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Computation method for analysis of sliding faults in power systems

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Abstract. Short-circuit analysis is conducted based on the nodal impedance matrix, which is the inversion of the nodal admittance matrix. If analysis is conducted for sliding faults, then for each fault location four elements of the nodal admittance matrix are subject to changes and the calculation of the admittance matrix inversion needs to be repeated many times. For large-scale networks such an approach is time consuming and unsatisfactory. This paper proves that for each new fault location a new impedance matrix can be found without recalculation of the matrix inversion. It can be found by a simple extension of the initial nodal impedance matrix calculated once for the input model of the network. This paper derives formulas suitable for such an extension and presents a flowchart of the computational method. Numerical tests performed for a test power system confirm the validity and usefulness of the proposed method.

Key words: power system; short-circuit analysis; sliding faults; computation methods.

Nomenclature

$\underline{I}_k$ – Short-circuit current at node k
$\underline{\boldsymbol{I}}$ – Vector of nodal currents
κ – Distance of a fault
$\Delta\kappa$ – Step length for analysis of sliding fault
$\underline{\boldsymbol{V}}$ – Vector of nodal voltages
$\underline{V}_k^0$ – Voltage at node k in the pre-fault state
$\underline{V}_k$ – Voltage at node k in the fault state
$\underline{\boldsymbol{Y}}$ – Nodal admittance matrix for network without sliding fault
$\underline{Y}_{ij}$ – Element of $\underline{\boldsymbol{Y}}$ matrix
$\underline{\boldsymbol{Z}}$ – Nodal impedance matrix (inversion of $\underline{\boldsymbol{Y}}$)
$\underline{Z}_{ij}$ – Element of $\underline{\boldsymbol{Z}}$ matrix
$\underline{Z}_{\mathrm{F}}$ – Impedance of the short circuit
$\underline{z}_{\mathrm{L}}$ – Impedance of a selected transmission line L
$\underline{Z}_{\mathrm{Th}}$ – Thevenin's impedance
$\underline{\boldsymbol{Y}}_{\mathrm{ext}}$ – Extended admittance matrix for the network including a sliding fault location
$\underline{\boldsymbol{Z}}_{\mathrm{ext}}$ – Extended nodal impedance matrix for the network including a sliding fault location

1. Introduction

1.1. Background and Related works. Fault analysis is one of the most frequently used analyses in power engineering [1, 2]. The fault analyses are performed in off-line studies for the bulk power system [3] and microgrids [4, 5] as well in the on-line systems [6, 7] and fault locators [8].

In off-line studies the fault analysis consists of the calculation of short-circuit currents and their flows in network branches. The short-circuit current at a given node is calculated using the Thevenin's theorem. Nodal voltages and current flows in the fault state are calculated using the nodal impedance matrix, which is the inversion of the nodal admittance matrix obtained for the network model with all voltage sources short-circuited to the reference node [9–11].

Generally, the impedance matrix can be obtained by the inversion of the admittance matrix [12].

In the past, when computer memories used to be small, the impedance matrix was generated using El-Abiad's method [13] based on the laws of the circuit theory. A detailed description of this method is in book [14]. In this method the impedance matrix is created step by step together with the network model using circuit operations such as connecting a radial branch to the network model with a new node and connecting a new branch between two nodes already existing in the network model.

Currently, columns of the impedance matrix are calculated by sparsity-oriented factorisation of the admittance matrix [11, 15–17].

In typical off-line analyses the faults are located at busbars of the substations. For certain topics (e.g., concerning power system protections [18] or voltage sags [19–22]), short-circuit currents and their flows in the network branches need to be calculated for sliding faults, in which the location of the fault can be moved across the entire length of the line.

Sliding fault analysis is offered as one of the options in power system analysis software [23]. Hence, fast computation methods are desirable. Although impedance matrix calculation for sliding fault analysis is a very important issue, only a limited research has been conducted on this topic. This paper addresses and develops this research.

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1.2. Motivation. Two simple approaches to analyse the sliding faults are illustrated in Fig. 1. In case of the first approach (Fig. 1a) one fictitious node c is inserted to the input network data placed on the selected line close to one of the end busbars. For such a modified network the nodal admittance matrix $\underline{\boldsymbol{Y}}$ is modified and extended to matrix $\underline{\boldsymbol{Y}}_{\text{ext}}$ including a fictitious node c and the impedance matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ is computed as the inversion of the nodal admittance matrix $\underline{\boldsymbol{Y}}_{\text{ext}}$. The calculation of the impedance matrix is repeated consecutively for each new location of the fictitious node c moved along the line. The calculation of the matrix inversion is the most time-consuming stage and the repetition of this operation many times slows down the analysis, which is particularly burdensome when the analysis must be performed for many lines of large-scale power systems. In the second approach (Fig. 1b) the extended matrix $\underline{\boldsymbol{Y}}_{\text{ext}}$ and its inversion $\underline{\boldsymbol{Z}}_{\text{ext}}$ are calculated considering many fictitious nodes c_1, c_2, ... c_N placed across the entire length of a selected line. When a fault analysis is applied to many lines of large-scale power systems, this method leads to a significant increase in dimension of matrices and computing time.

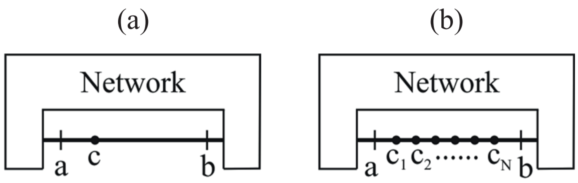

Fig. 1. Illustration of two simple approaches: a) with single fictitious node; b) with many fictitious nodes

The two simple approaches described above have their drawbacks. The method similar to El-Abiad's method would be the most suitable for the analysis of sliding faults. Such a method would allow us to modify the impedance matrix according to each location of the sliding fault without repetition of the matrix inversion. The idea of such an approach is mentioned in paper [21]. Unfortunately, that paper does not provide a suitable mathematical derivation of the relevant formulas. Moreover, El-Abiad's method [13, 14] does not cover such a case as the insertion of a new node to a branch previously defined in the input data.

1.3. Paper contributions and organization. It is proved that for the purpose of sliding faults analysis there is no need to repeat calculations of the inversion of the nodal admittance matrix for each consecutive fault location (Fig. 1a) or to create many fictitious nodes in input data of the network (Fig. 1b). It is sufficient to calculate one impedance matrix $\underline{\boldsymbol{Z}}$ for the network model including only the transmission lines and busbars of substations. Based on matrix $\underline{\boldsymbol{Z}}$ and formulas derived in this paper, it is possible to find the matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ concerning any location of the sliding fault. The flowchart of the computation algorithm is presented.

The paper consists of seven Sections. Section 2 focuses on the preliminaries. Section 3 presents problem formulation. Section 4 discusses a proposed method of determining of the extended impedance matrix. In Section 5 a proposed computation algorithm is presented. In Section 6 simulation results are provided to show the correctness of the proposed method. Section 7 concludes this paper.

2. Preliminaries

2.1. Calculation of short-circuit currents. In off-line fault analyses, each synchronous generator is modelled by electromotive force behind subtransient reactance [2, 9, 11, 12, 24] Lines and transformers are modelled by π-equivalent circuits. Composite loads are replaced by shunt admittances.

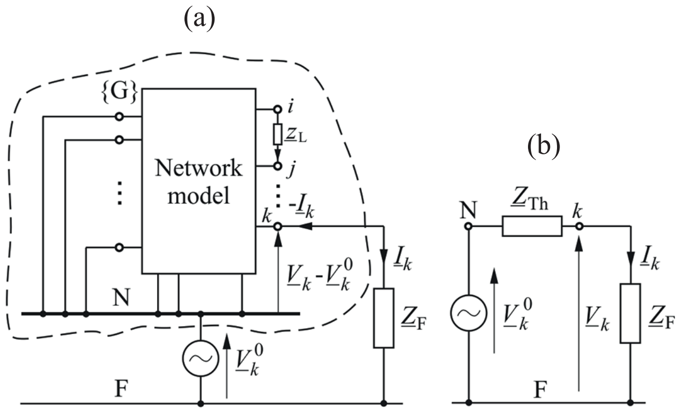

Fig. 2. Illustration of: a) supplementary network model; b) Thevenin's equivalent circuit

The method used to compute the short-circuit current at a given node and its flows in the network branches is derived from the superposition theorem. Node k is assumed to be short-circuited through impedance $\underline{Z}_{\text{F}}$. Two opposing voltage sources $+\underline{V}_k^0$ and $-\underline{V}_k^0$ are connected to this impedance, where $\underline{V}_k^0$ is voltage at node k in the pre-fault state (superscript 0 refers to the pre-fault state). Such modified network model is replaced by a sum of two models: a model for the pre-fault state and a supplementary model (Fig. 2a). The supplementary model is the difference between the models for the short-circuit state and the pre-fault state. In the supplementary model (Fig. 2a), all fictitious nodes {G} with subtransient electromotive forces of generators are short-circuited to the reference node N.

