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Application-oriented experiment design for model predictive control

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Abstract. The model predictive control (MPC) technique has been widely applied in a large number of industrial plants. Optimal input design should guarantee acceptable model parameter estimates while still providing for low experimental effort. The goal of this work is to investigate an application-oriented identification experiment that satisfies the performance objectives of the implementation of the model. A- and D-optimal input signal design methods for a non-linear liquid two-tank model are presented in this paper. The excitation signal is obtained using a finite impulse response filter (FIR) with respect to the accepted application degradation and the input power constraint. The MPC controller is then used to control the liquid levels of the double tank system subject to the reference trajectory. The MPC scheme is built based on the linearized and discretized model of the system to predict the system's succeeding outputs with reference to the future input signal. The novelty of this model-based method consists in including the experiment cost in input design through the objective function. The proposed framework is illustrated by means of numerical examples, and simulation results are discussed.

Key words: model predictive control, optimal input design, convex optimization, application-oriented identification.

1. Introduction

The efficiency of the model-based control scheme is highly influenced by the quality of the plant model and the accuracy of its parameters. The model parameters estimates depend on conditions under which the identification experiments are performed, including the proper choice of input signals. Optimal input design plays a substantial role in the task of establishing precise estimates of model parameters. Early results of optimal input design for system identification relied on minimization of the error of the parameters being estimated with respect to process constraints. The fundamental principle of system identification is to maximize the sensitivity of the state variable to unknown parameters [1–3]. The focus of the system identification theory is to design optimal inputs for parametric identification of linear time-invariant models.

Most recently, great effort has been made to develop identification methods for robust control [4, 5]. The identification for robust control relies on estimating the nominal model and imposing a limitation on the model's uncertainty set [6]. With the worst-case identification experiment model in mind, parameters are estimated with the established error bounds expressed in the form of the noise affecting the model of the system [7]. Considering the set membership uncertainty method, a set of models, including the true system, is identified [8]. Then the robust control design is used to provide acceptable control efficiency for the models within the set [9].

Effective analysis tools for the stochastic uncertainty set are based on the covariance matrix of the parameters to be estimated [10].

By applying these tools to the task of identification for robust control, it becomes possible to rate the performance of the real unknown system with the controller developed using an estimated model. However, such an approach does not provide the satisfactory performance of the control loop. This is because the conditions of the identification experiment cannot guarantee that the identified model is precise. This inconvenience can be eliminated by the concept of the least-costly identification experiment design for control. The objective of the least-costly design is to develop a strategy that results in an uncertainty set that is relatively small and guarantees the best control performance. Instead of putting constraints on the experiment cost (i.e. input energy, experiment duration), the experiment cost is integrated with input design through the cost function [11]. The application-oriented input design problem presented in this article is based on some results of the approach proposed by Bombois *et al.* [12].

The cost of the identification experiment is quantified by the power of a perturbation signal under real working conditions [13]. Another approach for experimental cost minimization, with a so-called 'plant-friendly' identification task, has been proposed in [14–17]. The plant-friendly excitation signal design is similar to the application-oriented system identification. The goal is to find a trade-off between minimum deviation of working conditions and the accuracy of the model parameters to be estimated [14, 18]. In [19], the concept of a robust plant-friendly input design task with the constraints imposed on the power of the input and the output trajectory was developed. This type of experiment consists in utilization of the sequential

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and robust problems solution. The techniques for computing the input signals in the economic, plant-friendly and application-oriented frameworks, where the purpose is to minimize the displacement from normal operating conditions, were described in [6, 17, 20].

It has been shown that the model development assimilates about 75% of the costs related to real-life control processes [21]. Therefore, one of the trends is to design tools which perform simultaneous identification and system control in a sequential manner. The goal is to sequentially ameliorate the system performance by means of more accurate estimates of model parameters. The problem can be solved by the model predictive control (MPC) procedure, where the input signal yielded by the MPC algorithm is designed to ensure acceptable control performance [22–24].

It has also been reported that fractional models provide a more exact description of the system dynamics than the models developed using ordinary differential equations [25, 26]. For an overview of the optimal input design for fractional-order system identification, see publications [27, 28].

A method presented in this article can be classified as an application-oriented input design for control. The goal of this paper is to design an input signal that minimizes the objective function which includes the experimental cost. The excitation signal of a given length is obtained by means of utilizing a finite impulse response filter (FIR), subject to input power constraints [19, 29]. The simulation experiments have been performed considering the non-linear gravitational water tanks system which has been disturbed with the additive white output noise. A- and the D-optimal criteria have been considered [2] as a measure of optimality. After obtaining the accepted model parameters, the MPC controller is adopted to control the water levels of the double-tank system.

