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A Simple Nonlinear Companding Transfrom for Nonlinear Compensation of Direct-Detection Optical OFDM Systems

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***Abstract*—In direct-detection OFDM systems, the nonlinear effects caused by optical modulation and fiber transmission can degrade the system performance severely. In this study, we propose a new nonlinear companding transform to improve the performance of direct detection optical OFDM transmission systems. The demonstration is realized by Monte-Carlo simulation of the intensity modulation and direct detection DCO-OFDM optical transmission system at 40 Gbps over a 80 km of standard single mode fiber link. The influence of the companding parameters on the performance of system in different nonlinear transmission conditions has been investigated via simulation.**

***Keywords*—nonlinear compensation, companding transform, DCO-OFDM, PAPR reduction, direct detection**

I. INTRODUCTION

THE orthogonal frequency division multiplexing (OFDM) has become a promising solution of next generation passive optical networks (NG-PONs) because its advantages such as highly spectral efficiency and highly chromatic dispersion tolerance allow the PONs to extend both the capacity and the range. Moreover, by transferring the complexity of the transmitters and the receivers from the analog to digital domain, OFDM helps the convergence between optical and wireless access networks to be more feasible [1]. To meet the requirements of simple design and cost efficiency for access networks, the intensity modulation and direct detection (IM-DD) are suitable techniques to be employed for these systems. However, one inherent drawback of the optical OFDM transmission systems is sensitive to the nonlinearity originated from the high peak-to-average power ratio (PAPR) that is similar to the wireless systems. The high peaks of the OFDM signal resulted by phase coherence of subcarriers cause the nonlinear impairments not only in electronic components in the transmitter and receiver of the OFDM systems but also in fiber transmission [2-4].

Finding an effective solution of nonlinear mitigation has been an attractive research topic for last decade [5-17]. For optical OFDM systems, the nonlinear compensation methods can be classified into two groups, namely optical domain and digital domain methods. Although the optical domain methods based on optical phase conjugation (OPC) show a remarkable performance improvement in long-range OFDM systems [5-7], they require a strict setup of the fiber link that is unsuitable for NG-PONs. Digital domain methods are implemented in digital domain to compensate the nonlinear impairments in both fiber transmission and electronic components [8-17]. In which many methods inherited from wireless communication systems have been investigated to improve the performance of the IM-DD OFDM communication systems by reducing the PAPR of the OFDM signal [8-12]. In general, these techniques provide a good PAPR reduction, but they also add more complexity to both the transmitter and the receiver of the OFDM system. For example, coding or selective mapping (SLM) schemes does not cause any distortion but they require the receiver to know the side information to recover the original signal [8-10]. Some other simpler solutions such as clipping and filtering introduce large distortion [11]. Additionally, the complexity and distortion will drastically depend on the modulation constellation size and the number of subcarriers.

Another simple method is based on nonlinear companding algorithms such as A-law and μ-law that are commonly used in audio transmission system. This method has been recently applied for the optical OFDM systems to improve the BER performance [13-17]. However, the performance improvement of the systems is unremarkable due to companding-induced distortion that depends strongly on the companding parameter. Therefore, if the companding parameter is inappropriately chosen, the system performance can be degraded. In this paper, we propose a new nonlinear companding transform for IM-DD optical OFDM system to enhance the efficiency of nonlinear compensation, which results in a better performance improvement. The influence of companding parameters on system performance is investigated via Monte-Carlo simulations. The results show that the proposed companding model improves the performance considerably compared to uncompanded system and A-law model. The efficient suppression of nonlinear impairments of the proposed companding model not only improves the bit-error-rate (BER) but also increases the average launched power for IM-DD optical OFDM system, which is very significant to NG-PONs.

The rest of this paper is structured as follows: Section 2 describes the proposed companding function applied in IM-DD OFDM system for PAPR reduction. In section 3, a simulation setup of the IM-DD optical OFDM system and its parameters are given. Section 4 shows the results and discussions. Finally, our conclusion is impressed in section 5.

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II. PROPOSED NONLINEAR COMPANDING MODEL

A. *Proposed companding functions*

The aim of companding technique is to reduce the PAPR of OFDM signals that results in better resilience to nonlinear effects. The proposed companding transform is based on Rapp nonlinear model, which is commonly used to model the nonlinearity of high power amplifiers (HPA) [18]. This model uses a parameter to limit the range of compressed amplitudes, which is only applied into high amplitudes. On the other words, the small signals are undistorted by companding transform.