The part of the supplementary model in Fig. 2a surrounded by the dashed line can be described by the following equation:

$$\begin{bmatrix} \vdots \\ 0 \\ 0 \\ -\underline{I}_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & \underline{Y}_{ii} & \underline{Y}_{ij} & \underline{Y}_{ii} & \cdots \\ \cdots & \underline{Y}_{ji} & \underline{Y}_{jj} & \underline{Y}_{jk} & \cdots \\ \cdots & \underline{Y}_{ki} & \underline{Y}_{kj} & \underline{Y}_{kk} & \cdots \\ & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \underline{V}_i - \underline{V}_i^0 \\ \underline{V}_j - \underline{V}_j^0 \\ \underline{V}_k - \underline{V}_k^0 \\ \vdots \end{bmatrix}, \tag{1}$$

or $\underline{\boldsymbol{I}} = \underline{\boldsymbol{Y}} \cdot \Delta\underline{\boldsymbol{V}}$, where $\underline{\boldsymbol{Y}}$ is the admittance matrix; $\Delta\underline{\boldsymbol{V}}$ is the vector of differences between voltages in the fault state and the pre-fault state; $\underline{\boldsymbol{I}}$ is vector with only one non-zero element equal to the negative value of the short-circuit current $\underline{I}_k$. For large-scale networks, the nodal admittance matrix is sparse and is memorized without zero elements [10, 15, 17].

The further considerations assume that the admittance matrix is symmetrical $\underline{\boldsymbol{Y}}^{\mathrm{T}} = \underline{\boldsymbol{Y}}$, which is typical for power system networks. Asymmetry of $\underline{\boldsymbol{Y}}$ occurs only when phase-shifting transformers are installed in the network. The asymmetry of $\underline{\boldsymbol{Z}} = \underline{\boldsymbol{Y}}^{-1}$ resulting from the phase-shifting transformers can be considered by method described in paper [25] and it is not considered here.

The considered part of the supplementary model (Fig. 2a) can be replaced by Thevenin's impedance $\underline{Z}_{\mathrm{Th}}$ seen at nodes k and N. This leads to the equivalent circuit shown in Fig. 2b. For this circuit, the short-circuit current $\underline{I}_k$ and voltage at node k can be calculated using the following formulas:

$$\underline{I}_k = \frac{\underline{V}_k^0}{\underline{Z}_{\mathrm{Th}} + \underline{Z}_{\mathrm{F}}}; \quad \underline{V}_k = \underline{V}_k^0 - \underline{Z}_{\mathrm{Th}} \underline{I}_k, \tag{2}$$

where $\underline{Z}_{\mathrm{F}}$ is the impedance of the short circuit (often assumed to be equal to zero).

2.2. Calculation of nodal voltages and branch currents. Equation (1) can be transformed into the following form:

$$\begin{bmatrix} \vdots \\ \underline{V}_i - \underline{V}_i^0 \\ \underline{V}_j - \underline{V}_j^0 \\ \underline{V}_k - \underline{V}_k^0 \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & \underline{Z}_{ii} & \underline{Z}_{ij} & \underline{Z}_{ik} & \cdots \\ \cdots & \underline{Z}_{ji} & \underline{Z}_{jj} & \underline{Z}_{jk} & \cdots \\ \cdots & \underline{Z}_{ki} & \underline{Z}_{kj} & \underline{Z}_{kk} & \cdots \\ & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ 0 \\ 0 \\ -\underline{I}_k \\ \vdots \end{bmatrix}, \tag{3}$$

or $\Delta\underline{\boldsymbol{V}} = \underline{\boldsymbol{Z}} \cdot \underline{\boldsymbol{I}}$, where $\underline{\boldsymbol{Z}} = \underline{\boldsymbol{Y}}^{-1}$.

Matrix $\underline{\boldsymbol{Z}}$ has an interesting and important property resulting from Eqs. (2) and (3). From Eq. (3) it is obtained:

$$\underline{V}_k = \underline{V}_k^0 - \underline{Z}_{kk} \underline{I}_k, \tag{4}$$

Comparison of Eqs. (4) and (2) leads to:

$$\underline{Z}_{\mathrm{Th}} = \underline{Z}_{kk}, \tag{5}$$

which means that each diagonal element of matrix $\underline{\boldsymbol{Z}}$ is equal to Thevenin's equivalent impedance. Therefore, when matrix $\underline{\boldsymbol{Z}}$ is known, short-circuit currents at all network nodes can be calculated by Eq. (2). Then voltages at any nodes can be calculated by Eq. (3):

$$\underline{V}_i = \underline{V}_i^0 - \underline{Z}_{ik} \underline{I}_k, \tag{6}$$

$$\underline{V}_j = \underline{V}_j^0 - \underline{Z}_{jk} \underline{I}_k. \tag{7}$$

The current flowing through a branch with impedance $\underline{z}_{\mathrm{L}}$ connecting nodes i, j can be calculated by Ohm's law:

$$\underline{I}_{i-j} = \frac{\underline{V}_i - \underline{V}_j}{\underline{z}_{\mathrm{L}}} = \underline{I}_{i-j}^0 - \frac{\underline{Z}_{ik} - \underline{Z}_{jk}}{\underline{z}_{\mathrm{L}}} \underline{I}_k, \tag{8}$$

where $\underline{I}_{i-j}^0 = \left(\underline{V}_i^0 - \underline{V}_j^0\right)/\underline{z}_{\mathrm{L}}$ corresponds to the current flowing through the branch in the pre-fault state; $\underline{Z}_{ik}$, $\underline{Z}_{jk}$ are elements of the nodal impedance matrix.

3. Problem formulation

3.1. Important equalities of submatrices. It is assumed that the nodes of the considered network model are divided into two sets: retained nodes $\{\mathrm{R}\}$ and eliminated nodes $\{\mathrm{E}\}$. The network model is described by the following nodal admittance equation:

$$\begin{bmatrix} \underline{\boldsymbol{I}}_{\mathrm{R}} \\ \underline{\boldsymbol{I}}_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{RR}} & \underline{\boldsymbol{Y}}_{\mathrm{RE}} \\ \underline{\boldsymbol{Y}}_{\mathrm{ER}} & \underline{\boldsymbol{Y}}_{\mathrm{EE}} \end{bmatrix} \begin{bmatrix} \Delta\underline{\boldsymbol{V}}_{\mathrm{R}} \\ \Delta\underline{\boldsymbol{V}}_{\mathrm{E}} \end{bmatrix} \quad \text{or} \quad \underline{\boldsymbol{I}} = \underline{\boldsymbol{Y}} \Delta\underline{\boldsymbol{V}}, \tag{9}$$

Equation (9) can be transformed into the following form referred to as the partial inversion:

$$\begin{bmatrix} \underline{\boldsymbol{I}}_{\mathrm{R}} \\ \Delta\underline{\boldsymbol{V}}_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} (\underline{\boldsymbol{Y}}_{\mathrm{RR}} - \underline{\boldsymbol{Y}}_{\mathrm{RE}} \underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \underline{\boldsymbol{Y}}_{\mathrm{ER}}) & \underline{\boldsymbol{Y}}_{\mathrm{RE}} \underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \\ -\underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \underline{\boldsymbol{Y}}_{\mathrm{ER}} & \underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \end{bmatrix} \begin{bmatrix} \Delta\underline{\boldsymbol{V}}_{\mathrm{R}} \\ \underline{\boldsymbol{I}}_{\mathrm{E}} \end{bmatrix}, \tag{10}$$

When at all nodes $\{\mathrm{E}\}$ nodal currents $\underline{\boldsymbol{I}}_{\mathrm{E}} = \boldsymbol{0}$ then:

$$\underline{\boldsymbol{I}}_{\mathrm{R}} = \underline{\boldsymbol{Y}}_{\mathrm{R}} \Delta\underline{\boldsymbol{V}}_{\mathrm{R}}; \quad \underline{\boldsymbol{Y}}_{\mathrm{R}} = \underline{\boldsymbol{Y}}_{\mathrm{RR}} - \underline{\boldsymbol{Y}}_{\mathrm{RE}} \underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \underline{\boldsymbol{Y}}_{\mathrm{ER}}. \tag{11}$$

Matrix $\underline{\boldsymbol{Y}}_{\mathrm{R}}$ in Eq. (11) determines the parameters of an equivalent network that directly connects nodes $\{\mathrm{R}\}$ and therefore is referred to as the transfer equivalent matrix [2].