2. Dynamic system identification

The goal of the application-oriented input signal design is to construct a model of the dynamic system from experimental data in a manner that reduces model uncertainty. The open-loop system structure, assumed for system identification, is presented in Fig. 1, where $u(t)$ is the input sequence, $y(t)$ is the measured output, and $e(t)$ is the white noise signal. The G and H transfer functions are parameterized by the parameter vector θ, and q is the unit delay operator. In this paper, the open-loop discrete-time LTI system is used. Therefore, the model response $y(t)$ is given by the following:

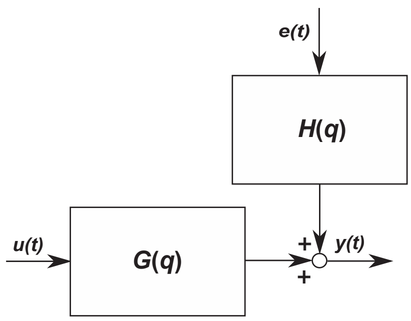

Fig. 1. System description

$$y(t) = G(q,\theta)\,u(t) + H(q,\theta)\,e(t). \tag{1}$$

There exists the vector θ^0 of real parameters that leads to the true system response $y_s(t)$ in the following form:

$$y_s(t) = G\left(q,\theta^0\right)u(t) + H\left(q,\theta^0\right)e(t). \tag{2}$$

The vector of model parameters estimated from the number of N measurements of the system input and output is denoted as $\hat{\theta}_N$.

3. Prediction error identification technique

The prediction error method (PEM) is used for the estimation of unknown parameters of the model described by equation (1). This technique relies on the minimization of a difference between the output of the real plant and the output of the estimated model [30]. The one-step-ahead predictor of model (1) is formulated as:

$$\hat{y}(t|\theta) = H^{-1}(q,\theta)\,G(q,\theta)\,u(t) + \left[I - H^{-1}(q,\theta)\right]y(t). \tag{3}$$

The operation of inversion (H^{-1}) is dedicated only for square systems (i.e. those with the same number of inputs and outputs) with an invertible feedthrough matrix, which handles both continuous- and discrete-time systems [30]. Hence, the one-step-ahead error estimator is expressed by:

$$\varepsilon(t,\theta) = y(t) - \hat{y}(t|\theta) = H^{-1}(q,\theta)\left[y(t) - G(q,\theta)\,u(t)\right]. \tag{4}$$

The parameter values, based on N observations, are evaluated by minimization of the cost function given by:

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon(t)^2. \tag{5}$$

The estimate of the parameter vector is defined as follows:

$$\hat{\theta}_N = \arg\min_{\theta} V_N(\theta). \tag{6}$$

In this paper, the performance function (5) will be reformulated to fit quadratic criterion, required by the Moose2 package, to allow for solution [31]. This criterion can be expressed as:

$$V_N(\theta) = \frac{1}{2N}\sum_{t=1}^{N}\varepsilon(t,\theta)\Lambda^{-1}\varepsilon(t,\theta)^T, \tag{7}$$

where Λ is a zero-mean, white noise covariance matrix. The parameters to be estimated converge to the real parameter values when the number of measurements tends to infinity. In general, the series of random variables is given by:

$$N\left(\hat{\theta}_N - \theta^0\right)\mathrm{I}_F\left(\hat{\theta}_N - \theta^0\right)^T, \tag{8}$$

where I_F is the Fisher information matrix (FIM). It converges to the χ^2 distribution with n-degrees of freedom. According to the Cramer-Rao inequality definition, when the number of measurements N appears to be large, the expected value E of the FIM can be obtained from:

$$\mathrm{I}_F = \frac{1}{N}\sum_{t=1}^{N} E\left\{\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\hat{y}(t,\theta)\right)\Lambda^{-1}\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\hat{y}(t,\theta)\right)^T\right\}. \tag{9}$$

For a large number of samples N, the parameter estimates $\hat{\theta}_N$ are, with probability α, internal elements of the identification confidence ellipsoid [30].

$$\varepsilon_{SI} = \left\{\theta|\left(\hat{\theta}_N-\theta^0\right)\mathrm{I}_F\left(\hat{\theta}_N-\theta^0\right)^T \leq \frac{\chi^2_\alpha(n)}{N}\right\}, \tag{10}$$

where the constant $\chi^2_\alpha(n)$ is the χ^2 distribution with n-degrees of freedom and with probability α. The confidence uncertainty region should confirm the advantages of optimal input design.

