Irreversible energy degradation in heat transfer between two fluids – analysis on manifolds

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Abstract   Thermodynamics deals with irreversible transformations of substances. Every thermodynamic property of a substance, as a function of two parameters describing its state, can be illustrated as a simply connected manifold. The term manifold stands for the Methods of Geometrical Representation of Thermodynamic Properties of Substances by Means of Surfaces. Generally, every transformation of a substance changes its energy (or enthalpy) by heat transfer and work done on it. All such changes (transformations) are considered to be irreversible and can be described using appropriate manifolds. Studies show that every transformation is associated with the degradation of energy. Such relations (between heat, work and other forms of energy or enthalpy) can be described by the Pfaff formulas and their integrations.

This article discusses the issue of irreversible energy degradation in heat transfer between two fluids. Irreversible heat transfer between separated fluids most often occurs through surface heat exchangers. All such processes are determined by convective heat transfer in thermal boundary layers and conduction through the wall. Consequently, entropy changes of fluids in heat and mass transfer can be observed in these layers. While the entropy rate of the heating fluid is negative and that of the heated medium is positive, the sum of entropy changes of all substances involved in the heat transfer process is always positive. These sums, known as entropy increase (entropy generation), can be interpreted as the measure of irreversible degradation of energy in heat transfer processes. The consequence of this degradation is that an arbitrary engine powered by the degraded (lower-temperature) heat flux will operate at a lower efficiency. The significance of this discussion relates especially to cases in power plants and cooling systems where

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surface heat exchangers are used. In the discussion proposed is the entropy increase as a criterion of irreversible energy degradation in heat transfer. Such introduced measure of effectiveness leads to an analysis of local overall heat transfer coefficient optimization on the cone-shaped manifold.

Keywords: Heat transfer; Analysis; Manifolds

Nomenclature

A –   area, surface, m^2
D –   characteristic diameters, m
h –   heat transfer coefficient, $\text{W/m}^2\text{K}$
k –   thermal conductivity, W/mK
l –   coordinate, m
p –   pressure, Pa
q –   heat flux, W/m^2
Q –   heat, J
S –   entropy, J / K
s –   specific entropy, J/kg K
t –   temperature, °C
T –   temperature, K
V –   volume, m^3
v –   specific volume, m^3/kg
u –   velocity, m/s

Greek symbols

η –   dynamic viscosity, Pa s
ρ –   density, kg/m^3
Π –   total entropy increase, W/K
π –   local entropy increase, W/m^2 K

Subscripts

a –   average
c –   cold fluid
h –   hot fluid
i –   inner
o –   outer
w –   wall

Dimensionless numbers

Nu   –   Nusselt number
Pr   –   Prandtl number
Re   –   Reynolds number

1 Introduction

The first step in every thermodynamic problem is a phenomenological model description with the system assumptions made. Also very important is a definition of the control volume (as an open set) and its boundary as a surface control (enclosure). Analysis of the problem begins with defining the simply connected control volume $V(l_1, l_2, l_3)$ for the analysed substance, which is then divided into elementary control volume elements $dV(l_1, l_2, l_3)$, where l_1, l_2, l_3 are geometrical coordinates of the entire control space $V(l_1, l_2, l_3)$. The element of the control volume $dV(l_1, l_2, l_3)$ is the open space and the boundary surface control $dA(l_1, l_2, l_3)$ is its enclosure. Let us analyse the fluid transformation in the element of the control volume $dV(l_1, l_2, l_3)$ with the boundary surface defined as $dA(l_1, l_2, l_3)$. The analysis is performed locally in the elements of the control volume $dV(l_1, l_2, l_3)$ and then applied to optimise the entire device by appropriate integration over the entire control volume $V(l_1, l_2, l_3)$. In technical considerations, all thermodynamic parameters are real functions describing distributions of these quantities in the space $V(l_1, l_2, l_3)$.

The basic principles in thermodynamics are Heat, Energy/Enthalpy and Work (HEW) balances known as the Law of Energy Conservation. Written with the coordinates $Q(l_1, l_2, l_3)$, $T(l_1, l_2, l_3)$, $v(l_1, l_2, l_3)$ this law takes the form:

$$
\begin{aligned}
dq_t(T, v) &= dq(T, v) + dq_f(T, v) \\
&= \left(\frac{\partial q_t(T, v)}{\partial T} \right)_v dT + \left(\frac{\partial q_t(T, v)}{\partial v} \right)_T dv \\
&= du(T, v) + p(T, v)dv,
\end{aligned}
\tag{1}
$$

where:

$dq_t(T, v)$ [J/kg] – total amount of heat accompanying fluid transformation in the control volume element dV,

$dq(T, v)$ [J/kg] – amount of heat transferred from an external source through the heat transfer surface element dA to the fluid control volume element dV,

$dq_f(T, v)$ [J/kg] – amount of heat heating the given fluid control volume element dV as a result of friction on the surface element dA in viscous fluid,

$q_t(T, v)$ [J/kg] – total amount of heat accompanying fluid transformation, related to one kilogram of its mass,

$u(T, v)$ [J/kg]   – specific internal energy of the fluid in the control volume element dV,

$c_v(T, v)$ [J/kg K] – specific heat capacity, determined for the substance being analysed at constant volume.

The above equation, when referred to the given fluid control volume element dV and written in the Q, T, v, space, which is also known as the Zeuner equation, is an inexact partial differential equation. This means that the result of integration depends on the path of integration.

Similarly, HEW balance coordinates $Q(l_1, l_2, l_3)$, $T(l_1, l_2, l_3)$, $p(l_1, l_2, l_3)$ take the form

$$
\begin{aligned}
dq_t(T, p) &= dq(Tp) + dq_f(Tp) \\
&= \left(\frac{\partial q_t(T, p)}{\partial T} \right)_p dT - \left(\frac{\partial q_t(T, p)}{\partial p} \right)_T dp \\
&= dh(T, p) - v(T, p)dp,
\end{aligned} \tag{2}
$$

where:

$h(T, v)$ [J/kg]   – specific enthalpy of the fluid in the control volume element dV,

$c_p(T, p)$ [J/kg K] – specific heat capacity, determined for the substance being analysed at constant pressure.

