $q(j_1, j_2)$, we calculate the CBP $B(k,l)$ of service-class $(k,l)$ using

$$B(k,l) = \sum_{\{(j_1+b_l>M)\cup(j_2+p_k b_l>P)\}} G^{-1} q(j_1, j_2), \tag{5}$$

and the mean number of in-service calls of service-class $(k,l)$, $E(k,l)$ using

$$E(k,l) = a_{kl}\,(1 - B(k,l)), \tag{6}$$

where $G$ is the normalization constant, determined via the formula $G = \sum_{j_1=0}^{M}\sum_{j_2=0}^{P} q(j_1, j_2)$.

Having determined the values of $E(k,l)$, we can also calculate the entire system blocking probability (BP), the subcarrier utilization (SU) and the power utilization (PU), using the formulas

$$BP = \sum_{k=1}^{K}\sum_{l=1}^{L} B(k,l)\lambda_{k,l}/\Lambda, \qquad \Lambda = \sum_{k=1}^{K}\sum_{l=1}^{L} \lambda_{k,l}, \tag{7}$$

$$SU = \sum_{k=1}^{K}\sum_{l=1}^{L} E(k,l) b_l/M, \tag{8}$$

$$PU = \sum_{k=1}^{K}\sum_{l=1}^{L} p_k E(k,l) b_l/P. \tag{9}$$

## 3. The proposed P-S/BR loss model

In the BR policy, a new service-class $(k,l)$ call requests $b_l$ subcarriers and has a reservation parameter $t_l$ similar to the MFCR policy. The call admission mechanism in the proposed P-S/BR model is as follows: a) if $(M - j_1 - t_l \geq b_l) \cap (j_2 + p_k b_l \leq P)$ then the call is accepted in the cell, b) if $(M - j_1 - t_l < b_l) \cup (j_2 + p_k b_l > P)$ then the call is blocked and lost.

The steady-state probabilities in the P-S/BR model do not have a PFS, since the BR policy destroys local balance (LB) between the adjacent states $\boldsymbol{n}_{kl}^{-}$ and $\boldsymbol{n}$, where $\boldsymbol{n}_{kl}^{-} = (n_{11},\ldots,n_{k1},\ldots,n_{K1},\ldots,n_{1l},\ldots,n_{kl}-1,\ldots,n_{Kl},n_{1L},\ldots,n_{kL},\ldots,n_{KL})$.

However, based on Section 2, it can be proved that the unnormalized values of all $q(j_1, j_2)$ are given by

$$q(j_1,j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \dfrac{1}{j_1}\displaystyle\sum_{k=1}^{K}\sum_{l=1}^{L} a_{kl}(j_1-b_l) b_l q(j_1-b_l, j_2-p_k b_l), \\ \quad \text{for } j_1 = 1,\ldots,M \text{ and } j_2 = 1,\ldots,P \end{cases} \tag{10}$$

where $a_{kl}(j_1 - b_l) = a_{kl}$, for $j_1 \leq M - t_l$.

Having obtained $q(j_1, j_2)$ we calculate $B(k,l)$ via

$$B(k,l) = \sum_{\{(j_1+b_l+t_l>M)\cup(j_2+p_k b_l>P\}} G^{-1} q(j_1, j_2), \tag{11}$$

while the values of $E(k,l)$, $BP$, $SU$ and $PU$ can be determined via (6), (7), (8) and (9), respectively.

## 4. The proposed qr-P-S loss model

Consider again the downlink of an OFDM-based cell that accommodates calls from $KL$ different service-classes. New calls of service-class $(k,l)$ come from a finite source population $N_{k,l}$. The mean arrival rate of service-class $(k,l)$ idle sources is $\lambda_{kl,\text{fin}} = (N_{kl} - n_{kl})\,v_{kl}$, where $n_{kl}$ is the number of in-service calls of service-class $(k,l)$ and $v_{kl}$ is the arrival rate per idle source. Based on the above, the offered traffic-load per idle source of service-class $(k,l)$ is determined by $a_{kl,\text{idle}} = v_{kl}/\mu$ (in Erlang). If $N_{kl} \to \infty$ for all service-classes and the total offered traffic-load is constant, then we have the Poisson process.

A new call of service-class $(k,l)$ requires $b_l$ subcarriers in order to be accepted in the cell. If these subcarriers are available then the call remains in the cell for a generally distributed service time with mean $\mu^{-1}$.