4. Input spectrum generation

The MPC technique considers optimal input design to minimize experimental effort while still ensuring adequate performance. In real-life applications, it is extremely important to disturb normal working conditions of a system as little as possible. Considering the experimental expense, the following measures should be included: the input power employed for the experiment, the excitation time duration and the length of the experiment [12]. Thus, an optimal input signal can be designed in the frequency domain by means of computing its spectrum. And spectral density of the input signal can be defined as:

$$\Phi_u(\omega) = \sum_{k=-\infty}^{\infty} c_k\mathfrak{R}_k\left(e^{j\omega}\right), \tag{11}$$

where the scalar basis functions $\left\{\mathfrak{R}_k\left(e^{j\omega}\right)\right\}_{k=0}^{\infty}$ are proper, stable and rational so that $\mathfrak{R}_{-k}\left(e^{j\omega}\right) = \mathfrak{R}_k\left(e^{-j\omega}\right)$ and the factors $c_{-k} = c_k^T$. Since the Fisher information matrix is associated with input spectrum Φ_u, the model parameter estimates are disrupted by the input spectrum to be designed. The Moose2 toolbox uses the FIR filter parametrization of the input spectrum with a predetermined number of coefficients in the spectral density function. Hence, the basis functions are exponentials $\mathfrak{R}_k\left(e^{j\omega}\right) = e^{-j\omega k}$. The optimal input design problem can sometimes be solved by means of convex optimization with respect to the decision factors $c_k = \mathrm{E}\left\{u(t)u(t-k)^T\right\}$. Consequently, c_k factors should be searched such that:

$$\Phi_u(\omega) \succeq 0, \ \forall\, \omega. \tag{12}$$

Finally, only m initial factors of (11) are considered in order to identify the input signal spectrum:

$$\Phi_u(\omega) = \sum_{k=-(m-1)}^{m-1} c_k\mathfrak{R}_k\left(e^{j\omega}\right). \tag{13}$$

Finite-dimensional parametrization can be executed using the positive real lemma which arises from the Kalman-Yakubovich-Popov lemma [32]. The above partial expansion formulation will be used in the experimental part of this work.

The objective of optimal input design is to create an exact model of the system which provides for acceptable performance during system identification.

5. Input design constraints

The application cost function in the form given by equation (5) depends on the established vector of model parameters, θ. Let us denote the performance index to be minimized as $V_{app}(\theta)$. When the value of the cost function V_{app} is zero, then the parameters θ are equal to real parameters, θ^0. If the performance index is differentiable in a θ^0 region, the application cost has the following attributes:

$$V_{app}\left(\theta^0\right) = 0, \quad V'_{app}\left(\theta^0\right) = 0, \quad V''_{app}\left(\theta^0\right) \succeq 0. \tag{14}$$

When the application cost increases its value, the performance degradation ratio also increases. The maximum permitted performance degradation can be defined as:

$$V_{app}(\theta) \leq \frac{1}{\gamma}, \tag{15}$$

where γ is at least some high pre-specified value, e.g. the probability that the performance degradation cost is less than $1 = (2\gamma)$ is at least 99% when using the identified model [33]. The estimates of model parameters that satisfy inequality (15) are included in an acceptable application performance set Θ_{app}, given by:

$$\Theta_{app}(\gamma) = \left\{\theta|V_{app}(\theta) \leq \frac{1}{\gamma}\right\}. \tag{16}$$

The fundamental principle of model parameter estimation is to maximize sensitivity of the state variables to unidentified parameters [1]. Applying definition (16) for input design purposes, we ensure that, with high probability, the parameter estimates are adequate to the real parameters. This requirement can be formulated as:

$$\varepsilon_{SI}(\alpha) \subseteq \Theta_{app}(\gamma), \tag{17}$$

for selected values of α and γ. It is assumed that an application-oriented input design relies on an input construction with probability α that satisfies inequality (15).

The method presented in this paper is similar to the least-costly identification experiment and is referred to as the application-oriented experiment design. The cost function based on the application-oriented experiment assumption can be defined as:

$$\begin{array}{ll} \underset{\text{input}}{\text{minimize}} & \text{Experimental effort} \\ \text{subject to} & \mathrm{I}_F\left\{\theta \in \Theta_{app}(\gamma)\right\} \geq \alpha. \end{array} \tag{18}$$

Using the optimal input design technique for the model parameters estimation, an adequate criterion based on the Fisher information matrix I_F (FIM) should be selected:

- A-optimality: tr $\left(I_F^{-1}\right)$, minimizes the total variance of parameter estimates,
- D-optimality: det $\left(I_F^{-1}\right)$, minimizes the generalized variance of parameter estimates.

For an overview of available criteria, see [2]. Instead of only implementing the assumptions arising from the Cramer-Rao definition [1], the difference between system performance and the performance obtained from the model is also verified [33].

6. Interacting liquid tanks process

The application-oriented input design for the non-linear system identification is executed on the water double-tank process.

The system consisting of two interconnected cylindrical water tanks is presented in Fig. 2. The system is determined by the ratio of the volumetric flow $Q_{\text{in}}(t)$ into the upper tank to the water outflow $Q_{\text{out}}(t)$ through the valve of the lower tank. The balance of the liquid flow in the tank can be expressed as follows:

$$A\frac{\mathrm{d}h(t)}{\mathrm{d}t} = Q_{\text{in}}(t) - Q_{\text{out}}(t), \tag{19}$$

where: A is the cross-sectional area of the tank and $h(t)$ is the liquid level in the tank.