It can be easily shown that vector pairs $c_v(T, v)$, $p(T, v)$ and $c_p(T, p)$, $-v(T, p)$ determining rotational vector fields do not allow for the calculation of integrals. When written in differential form, the HEW balance equations are inexact differential equations. Thus, in the analysed spaces $(Q$, T, $v)$ or $(Q$, T, $p)$ no surfaces or manifolds provide solutions to HEW balances. There are no manifolds describing the solution of these equations defined in these spaces. To solve and optimize the problem, new equations are needed. After determining the integrating factor and its application, converted law equations become an exact differential Pfaff's form. Consequently, solutions exist for new equations written in the new converted spaces $(Q$, T, v, $s)$ or $(Q$, T, p, $s)$. These solutions have the form of surfaces obtained from integration, which are independent of the path of integration. These surfaces meet conditions of differentiable potential manifolds. The results of integration are differentiable manifolds and corresponding vector fields.

In thermodynamics, converting inexact differential equations into exact differential equations allowed Clausius to define the specific entropy of a substance [3]. Thermodynamically, this operation involves defining

manifolds (vector fields), which make it possible to solve equations describing real processes in fluids. Entropy defined in the above way is an additional state parameter of the fluid, which can be used to describe its real irreversible processes. The new form of equations written in the spaces Q, T, v, s or Q, T, p, s can be integrated, and the results of this operation exist in the form of differential, potential manifolds. The equations describing the specific entropy of the fluid are converted equations of HEW balances.

For parameters $T(l_1, l_2, l_3)$, $v(l_1, l_2, l_3)$, $s(l_1, l_2, l_3)$ the following equation is true:

$$
\begin{aligned}
ds(T, v) &= \frac{dq_t(T, v)}{T} = \frac{dq(T, v) + dq_f(T, v)}{T} \\
&= \left[\left(\frac{\partial q_t(T, v)}{\partial T} \right)_v dT + \left(\frac{\partial q_t(T, v)}{\partial v} \right)_T dv \right] \frac{1}{T} \\
&= \left[c_v(T, v) dT + p(T, v) dv \right] \frac{1}{T}.
\end{aligned}
\tag{3}
$$

Similarly, for coordinates $T(l_1, l_2, l_3)$, $p(l_1, l_2, l_3)$, $s((l_1, l_2, l_3)$ we have

$$
\begin{aligned}
ds(T, p) &= \frac{dq_t(T, p)}{T} = \frac{dq(T, p) + dq_f(T, p)}{T} \\
&= \left[\left(\frac{\partial q_t(T, p)}{\partial T} \right)_p dT + \left(\frac{\partial q_t(T, p)}{\partial p} \right)_T dp \right] \frac{1}{T} \\
&= \left[c_p(T, p) dT - v(T, p) dp \right] \frac{1}{T}
\end{aligned}
\tag{4}
$$

provided that $T \neq 0$.

The definition of the specific entropy of a substance makes it possible to balance the heat flux delivered to (or removed from) one kilogramme of the substance to allow performing (or being subjected to) certain work by changing its specific energy or enthalpy in real irreversible transformations.

The last two equations (Pfaff formulas) are described in defined spaces and conditions, i.e. for specific substances. Therefore their illustration first requires the specification of the substance being analyzed. In the following study, water is chosen as the exemplar substance. Similarly, charts can be drawn for an arbitrary substance as a function of two of its parameters of state. Each real substance has a set of its unique surfaces, manifolds (Fig. 1 to Fig. 9), on which all possible thermodynamic processes can be described and analysed [1].
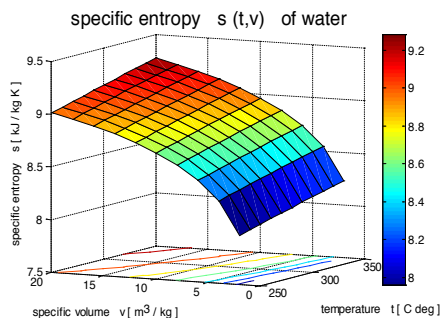
Figure 1: Surface illustrating specific entropy of water as a function of temperature and specific volume.
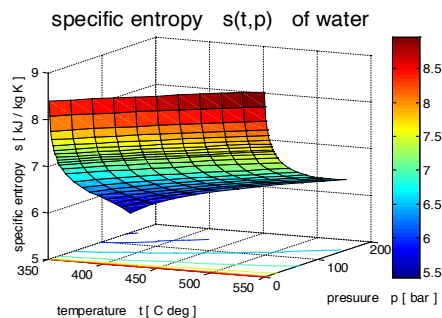

Figure 2: Surface illustrating specific entropy of water as a function of temperature and pressure.
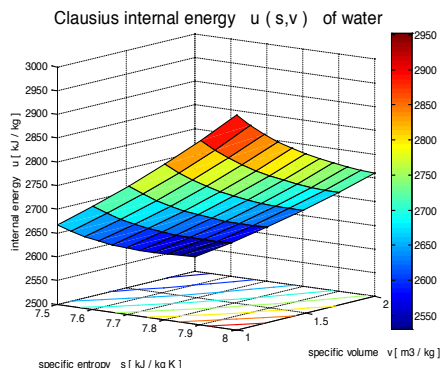

Figure 3: Surface illustrating Clausius internal energy of water as a function of specific entropy and specific volume.
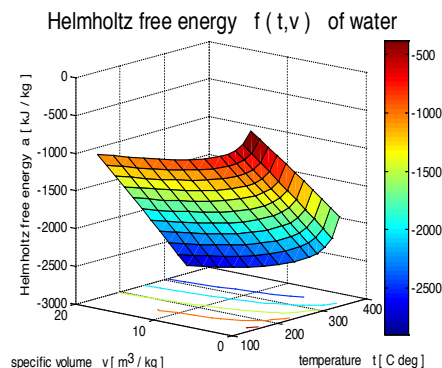