To analyze the qr-P-S model, we show that the steady-state probability, $\pi(\boldsymbol{n})$, has a PFS. Based on the steady-state transition rates of the proposed model, the global balance equation (*rate in* = *rate out*) for state $\boldsymbol{n}$ is

$$\sum_{k=1}^{K}\sum_{l=1}^{L}(N_{kl}-n_{kl}+1)v_{kl}\pi(\boldsymbol{n}_{kl}^{-})+ + \sum_{k=1}^{K}\sum_{l=1}^{L}(n_{kl}+1)\mu\pi(\boldsymbol{n}_{kl}^{+}) = = \sum_{k=1}^{K}\sum_{l=1}^{L}(N_{kl}-n_{kl})v_{kl}\pi(\boldsymbol{n})+\sum_{k=1}^{K}\sum_{l=1}^{L}n_{kl}\mu\pi(\boldsymbol{n}), \tag{12}$$

where

$$\boldsymbol{n}_{kl}^{-}=\big(n_{11},\ldots,n_{k1},\ldots,n_{K1},\ldots,n_{1l},\ldots,n_{kl}-1,\ldots,n_{Kl}, n_{1L},\ldots,n_{kL},\ldots,n_{KL}\big),$$
$$\boldsymbol{n}_{kl}^{+}=\big(n_{11},\ldots,n_{k1},\ldots,n_{K1},\ldots,n_{1l},\ldots,n_{kl}+1,\ldots,n_{Kl}, n_{1L},\ldots,n_{kL},\ldots,n_{KL}\big)$$

and $\pi(\boldsymbol{n})$, $\pi(\boldsymbol{n}_{kl}^{-})$, $\pi(\boldsymbol{n}_{kl}^{+})$ are the probability distributions of the corresponding states $\boldsymbol{n}$, $\boldsymbol{n}_{kl}^{-}$, $\boldsymbol{n}_{kl}^{+}$, respectively.

Let us assume the existence of LB between adjacent states. Equations (13) and (14) are the LB equations which exist because the Markov chain of the proposed model is reversible

$$(N_{kl}-n_{kl}+1)\,v_{kl}\pi(\boldsymbol{n}_{kl}^{-})=n_{kl}\mu\pi(\boldsymbol{n}), \tag{13}$$
$$(N_{kl}-n_{kl})\,v_{kl}\pi(\boldsymbol{n})=(n_{kl}+1)\mu\pi(\boldsymbol{n}_{kl}^{+}), \tag{14}$$

for $k=1,\ldots,K$, $l=1,\ldots,L$ and $\boldsymbol{n}\in\boldsymbol{\Omega}$.

Based on the LB assumption, the probability distribution $\pi(\boldsymbol{n})$ has the following PFS

$$\pi(\boldsymbol{n})=G^{-1}\left(\prod_{k=1}^{K}\prod_{l=1}^{L}\binom{N_{kl}}{n_{kl}}a_{kl,\mathrm{idle}}^{n_{kl}}\right), \tag{15}$$

where $a_{kl,\mathrm{idle}}=v_{kl}/\mu$ is the offered traffic-load per idle source of service-class $(k,l)$ and

$$G=\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}}\left(\prod_{k=1}^{K}\prod_{l=1}^{L}\binom{N_{kl}}{n_{kl}}a_{kl,\mathrm{idle}}^{n_{kl}}\right).$$

Let $q_{\mathrm{fin}}(\vec{j})=q_{\mathrm{fin}}(j_1,j_2)$ be the occupancy distribution in the proposed model, given by

$$q_{\mathrm{fin}}(\vec{j})=q_{\mathrm{fin}}(j_1,j_2)=\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}\pi(\boldsymbol{n}). \tag{16}$$

To prove a recursive formula for the determination of $q_{\mathrm{fin}}(\vec{j})$ we initially sum both sides of (13) over $\boldsymbol{\Omega}_{\vec{j}}$

$$N_{kl}a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}\pi(\boldsymbol{n}_{kl}^{-})-a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}(n_{kl}-1)\,\pi(\boldsymbol{n}_{kl}^{-}) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}n_{kl}\pi(\boldsymbol{n}), \tag{17}$$

We initially examine the left hand side of (17) (whose final form is given in (20)) and then the right hand side of (17) (whose final form is given in (21)). The left hand side of (17) can be written as

$$N_{kl}a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}\pi(\boldsymbol{n}_{kl}^{-})-a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}(n_{kl}-1)\,\pi(\boldsymbol{n}_{kl}^{-}) = N_{kl}a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}\cap\{n_{kl}\geq1\}}\pi(\boldsymbol{n}_{kl}^{-}) - a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}\cap\{n_{kl}\geq1\}}(n_{kl}-1)\,\pi(\boldsymbol{n}_{kl}^{-}), \tag{18}$$