When the entire admittance matrix $\underline{\boldsymbol{Y}}$ is inverted, the following is obtained from Eq. (9):

$$\begin{bmatrix} \Delta\underline{\boldsymbol{V}}_{\mathrm{R}} \\ \Delta\underline{\boldsymbol{V}}_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{Z}}_{\mathrm{RR}} & \underline{\boldsymbol{Z}}_{\mathrm{RE}} \\ \underline{\boldsymbol{Z}}_{\mathrm{ER}} & \underline{\boldsymbol{Z}}_{\mathrm{EE}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{I}}_{\mathrm{R}} \\ \underline{\boldsymbol{I}}_{\mathrm{E}} \end{bmatrix} \quad \text{or} \quad \Delta\underline{\boldsymbol{V}} = \underline{\boldsymbol{Z}} \underline{\boldsymbol{I}}, \tag{12}$$

where $\underline{\boldsymbol{Z}} = \underline{\boldsymbol{Y}}^{-1}$ is the nodal impedance matrix. Equation (12) can be transformed into the following form:

$$\begin{bmatrix} \underline{\boldsymbol{I}}_{\mathrm{R}} \\ \Delta\underline{\boldsymbol{V}}_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{Z}}_{\mathrm{RR}}^{-1} & -\underline{\boldsymbol{Z}}_{\mathrm{RR}}^{-1} \underline{\boldsymbol{Z}}_{\mathrm{RE}} \\ \underline{\boldsymbol{Z}}_{\mathrm{ER}} \underline{\boldsymbol{Z}}_{\mathrm{RR}}^{-1} & (\underline{\boldsymbol{Z}}_{\mathrm{EE}} - \underline{\boldsymbol{Z}}_{\mathrm{ER}} \underline{\boldsymbol{Z}}_{\mathrm{RR}}^{-1} \underline{\boldsymbol{Z}}_{\mathrm{RE}}) \end{bmatrix} \begin{bmatrix} \Delta\underline{\boldsymbol{V}}_{\mathrm{R}} \\ \underline{\boldsymbol{I}}_{\mathrm{E}} \end{bmatrix}. \tag{13}$$

Four equalities result from the comparison of submatrices in Eqs. (10) and (13). One of them is important for further considerations:

$$\underline{\boldsymbol{Y}}_{\mathrm{R}} = (\underline{\boldsymbol{Y}}_{\mathrm{RR}} - \underline{\boldsymbol{Y}}_{\mathrm{RE}} \underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \underline{\boldsymbol{Y}}_{\mathrm{ER}}) = \underline{\boldsymbol{Z}}_{\mathrm{RR}}^{-1}, \tag{14}$$

where $\underline{\boldsymbol{Y}}_{\mathrm{R}}$ describes the equivalent network obtained by elimination of all nodes $\{\mathrm{E}\}$.

In the particular case when the set of retained nodes $\{\mathrm{R}\}$ contains only one node k and all remaining nodes are eliminated, Eq. (14) yields the following:

$$\underline{Y}_{k} = (\underline{Y}_{kk} - \underline{\boldsymbol{Y}}_{k\mathrm{E}} \underline{\boldsymbol{Y}}_{\mathrm{EE}}^{-1} \underline{\boldsymbol{Y}}_{\mathrm{E}k}) = \underline{Z}_{kk}^{-1}, \tag{15}$$

where $\underline{Y}_{k} = 1/\underline{Z}_{\mathrm{Th}}$ is the equivalent shunt admittance replacing the whole network and $\underline{Z}_{\mathrm{Th}}$ is Thevenin's impedance seen from node k.

Equations (10), (13) and (14) lead to the following conclusions:

(i) Elimination of nodes in the network model in the nodal admittance matrix $\underline{\boldsymbol{Y}}$ is equivalent to the elimination of relevant rows and columns and in the nodal impedance matrix is equivalent to the removal of the relevant row and column.

(ii) The transfer equivalent matrix $\underline{\boldsymbol{Y}}_{\mathrm{R}}$ can be calculated as the inverse of $\underline{\boldsymbol{Z}}_{\mathrm{RR}}$ which is the submatrix of the nodal impedance matrix $\underline{\boldsymbol{Z}}$.

In large-scale power systems the fault analysis is not carried out for the entire network, but for a selected area of interest defined as a set of nodes $\{\mathrm{R}\}$. Rows (or columns) of the nodal impedance matrix are only calculated for this set of nodes, i.e. only $\underline{\boldsymbol{Z}}_{\mathrm{RR}}$ is calculated. For the sake of simplicity, subscript R is removed from all symbols below.

3.2. Task formulation. The aim of this paper is to find mathematical formulas which enable to find the extended impedance matrix $\underline{\boldsymbol{Z}}_{\mathrm{ext}}$ without the calculation of additional matrix inversion and only on the basis of the impedance matrix $\underline{\boldsymbol{Z}}$ and impedance $\underline{z}_{\mathrm{L}}$ of the considered transmission line $\underline{z}_{\mathrm{L}}$ and the fault distance κ.

4. Problem solution

4.1. Input data and initial impedance matrix. The input data fed into the short-circuit program contain only the nodes representing the busbars of substations and the branches representing the lines and transformers connecting these nodes. No fictitious nodes representing the sliding faults are defined in the input data. For such an input model, the nodal impedance matrix $\underline{\boldsymbol{Z}}$ is computed. Further such a matrix is referred to as the initial impedance matrix.

An input system model is illustrated in Fig. 3. Assuming that a short circuit is located at any node i, the following nodal impedance equation can be written for this model:

$$\begin{bmatrix} \vdots \\ \underline{V}_i - \underline{V}_i^0 \\ \vdots \\ \underline{V}_{\mathrm{a}} - \underline{V}_{\mathrm{a}}^0 \\ \underline{V}_{\mathrm{b}} - \underline{V}_{\mathrm{b}}^0 \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \cdots & \vdots & \vdots \\ \cdots & \underline{Z}_{ii} & \cdots & \underline{Z}_{i\mathrm{a}} & \underline{Z}_{i\mathrm{b}} \\ \cdots & \vdots & \ddots & \vdots & \vdots \\ \cdots & \underline{Z}_{\mathrm{a}i} & \cdots & \underline{Z}_{\mathrm{aa}} & \underline{Z}_{\mathrm{ab}} \\ & \underline{Z}_{\mathrm{b}i} & \cdots & \underline{Z}_{\mathrm{ba}} & \underline{Z}_{\mathrm{bb}} \end{bmatrix} \begin{bmatrix} \vdots \\ -\underline{I}_i \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \tag{16}$$

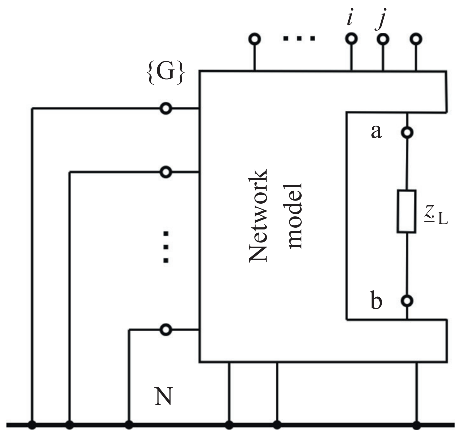

Fig. 3. Illustration of the input system model

or $\Delta\underline{\boldsymbol{V}} = \underline{\boldsymbol{Z}} \cdot \underline{\boldsymbol{I}}$. Based on Eq. (16), voltages at nodes a, b are determined by the following equations:

$$\underline{V}_{\mathrm{a}} = \underline{V}_{\mathrm{a}}^0 - \underline{Z}_{\mathrm{a}i} \underline{I}_i, \tag{17}$$

$$\underline{V}_{\mathrm{b}} = \underline{V}_{\mathrm{b}}^0 - \underline{Z}_{\mathrm{b}i} \underline{I}_i, \tag{18}$$

where $\underline{I}_i = V_i^0 / \left(\underline{Z}_{\mathrm{Th}} + \underline{Z}_{\mathrm{F}}\right)$ is the short-circuit current.