The proposed companding characteristics based on Rapp model is defined as follow

$$y = \frac{\kappa x}{\left[1+\left(\frac{|x|}{a_{sat}}\right)^{2\alpha}\right]^{1/2\alpha}} \tag{1}$$

$$x = \frac{y}{\kappa\left[1-\left(\frac{|y|}{a_{sat}}\right)^{2\alpha}\right]^{1/2\alpha}} \tag{2}$$

where x is the normalized instantaneous amplitude of input signal $(-1 \le x \le 1)$, κ is a scaling factor to maintain a output power same as input power, a_{sat} is the saturation level parameter determining the limited output and the position of companding curve, α is the smoothness parameter defining the curvature of companding profile or transition smoothness from the linear region to the saturated region. Thus, the compression process is done by (1) in the transmitter and the expanding process is done by (2) in the receiver. Figure 1 shows the compressor input-output transfer characteristics of proposed companding model with different saturation level and smoothness parameters. In this figure, the output is normalized to compare between the proposed companding profile with another conventional companding profile such as A-law. By selecting appropriate parameters, the proposed companding profile similar to A-law and μ-law profiles can be obtained.

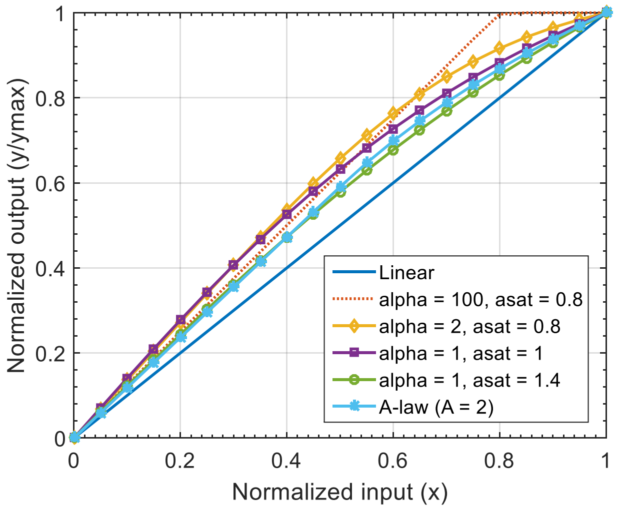

Fig.1. Transfer characteristics of the proposed companding mode with normalized output by the output maximum.

However, there is a difference between the proposed algorithm and other companding algorithms such as A-law and μ-law. Firstly, the input-output transfer characteristic of A-law or μ-law depends only on a companding parameter while the proposed model depends on two parameters: saturation level a_{sat} and smoothness α. The parameter α defines how smoothly the characteristic of the companding model change from linear region to the nonlinear region. At small values of α, the range of compressed amplitudes is expanded. This range is narrowed when the parameter α increases. Parameter a_{sat} defines the saturation level that limits the amplitude of the compressed output. Hence the proposed compander allows the maximum output value to be smaller than the maximum input. This is another difference between the proposed algorithm with traditional algorithms. Figure 2 shows the characteristics of different companding algorithms with the output that is normalized by the input maximum. From the curves in Fig. 2, we can clearly see the differences between different algorithms as mentioned above. For large enough values of α, the proposed companding curve becomes a clipping mode with the clipping threshold given by a_{sat}. For parameters $\alpha = 1$ and $a_{sat} = 1$, the companding functions (1) & (2) become simple to implement.

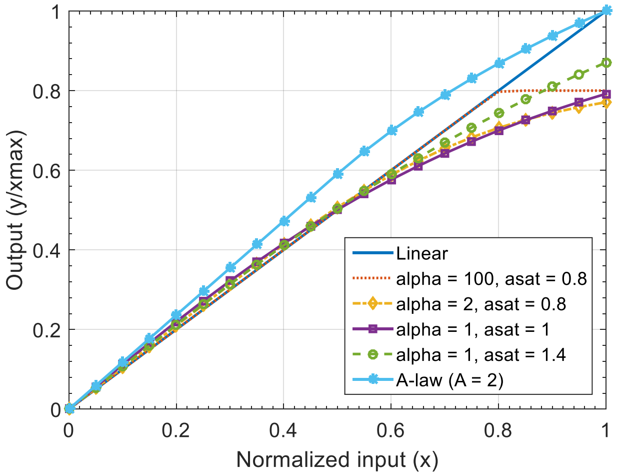

Fig.2. Transfer characteristics of the proposed companding mode with normalized output by the input maximum.