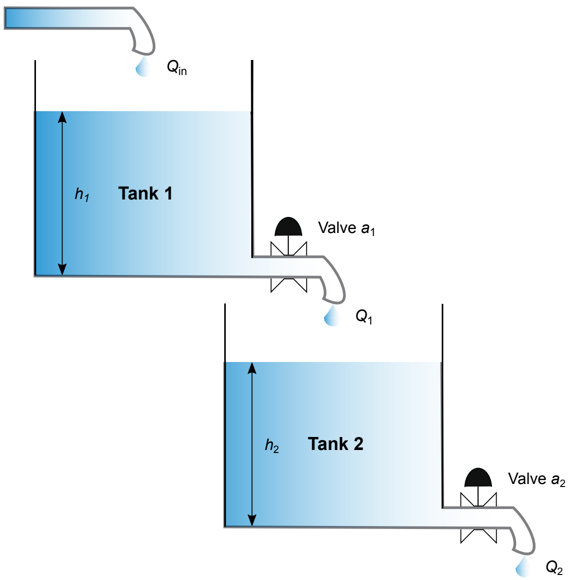

Fig. 2. Interacting water tanks process diagram

It has been assumed that the outlet hole has an ideal sharp-edged orifice. Liquid outflow from the interconnected tanks can be determined by means of the Torricelli's law, given by the following equation:

$$Q_{\text{out}}(t) = a\cdot\sqrt{2gh(t)}\,. \tag{20}$$

where: a is the cross-sectional surface of the hole and g is the gravitational constant value (9.8 m/s^2).

Substituting equation (20) with (19) and supposing that the tanks are connected, as in Fig. 2, it becomes possible to formulate the following nonlinear differential equations:

$$\begin{cases} A_1\dfrac{\mathrm{d}h_1(t)}{\mathrm{d}t} = -a_1\cdot\sqrt{2gh_1(t)} + Q_{\text{in}}(t), \\ \hfill h_1(0) = h_{10}\,, \\ A_2\dfrac{\mathrm{d}h_2(t)}{\mathrm{d}t} = a_1\cdot\sqrt{2gh_1(t)} - a_2\cdot\sqrt{2gh_2(t)}, \\ \hfill h_2(0) = h_{20}\,. \end{cases} \tag{21}$$

The pointer $n = 1,\ 2$ represents one of the system's tanks. After the following substitution, the nonlinear differential equation (21) could be presented in the standard form of the state-space equations: $Q_{\text{in}}(t) = u(t)$, $x_1(t) = h_1(t)$, $x_2(t) = h_2(t)$, $y(t) = h_1(t)$.

$$\begin{cases} \dot{x}_1 = -\dfrac{a_1}{A_1}\cdot\sqrt{2gx_1} + \dfrac{1}{A_1}u, & x_1(0) = h_{10}\,, \\ \dot{x}_2 = \dfrac{a_1}{A_2}\cdot\sqrt{2gx_1} - \dfrac{a_2}{A_2}\cdot\sqrt{2gx_2}, & x_2(0) = h_{20}\,, \end{cases} \tag{22}$$

where: $x_1 = x_1(t,a_1)$, $x_2 = x_2(t,a_1,a_2)$. The liquid level in the upper tank $h_1(t)$ is to be predicted using the MPC method. The water levels have real constraints:

$$h_{i,\max} \geq x_i(t) \geq 0, i = 1,\ 2. \tag{23}$$

The physical constraints and the model parameters of the water tanks process are displayed in Table 1.

Table 1
Physical constraints and model parameters

Parameter	Value	Unit	Description
$h_{1,\max}$	4.00	[m]	Max. water level in tank 1
$h_{1,\min} = h_{2,\min}$	0.00	[m]	Min. water level in tanks 1 and 2
$h_{2,\max}$	2.00	[m]	Max. water level in tank 2
h_{10}	0.60	[m]	Initial condition of tank 1
h_{20}	0.50	[m]	Initial condition of tank 2
$a_1 = a_2$	0.05	[m^2]	Area of water outlet holes
A_1	1.50	[m^2]	Cross-section of tank 1
A_2	0.75	[m^2]	Cross-section of tank 2
u_0	0.05	[m^3/s]	Initial water inflow

To control the water level in the upper tank (Fig. 2), the model predictive control method can be applied. In the case of the MPC application, the controlled system should be linear and discrete-time-guided. Generally, the model of the discrete system can be written as [23]:

$$\begin{aligned} x(t+1) &= A_d x(t) + B_d u(t) + v(t), \\ y(t) &= C_d x(t) + w(t), \end{aligned} \tag{24}$$

where: $x(t) \in \Re^n$ denotes a state vector, $u(t) \in \Re^m$ is an input signal, $y(t)$ signifies the output measured, whereas $v(t)$ and $w(t)$

represent stationary, zero-mean process noise and measurement noise, respectively.