The current flowing through the branch connecting nodes a, b results from Ohm's law:

$$\underline{I}_{\mathrm{a-b}} = \frac{\underline{V}_{\mathrm{a}} - \underline{V}_{\mathrm{b}}}{\underline{z}_{\mathrm{L}}} = \underline{I}_{\mathrm{a-b}}^0 - \frac{\underline{Z}_{\mathrm{a}i} - \underline{Z}_{\mathrm{b}i}}{\underline{z}_{\mathrm{L}}} \underline{I}_i, \tag{19}$$

where $\underline{z}_{\mathrm{L}}$ is the impedance of this branch.

4.2. Insertion of an additional node. Now it is assumed that the additional node c must be inserted inside the line connecting nodes a, b as shown in Fig. 4. The impedance of this line is $\underline{z}_{\mathrm{L}}$. The location of node c is determined by coefficient κ such that the impedance between nodes a, c is $\kappa\underline{z}_{\mathrm{L}}$ and between nodes b, c it is $(1-\kappa)\underline{z}_{\mathrm{L}}$. The admittance matrix considering an additional

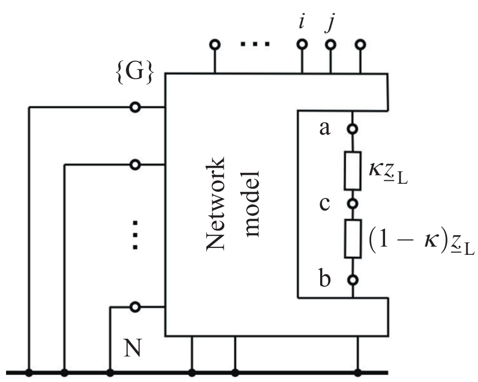

Fig. 4. Illustration of the extended system model

node representing a given fault location is further denoted as $\underline{\boldsymbol{Y}}_{\text{ext}}$ and referred to as the extended admittance matrix.

The considered extended model (Fig. 4) can be described by the following equation (similar to Eq. (16)):

$$\begin{bmatrix} \vdots \\ \underline{V}_i - \underline{V}_i^0 \\ \vdots \\ \underline{V}_\text{a} - \underline{V}_\text{a}^0 \\ \underline{V}_\text{b} - \underline{V}_\text{b}^0 \\ \underline{V}_\text{c} - \underline{V}_\text{c}^0 \end{bmatrix} = \left[\begin{array}{cccccc|c} \ddots & \vdots & \cdots & \vdots & \vdots & & \vdots \\ \cdots & \underline{Z}_{ii} & \cdots & \underline{Z}_{i\text{a}} & \underline{Z}_{i\text{b}} & & \underline{Z}_{i\text{c}} \\ \cdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ \cdots & \underline{Z}_{\text{a}i} & \cdots & \underline{Z}_\text{aa} & \underline{Z}_\text{ab} & & \underline{Z}_\text{ac} \\ \cdots & \underline{Z}_{\text{b}i} & \cdots & \underline{Z}_\text{ba} & \underline{Z}_\text{bb} & & \underline{Z}_\text{bc} \\ \hline \cdots & \underline{Z}_{\text{c}i} & \cdots & \underline{Z}_\text{ca} & \underline{Z}_\text{cb} & & \underline{Z}_\text{cc} \end{array}\right] \left[\begin{array}{c} \vdots \\ -\underline{I}_i \\ \vdots \\ 0 \\ 0 \\ \hline 0 \end{array}\right], \tag{20}$$

or $\Delta\underline{\boldsymbol{V}}_{\text{ext}} = \underline{\boldsymbol{Z}}_{\text{ext}} \cdot \underline{\boldsymbol{I}}_{\text{ext}}$, where the last row and the last column of matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ concerning the additional node c are unknown. The remaining part of matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ is the same as in Eq. (16), i.e. it is equal to the initial matrix $\underline{\boldsymbol{Z}}$ computed for the input model. This can be justified by two facts:

(i) Inserting node c inside the line connecting nodes a, b does not change the electrical state of any of the remaining nodes and the short-circuit currents and nodal voltages in the network.

(ii) As demonstrated in Section 3, the elimination of any node from the network is equivalent to the removal of the relevant row and column from the nodal impedance matrix. This also means, that elimination of node c from the model shown in Fig. 4 reduces the extended matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ to the initial matrix $\underline{\boldsymbol{Z}}$ describing the model shown in Fig. 3.

4.3. Off-diagonal elements of the extended matrix. To determine the off-diagonal elements of matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$, voltage $\underline{V}_\text{c}^0$ needs to be found at additional node c in the pre-fault state and voltage $\underline{V}_\text{c}$ in the fault state in the case of the short-circuit at node *i*. The relevant values result directly from Kirchhoff's and Ohm's laws applied to the circuit in Fig. 4. For the pre-fault state the following can be written:

$$\underline{V}_\text{c}^0 = \underline{V}_\text{a}^0 - \kappa \underline{z}_\text{L} \underline{I}_\text{a-b}^0, \tag{21}$$

$$\underline{V}_\text{c} = \underline{V}_\text{a} - \kappa \underline{z}_\text{L} \underline{I}_\text{a-b}, \tag{22}$$

where $\underline{I}_{\text{a}-\text{b}}^0 = \left(\underline{V}_\text{a}^0 - \underline{V}_\text{b}^0\right)/\underline{z}_\text{L}$ and $\underline{I}_{\text{a}-\text{b}}$ are the pre-fault and fault currents in the line a – b. Substituting the fault current $\underline{I}_{\text{a}-\text{b}}$ resulting from (19) into Eq. (22) leads to:

$$\underline{V}_\text{c} = \underline{V}_\text{a} - \kappa \underline{z}_\text{L}\left[\underline{I}_{\text{a}-\text{b}}^0 - \frac{\underline{Z}_{\text{a}i} - \underline{Z}_{\text{b}i}}{\underline{z}_\text{L}} \underline{I}_i\right],$$

and hence

$$\underline{V}_\text{c} = \underline{V}_\text{a} - \kappa \underline{z}_\text{L} \underline{I}_\text{a-b}^0 + \kappa(\underline{Z}_{\text{a}i} - \underline{Z}_{\text{b}i})\underline{I}_i. \tag{23}$$

From Eq. (21) it results that $-\kappa \underline{z}_\text{L} \underline{I}_{\text{a}-\text{b}}^0 = \underline{V}_\text{c}^0 - \underline{V}_\text{a}^0$. Substituting this into Eq. (23) gives:

$$\underline{V}_\text{c} = \underline{V}_\text{a} + \underline{V}_\text{c}^0 - \underline{V}_\text{a}^0 + \kappa\left(\underline{Z}_{\text{a}i} - \underline{Z}_{\text{b}i}\right)\underline{I}_i,$$

or

$$\underline{V}_\text{c} - \underline{V}_\text{c}^0 = \underline{V}_\text{a} - \underline{V}_\text{a}^0 + \kappa(\underline{Z}_{\text{a}i} - \underline{Z}_{\text{b}i})\underline{I}_i. \tag{24}$$