B. *PAPR reduction and noise enhancement by companding transform*

An OFDM signal in the time domain is described by

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} S_k e^{j2\pi \frac{nk}{N}} \tag{3}$$

where N is the number of subcarriers in an OFDM symbol and S_k is the k^{th} complex modulated symbol. For an IM-DD optical system in which DCO-OFDM scheme is commonly used, the real output of the IFFT block is required, hence the input has Hermitian symmetry where the S_m is the complex conjugate of S_{N-m}. When the number of subcarriers is large enough, the discrete time samples s_n have Gaussian distribution with probability density function (pdf) given by

$$f_{s_n}(s) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{s^2}{2\sigma^2}} \tag{4}$$

where σ^2 is the variance of the original DCO-OFDM signals. Then, the PAPR of OFDM signals s_n can be described as [17]

$$PAPR(s_n) = 10\log_{10}\frac{\max\left\{|s_n|^2\right\}}{\sigma^2} \tag{5}$$

By using companding transform, the s_n samples are compressed to reduce PAPR. The compressed samples can be given by

$$s_n^c = C(s_n) \tag{6}$$

where $C(.)$ is the companding function defined in (1). For traditional companding functions such as A-law, the peak of the companded output remains unchanged, but the variance of s_n^c is increased that leads to reduce PAPR. While, for proposed companding algorithm, the power of signals before and after companding remain unchanged ($E\left\{|s_n^c|^2\right\} = E\left\{|s_n|^2\right\}$), but the peak power after companding transform is compressed to reduce the PAPR of the DCO-OFDM signals. Figure 3 shows the complementary cumulative distribution function (CCDF) of the PAPR for original OFDM signal and companded signals. The reduction of the PAPR depends on the parameters of companding function. In case of α = 100, the companding function similar to a clipping mode, the PAPR reduction is small that is only 0.8 dB at the CCDF of 10^{-3}. While the reduction is about 1.5 dB for the A-law case with $A = 2$. For α = 1 and a_{sat} = 1, the PAPR reduction of 2 dB can be achieved. The higher reduction of the PAPR can be obtained by smaller values of α and a_{sat}. Although the companding profiles of two cases: α = 1, a_{sat} = 1 and α = 2, a_{sat} = 0.8 are almost similar as shown in Fig. 2, their PAPR curves exhibit a clear difference at the CCDF of higher 10^{-2} that can significantly affect to the nonlinear compensation.

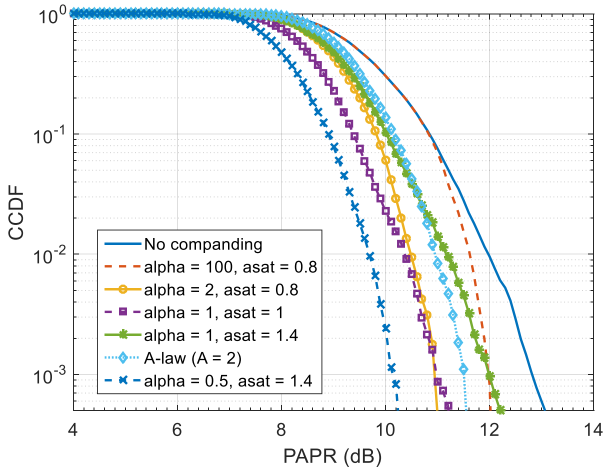

Fig.3. Complementary cumulative distribution function (CCDF) of PAPR for original OFDM signals and companded signals (N=256, 64-QAM).