Since

$$\boldsymbol{\Omega}_{\vec{j}}\cap\{n_{kl}\geq1\}=\Bigg\{\boldsymbol{n}:\ \sum_{m\neq k}\sum_{t\neq l}n_{mt}b_t+(n_{kl}-1)b_l=j_1-b_l\,, \sum_{m\neq k}\sum_{t\neq l}p_m n_{mt}b_t+(n_{kl}-1)b_l=j_2-p_kb_l, n_{kl}\geq1, n_{mt}\geq0\Bigg\},$$

we may write the right hand side of (18) as follows

$$N_{kl}a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}\cap\{n_{kl}\geq1\}}\pi(\boldsymbol{n}_{kl}^{-})-a_{kl,\mathrm{idle}}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}\cap\{n_{kl}\geq1\}}(n_{kl}-1)\pi(\boldsymbol{n}_{kl}^{-})= N_{kl}a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\pi(\hat{\boldsymbol{n}})-a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\hat{n}_{kl}\pi(\hat{\boldsymbol{n}}), \tag{19}$$

where

$$\hat{n}_{mt}=\begin{cases}n_{kl} & \text{if } m\neq k \text{ and } t\neq l\\ n_{kl}-1 & \text{if } m=k \text{ and } t=l\end{cases}.$$

The term

$$N_{kl}a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\pi(\hat{\boldsymbol{n}})$$

can be written as $N_{kl}a_{kl,\mathrm{idle}}q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)$ while the term

$$a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\hat{n}_{kl}\pi(\hat{\boldsymbol{n}})$$

as

$$a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\frac{\hat{n}_{kl}\pi(\hat{\boldsymbol{n}})}{q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)}q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)= = a_{kl,\mathrm{idle}}y_{kl,\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)$$

where $y_{kl,\mathrm{fin}}(j_1-b_l, j_2-p_kb_l)$ is the mean number of service-class $(k,l)$ calls in state $(j_1-b_l, j_2-p_kb_l)$.

The right hand side of (19) can now be written as

$$N_{kl}a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\pi(\hat{\boldsymbol{n}})-a_{kl,\mathrm{idle}}\sum_{\hat{\boldsymbol{n}}\in\boldsymbol{\Omega}_{(j_1-b_l,j_2-p_kb_l)}}\hat{n}_{kl}\pi(\hat{\boldsymbol{n}})=$$
$$=N_{kl}a_{kl,\mathrm{idle}}q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)+ \tag{20}$$
$$+a_{kl,\mathrm{idle}}y_{kl,\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l).$$

The right hand side of (17) is written as

$$\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}n_{kl}\pi(\boldsymbol{n})=\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}}n_{kl}\frac{\pi(\boldsymbol{n})}{q_{\mathrm{fin}}(j_1,j_2)}q_{\mathrm{fin}}(j_1,j_2)= \tag{21}$$
$$=y_{kl,\mathrm{fin}}(j_1,j_2)q_{\mathrm{fin}}(j_1,j_2).$$

By equating (20) and (21) we have

$$\big(N_{kl}-y_{kl,\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)\big)$$
$$a_{kl,\mathrm{idle}}q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)= \tag{22}$$
$$=y_{kl,\mathrm{fin}}(j_1,j_2)q_{\mathrm{fin}}(j_1,j_2).$$

Multiplying both sides of (22) by $b_l$ and summing over $k$ and $l$ we have

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\big(N_{kl}-y_{kl,\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)\big)$$
$$a_{kl,\mathrm{idle}}b_lq_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)= \tag{23}$$
$$=j_1q_{\mathrm{fin}}(j_1,j_2).$$

The value of $y_{kl,\mathrm{fin}}(j_1-b_l, j_2-p_kb_l)$ in (23) is not known. To determine it, we use the following lemma [41]: two stochastic systems are equivalent and result in the same CBP, if they have: a) the same traffic description parameters $(K, L, N_{kl}, a_{kl,\mathrm{idle}})$ where $k=1,\ldots,K$, $l=1,\ldots,L$ and b) exactly the same set of states.