The nonlinear process of level control in water tanks (22) was linearized around the steady-state values of x^0 and u^0 and subsequently discretized. The state-space matrices obtained using a first-order Taylor expansion are:

$$A_l = \begin{bmatrix} -\tau_1 & 0 \\ \tau_3 & -\tau_4 \end{bmatrix}, \quad B_l = \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}, \quad C_l = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$\tau_1 = \frac{a_1}{A_1}\sqrt{\frac{g}{2x_1^0}}, \quad \tau_3 = \frac{a_1}{A_2}\sqrt{\frac{g}{2x_1^0}}, \quad \tau_4 = \frac{a_2}{A_2}\sqrt{\frac{g}{2x_2^0}}. \tag{25}$$

The discretization utilizing a zero-order hold on the input and a 1 Hz sampling rate according to (24) yields:

$$A_d = e^{A_l}, \quad B_d = \int_0^1 e^{A_l(1-t)} B_l \, \mathrm{d}t, \quad C_d = C_l. \tag{26}$$

The above matrices are then used by the model predictive control algorithm to predict the future system output.

7. Simulation results of water tanks system

The optimal input design and system identification methods are verified in this section on the two water tanks system described in the previous section. The system, originally non-linear, was converted to a linear process close to an operating point.

All computations were performed using Matlab dedicated toolbox Moose2 [34]. To run some of the applications, the YALMIP and SDPT3 packages should also be installed [35, 36].

7.1. Application-oriented input design. Optimal input signal is the result of an application-oriented input design task. The parameters estimated should provide a model with adequate application performance and the experimental cost should be as low as possible.

The Moose2 package supports the models of the type used by the *idpoly* function from the Matlab System Identification Toolbox [37]. It is a polynomial model of the system with the input vector $u(t)$, output $y(t)$, and the white Gaussian noise process $e(t)$ with covariance λ. The single-input and single-output linearized system (25) was implemented in the form of a polynomial model with identifiable coefficients:

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t), \tag{27}$$

where: A, B, C, D, and F are polynomials, and q is the shift forward operator, so q^{-1} corresponds to a unit time decrement. Optimal input is assumed to be designed using the FIR filter parameterization of the spectrum with 20 lags (i.e. 20 factors in the spectral density function (13)). The problem specification is: $A = 1.0$; $B = [0.0 \quad \theta_1]$; $F = [1.0 \quad \theta_2]$; $C = 1.0$; $D = 1.0$; $\theta^0 = [0.6667 \quad 0.0851]$ – initial estimate; $\lambda = 1.0$ – noise variance; $T_s = 1.0$ – sampling time [s]; $N = 401$ – number of samples; $\gamma = 100$ – acceptable application degradation; $\alpha = 0.95$ – confidence region of degradation. The optimal experiment has been performed in the open-loop system using A- and D-optimality criteria. The objective function has been formulated as:

$$\begin{aligned} \underset{\Phi_u(\omega)}{\text{minimize}} \quad & \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u(\omega)\,\mathrm{d}(\omega) \\ \text{subject to} \quad & \varepsilon_{\mathrm{SI}}(0.95) \subseteq \Theta_{app}(100) \\ & \Phi_u(\omega) \leq 1, \ \ \forall\omega. \end{aligned} \tag{28}$$

The employment of approximation with $\varepsilon_{\mathrm{SI}}$ allows to solve the convex optimization problem.

The resulting optimal input signal is displayed in Fig. 3.

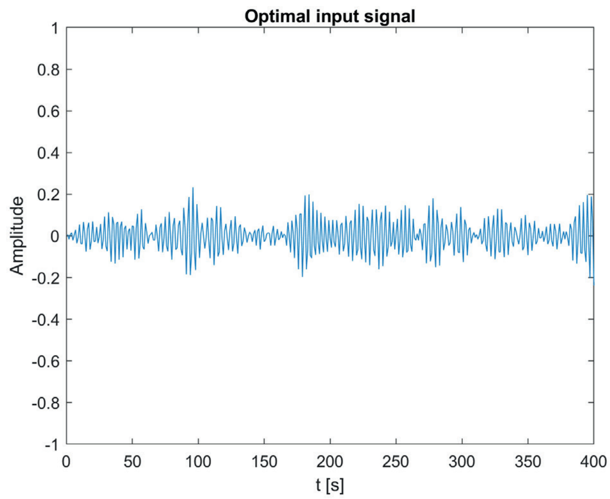

Fig. 3. Optimal input design using D-optimality criteria

Parameters of the white input signal are selected in such a way that the pertinence of the estimates is provided with an identical probability regarding optimal excitation. White Gaussian input with the zero mean and the variance one (Fig. 4) has