However, there is a trade-off between the PAPR and the noise enhancement by companding operation. The signal r_n received at the output of the communication channel can be given by

$$r_n = h_n * s_n^c + w_n = h_n * C(s_n) + w_n \tag{7}$$

where h_n is the impulse response of the channel which is responsible for nonlinear distortions, w_n is the additive linear noise. Then, the OFDM signal at the receiver can be recovered by de-companding operation as follows

$$s_n' = C^{-1}(r_n) = C^{-1}\left(h_n * C(s_n) + w_n\right) \tag{8}$$

where C^{-1} is the de-companding function given by (2). For an optical fiber channel, we can assume that the nonlinear impairments can be considered as an equivalent additive noise [3,4]. Therefore the recovered OFDM signal after de-companding operation can be given by

$$\begin{aligned} s_n' &= C^{-1}\left(C(s_n) + x_n^{nonlinear} + w_n\right) \\ &= s_n + C^{-1}\left(x_n^{nonlinear}\right) + C^{-1}(w_n) \\ &= s_n + n_{non}^c + n_{lin}^c \end{aligned} \tag{9}$$

where $x_n^{nonlinear}$ is the nonlinear noise of the channel, n_{nonl}^c and n_{lin}^c are respectively the nonlinear and linear noises enhanced by companding operation.

Hence, the performance of the optical OFDM system is only improved when the nonlinear noise is significantly reduced by companding operation. Otherwise, the companding process can degrade the system performance by the noise enhancement. In optical communication systems, the nonlinear noises are mainly caused by intensity dependence of refractive index of fiber channel, therefore the compression of optical power-peaks through companding operation allows a performance improvement of the optical OFDM system.

III. SIMULATION MODEL

In order to verify the performance of the proposed companding scheme, a MATLAB simulation model of IM-DD optical OFDM system is developed to investigate the system performance in terms of BER. The block diagram of the IM-DD optical OFDM system is shown in Fig. 4.

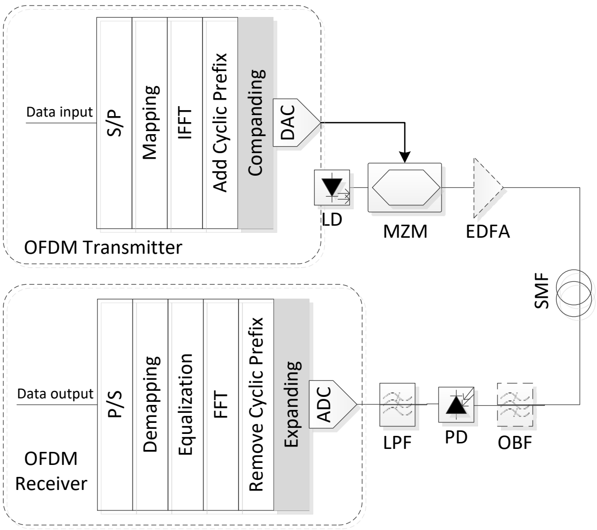

Fig.4. Block diagram of IM-DD OFDM system using nonlinear companding transform. S/P: serial to parallel, IFFT/FFT: Inverse Fourier Transform, ADC/DAC: analog-to-digital/digital-to-analog converters, MZM: Mach-Zehnder modulator, LD: laser diode, PD: photo-detector, EDFA: Erbium doped fiber amplifier, SMF: single mode fiber, OBF: optical bandpass filter, LPF: lowpass filter. (Dashed boxes are optional in the system.)

At the transmitter, the random bit sequence data stream is serial-to-parallel converted and mapped onto M-QAM complex symbols and then fed into 256-point inverse fast Fourier transform (IFFT) section. DCO-OFDM scheme is imposed to obtain the real and positive time domain signal for IM-DD optical system. Among 256 subcarriers of OFDM signal, only 95 subcarriers carry data, 95 symmetrical subcarriers carry conjugated data and the others are zero padded. By adding cyclic prefix (CP), inter-symbol and inter-carrier interference caused by the transmitter, optical channel and receiver filtering effects are combated. The nonlinear companding operation is applied to the OFDM signal at this point before digital to analog conversion. Because nonlinear phase shift mainly occurs at high optical power peaks where carry the positive parts of original OFDM samples, a DC-bias is optimally applied before companding operation to move the positive peaks of the OFDM samples to the nonlinear region of companding profile, while the negative parts of OFDM samples represented by low optical power levels are in the linear region to minimize the noise enhancement. External modulation using the MZM biased at quadrature point $V_\pi/2$ is chosen to modulate the companded OFDM signal on the optical carrier. Figure 5 shows the waveforms of uncompanded and companded optical OFDM signals in time domain. The modulated signal is transmitted via the standard single mode fiber, which is modeled by the nonlinear Schrodinger equation [19]. In the transmission link, the EDFA and optical filter are optional components and they are not employed in this study.