The purpose is therefore to find a new stochastic system, whereby we can determine the values of $y_{kl,\mathrm{fin}}(j_1-b_l, j_2-p_kb_l)$. The subcarriers requirements of calls of all service-classes and the values of $M$ and $P$ in the new stochastic system are chosen according to the following two criteria: 1) conditions (a) and (b) are valid and 2) each state $\vec{j}$ has a unique occupancy $(j_1, j_2)$.

Now, state $\vec{j}=(j_1,j_2)$ can be reached only via the state $(j_1-b_l, j_2-p_kb_l)$. Thus, $y_{kl,\mathrm{fin}}(j_1-b_l, j_2-p_kb_l)=n_{kl}-1$.

Based on the above, (23) can be written as

$$q_{\mathrm{fin}}(j_1,j_2)=\begin{cases}1, & \text{for } j_1=j_2=0\\[4pt] \dfrac{1}{j_1}\displaystyle\sum_{k=1}^{K}\sum_{l=1}^{L}(N_{kl}-n_{kl}+1) & \\ \qquad a_{kl,\mathrm{idle}}b_lq_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l) & \\ \qquad \text{for } j_1=1,\ldots,M \text{ and } j_2=1,\ldots,P & \end{cases}. \tag{24}$$

Note that if $N_{kl}\to\infty$ for all service-classes and the total offered traffic-load remains constant, then we have (4) of the P-S model.

Having obtained the unnormalized values of $q_{\mathrm{fin}}(j_1, j_2)$, we can calculate the TC probabilities of service-class $(k,l)$ calls, $B_{TC}(k,l)$, via the formula

$$B_{TC}(k,l)=\sum_{\{(j_1+b_l>M)\cup(j_2+p_kb_l>P)\}}G^{-1}q_{\mathrm{fin}}(j_1,j_2), \tag{25}$$

and the CC probabilities of service-class $(k,l)$ calls via (25) but for a system with $N_{kl}-1$ traffic sources.

Also, we determine the average number of in-service calls of service-class $(k,l)$, $E_{\mathrm{fin}}(k,l)$, via the formula

$$E_{\mathrm{fin}}(k,l)=\sum_{j_1=1}^{M}\sum_{j_2=1}^{P}G_{\mathrm{fin}}^{-1}y_{kl,\mathrm{fin}}(j_1,j_2)q_{\mathrm{fin}}(j_1,j_2), \tag{26}$$

where $G_{\mathrm{fin}}$ is the normalization constant, determined via

$$G_{\mathrm{fin}}=\sum_{j_1=0}^{M}\sum_{j_2=0}^{P}q_{\mathrm{fin}}(j_1,j_2)$$

and $y_{kl,\mathrm{fin}}(j_1-b_l, j_2-p_kb_l)$ is the mean number of service-class $(k,l)$ calls in state $(j_1-b_l, j_2-p_kb_l)$ calculated via

$$y_{kl,\mathrm{fin}}(j_1,j_2)=$$
$$=\frac{(N_{kl}-n_{kl}+1)\,a_{kl,\mathrm{idle}}q_{\mathrm{fin}}(j_1-b_l,j_2-p_kb_l)}{q_{\mathrm{fin}}(j_1,j_2)}. \tag{27}$$

Having determined the values of $E_{\mathrm{fin}}(k,l)$, we can also calculate the entire system BP based on the TC probabilities of all service-classes, $\mathrm{BP}_{TC}$, the $\mathrm{SU}_{\mathrm{fin}}$ and the $\mathrm{PU}_{\mathrm{fin}}$, using the formulas

$$\mathrm{BP}_{TC}=\sum_{k=1}^{K}\sum_{l=1}^{L}B_{TC}(k,l)N_{k,l}v_{k,l}\Big/\Lambda_{\mathrm{fin}},$$
$$\Lambda_{\mathrm{fin}}=\sum_{k=1}^{K}\sum_{l=1}^{L}N_{k,l}v_{k,l}, \tag{28}$$

$$\mathrm{SU}_{\mathrm{fin}}=\sum_{k=1}^{K}\sum_{l=1}^{L}E_{\mathrm{fin}}(k,l)b_l\Big/M, \tag{29}$$

$$\mathrm{PU}_{\mathrm{fin}}=\sum_{k=1}^{K}\sum_{l=1}^{L}p_kE_{\mathrm{fin}}(k,l)b_l\Big/P. \tag{30}$$