From Eq. (17) it results $\underline{V}_\text{a} - \underline{V}_\text{a}^0 = -\underline{Z}_{\text{a}i}\underline{I}_i$. Substituting this into Eq. (24) gives:

$$\begin{aligned} \underline{V}_\text{c} - \underline{V}_\text{c}^0 &= -\underline{Z}_{\text{a}i}\underline{I}_i + \kappa\underline{Z}_{\text{a}i}\underline{I}_i - \kappa\underline{Z}_{\text{b}i}\underline{I}_i, \\ \underline{V}_\text{c} - \underline{V}_\text{c}^0 &= -\left[(1-\kappa)\underline{Z}_{\text{a}i} + \kappa\underline{Z}_{\text{b}i}\right]\underline{I}_i. \end{aligned} \tag{25}$$

However, from Eq. (20) it results that:

$$\underline{V}_\text{c} - \underline{V}_\text{c}^0 = -\underline{Z}_{\text{c}i}\,\underline{I}_i. \tag{26}$$

Comparison of Eqs. (25) and (26) leads to:

$$\underline{Z}_{\text{c}i} = (1-\kappa)\underline{Z}_{\text{a}i} + \kappa\underline{Z}_{\text{b}i}. \tag{27}$$

Equation (27) applies to any node *i* in the input model and at the same time to $i = \text{a}$ and $i = \text{b}$. Hence:

$$\underline{Z}_\text{ca} = (1-\kappa)\underline{Z}_\text{aa} + \kappa\underline{Z}_\text{ba}, \tag{28}$$

$$\underline{Z}_\text{cb} = (1-\kappa)\underline{Z}_\text{ab} + \kappa\underline{Z}_\text{bb}. \tag{29}$$

Admittance and impedance matrices describing the network model used for short-circuit analysis are symmetrical and therefore $\underline{Z}_{i\text{c}} = \underline{Z}_{\text{c}i}$.

4.4. Diagonal element of the extended matrix. A different approach is needed to derive the formula determining the diagonal element $\underline{Z}_\text{cc}$. The simplest way is to use the fact that $\underline{\boldsymbol{Z}}_{\text{ext}} \cdot \underline{\boldsymbol{Y}}_{\text{ext}} = \mathbf{1}$, where $\mathbf{1}$ is the identity matrix. As shown in Fig. 4, node c is directly connected only to nodes a, b. Hence, matrix $\underline{\boldsymbol{Y}}_{\text{ext}}$ in the last row and the last column has only three non-zero elements:

$$\underline{Y}_\text{ca} = \underline{Y}_\text{ac} = -\frac{1}{\kappa\underline{z}_\text{L}}; \quad \underline{Y}_\text{cb} = \underline{Y}_\text{bc} = -\frac{1}{(1-\kappa)\underline{z}_\text{L}}, \tag{30}$$

$$\underline{Y}_\text{cc} = \frac{1}{\kappa\underline{z}_\text{L}} + \frac{1}{(1-\kappa)\underline{z}_\text{L}}. \tag{31}$$

Considering these non-zero elements, the following equation can be written:

$$\left[\begin{array}{ccc|c} \ddots & \vdots & \vdots & \vdots \\ \cdots & \underline{Z}_\text{aa} & \underline{Z}_\text{ab} & \underline{Z}_\text{ac} \\ \cdots & \underline{Z}_\text{ba} & \underline{Z}_\text{bb} & \underline{Z}_\text{bc} \\ \hline \cdots & \underline{Z}_\text{ca} & \underline{Z}_\text{cb} & \underline{Z}_\text{cc} \end{array}\right] \cdot \left[\begin{array}{ccc|c} \ddots & \vdots & \vdots & \mathbf{0} \\ \cdots & \underline{Y}_\text{aa} & \underline{Y}_\text{ab} & \underline{Y}_\text{ac} \\ \cdots & \underline{Y}_\text{ba} & \underline{Y}_\text{bb} & \underline{Y}_\text{bc} \\ \hline \mathbf{0} & \underline{Y}_\text{ca} & \underline{Y}_\text{cb} & \underline{Y}_\text{cc} \end{array}\right] = \left[\begin{array}{ccc|c} \ddots & & & \\ & 1 & & \\ & & 1 & \\ \hline & & & 1 \end{array}\right]$$

By multiplying the last column of matrix $\underline{\boldsymbol{Y}}_{\text{ext}}$ by the last row of matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ in the above equation, the following result is obtained:

$$\underline{Z}_{ca}\underline{Y}_{ac} + \underline{Z}_{cb}\underline{Y}_{bc} + \underline{Z}_{cc}\underline{Y}_{cc} = 1, \tag{32}$$

Admittances $\underline{Y}_{ac}$, $\underline{Y}_{bc}$, $\underline{Y}_{cc}$ in Eq. (32) can be replaced by admittances determined by Eqs. (30) and (31). After such an operation it is obtained:

$$-\frac{\underline{Z}_{ca}}{\kappa \underline{z}_L} - \frac{\underline{Z}_{cb}}{(1-\kappa)\underline{z}_L} + \underline{Z}_{cc}\frac{1}{\kappa \underline{z}_L} + \frac{1}{(1-\kappa)\underline{z}_L} = 1,$$

$$\underline{Z}_{cc} = (1-\kappa)\underline{Z}_{ca} + \kappa \underline{Z}_{cb} + \kappa(1-\kappa)\underline{z}_L.$$

Impedances $\underline{Z}_{ca}$, $\underline{Z}_{cb}$ in the above equation can be replaced by impedances determined by Eqs. (28) and (29). Then the following equation is obtained:

$$\begin{aligned}\underline{Z}_{cc} &= (1-\kappa)^2\underline{Z}_{aa} + 2\kappa(1-\kappa)\underline{Z}_{ab} + \\ &+ \kappa^2\underline{Z}_{bb} + \kappa(1-\kappa)\underline{z}_L,\end{aligned} \tag{33}$$

where $\underline{Z}_{cc}$ in the above equation is the diagonal element of the extended impedance matrix and the same time the Thevenin's equivalent impedance for node c.

4.5. Features of extended impedance matrix. The consideration regarding the extended impedance matrix $\underline{\boldsymbol{Z}}_{ext}$ presented above can be summarized as follows:

(i) The extended impedance matrix $\underline{\boldsymbol{Z}}_{ext}$ can be found without the need to calculate the inversion of the extended admittance matrix $\underline{\boldsymbol{Y}}_{ext}$.

(ii) The unknown diagonal and off-diagonal elements of the extended impedance matrix $\underline{\boldsymbol{Z}}_{ext}$ are the functions of appropriate elements of the initial impedance matrix $\underline{\boldsymbol{Z}}$ and impedances $\kappa\underline{z}_L$ and $(1-\kappa)\underline{z}_L$.

(iii) The calculation of the diagonal and off-diagonal elements of the extended impedance matrix $\underline{\boldsymbol{Z}}_{ext}$ based on Eqs. (27–29, and 33) is very simple.

5. Computation algorithm

A flowchart of the proposed computational algorithm is shown in Fig. 5. It is assumed that, based on the initial impedance matrix $\underline{\boldsymbol{Z}}$ computed for the area of interest, a computer program based on the proposed method computes the extended impedance matrix $\underline{\boldsymbol{Z}}_{ext}$ for all lines and all sliding faults without repetition of the inversion or factorisation of the nodal admittance matrix.

In the flowchart shown in Fig. 5 the blocks 1–3 relate to the input data of considered network and determination of the area of interest, which is the part of typical short-circuit analysis. The step length for analysis of sliding fault $\Delta\kappa$ is determined in block 4 and the line for such an analysis is selected in block 5. Blocks 6–9 perform calculations of diagonal and off-diagonal elements of the extended matrix based on the proposed method. A short-circuit analysis for the consecutive fault location is performed in block 10.