At the receiver, the photodiode PIN is used to detect the optical OFDM signal. The OFDM samples are de-companded before the data recovery is done by the OFDM demodulator with the reverse process of the transmitter. EDFA can be used in case of long-range system. The bandwidth of the receiver is limited by the low pass filter with a setup similar to that of conventional direct detection optical transmission systems. All noise sources such as shot noise and thermal noise at the receiver are included in the simulation model.

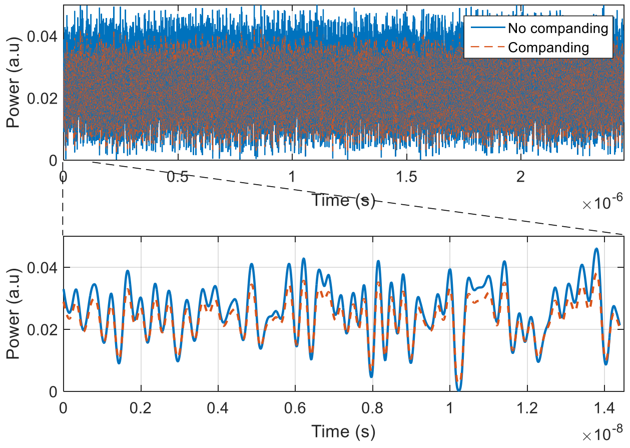

Fig.5. Waveforms of original and companded optical OFDM signals.

The BER performance of the IM-DD optical OFDM system using proposed nonlinear companding transform is evaluated by the Monte-Carlo simulations. The bit-error-rate (BER) is calculated by comparing the recovered bit data and the original bit data. In this paper, the BER performance of the system is investigated under the influence of the saturation level and smoothness parameter of the proposed companding model, the optical power and the modulation index of the system. The important system parameters used in the simulations are shown in Table I.

IV. RESULTS AND DISCUSSIONS

In proposed companding model, the PAPR reduction depends on both parameters: smoothness parameter α and saturation level a_{sat}. Firstly, the BER performance of the IM-DD OFDM system as a function of α at different saturation levels is shown in Fig. 6. In our investigation, the link length is set to 80 km and the average launched power is set to about 14 dBm, which can be required for a long range PON system. For the a_{sat} values higher than 1, the best performance of the system in terms of BER is obtained at $\alpha = 1$. At values $\alpha < 1$, the higher PAPR reduction can achieve as shown in Fig. 3, but the enhanced distortion noise becomes large enough to degrade the performance. However, when the parameter α increases from 1 to larger values, the BER performance of the system is degraded about two orders of magnitude due to a decrease in the PAPR reduction by companding operation. It then attains saturation value at the large enough α values that is corresponding to the BER of the system without using companding method. For the a_{sat} values smaller than 1, the minimum BER moves to the higher α values of 2.5. However, the BER curve in this case is increased due to the stronger noise enhancement.

TABLE I
SIMULATION PARAMETERS FOR DCO-OFDM OPTICAL SYSTEM

Parameters	Symbol	Value
Fiber attenuation coefficient	α_f	0.2 dB/km
Fiber dispersion coefficient	D	17 ps/nm.km
Nonlinear index	n_2	$2.6\text{x}10^{-20}$ m^2/W
Effective area	A_{eff}	76 μm^2
Optical frequency carrier	f	193.1 THz
Optical spectral linewidth	Δf	10 MHz
PD responsivity	R	0.6 A/W
Dark current	I_d	2 nA
Thermal noise spectrum density	S_T	$2\text{x}10^{-23}$ $\text{A/(Hz)}^{1/2}$
M-ary	M	64
Data rate	R_b	40 Gb/s

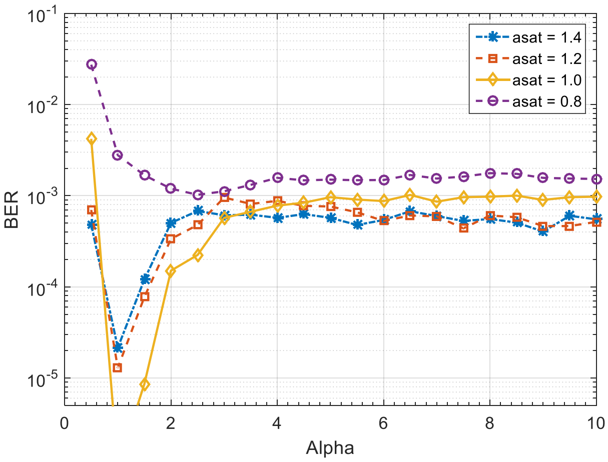

Fig.6. BER vs the companding parameter α with fiber length L=80km, $P_{avg} = 14$dBm.