In order to determine the values of $q_{\mathrm{fin}}(j_1, j_2)$ according to (24), it is required to know the values of $n_{kl}$ which are unknown. These values can be obtained via an equivalent stochastic system, with the same traffic parameters and the same set of states as already described for the proof of (24). However, the state space determination of the equivalent system becomes complex due to the large number of service-classes. To this end and contrary to (24), which provides the exact values of $q_{\mathrm{fin}}(j_1, j_2)$ at the cost of state space enumeration and processing, we propose an algorithm which provides approximate values but is much simpler and easy to implement:

a) Determine the values of $q(j_1, j_2)$ according to (4).

b) Determine the values of $y_{kl}(j_1, j_2)$ via the formula

$$y_{kl}(j_1, j_2) = a_{kl} q(j_1 - b_l, j_2 - p_k b_l)/q(j_1, j_2). \quad (31)$$

c) Modify (24) to the following recursive formula where the values of $y_{kl}(j_1, j_2)$ have been determined via (31),

$$q_{\text{fin}}(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{j_1} \sum_{k=1}^{K} \sum_{l=1}^{L} (N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l)) \\ \quad a_{kl,\text{idle}} b_l q_{\text{fin}}(j_1 - b_l, j_2 - p_k b_l) \\ \quad \text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P \end{cases}. \quad (32)$$

d) Determine the average number of in-service calls of service-class $(k,l)$, $E_{\text{fin}}(k,l)$ via (26), where the values of $y_{kl,\text{fin}}(j_1, j_2)$ are given by

$$y_{kl,\text{fin}}(j_1, j_2) = \\ = \frac{(N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l))\, a_{kl,\text{idle}} q_{\text{fin}}(j_1 - b_l, j_2 - p_k b_l)}{q_{\text{fin}}(j_1, j_2)}. \quad (33)$$

e) Determine: 1) the TC probabilities of service-class $(k,l)$ calls, $B_{TC}(k,l)$, via (25), and 2) the $BP_{TC}$, the $\text{SU}_{\text{fin}}$ and the $\text{PU}_{\text{fin}}$, via (28), (29) and (30), respectively.

## 5. The proposed qr-P-S/BR loss model

The call admission mechanism in the proposed qr-P-S/BR model is the same with that of the P-S/BR model.

The steady-state probabilities in the qr-P-S/BR model do not have a PFS, since the BR policy destroys LB between the adjacent states $\boldsymbol{n}_{kl}^{-}$ and $\boldsymbol{n}$. However, based on Section 3 and the algorithm proposed for the determination of $q_{\text{fin}}(j_1, j_2)$ in the qr-P-S model (Section 4), we propose the following algorithm for the determination of $q_{\text{fin}}(j_1, j_2)$ in the qr-P-S/BR model:

a) Determine $q(j_1, j_2)$ according to (10).

b) Determine $y_{kl}(j_1, j_2)$, for $j_1 \leq M - t_l$, via (31).

c) Modify (32) to the following recursive formula where the values of $y_{kl}(j_1, j_2)$ have been determined in step (b)

$$q_{\text{fin}}(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{j_1} \sum_{k=1}^{K} \sum_{l=1}^{L} (N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l)) \\ \quad a'_{kl,\text{idle}} b_l q_{\text{fin}}(j_1 - b_l, j_2 - p_k b_l) \\ \quad \text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P \end{cases}, \quad (34)$$

where $a'_{kl,\text{idle}} \equiv a_{kl,\text{idle}}(j_1 - b_l) = a_{kl,\text{idle}}$, for $j_1 \leq M - t_l$.

d) Determine the average number of in-service calls of service-class $(k,l)$, $E_{\text{fin}}(k,l)$ via (26), where the values of $y_{kl,\text{fin}}(j_1, j_2)$ are calculated for $j_1 \leq M - t_l$, via

$$y_{kl,\text{fin}}(j_1, j_2) = \\ = \frac{(N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l))\, a_{kl,\text{idle}} q_{\text{fin}}(j_1 - b_l, j_2 - p_k b_l)}{q_{\text{fin}}(j_1, j_2)}. \quad (35)$$

e) Determine: 1) the $BP_{TC}$, the $\text{SU}_{\text{fin}}$ and the $\text{PU}_{\text{fin}}$, via (28), (29) and (30), respectively and 2) the TC probabilities of service-class $(k,l)$ calls, $B_{TC}(k,l)$, via

$$B_{TC}(k,l) = \sum_{\{(j_1 + b_l + t_l > M) \cup (j_2 + p_k b_l > P)\}} G^{-1} q_{\text{fin}}(j_1, j_2). \quad (36)$$