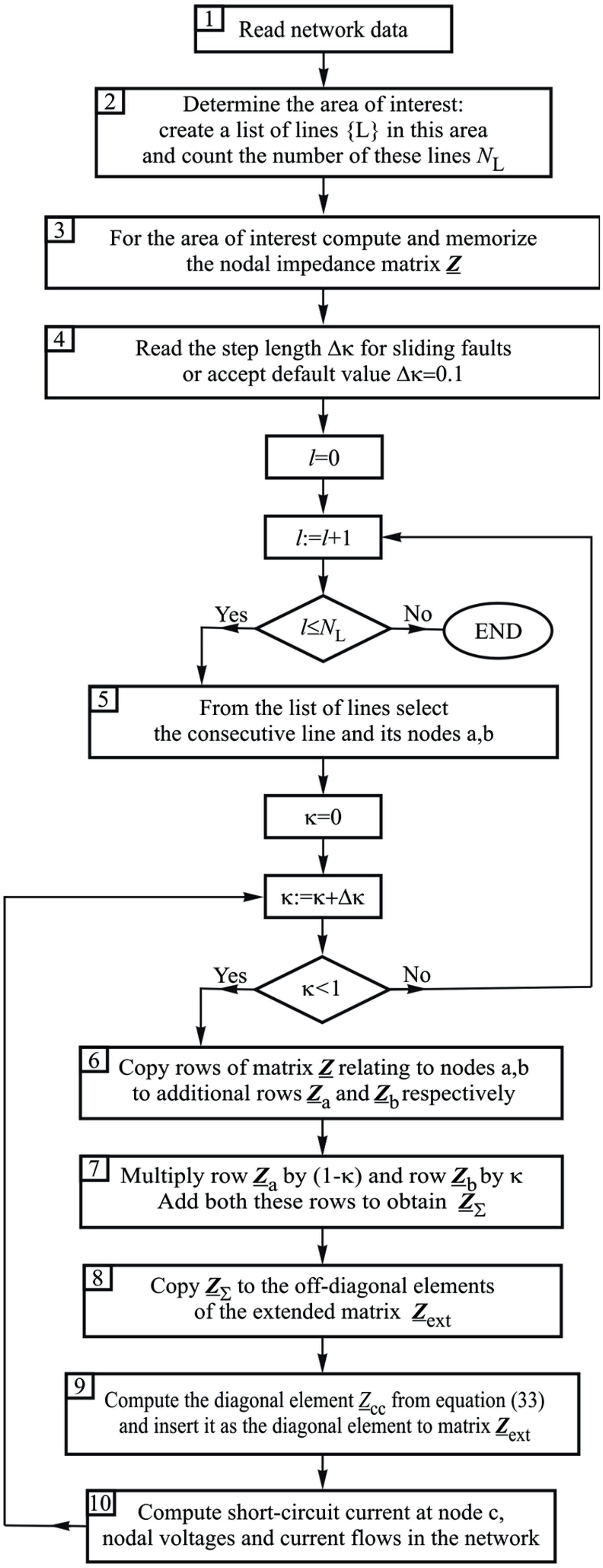

Fig. 5. Flowchart of computation algorithm

6. Case studies

6.1. Test system. There are various test systems. It is sufficient to use the test system shown in Fig. 6 to verify the correctness of the proposed method. The network of the test system consists of six transmission lines with nominal voltage 220 kV, two generating units G1, G2 and an equivalent generator G3, which is an equivalent source for the remaining part of a larger system. Network data for the considered test system are given in Table 1.

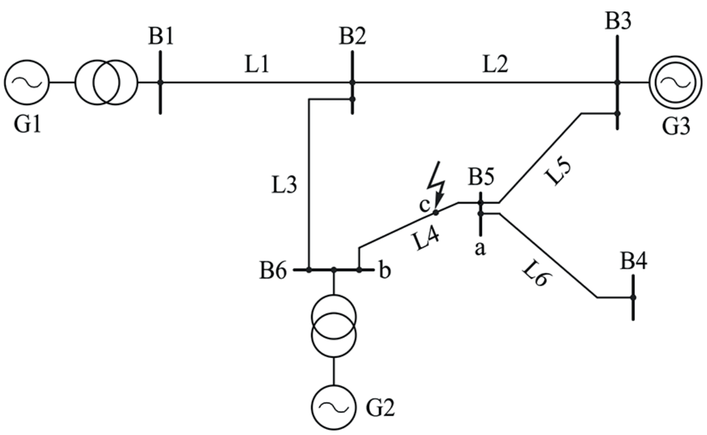

Fig. 6. Circuit diagram of 3G test system

Table 1
Data of the 3G test system from Fig. 6

Branch	Nodes		Impedance	Admittance
L1	B1	B2	6.0 + 59.5i	$(1.68 - 16.64\text{i}) \cdot 10^{-3}$
L2	B2	B3	10.7 + 90.0i	$(1.30 - 10.96\text{i}) \cdot 10^{-3}$
L3	B2	B6	3.5 + 30.8i	$(3.64 - 32.05\text{i}) \cdot 10^{-3}$
L5	B3	B5	4.2 + 47.0i	$(1.89 - 21.11\text{i}) \cdot 10^{-3}$
L6	B4	B5	3.5 + 30.8i	$(3.64 - 32.05\text{i}) \cdot 10^{-3}$
L4	B5	B6	5.3 + 56.0i	$(1.68 - 17.70\text{i}) \cdot 10^{-3}$
G1	B1	N	3.5 + 70.2i	$(0.71 - 14.21\text{i}) \cdot 10^{-3}$
G2	B6	N	2.2 + 40.3i	$(1.35 - 24.74\text{i}) \cdot 10^{-3}$
G3	B3	N	0.4 + 6.1i	$(10.70 - 163.23\text{i}) \cdot 10^{-3}$

6.2. Calculation results. The admittance matrix (A1) shown in Appendix can be obtained based on the data from Table 1 and circuit diagram from Fig. 6 without additional node c. The dimension of this matrix is 6×6. In this matrix the numbering of rows and columns is consistent with the numbering of the nodes. Calculations of inversion of matrix (A1) yields the initial impedance matrix (A2).

In the considered example it is assumed that a sliding fault occurs in line L4 at node c at a distance κ from node B5 and $(1-\kappa)$ from node B6. It is assumed now that $\kappa = 0.2$. In accordance with the flowchart (Fig. 5) and Eq. (27), the fifth row of matrix $\underline{\boldsymbol{Z}}$ is multiplied by $(1-\kappa) = 0.8$, and the sixth row is multiplied by $\kappa = 0.2$. As a result, the row matrices (A3) and (A4) are obtained. Sum of them is equal to the row matrix (A5). For $\kappa = 0.2$ it is obtained from Eq. (33):

$$\underline{Z}_{\text{cc}} = 0.64 \cdot \underline{Z}_{5.5} + 0.32 \cdot \underline{Z}_{5.6} + 0.04 \cdot \underline{Z}_{6.6} + 0.16 \cdot \underline{z}_{\text{L}} = \\ = 2.96 + 34.51\text{i}.$$

Finally, the extended impedance matrix (A6) can be written in which the off-diagonal elements of the seventh row are equal to the row matrix $\underline{\boldsymbol{Z}}_{\Sigma}$ from (A5) and $\underline{Z}_{7.7} = \underline{Z}_{\text{cc}}$, which is obtained above.

The following additional calculations are performed to verify the correctness of the proposed method. In line L4, at a distance $\kappa = 0.2$, additional node c is inserted, and for such a model, the extended admittance matrix (A7) is created. The calculation of the inversion of this matrix yields the impedance matrix (A8). A comparison of matrices (A6) and (A8) shows that $\underline{\boldsymbol{Z}}_{\text{ext}} = \underline{\boldsymbol{Y}}_{\text{ext}}^{-1}$, i.e. the extended impedance matrix $\underline{\boldsymbol{Z}}_{\text{ext}}$ computed by the proposed method is the same as the extended impedance matrix obtained by the inversion $\underline{\boldsymbol{Y}}_{\text{ext}}^{-1}$ of the extended admittance matrix.

Figure 7 shows the plot of functions $I_{\text{c}}(\kappa)$ and $\Delta V_{\text{a}}(\kappa)$, $\Delta V_{\text{b}}(\kappa)$, where the short-circuit current $\underline{I}_{\text{c}}$ is calculated from Eq. (2) for the pre-fault voltage $V_{\text{c}}^0 = 1.05 \cdot 220/\sqrt{2}$ and voltage sags at nodes a, b from $\Delta V_{\text{a}} = |\underline{Z}_{\text{ac}} \underline{I}_{\text{c}}|$ and $\Delta V_{\text{b}} = |\underline{Z}_{\text{cb}} \underline{I}_{\text{c}}|$.