Figure 7 demonstrates the performance of the IM-DD OFDM system versus the saturation level at different α values.

For a given α value, there is an optimum range of a_{sat} to get the best performance of the IM-DD OFDM system. The optimum range of a_{sat} has a tendency to move toward higher values when the α value decreases. Similar to above obtained results, the best BER performance is attained at $\alpha = 1$ with the a_{sat} in the range from 1 to 1.2 where the nonlinear noise is significantly reduced by the PAPR reduction and the noises are insignificantly enhanced by companding process.

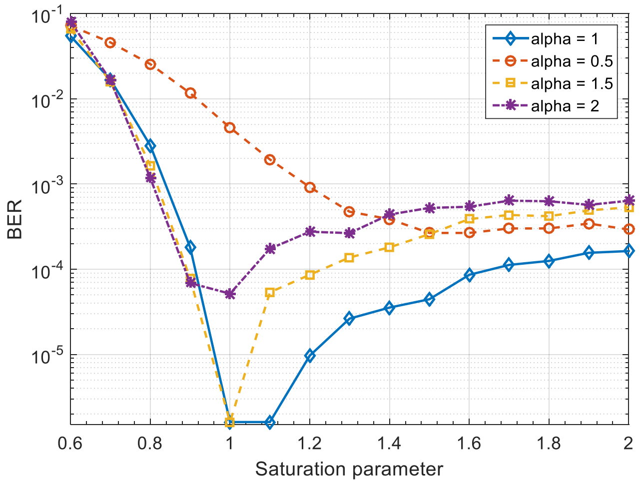

Fig.7. BER vs the saturation level a_{sat} with fiber length L=80km, P_{avg} = 14dBm.

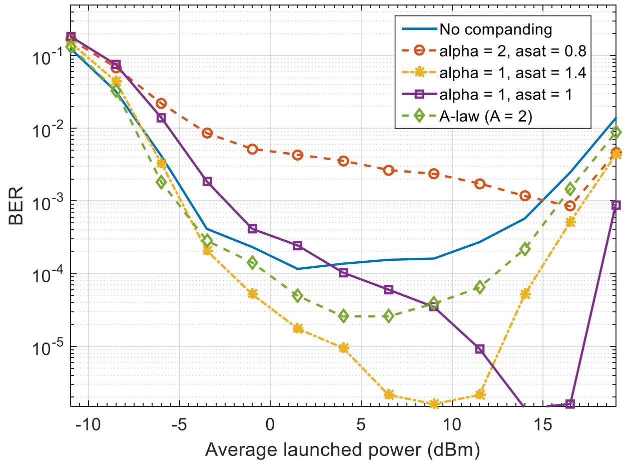

Fig.8. BER vs the launched power with fiber length L=80km.

In order to investigate the effectiveness of the proposed companding transform, the performance of the IM-DD optical OFDM system with the fiber length of 80 km versus the average launched power is shown in Fig. 8. In this figure, the performance curve of A-law companding model with optimum compression parameter is also plotted to compare with that of the proposed model at different parameters. When the parameters are optimally chosen, the BER performance can be improved two orders of magnitude compared to that of the system without using companding transform. In particular, the best performance is attained in the range of the average optical power from 14 dBm to 16 dBm for α = 1 and $a_{sat} = 1$, and from 6 dBm to 11 dBm for α = 1 and $a_{sat} = 1.4$. For uncompanded system, the best performance is attained at the average optical power of 1.5 dBm. For the case of A-law companding transform, the system performance is also improved in the range of the average optical power from 4 dBm to 6 dBm. Thus, the system performance using the proposed companding model can be improved by 12 dB compared to the uncompanded system and by 10 dB compared to the A-law companded system. This increase in average launched power allows a significant extension of power budget of IM-DD optical OFDM system that plays an important role in designing NG-PONs.