## 6. Evaluation

We consider the downlink of an OFDM-based cell and provide analytical and simulation congestion probabilities results for the P-S the P-S/BR and the qr-P-S models. The input parameters for the abovementioned models are: $B = 20$ MHz, $P = 25$ Watt, $M = 256$, $R = 329.6$ kbps, $L = 64$, $b_l = l$, $l = 1, \ldots, 64$, and the values of $b_l$ are uniformly distributed. In addition, let $K = 3$ which results in $LK = 192$ service-classes. In the case of the qr-P-S and the qr-P-S/BR models, we assume that $N_{kl} = 20$ for all service-classes. Let the integer representations of $p_k$ ($k = 1, 2, 3$) and $P$ be: $p'_1 = 6$, $p'_2 = 10$, $p'_3 = 16$, $P' = 2500$. The values of $p'_k$ require: $p_1 \approx 0.06$, $p_2 \approx 0.01$, $p_3 \approx 0.16$ achieved via $\gamma_1 = 24.679$ dB, $\gamma_2 = 22.460$ dB, $\gamma_3 = 20.419$ dB. We further assume that the probability an arriving call has an average channel gain $\gamma_k$ is given by two sets: 1) set 1: $r_k = 1/3$ ($k = 1, 2, 3$) and 2) set 2: $r_1 = 1/4$, $r_2 = 1/4$, $r_3 = 1/2$. According to set 2, the amount of power assigned to calls is larger compared to set 1. Also, let $\lambda_{kl} = \Lambda r_k / L$ be the arrival rate of Poisson arriving service-class $(k,l)$ calls, where $\Lambda$ is the total arrival rate in the cell given by $\Lambda = aM\mu/\hat{g}$, $a$ is the traffic intensity of the cell, $\mu = 0.00625$ and $\hat{g} = 32.5$ is the average subcarrier requirement of a new call (since $b_l$ is uniformly distributed). As far as the BR parameters are concerned, let $t_l = 64 - l$, $l = 1, \ldots, 64$, so that $b_1 + t_1 = \ldots = b_{64} + t_{64}$.

Simulation results, based on Simscript III [42], are mean values of 7 runs, while each run is based on the generation of 10 million calls. To account for a warm-up period, the blocking events of the first 3% of these generated calls are not considered in the results. In all figures of this section, analytical results are quite close to the corresponding simulation results.

In Figs. 1, 2, we consider the qr-P-S and the P-S models for both sets of $r_k$. In the $x$-axis of Figs. 1, 2, the value of $a$ increases from 0.05 to 0.2 in steps of 0.025. Figures 1 and 2 show the analytical and simulation TC probabilities of service-classes (3, 16) and (3, 64), respectively. We observe that: 1) in the qr-P-S model the TC probabilities are lower compared to those obtained in the P-S model, which is due to the quasi-random process and 2) the selection of set 2 for the values of $r_k$, increases the TC probabilities since the amount of power assigned to calls in the case of set 2 is larger compared to set 1.

In Figs. 3–5, we consider the P-S and the P-S/BR models assuming that $r_k = 1/3$ ($k = 1, 2, 3$). In the $x$-axis of Figs. 3–5, the