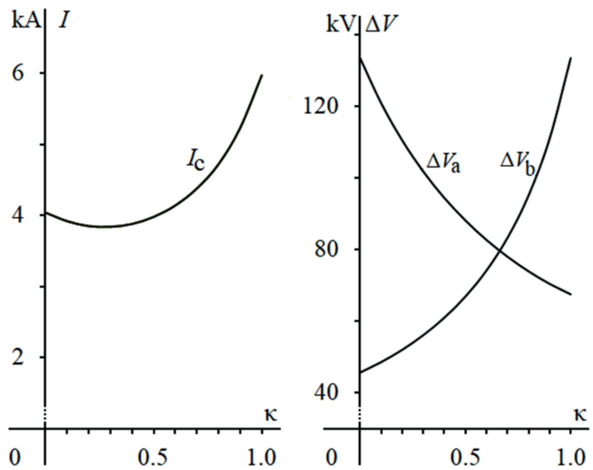

Fig. 7. Plots of short-circuit current and voltage sags

7. Conclusions

Mathematical considerations provided in this paper show that the analysis of the sliding faults can be performed without recalculation of the matrix inversion and without extension of the network model with many additional nodes. This paper demonstrates that the impedance matrix incorporating any location of the sliding fault can be easily created by extension of the initial impedance matrix using simple formulas. A typical short-circuit computer program can be easily modified using the flowchart presented in this paper.

The proposed method has two advantages: (i) all quantities calculated for the sliding faults can be explicitly expressed as functions of the fault distance, which simplifies the analysis and plotting of the relevant diagrams; (ii) it makes the sliding faults analysis very fast, which is important for large-scale networks. As a result, the proposed method reduces the computational complexity, and simplifies the analysis of the sliding faults.

References

[1] IEEE Recommended Practice for the Design of Reliable Industrial and Commercial Power Systems, *IEEE Std P493/D4*, 2006.
[2] J. Machowski, Z. Lubośny, J. Bialek, and J. Bumby, *Power System Dynamics. Stability and Control*, 3rd ed., John Wiley & Sons, Chichester, New York, 2020.
[3] C. Fan, K. Xu, and Q. Liu, "Short-circuit current calculation method for partial coupling transmission lines under different voltage levels", *Int. J. Electr. Power Energy Syst.* 78, 647–654 (2016).
[4] B. Dağ, A.R. Boynueğri, Y. Ateş, A. Karakaş, A. Nadar, and M. Uzunoğlu, "Static Modeling of Microgrids for Load Flow and Fault Analysis," *IEEE Trans. Power Syst.* 32(3), 1990–2000 (2017).
[5] Ł. Nogal, S. Ribak, and J. Bialek: "Advances in electrical power engineering", *Bull. Pol. Ac.: Tech.* 68(4), 647–649 (2020).
[6] H. Li, A. Bose, and Y. Zhang, "On-line short-circuit current analysis and preventive control to extend equipment life", *IET Gener. Transm. Distrib.* 7(1), 69–75 (2013).
[7] S. Azizi and M. Sanaye-Pasand, "From Available Synchrophasor Data to Short-Circuit Fault Identity: Formulation and Feasibility Analysis", *IEEE Trans. Power Syst.* 32(3), 2062–2071 (2017).
[8] V.A. Stanojević, G. Preston, and V. Terzija, "Synchronised Measurements Based Algorithm for Long Transmission Line Fault Analysis", *IEEE Trans. Smart Grid* 9(5), 4448–4457 (2018).
[9] T. Gonen, *Modern Power System Analysis*, 2nd ed., CRC Press, 2013.
[10] A.R. Bergen and V. Vittal, *Power System Analysis*, 2nd ed., Englewood Cliffs, NJ, USA: Prentice-Hall, 2000.
[11] P. Kacejko and J. Machowski, *Short-circuits in power systems*, PWN/WNT Warszawa 2017, [in Polish].
[12] P.M. Anderson, *Analysis of Faulted Power Systems*, New York: IEEE Press, 1995.
[13] A.H. El-Abiad, "Digital Calculation of Line-to-Ground Short Circuits by Matrix Method", *rans. Am. Inst. Electr. Eng. Part III: Power Apparatus and Systems* 79(3), 323–331 (1960).
[14] J.J. Grainger and W.D. Stevenson, JR, *Power System Analysis*, McGraw-Hill, New York, 1994
[15] T.A. Davis, *Direct methods for sparse linear systems*, Society for Industrial and Applied Mathematics, 2006.
[16] X. Luo *et al*., "An Efficient Second-Order Approach to Factorize Sparse Matrices in Recommender Systems", *IEEE Trans. Ind. Inform.* 11(4) 946–956 (2015).
[17] X. Luo, M. Zhou, S. Li, and M. Shang, "An Inherently Nonnegative Latent Factor Model for High-Dimensional and Sparse Matrices from Industrial Applications", *IEEE Trans. Ind. Inform.* 14(5), 2011–2022 (2018).
[18] M.R. Araújo and C.R. Pereira, "A practical first-zone distance relaying algorithm for long parallel transmission lines", *Electr. Power Syst. Res.* 146, 17–24 (2017).
[19] N. Abu Bakar, A. Mohamed, M. Ismail, and N. Hamzah, "A voltage sag analysis software tool for determine areas of vulnerability," *2004 IEEE Region 10 Conference TENCON 2004.*, Chiang Mai, 2004, pp. 299–302.
[20] S.R. Naidu, G.V. de Andrade, and E.G. da Costa, "Voltage Sag Performance of a Distribution System and Its Improvement", *IEEE Trans. Ind. Appl.* 48(1), 218–224 (2012).
[21] D. Ma and L. Tian, "Practical fault location estimation based on voltage sags magnitude," *2016 China International Conference on Electricity Distribution (CICED)*, Xi'an, 2016, pp. 1–5.
[22] R.J. Gopi, V.K. Ramachandaramurthy, and M.T. Au, "Analytical approach to stochastic assessment for balanced voltage sags and duration on transmission networks", *2009 10th International Conference on Electrical Power Quality and Utilisation*, Lodz, 2009, pp. 1–6.
[23] NEPLAN Smarter Tools "Power System Analysis Software" NEPLAN AG Oberwachtstrasse 2 CH 8700 Küsnacht ZH, [Online]. Available https://www.neplan.ch/wp-content/uploads/2015/01/Electricity.pdf
[24] A. Boboń, A. Nocoń, S. Paszek, and P. Pruski, "Determination of synchronous generator nonlinear model parameters based on power rejection tests using a gradient optimization algorithm", *Bull. Pol. Ac.: Tech.* 65(4), 479–488 (2017).
[25] P. Kacejko and J. Machowski, "Application of the Sherman-Morrison formula to short-circuit analysis of transmission networks with phase-shifting transformers", *Electr. Power Syst. Res.* 155, 289–295 (2018).

Appendix

$$\underline{Y} = 10^{-3} \cdot \left[\begin{array}{cccc:cc} 2.39-30.85i & -1.68+16.64i & 0 & 0 & 0 & 0 \\ -1.68+16.64i & 6.62-59.65i & -1.30+10.96i & 0 & 0 & -3.64+32.05i \\ 0 & -1.30+10.96i & 13.89-195.30i & 0 & -1.89+21.11i & 0 \\ 0 & 0 & 0 & 3.64-32.05i & -3.64+32.05i & 0 \\ \hdashline 0 & 0 & -1.89+21.11i & -3.64+32.05i & 7.20-70.86i & -1.68+17.70i \\ 0 & -3.64+32.05i & 0 & 0 & -1.68+17.70i & 6.67-74.49i \end{array}\right], \quad \text{(A1)}$$