Figure 4: Surface representing Helmholtz free energy of water as a function of specific volume and temperature.
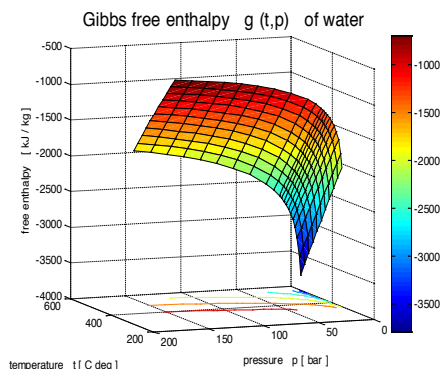

Figure 5: Surface illustrating Gibbs free enthalpy of water as a function of temperature and pressure.
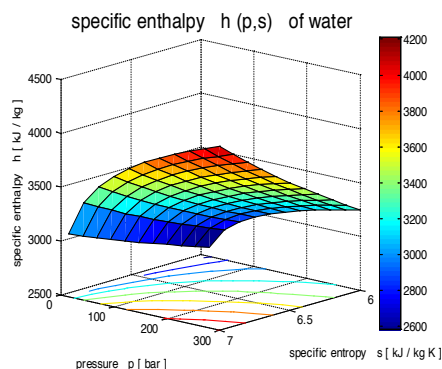

Figure 6: Surface illustrating specific enthalpy of water as a function of pressure and specific entropy.
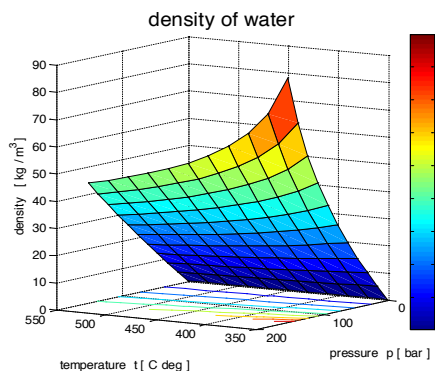
Figure 7: Surface illustrating density of water as a function of temperature and pressure.
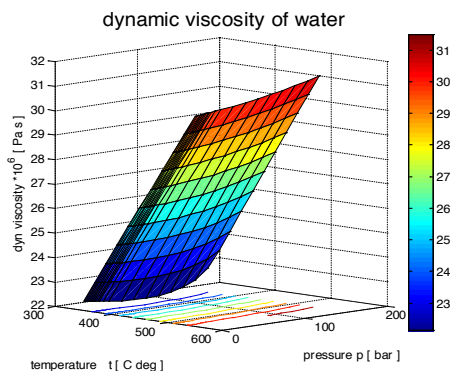

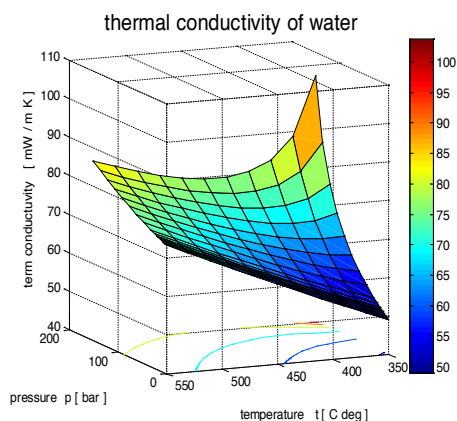Figure 8: Surface illustrating dynamic viscosity of water as a function of temperature and pressure.

Figure 9: Surface illustrating thermal conductivity of water as a function of temperature and pressure.

The last two exact differentiable equations can be taken to solve the Navier Stokes set of equations expressing the problem of irreversible transformations.

The above equations allow for another interpretation of entropy. As can be noticed from the structure of Eqs. (1), (2) and their converted versions (3), (4), the duality of entropy is expressed in the fact that, on one hand, it is a function of the temperature field and, on the other, it is a volumetric/mass phenomenon.

The above diagrams (Fig. 1 and Fig. 2) of potential differentiable manifolds are perfect illustrations for interpretation of Caratheodory's theorem.

Specific entropy has made it possible to define and generate other manifolds and their inducted vector fields, such as Clausius internal energy,

Helmholtz free energy, Gibbs free enthalpy, or specific enthalpy of a given substance [4–7].

The above four manifolds (presented in Fig. 3 to Fig. 6) can only be defined using the coordinates introduced by Clausius, which are the specific entropy of the substance $s(T, v)$ or $s(T, p)$. When determining these manifolds, preserving the Maxwell relations is essential. All the above functions, determined as integrals of exact partial differential equations, have the properties of potential manifolds. Each real thermodynamic substance has its individual, unique set of surfaces, potential manifolds, on which all possible thermodynamic processes can be described, calculated and analysed. Knowing these manifolds (or equations) and the HEW balance equations, we can solve the problem of irreversible heat flow, i.e. analyse its degradation. The above manifolds are generated for water as an example. Similarly, charts can be drawn for an arbitrary substance as a function of two its parameters of state.

The processes taking place in the surface heat exchangers, analysed below, illustrate the most frequently used method of heat transfer. The flows of both fluids are separated by a wall. The analysis assumes convective heat transfer in the thermal boundary layers (TBL) of the fluids on both sides and heat conduction through the wall. Fluid on one side of the wall has fully defined initial and final properties, including mass flow rate, whereas the fluid on the other side of the wall can be the object of optimisation in terms of the type of substance, mass flow rate and its initial thermodynamic parameters. Another object of optimisation can be the arrangement of heat transfer surfaces. The geometry of heat transfer surfaces, as well as the mass flow rate and thermodynamic parameters of the second fluid need to be selected in such a way that the irreversible degradation of energy in the entire process is minimal. The analysis requires an explicit definition of a criterion of the process optimisation. It is proposed to be the minimisation of entropy increase in heat transfer, i.e. minimisation of irreversible degradation of energy [2]. The dominating processes, which is the object of this analysis, is the entropy increase caused by heat transfer in the TBL of the fluids and heat conduction through the wall. The analysis requires information on the basic properties of the fluids involved in the process. The fluid properties, defined as real functions of two parameters relating to both fluids, are available in the form of manifolds limited by the extreme values of their parameters.