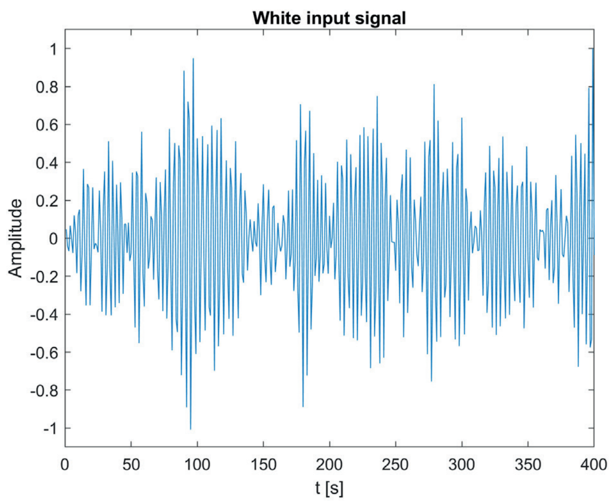

Fig. 4. White Gaussian input signal using the above framework

approximately five times larger peak to peak values in comparison to the D-optimal input signal. The A- and D-optimal input signals have then been used for the system (27) parameter estimation. The optimal input task has been configured to provide 1% performance degradation with 95% probability.

For identification of the water tanks system parameters, 100 Monte-Carlo (MC) independent attempts have been made. The ellipsoidal confidence regions of the identified parameters of the water tanks system are displayed in Figs. 5 and 6.

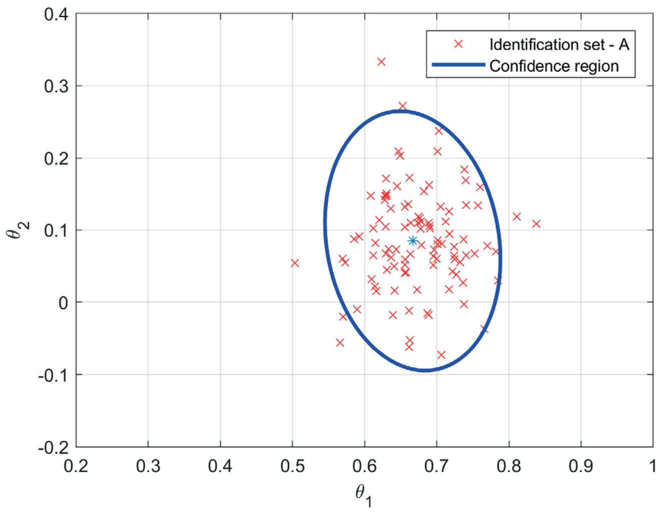

Fig. 5. A-optimal experiment application set

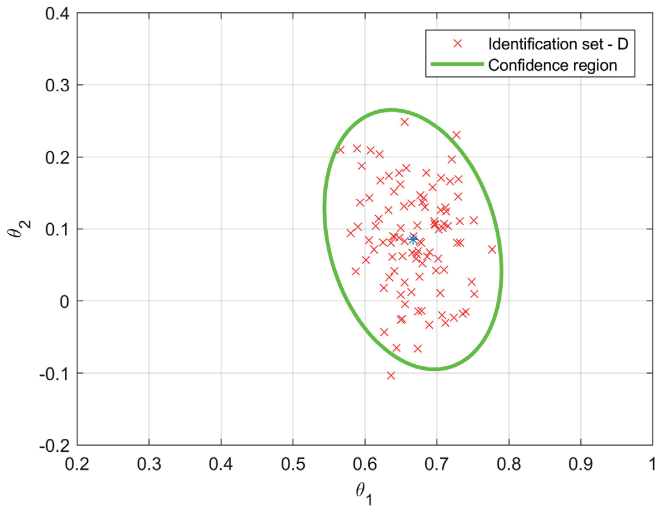

Fig. 6. D-optimal experiment application set

A comparison of the ellipsoidal confidence regions (Fig. 5, 6) of the water tanks parameters identification confirms that the spaces occupied by the estimated parameters are similar. But the values of the estimates obtained using D-optimal input signal have less dispersion. The values of objective functions, identification costs and estimated values of parameters are presented in Table 2.

The results of the identification process are based on Monte-Carlo simulations and have been used for the calculation of experimental indices. The average application cost was obtained using the 2-norm Matlab function as the difference between the model parameter estimate θ (obtained in the minimization procedure) and the true system parameter θ^0 is displayed in Table 2. Based on MC data, the mean values of the estimated model parameters have been also presented.

Table 2
Results of A- and D-optimal experiments

	A-optimality	D-optimality
Initial objective value	10.000	5.394
Final objective value	3.150	1.270
Average cost for θ_1	0.041	0.039
Average cost for θ_2	0.059	0.055
Mean value of θ_1	0.681	0.675
Mean value of θ_2	0.084	0.086

Several disadvantages arise when implementing the application-oriented input design under industrial conditions. First of all, an application cost must contain operating costs related to the production process. Secondly, the identification experiment, in reality, must be executed during the normal operation mode. Thirdly, the closed loop cannot be cut off during the normal production process.

7.2. MPC implementation for water level control. To control the water level of the first system tank, the MPC controller has been introduced [23]. The MPC scheme constructed using a Kalman filter (KF) is shown in the figure below.