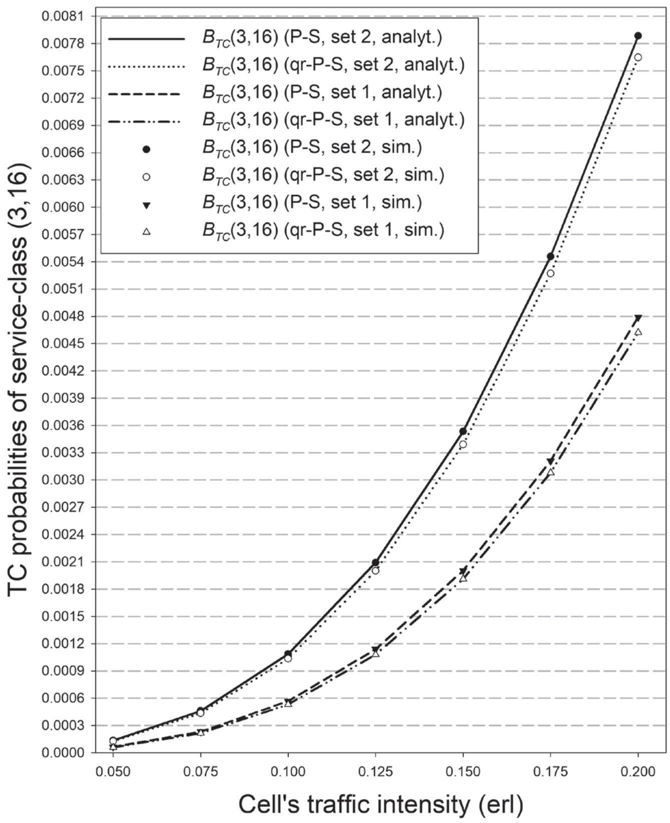


Fig. 1. TC probabilities of service-class (3, 16)

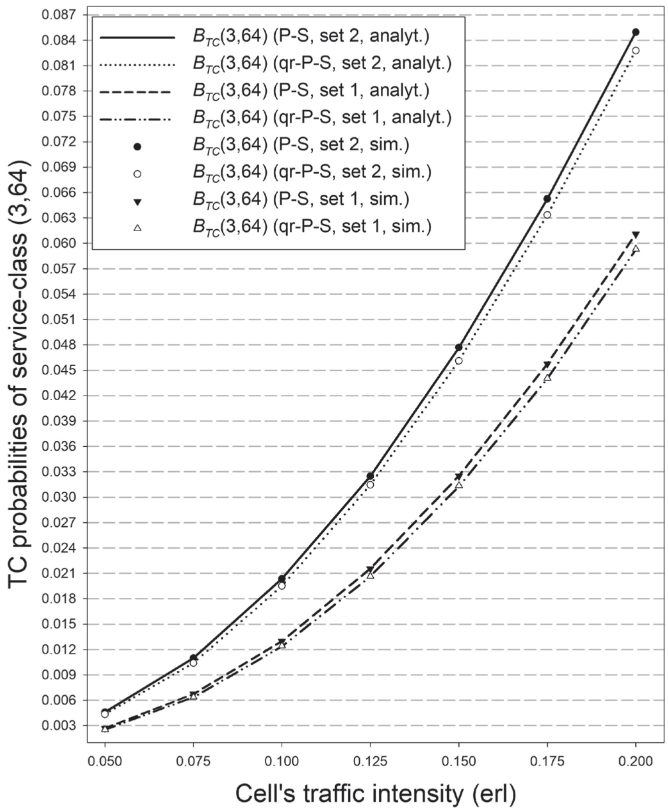


Fig. 2. TC probabilities of service-class (3, 64)

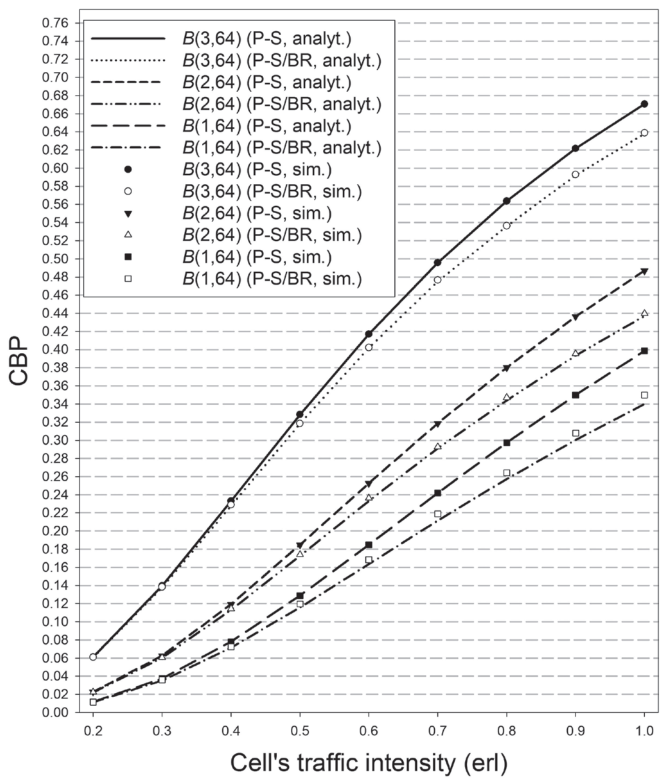


Fig. 3. CBP of service-classes (1, 64), (2, 64) and (3, 64)