The present work is a continuation of ideas described in articles [2, 11, 15]. The paper [2] defines and discusses entropy increase as a measure of

energy degradation in heat transfer. It also shows the influence of entropy increments calculated for upper and lower energy sources on the efficiency of the whole cycle. This influence means that an arbitrary engine powered by a degraded (lower-temperature) heat flux will operate at a lower efficiency. Paper [15] discusses the effects of heat transfer irreversibility in condensers working as a lower heat source of a power plant cycle. For the first time discussed is the analysis of steam condensation in the case of inertial gases, including processes in the vapour-gas sublayer, heat conduction through condensate film and fouled wall as well as an analysis of heat transfer in TBL of water. An analysis of irreversibility sources is carried out by determining the local entropy increase in individual TBLs. The results of the analyses published in [15] help explain how heat transfer irreversibility occurs and assess its scope. This article presents the possibilities of heat transfer optimization using manifolds.

2 Assumptions for heat transfer analysis in surface heat exchangers

To make the optimisation process as clear and transparent as possible, the following assumptions and simplifications have been adopted:

- Heat flow is only analysed between two separated (non-mixing) mass flows of fluids m_h and m_c with different temperatures (averaged in the channel cross-section): $T_h(l) > T_c(l)$;

- The phenomenon is steady and time independent;

- The phenomenon is one-dimensional, i.e. the averaged temperatures of fluid change along the channel length $0 < l < L$ as a result of heat transfer between the fluids, but the heat flow is mainly driven by what takes place in the fluid's TBL in the vicinity of the heat transfer surface $dA(l)$;

- For simplification, it is assumed that the fluid flow areas are simply-connected spaces

- Fluids flowing through the channels require optimisation. Geometrical properties described as $F_h(l)$ and $F_c(l)$ (where F is the cross-section area of the channels) and their characteristic diameters $D_h(l)$ and $D_c(l)$ can be optimised. These properties make it possible to calculate

explicitly the Reynolds and Nusselt numbers which determine the
nature of the analysed phenomena;

- Channels are separated by a single-layer wall with heat transfer
  area $A(l)$;

- The wall thickness $\delta(l) = \delta$ is constant along the heat transfer area;

- The wall is made of a material with known thermal conductivity,
  $k(T) = k$.

Control volumes $dV_h(l)$ and $dV_c(l)$ for both fluids are marked in such
a way that their common intersection is the control area segment $dA(l)$.
The coordinate l defines the position of heat transfer control element $dA(l)$
in the device. All thermodynamic parameters are functions of the channel
length l. For calculations let us generally assume that the fluid undergoing
the thermal process is hot with initial parameters $T_h(l = 0) = T_{h0}$, $p_h(l =
0) = p_{h0}$, and final parameters $T_h(l = L) = T_{hL}$, $p_h(l = L) = p_{hL}$ as well
as a mass flow rate of m_h. Also all the thermodynamic and thermokinetic
properties of the fluids necessary to determine the heat transfer process
are known. These characteristics are explicitly given as functions of two
parameters (e.g., T – temperature and p – pressure of the fluid). They are
differentiable manifolds:

- fluid densities $\rho_h(T_h, p_h)$ and $\rho_c(T_c, p_c)$,

- specific fluid enthalpies $h_h(T_h, p_h)$ and $h_c(T_c, p_c)$ or values of an arbi-
  trary thermodynamic potential,

- fluid dynamic (or kinematic) viscosity coefficients $\eta_h(T_h, p_h)$ and
  $\eta_c(T_c, p_c)$,

- fluid thermal conductivities $k_h(T_h, p_h)$ and $k_c(T_c, p_c)$,

- Prandtl numbers $\mathrm{Pr}_h(T_h, p_h)$ and $\mathrm{Pr}_c(T_c, p_c)$,

where:

$$T_{h\,\min} < T_h < T_{h\,\max} \ \text{ and } \ T_{c\,\min} < T_c < T_{c\,\max}\,,$$
$$p_{h\,\min} < p_h < p_{h\,\max} \ \text{ and } \ p_{c\,\min} < p_c < p_{c\,\max}\,,$$

T_h and T_c as well as p_h and p_c – temperature and pressure of the fluids are
within the changeability range of these parameters. The fluid properties
defined (within appropriate ranges) in the above way can be treated as
differentiable manifolds and as such used in calculations.

Figures 1 to Fig. 9 present various differentiable potential surfaces, or manifolds representing the basic properties of water to calculate heat transfer. For greater clarity, the entire analysis will be presented in the flat geometry of the wall. The prerequisite formal assumption for the analysis refers to the continuity of temperature distributions.

3 Temperature distribution in the heat transfer process

The analysis is performed locally on control volume elements $dV(l)$ and then applied to the general heat exchange process by appropriate integration over the entire control volume $V(l)$. The first step in every thermodynamics problem is a phenomenological model description with system assumptions made. Also very important is a definition of the control volume and its boundary surface control.
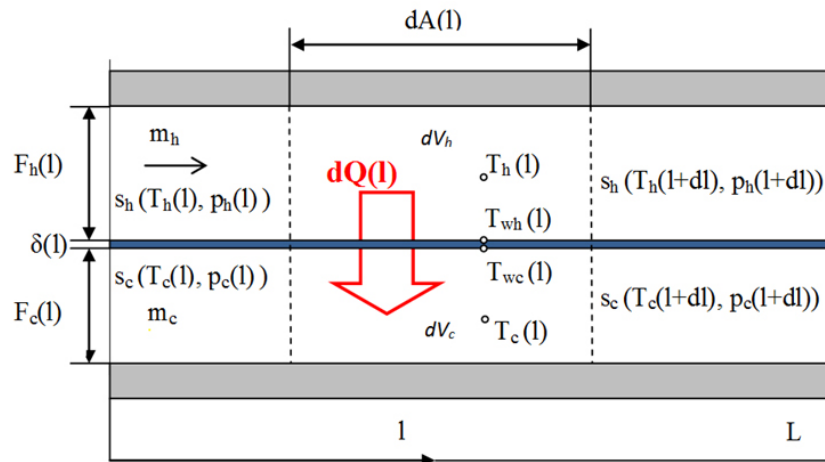