$$\underline{\boldsymbol{Z}}=\left[\begin{array}{cccc|cc} 2.88+40.80\mathrm{i} & 1.09+15.84\mathrm{i} & 0.03+1.36\mathrm{i} & 0.16+4.33\mathrm{i} & 0.16+4.33\mathrm{i} & 0.34+7.87\mathrm{i} \\ 1.09+15.84\mathrm{i} & 2.71+29.26\mathrm{i} & 0.11+2.52\mathrm{i} & 0.49+8.01\mathrm{i} & 0.49+8.01\mathrm{i} & 0.97+14.55\mathrm{i} \\ 0.03+1.36\mathrm{i} & 0.11+2.52\mathrm{i} & 0.38+5.67\mathrm{i} & 0.25+4.02\mathrm{i} & 0.25+4.02\mathrm{i} & 0.08+2.04\mathrm{i} \\ 0.16+4.33\mathrm{i} & 0.49+8.01\mathrm{i} & 0.25+4.02\mathrm{i} & 6.32+63.68\mathrm{i} & 2.82+32.88\mathrm{i} & 0.75+11.28\mathrm{i} \\ \hline 0.16+4.33\mathrm{i} & 0.49+8.01\mathrm{i} & 0.25+4.02\mathrm{i} & 2.82+32.88\mathrm{i} & 2.82+32.88\mathrm{i} & 0.76+11.28\mathrm{i} \\ 0.34+7.87\mathrm{i} & 0.97+14.55\mathrm{i} & 0.08+2.04\mathrm{i} & 0.75+11.28\mathrm{i} & 0.76+11.28\mathrm{i} & 1.63+22.28\mathrm{i} \end{array}\right], \tag{A2}$$

$$(1-\kappa)\cdot\underline{\boldsymbol{Z}}_5=\left[0.13+3.47\mathrm{i} \quad 0.39+6.41\mathrm{i} \quad 0.20+3.21\mathrm{i} \quad 2.25+26.30\mathrm{i} \quad 2.25+26.30\mathrm{i} \quad 0.61+9.02\mathrm{i}\right], \tag{A3}$$

$$\kappa\cdot\underline{\boldsymbol{Z}}_6=\left[0.07+1.57\mathrm{i} \quad 0.19+2.91\mathrm{i} \quad 0.01+0.41\mathrm{i} \quad 0.15+2.26\mathrm{i} \quad 0.15+2.26\mathrm{i} \quad 0.33+4.46\mathrm{i}\right], \tag{A4}$$

$$\underline{\boldsymbol{Z}}_\Sigma=\left[0.20+5.04\mathrm{i} \quad 0.58+9.32\mathrm{i} \quad 0.21+3.62\mathrm{i} \quad 2.40+28.56\mathrm{i} \quad 2.40+28.56\mathrm{i} \quad 0.93+13.48\mathrm{i}\right], \tag{A5}$$

$$\underline{\boldsymbol{Z}}_{\mathrm{ext}}=\left[\begin{array}{cccc|cc|c} 2.88+40.80\mathrm{i} & 1.09+15.84\mathrm{i} & 0.03+1.36\mathrm{i} & 0.16+4.33\mathrm{i} & 0.16+4.33\mathrm{i} & 0.34+7.87\mathrm{i} & 0.20+5.04\mathrm{i} \\ 1.09+15.84\mathrm{i} & 2.71+29.26 & 0.11+2.52\mathrm{i} & 0.49+8.01\mathrm{i} & 0.49+8.01\mathrm{i} & 0.97+14.55\mathrm{i} & 0.56+9.32\mathrm{i} \\ 0.03+1.36\mathrm{i} & 0.11+2.52\mathrm{i} & 0.38+5.67\mathrm{i} & 0.25+4.02\mathrm{i} & 0.25+4.02\mathrm{i} & 0.08+2.04\mathrm{i} & 0.21+3.62\mathrm{i} \\ 0.16+4.33\mathrm{i} & 0.49+8.01\mathrm{i} & 0.25+4.02\mathrm{i} & 6.32+63.68\mathrm{i} & 2.82+32.88 & 0.75+11.28\mathrm{i} & 2.40+28.56\mathrm{i} \\ \hline 0.16+4.33\mathrm{i} & 0.49+8.01\mathrm{i} & 0.25+4.02\mathrm{i} & 2.82+32.88 & 2.82+32.88\mathrm{i} & 0.76+11.28\mathrm{i} & 2.40+28.56\mathrm{i} \\ 0.34+7.87\mathrm{i} & 0.97+14.55\mathrm{i} & 0.08+2.04\mathrm{i} & 0.75+11.28\mathrm{i} & 0.76+11.28\mathrm{i} & 1.63+22.28\mathrm{i} & 0.93+13.48\mathrm{i} \\ \hline 0.20+5.04\mathrm{i} & 0.58+9.32\mathrm{i} & 0.21+3.62\mathrm{i} & 2.40+28.56\mathrm{i} & 2.40+28.56\mathrm{i} & 0.93+13.48\mathrm{i} & 2.96+34.51\mathrm{i} \end{array}\right], \tag{A6}$$

$$\underline{\boldsymbol{Y}}_{\mathrm{ext}}=10^{-3}\cdot\left[\begin{array}{cccc|cc|c} 2.39-30.85\mathrm{i} & -1.68+16.64\mathrm{i} & 0 & 0 & 0 & 0 & 0 \\ -1.68+16.64\mathrm{i} & 6.62-59.65\mathrm{i} & -1.30+10.96\mathrm{i} & 0 & 0 & -3.64+32.05\mathrm{i} & 0 \\ 0 & -1.30+10.96\mathrm{i} & 13.89-195.30\mathrm{i} & 0 & -1.89+21.11\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 3.64-32.05\mathrm{i} & -3.64+32.05\mathrm{i} & 0 & 0 \\ \hline 0 & 0 & -1.89+21.11\mathrm{i} & -3.64+32.05\mathrm{i} & 13.90-141.65\mathrm{i} & 0 & -8.38+88.49\mathrm{i} \\ 0 & -3.64+32.05\mathrm{i} & 0 & 0 & 0 & 7.09-78.92\mathrm{i} & -2.09+22.12\mathrm{i} \\ \hline 0 & 0 & 0 & 0 & -8.38+88.49\mathrm{i} & -2.09+22.12\mathrm{i} & 10.47-110.62\mathrm{i} \end{array}\right], \tag{A7}$$

$$\underline{\boldsymbol{Y}}_{\mathrm{ext}}^{-1}=\left[\begin{array}{cccc|cc|c} 2.88+40.80\mathrm{i} & 1.09+15.84\mathrm{i} & 0.03+1.36\mathrm{i} & 0.16+4.33\mathrm{i} & 0.16+4.33\mathrm{i} & 0.34+7.87\mathrm{i} & 0.20+5.04\mathrm{i} \\ 1.09+15.84\mathrm{i} & 2.71+29.26 & 0.11+2.52\mathrm{i} & 0.49+8.01\mathrm{i} & 0.49+8.01\mathrm{i} & 0.97+14.55\mathrm{i} & 0.56+9.32\mathrm{i} \\ 0.03+1.36\mathrm{i} & 0.11+2.52\mathrm{i} & 0.38+5.67\mathrm{i} & 0.25+4.02\mathrm{i} & 0.25+4.02\mathrm{i} & 0.08+2.04\mathrm{i} & 0.21+3.62\mathrm{i} \\ 0.16+4.33\mathrm{i} & 0.49+8.01\mathrm{i} & 0.25+4.02\mathrm{i} & 6.32+63.68\mathrm{i} & 2.82+32.88 & 0.75+11.28\mathrm{i} & 2.40+28.56\mathrm{i} \\ \hline 0.16+4.33\mathrm{i} & 0.49+8.01\mathrm{i} & 0.25+4.02\mathrm{i} & 2.82+32.88 & 2.82+32.88\mathrm{i} & 0.76+11.28\mathrm{i} & 2.40+28.56\mathrm{i} \\ 0.34+7.87\mathrm{i} & 0.97+14.55\mathrm{i} & 0.08+2.04\mathrm{i} & 0.75+11.28\mathrm{i} & 0.76+11.28\mathrm{i} & 1.63+22.28\mathrm{i} & 0.93+13.48\mathrm{i} \\ \hline 0.20+5.04\mathrm{i} & 0.58+9.32\mathrm{i} & 0.21+3.62\mathrm{i} & 2.40+28.56\mathrm{i} & 2.40+28.56\mathrm{i} & 0.93+13.48\mathrm{i} & 2.96+34.51\mathrm{i} \end{array}\right]. \tag{A8}$$