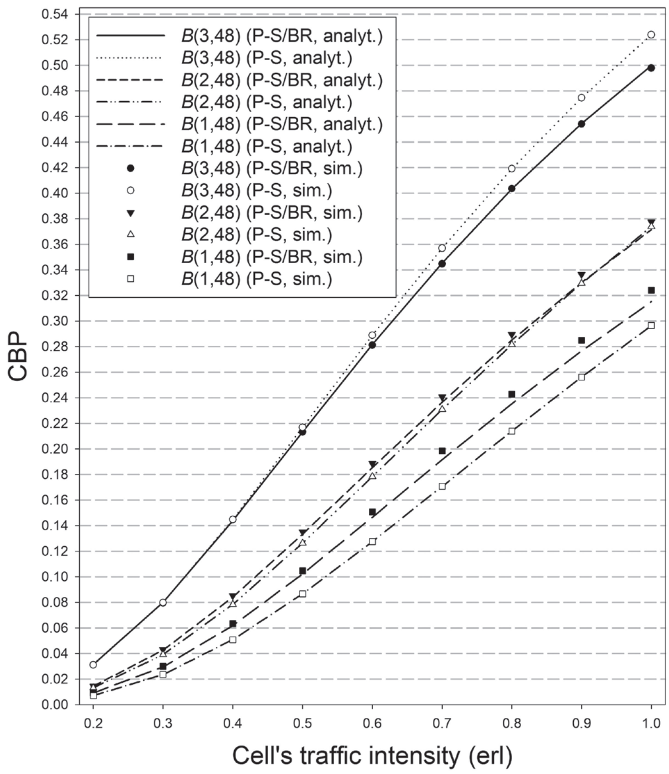


Fig. 4. CBP of service-classes (1, 48), (2, 48) and (3, 48)

value of $a$ increases from 0.2 to 1.0. Figure 3 shows the analytical and simulation CBP of service-classes (3, 64), (2, 64) and (1, 64), which require the highest number of subcarriers. We see that the BR policy reduces the CBP of these service-classes compared to the values of the P-S model. Figure 4 shows the analytical and simulation CBP of service-classes (3, 48), (2, 48) and (1, 48). We observe that, in most of the cases, the BR policy increases the CBP of these service-classes compared to the values of the P-S model. The same behavior (CBP increase) appears in most of the service-classes whose calls require less than 64 subcarriers. On the other hand, the BP of the entire system increases for both sets of $r_k$ (Fig. 5) since the $t_l$ parameters are chosen to benefit service-classes with high subcarrier requirements. Regarding the BR policy, a similar behavior is observed in the case of the qr-P-S and the qr-P-S/BR models.

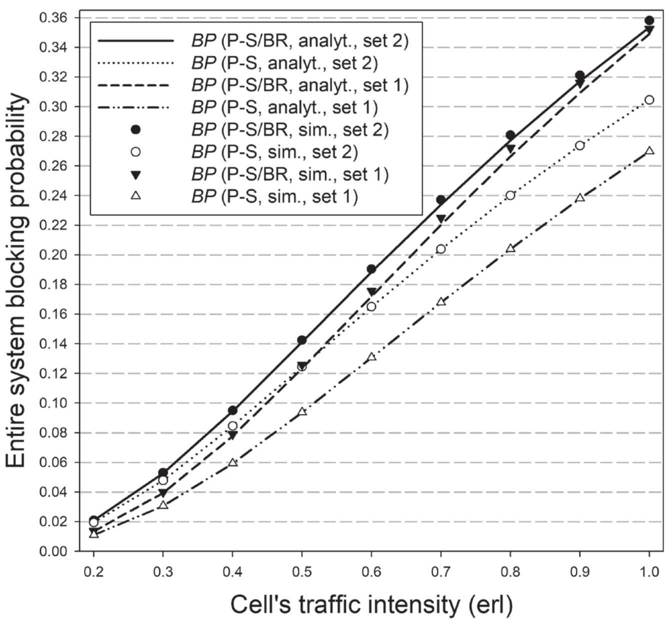


Fig. 5. BP of the entire system

## 7. Conclusion

We propose teletraffic loss models for the analysis of the downlink of an OFDM cell that accommodates quasi-random generated calls from different service-classes under the CS and the BR policies. The cell is analysed as a loss system which leads to a PFS for the steady-state probabilities in the case of the CS policy. Based on the PFS, recursive formulas are proposed for the determination of all performance measures. Modifications of these formulas result in approximate but recursive formulas in the case of the BR policy. The proposed formulas are quite accurate compared to simulation and can be used in network planning procedures.

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