Figure 10: Diagram of heat transfer between the flows of two fluids separated by a wall.
where: $T_h(l)$ [K]   – local averaged temperature in the hot fluid flow,
        $T_{wh}(l)$ [K]   – local wall temperature on the hot fluid side,
        $T_{wc}(l)$ [K]   – local wall temperature on the cold fluid side,
        $T_c(l)$ [K]   – local averaged temperature in the cold fluid flow,
        $dQ(l)$ – local heat transfer between the fluids.

The mass flow rate m_h of the medium is known and the optimised mass flow rate m_c is assumed in the optimization process. The above assumptions make it possible to determine the homogeneous flow velocities $w_h(l)$ and

$w_c(l)$ of each fluid in an arbitrary cross-section. The analysed heat transfer between the two fluid flows is composed of [8–10]:

- convection in the TBL of the hot fluid determined by the *local convective heat transfer coefficient* $h_{conv\,h}(l)$,

- local conduction through the wall with thickness $\delta(l) = \delta$ and thermal conductivity k,

- convection in the TBL of the cold fluid determined by the local *convective heat transfer coefficient* $h_{conv\,c}(l)$.

From the Law of Heat Conservation, we have:

$$
\begin{aligned}
dQ(l) &= h_{conv\,h}(l)\big(T_h(l) - T_{wh}(l)\big)dA(l) \\
&= \frac{k}{\delta}\big(T_{wh}(l) - T_{wc}(l)\big)dA(l) \\
&= h_{conv\,c}(l)\big(T_{wc}(l) - T_c(l)\big)dA(l),
\end{aligned}
\tag{5}
$$

where: $h_{conv\,h}(l)$ [W/(m^2 K)] – local convective heat transfer coefficient in the TBL of the hot fluid as a function of local *Reynolds* and *Nusselt* numbers for the geometry:

$$
\begin{aligned}
\mathrm{Re}_h(l) &= f_1\big(F_h(l), D_h(l), m_h, \rho_h(T_h(l), p_h(l)), \eta_h(T_h(l), p_h(l))\big), \\
\mathrm{Nu}_h(l) &= f_2\big(\mathrm{Re}_h(l), \mathrm{Pr}_h(T_h(l), p_h(l))\big), \\
h_{conv\,h}(l) &= f_h\big(D_h(l), \mathrm{Nu}_h(l), k_h(T_h(l), p_h(l))\big),
\end{aligned}
\tag{6}
$$

$h_{conv\,c}(l)$ [W/(m^{2}K)] – local convective heat transfer coefficient in the TBL of the cold fluid as a function of local Reynolds and Nusselt numbers for the geometry:

$$
\begin{aligned}
\mathrm{Re}_c(l) &= f_3\big(F_c(l), D_c(l), m_c, \rho_c(T_c(l), p_c(l)), \eta_c(T_c(l), p_c(l))\big), \\
\mathrm{Nu}_c(l) &= f_4\big(\mathrm{Re}_c(l), \mathrm{Pr}_c(T_c(l), p_c(l))\big), \\
h_{conv\,c}(l) &= f_c\big(D_c(l), \mathrm{Nu}_c(l), k_c(T_h(l), p_h(l))\big),
\end{aligned}
\tag{7}
$$

$dQ(l)$ [W] – heat flux between the flowing media through the heat transfer surface element dA.

The Nusselt numbers describe free or forced convection, depending on the Reynolds number determined for the given fluid flowing in the local geometry. The above equation system, along with the determined Reynolds

and Nusselt numbers, makes it possible to determine the missing local temperatures $T_{wh}(l)$, $T_{wc}(l)$ on the wall and the local heat transfer $dQ(l)$ passing between the fluids in the vicinity of the heat transfer area $dA(l)$. In this way, local temperature distributions $T_h(l)$, $T_{wh}(l)$, $T_{wc}(l)$, $T_c(l)$, and local heat fluxes $dQ(l)$ along the heat transfer area $A(1)$ can be calculated. Mathematically, the problem can be presented with a set of nonlinear equations [11] and [15].

For the given channel coordinate l, local temperature differences in fluids can be determined as:

- $(T_h(l) - T_c(l))$ – difference between averaged fluid temperatures in flows,

- $(T_h(l) - T_{wh}(l))$ – temperature difference in TBL of the hot fluid,

- $(T_{wh}(l) - T_{wc}(l))$ – temperature difference in the wall,

- $(T_{wc}(l) - T_c(l))$ – temperature difference in TBL of the cold fluid.

The fact that the sum of temperature differences in the layers and wall is equal to the temperature difference between the fluids provides a simple and well-known formula for local overall (resultant) heat transfer coefficient $h_{overall}(l)$. For the case of flat geometry:

$$h_{overall}(l) = \frac{h_{conv\,h}(l)h_{conv\,c}(l)k}{h_{conv\,c}(l)k + h_{conv\,h}(l)h_{conv\,c}(l)\delta + h_{conv\,h}k} . \tag{8}$$

Written for a plane wall, Eq. (8) illustrates the rule for a potential field of temperature. According to this rule (see analysis assumption and Fig. 10), the sum of temperature differences in boundary layers and the wall is equal to the temperature difference between substances in the vicinity of heat transfer field $dA(l)$. Equations following this rule can be written for walls of any geometry and their interpretations will always produce similar cone-shaped manifolds. Analyzes of these similarly cone-shaped manifolds lead to the same results, as described in Section 5 of this work.