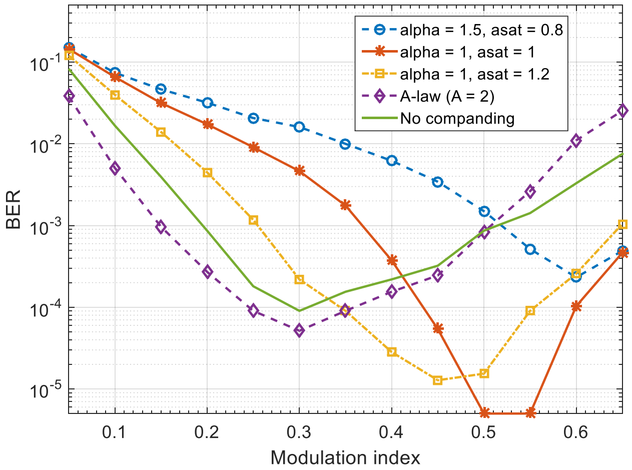

Fig.9. BER vs the modulation index with fiber length L=80km, P_{avg}=14dBm.

Figure 9 shows the improvement of MZM linearity performance through proposed companding model that is represented by the BER performance of the system as a function of the modulation index. In this study, the modulation index of MZM is defined as

$$m = \frac{V_{rms}}{V_{\pi}} \tag{10}$$

where V_{rms} is the rms driving voltage of MZM, V_{π} is the half-voltage. The obtained results as shown in Fig. 9 demonstrate the ability of the MZM nonlinearity suppression to extend the modulation index. While the optimal modulation index of the systems using A-law companding and without using companding is only 0.3, it can be extended up to 0.55 for the system using the proposed model. The extension of MZM modulation index is very significant to improve the linear noise resilience of the system.

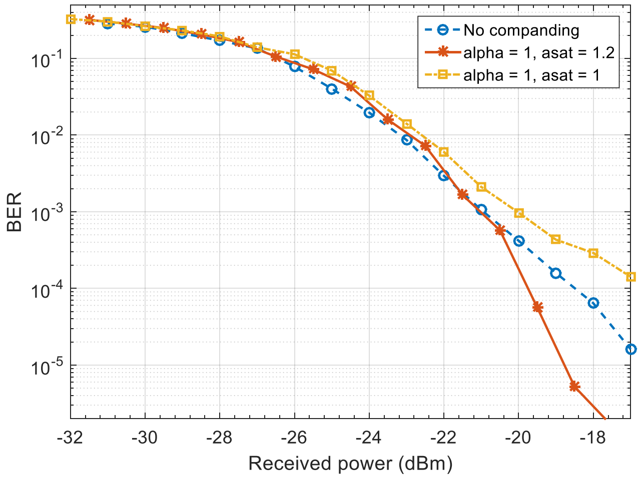

Fig.10. BER vs the optical received power in case of back-to-back.

The noise enhancement of companding transform process can cause a sensitivity penalty of optical direct detection. Figure 10 shows the BER performance of the back-to-back systems as a function of optical received power. For the system with $\alpha = 1$ and $a_{sat} = 1$ that attains the best performance of nonlinear compensation, there is a sensitivity penalty of 1 dB at BER of 10^{-3}. However, there is almost no penalty for the system with $\alpha = 1$ and $a_{sat} = 1.2$ and even the sensitivity is improved at the BER thresholds lower than 10^{-4}. This improvement can be resulted by smaller noise enhancement and improvement of linear noise resilience at higher modulation index.

CONCLUSION

We have proposed a nonlinear companding transform based on Rapp nonlinear model to improve the performance of IM-DD optical OFDM system. The proposed model compresses the peaks of optical OFDM signal while it keeps the average power unchanged to reduce the PAPR of OFDM signal that results in a compensation for the nonlinear impairments in the MZM and optical fiber. The PAPR reduction level can be adjusted by two parameters of the proposed model, which affect to the companding characteristics. However, there is always a tradeoff between the PAPR reduction and the system performance, which can be degraded by the noise enhancement from companding transforms. The influence of parameters of the proposed companding model has been investigated via Monte-Carlo simulations. By suitably choosing the companding parameters at which the noise enhancement is much smaller than the nonlinear noise reduction by proposed companding transform, the system performance is therefore significantly improved in comparison with A-law companding transform and uncompanded systems. The simulation results show that the BER performance of the IM-DD optical OFDM system can be improved up to two orders of magnitude and the average launched power can be 10 dB higher than that of uncompanded system. For parameters $\alpha = 1$ and $a_{sat} = 1$ at which the best performance of the system is obtained, the proposed companding function becomes simple to implement in practice.

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