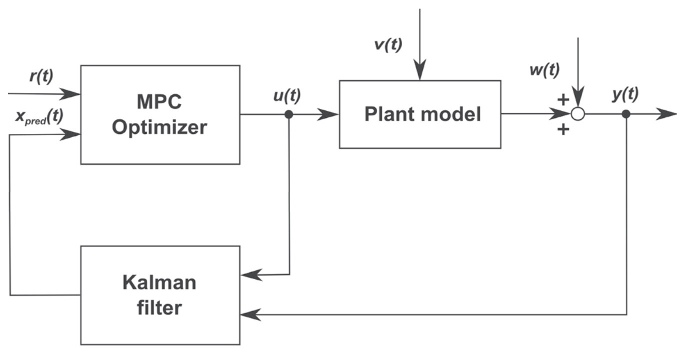

Fig. 7. Block diagram of the system and controller

The MPC takes advantage of the linearized and discrete-time plant model around its working point x^0 and u^0 in the form (24). The discrete-time matrices of state-space system are given by:

$$A_d = \begin{bmatrix} 0.918 & 0 \\ 0.147 & 0.812 \end{bmatrix}, \qquad B_d = \begin{bmatrix} 0.639 \\ 0.052 \end{bmatrix}, \qquad C_d = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{29}$$

The MPC is the recursive method that uses the model of the system and incoming data to predict the system's future output with respect to future input signals. To estimate the system output, full knowledge of the state variable is required:

$$\hat{y}(t+k\,|\,t) = C_d A_d^k x(t) + \sum_{i=0}^{k-1} A_d^{k-i-1} B_d u(t+i). \tag{30}$$

The model performance should be obtained by means of cost function minimization in the following form:

$$J(t) = \sum_{i=0}^{N_y} \|\hat{y}(t+i|t) - r(t+i)\|_Q^2 + \sum_{i=0}^{N_u} \|\Delta\hat{u}(t+i)\|_R^2, \tag{31}$$

where: $\hat{y}(t+i|t)$ is the predicted output, $r(t)$ is the reference signal and $\Delta\hat{u}(t)$ is the input variation over time. Q and R are both weighting matrices, N_y denotes the prediction region and N_u is the control horizon. The MPC input sequence is obtained as a solution to the minimization problem with the control performance index (31):

$$\begin{aligned} \underset{u(t)}{\text{minimize}} \quad & J(t) \\ \text{subject to} \quad & \hat{y} \in Y \\ & \hat{u} \in U. \end{aligned} \tag{32}$$

where Y and U are the constraint sets of the outputs and inputs, respectively. To provide reliable performance of the MPC algorithm, the model has to be constructed very precisely, including valid properties of the system.

The MPC issue of the water level control can be numerically solved using the Matlab-Simulink package. The Kalman estimator of state variables is working in the system presented in Fig. 7, where the process and the observation zero-mean Gaussian noise signals $v(t)$ and $w(t)$ have the same arbitrarily selected variances, in the range from 1e-5 to 1e-1, respectively. The prediction of state variables is executed for fixed values of parameters: $a_1 = 0.05$ and $a_2 = 0.05$. The identification experiment is performed using the sequential quadratic programming method with a time duration of 20 sec. Meanwhile, equations describing the dynamics of water tanks were simulated under the following initial conditions: $x_1(0) = 0.75$, $x_2(0) = 0.5$. The KF response sequence was computed using the fixed-step, 4-th order Runge-Kutta algorithm with a grid interval of 0.1 sec.

The reference trajectory, chosen similarly to [6, 24], has the shape of the zero-one square wave with a period of 10 [s] and duration of the high value (1 [m]) equal to 4 [s]. Figs. 8 and 9 show the nominal responses of model (29) controlled with the MPC developed for the twin system.

The goal of the MPC algorithm was to ensure the reference tracking of the water level in the double-tank system. The water levels in the upper and lower tanks for the plant model are

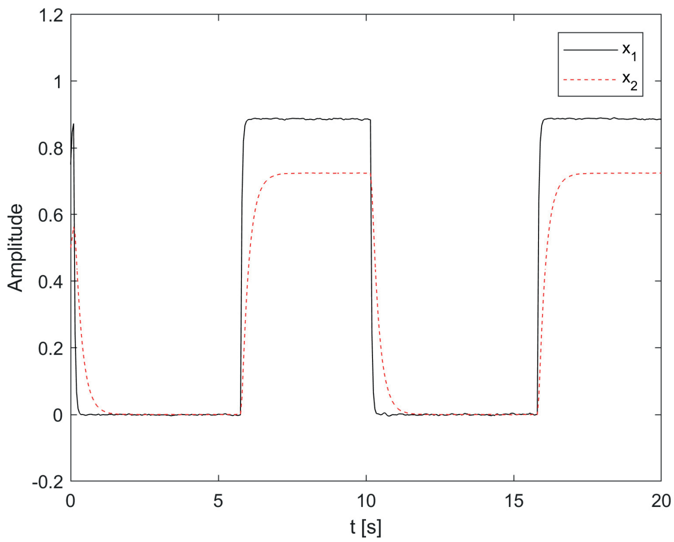

Fig. 8. States of the first (x_1) and second (x_2) tank of the system controlled with the MPC algorithm