The set of Eqs. (5)–(7) allows for an accurate designation of temperature distribution [15] needed in further calculations. However, a new equation criterion is needed to optimize the problem of heat transfer.

4   Irreversibility of heat transfer

Local entropy changes occur naturally in fluid flows due to irreversible heat transfer through the wall $dQ(l)$ and friction heat $dQ_f(l)$ resulting from turbulent fluid flow in the exchanger passages. For individual fluids:

$$dS_h(l) = \frac{dQ_{ht}(l)}{T_h(l)} = \frac{dQ_h(l) + dQ_{fh}(l)}{T_h(l)} \, , \tag{9}$$

$$dS_c(l) = \frac{dQ_{ct}(l)}{T_c(l)} = \frac{dQ_c(l) + dQ_{fc}(l)}{T_c(l)} \, . \tag{10}$$

It is also easy to determine the thermokinetic effects of mass flows [2, 11, 15]

$$dQ_{fh}(l) = f_5(F_h, dl),$$
$$dQ_{fc}(l) = f_6(F_c, dl).$$

The Second Law of Thermodynamics determines the direction of heat transfer from the hotter fluid to a lower temperature domain.

$$dQ_c(l) = -dQ_h(l) = dQ(l). \tag{11}$$

The entropy change of the warmer fluid is negative $(dS_h(l) < 0)$, while that of the cooler fluid is positive $(dS_c(l) > 0)$. Heat transfer $dQ(l)$ through the wall and friction heat $dQ_f(l)$ observed in the fluid flows in the exchanger passages are interdependent. The higher the local Reynolds numbers $(\mathrm{Re}_c(l), \mathrm{Re}_h(l))$ describing the turbulence of the fluid flows in the passages, the higher the corresponding Nusselt numbers $(\mathrm{Nu}_c(l), \mathrm{Nu}_h(l))$ and, as a consequence, the higher the local heat transfer coefficients $h_{conv\,h}(l)$, $h_{conv\,c}(l)$ describing the heat transfer between the fluids. Along with this rule, however, the increasing Reynolds number intensifies the effects of friction heat, which contributes to the additional entropy increase. The sum of entropy changes of substances involved in the heat transfer process is equal to the entropy increase (known in literature as the entropy generation) [2]

$$\begin{aligned}
d\pi(l) &= dS_c(l) + dS_h(l) \\
&= \frac{dQ(l)}{T_c(l)} \left[1 - \frac{T_c(l)}{T_h(l)} \right] + \frac{dQ_{fc}(l)}{T_c(l)} + \frac{dQ_{fh}(l)}{T_h(l)} \\
&= \frac{dQ(l)}{T_h(l)} \left[\frac{T_h(l)}{T_c(l)} - 1 \right] + \frac{dQ_{fc}(l)}{T_c(l)} + \frac{dQ_{fh}(l)}{T_h(l)} \\
&= d\pi_Q(l) + d\pi_f(l) \geq 0.
\end{aligned} \tag{12}$$

The local entropy increase is equal to the sum of entropy increase $d\pi_Q(l)$ resulting from irreversible heat transfer $dQ(l)$ through the wall and to the entropy increase $d\pi_f(l)$ caused by the friction of fluids in the passages. The sum of the entropy changes of the fluids involved in the heat transfer process is always positive. The entropy increment is equal to zero ($d\pi(l) = 0$) if and only if $dQ(l) = 0$ and $dQ_{f...}(l) = 0$ (no thermodynamic transformations). The entropy increments are always positive ($d\pi(l) > 0$) when $dQ(l) > 0$ or $dQ_{f...}(l) > 0$ (for all other thermodynamic transformations). In thermodynamics, there has been no explicit method for calculating the efficiency of heat transfer [12–14], and certainly the Second Law of Thermodynamics is not applicable in this case. The above defined entropy increase is the expected solution. When evaluating the heat transfer, we should determine in which process (respective convection or conduction) the largest entropy increases occur. For this purpose, the temperature distribution is to be determined in successive layers, and then in the wall. After complementing these equations with the temperature distributions on the wall surfaces ($T_{wh}(l)$, $T_{wc}(l)$) obtained from numerical or experimental analyses, we can analyse the entropy increase in the TBL on both sides of the heat transfer wall [2, 11, 15]

$$
\begin{aligned}
d\pi_Q(l) &= \frac{dQ(l)}{T_c(l)} \left[1 - \frac{T_c(l)}{T_h(l)} \right] \\
&= dQ(l) \left\{ \frac{1}{T_c(l)} \left[1 - \frac{T_c(l)}{T_{wc}(l)} \right] + \frac{1}{T_{wc}(l)} \left[1 - \frac{T_{wc}(l)}{T_{wh}(l)} \right] \right. \\
&\quad \left. + \frac{1}{T_{wh}(l)} \left[1 - \frac{T_{wh}(l)}{T_h(l)} \right] \right\} \\
&= d\pi_{T_{wc},T_c}(l) + d\pi_{T_{wh},T_{wc}}(l) + d\pi_{T_h,T_{wh}}(l) \\
&= d\pi_{T_h,T_c}(l) \geq 0.
\end{aligned}
\tag{13}
$$

The local entropy increase $d\pi_Q(l)$ caused by irreversible heat transfer $dQ(l)$ through infinitesimal surface element $dA(l)$ of the heat exchanger surface is equal to the sum of entropy increases in TBL of the fluids and the wall. The above differential form of the relations can be used to calculate the total entropy increase caused by the heat transfer between the flowing fluids in the entire heat exchanger. By integrating the local entropy increase $d\pi_Q(l)$ along the fluid flow path l (or over the heat transfer surface $A(l)$), we can obtain the total entropy increase Π_Q connected with heat transfer in the entire heat exchanger [15].

In order to obtain relevant heat transfer, the process should be optimized to minimize entropy increase, which involves minimizing all the terms in the above equation. As technical analyses have shown, the entropy increase connected with the irreversible heat transfer is far larger than the entropy increase caused by the turbulent fluid flow

$$d\pi_Q(A) \gg d\pi_f(A). \tag{14}$$

According to (13), calculating the entropy increase connected with the irreversible process of heat transfer through the infinitesimal surface element $dA(l)$ requires knowledge of the local specific entropy change in the heated fluid and the dimensionless local Thermodynamic Potential Coefficient, TPC, expressed by relevant temperature values

$$0 < \text{TPC}_{T_h,T_c}(A) = \left[1 - \frac{T_c(A)}{T_h(A)}\right] < 1. \tag{15}$$

By knowing the local heat transfer $dQ(A)$ as well as real distributions $T_c(A)$ and $T_h(A)$ we can calculate the entropy increase as a measure of energy degradation in the irreversible heat transfer process of the entire heat exchanger. As has been shown in [2], a consequence of this degradation is that an engine powered by the degraded (lower-temperature) heat flux will be less efficient.

In order to obtain relevant heat transfer, the process should be optimized to minimize the entropy increase

$$d\pi_Q(l) = \frac{dQ(l)}{T_c(l)}\left[1 - \frac{T_c(l)}{T_h(l)}\right] = \frac{dQ(l)}{T_h(l)}\left[\frac{T_h(l)}{T_c(l)} - 1\right] \geq 0 \to \min. \tag{16}$$

Optimization of the process, i.e. minimization of irreversible energy degradation (inflation) in the heat transfer process, involves minimizing entropy increase. For the entropy increase to be as small as possible the temperature difference between the fluids should also be minimal

$$\left[T_h(l) - T_c(l)\right] \to \min. \tag{17}$$

Generally, the smaller temperature difference between the heat transfer fluids, the smaller the entropy increase. On the other hand,

$$dQ(l) = dA(l)\, h_{overall}(l)\left[T_h(l) - T_c(l)\right]. \tag{18}$$

To reduce costs, we aim to obtain the smallest possible heat transfer surface

$$dA(l) \to \min. \tag{19}$$

To ensure the required effect of heat transfer, we should maximally intensify hydrodynamic conditions by

$$h_{overall}(l) \to \max. \qquad (20)$$

A geometric illustration of the local overall heat transfer coefficient $h_{overall}(l)$ as a function (8) of varying local convective heat transfer values $h_{conv\,h}(l)$, $h_{conv\,c}(l)$, with heat conduction across the wall (determined by its thickness δ, and thermal conductivity k) can be presented as a manifold in Fig. 11. Of course, the presented section of the cone-shaped manifolds is defined in an open domain.
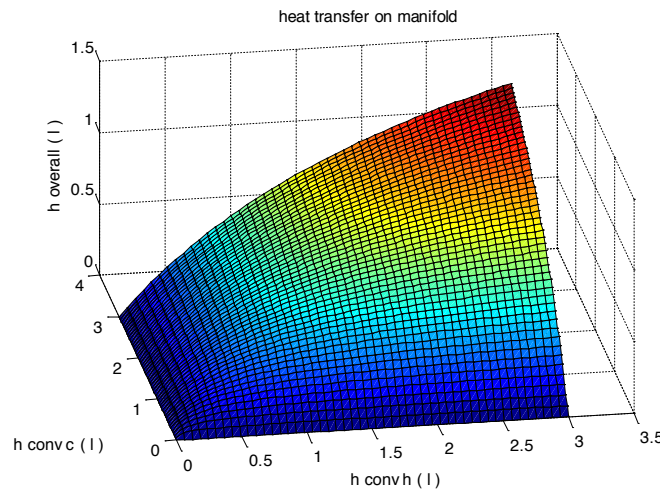

Figure 11: Manifold illustrating local overall heat transfer coefficient ($h_{overall}(l)$) values generated for variable local convective heat transfer parameters ($h_{conv\,h}(l)$, $h_{conv\,c}(l)$) and conduction conditions in the wall (δ and k), as well as for assumed initial fluid conditions.

Equipped with the optimization criterion (13)–(16), the system of equations (5)–(8) and necessary data specifying the properties of substances (manifolds) involved in the process, the optimization can begin. The analysis can be performed during the heat exchanger construction as well as during their operation. By executing step-by-step calculations for different types of substances, mass flow rates, and initial thermodynamic conditions, one can determine the local and total entropy increase. Additionally, by changing the geometrical arrangement (co- and counter-flow devices), one can examine the effect of channel geometries on the efficiency of the

entire process. The analysis of local overall heat transfer coefficients on cone-shaped manifolds is invaluable to the optimization of heat transfer efficiency.
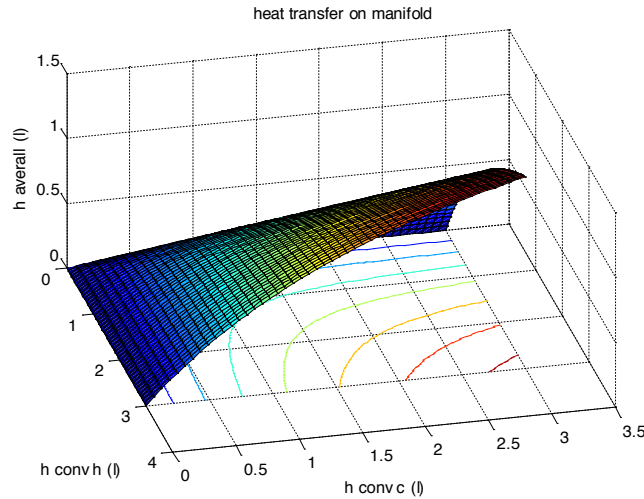

Figure 12: Cone-shaped manifold properties. Surface $h_{overall}(l)(h_{conv\,h}(l),\,h_{conv\,c}(l))$ with projections of its intersections with planes $dh_{overall}(l) = 0$.

5   Conclusions from manifold-based analysis

The aim of this paper was to analyse and then optimise heat transfer, i.e. minimise irreversible energy degradation in heat exchangers. The adopted criterion (17) provided the highest overall heat transfer coefficient value (20). The latter is defined as a function of two variable local convective heat transfer coefficients (8) and can be illustrated as a surface. This surface fulfils the baseline conditions for a cone-shaped manifold.