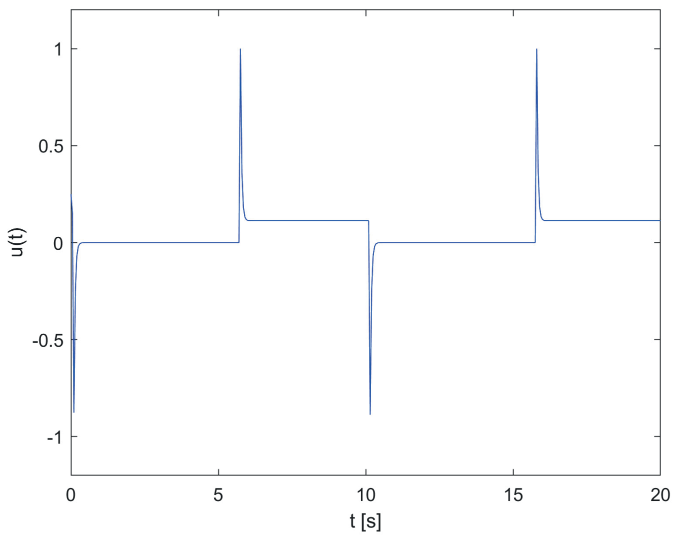

Fig. 9. Input signal to water tanks system controlled with MPC

displayed in Fig. 8. The black (solid) curve represents the water level in the first tank and the red (dashed) line illustrates the water level in the second tank. The input signal obtained for the system controlled by the MPC algorithm is shown in Fig. 9. These plots are obtained for the noise variance values $v(t) = $ 1e-5 and $w(t) = $ 1e-5, respectively. For greater variance values the charts are similar but the curves are more affected by noise. Fig. 10 shows the curves of the first water tank level measurement uncertainty (dashed) black line and the estimation error (solid) red curve obtained for noise variance $v(t) = $ 1e-2. The measurements and KF estimates errors were computed as a difference of values between estimated states and the real states normalized by the number of data points.

As can be noticed from Fig. 10, the KF estimates have about a 35% lower error rate comparing to the raw data.

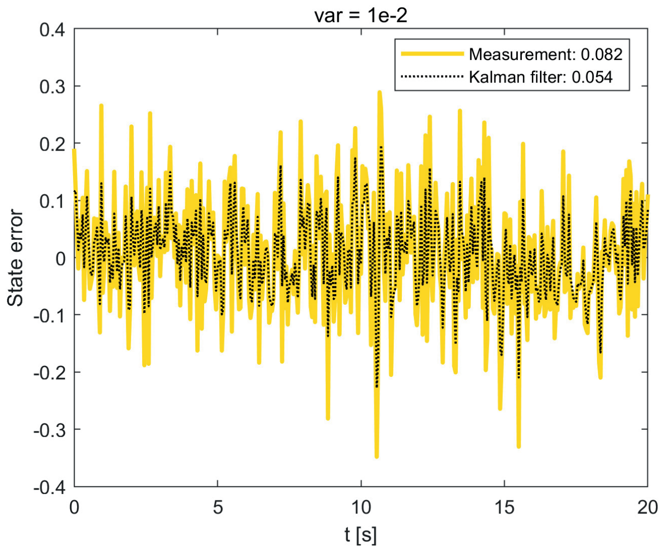

Fig. 10. Plot legend indicates water level measurement and the estimation errors normalized by the number of obtained data

8. Conclusions

The application-oriented input design framework and the MPC method were used for identification of the nonlinear water tanks process. The presented input design formulation was utilized in the identification of system parameters with the intended model application. The aim of this experiment was to predict the water level in the two-tank system, with respect to a reference signal, taking advantage of model prediction properties. The MPC technique requires the linearized and discretized model of the system to be controlled.

In this paper, the novel A- and D-optimal experiments were performed to estimate the system parameters in the presence of white noise disturbing the plant model. The experimental results show that the D-optimal input design yields more precise model parameter estimates. The D-optimal identification experiment shows that the estimated errors for the first and second parameter do not exceed 1.23% and 1.17%, respectively. Performance of the MPC scheme depends strictly on the accuracy of the model to be used by the controller. The ARMAX polynomial model was used for the precise estimation of the linearized and discretized system coefficients. The resulting plant model was then applied for the MPC output sequence prediction. The numerical example confirms that following completion of the intended model application experiment the cost of the identification decreases.

Application-oriented input design was executed using the Matlab-based Moose2 toolbox. From the user's point of view, the capabilities of the package should be extended to include spectrum forms (i.e. discrete spectra, a sum of the sinusoids, etc.), physically parameterized state-space models and different application costs.

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