- Each real substance has a set of unique surfaces, or simply connected manifolds, on which all possible thermodynamic processes can be described and analysed [1].

- The sum of entropy changes in heat transfer process substances equals the entropy increase (entropy generation), which is always positive and can be interpreted as a measure of irreversible energy degradation [2].

- The optimisation of the process, i.e. minimisation of irreversible energy degradation in the heat transfer process, involves minimising entropy increase.

- Every heat transfer process defined between two fluids separated by a wall (with a known geometry) can be determined as a resultant local overall heat transfer coefficient. This is a function of two local convective heat transfer coefficients and conductivity in the wall. Local overall heat transfer coefficient $h_{overall}(h_{conv\,h}(l),\, h_{conv\,c}(l))$ can be illustrated and analyzed on a cone-shaped manifold.

- Since for $0 \leq l \leq L$, the second mixed derivatives of the local overall heat transfer coefficient are equal

$$
\frac{\partial^2 h_{overall}(l)}{\partial h_{conv\,h}(l)\partial h_{conv\,c}(l)}
$$
$$
= \frac{2h_{conv\,c}(l)h_{conv\,h}(l)k^3}{[h_{conv\,c}(l)k + h_{conv\,h}(l)h_{conv\,c}(l)\delta + h_{conv\,h}(l)k]^3}
$$
$$
= \frac{\partial^2 h_{overall}(l)}{\partial h_{conv\,c}(l)\partial h_{conv\,h}(l)}\,, \tag{21}
$$

where surface $h_{overall}(h_{conv\,h}(l),\, h_{conv\,c}(l))$ is a differentiable manifold. This mathematical manifold has some very interesting properties regarding the theory of heat transfer.

- Apparently, to achieve a maximum overall heat transfer coefficient (Eq. (20)) both convective heat transfer coefficients should be as large as possible, where $h_{conv\,h}(l)$ and $h_{conv\,c}(l)$ can be calculated from known procedures (6) and (7).

- Manifold $h_{overall}(h_{conv\,h}(l),\, h_{conv\,c}(l))$ describing the overall heat transfer coefficient has the local maximum for $h_{conv\,h}(l) = h_{conv\,c}(l)$.

- At an arbitrary point of the manifold, we can determine changes $dh_{overall}(l)$ as a Pfaff's form

$$
dh_{overall}(l) = \frac{\partial h_{overall}(l)}{\partial h_{conv\,h}(l)}dh_{conv\,h}(l) + \frac{\partial h_{overall}(l)}{\partial h_{conv\,c}(l)}dh_{conv\,c}(l)
$$
$$
= \boldsymbol{B}\left[dh_{conv\,c}(l)^2 dh_{conv\,h}(l) + h_{conv\,h}(l)^2 dh_{conv\,c}(l)\right], \tag{22}
$$

where

$$B = \frac{k^2}{[h_{conv\,c}(l)k + h_{conv\,h}(l)h_{conv\,c}(l)\delta + h_{conv\,h}(l)k]^2} \; .$$

To intensify the heat transfer (i.e. to increase the overall heat transfer coefficient $h_{overall}(l)$), it is more effective to increase the smaller convective heat transfer coefficient.

- With equations (5) and (18), it is easy to show that the resultant overall heat transfer coefficient $h_{overall}(l)$ is always smaller than the smallest element of the set $\{h_{conv\,h}(l),\, h_{conv\,c}(l),\, k/\delta\}$

$$\min\{h_{conv\,h}(l),\, h_{conv\,c}(l),\;\; k/\delta\} > h_{overall}(l), \tag{23}$$

where $\min\{h_{conv\,h}(l),\, h_{conv\,c}(l),\, k/\delta\}$ – is the smallest element of the set for $0 < l < L$. This is another property of the cone-shaped manifold, describing the local overall heat transfer coefficient.

- The analysis should be first performed locally on control volume elements $dV(l_1, l_2, l_3)$ and then applied to the general heat exchange process by appropriate integration over the entire control volume $V(l_1, l_2, l_3)$ [15].

- The analysis can be performed during the construction of heat exchangers as well as during their operation.

- Geometric illustrations of equations defining the local overall heat transfer coefficient as a function of two local convective heat transfer parameters written for any wall geometry will always produce similar cone-shaped manifolds.

As mentioned before, plane wall Eq. (8) illustrates the general rule for the potential field of temperature. Analogous equations written in a cylindrical coordinate system for warmer fluids flowing in the pipe takes the form:

$$h_{overall}(l) = \frac{h_{conv\,h}(l)h_{conv\,c}(l)kC_\delta}{h_{conv\,c}(l)kC_o + h_{conv\,h}(l)h_{conv\,c}(l)\delta C_i C_o + h_{conv\,h}kC_i} \,, \tag{24}$$

where $C_o = D_o/D_r$, $C_i = D_i/D_r$, $C_\delta = (D_r \ln(D_o/D_i))/2$ and D_r – known in literature as an averaged diameter of the pipe.

Of course, Eqs. (5) and all dimensionless numbers (6), (7) take into account the new, cylindrical geometry of the wall. As can be seen in the

last Eq. (24), describing heat transfer on a different wall geometry takes the same functional form as Eq. (8). In spite of its different wall geometry, Eq. (24) still produces a similar cone-shaped manifold. The analysis of results for similar manifolds in Section 4 lead to the same conclusions. Additionally, for reasonable thermodynamics conditions, it can be shown that $C_o \to 1$, $C_i \to 1$ and $C_\delta \to \delta$. With equations valid for various conditions (24), it is possible to analyze the impact of wall geometry on irreversible heat transfer.

The obtained results present the general idea of applying the manifold-based approach for irreversibility of heat transfer process analysis. Based on the above analysis, it is easy to show that when the measure of irreversibility increases ($d\pi \uparrow$), the time of transformation ($d\tau \downarrow$) decreases.

All manifolds presented above can be generated using Matlab and REF-PROP.

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