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Maximization of generated electric power in the TEG module at various heat exchange conditions

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Thermoelectric generators using the Seebeck effect to generate electricity are increasingly used in various areas of human activity, especially in cases where a cheap high-temperature heat source is available. Despite many advantages, TEG generators have one major disadvantage: very low efficiency of heat conversion into electrical power which strongly depends on the applied load resistance. There is a maximum of generated power between the short and the open circuit in which it is zero. That is why optimization of TEG modules is particularly important. In this paper a method of maximization of generated power in a single TEG module is presented for two cases. The first case concerns a problem with fixed heat flux flow into the hot side of the module whereas the second one concerns a problem with fixed heat transfer parameters in hot heat exchanger i.e. supply gas temperature and heat transfer coefficient. A number of optimization results performed for various values of these parameters are presented and discussed.

Keywords: optimization, thermoelectric, TEG, heat transfer, electric power

1. INTRODUCTION

Strong social and political pressure related to environmental protection and reduction of pollutant emissions triggers the search for new ways of how to best utilize material and energy resources. In fact, many industrial processes generate hot waste streams. The use of energy from these streams is one of the most important tasks for engineers, designers and modernizers of these processes. One of the ways to recover energy from waste streams is to use thermoelectric generators (TEGs), which because of their reliability, small size and weight, and modular scalability are recently utilized in multitude applications. These include low power (e.g. sensors and battery charging) (Elefsiniotis et al., 2014; Ferrari et al., 2007; Kinsella et al., 2014; Ramadass and Chandrakasan, 2010; Wang et al., 2013; Yu et al., 2008), medium power (automotive (Anatychuk et al., 2011; Crane et al., 2012; Haidar and Ghojel 2001; Risse and Zellbeck, 2013)), ships (Hoang and Vinogradov, 2018), ships (Hoang and Vinogradov, 2018), Combined Heat and Power systems (CHP systems) (Chen et al., 2010)), high power (Kaibe et al., 2011) and geothermal industry (Suter et al., 2012)).

In thermoelectric generators (TEG), as a result of imposition a temperature difference on the both sides of the TEG, voltage difference occurs on the external TEG connectors. The magnitude of the voltage difference, ΔV, is determined by temperature difference, ΔT, whereas a direction of heat transfer determines voltage polarity.

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The variety of electricity applications requires different voltages, currents and electrical power ranging from milliwatts to kilowatts. To obtain the required electrical power single TEG modules can be connected to each other in an appropriate manner (series and/or parallel interconnections) to ensure the appropriate parameters of the electric current and voltage.

The single TEG module can be electrically modelled as a voltage source with an internal resistance (Lineykin and Ben-Yaakov, 2007; Rowe and Min, 1998). The values of both the voltage produced and the internal resistance strongly depend on imposed temperatures on both sides of the generator.

The available literature states that for a constant temperature difference the maximum of electrical energy extracted from TEG occurs when the load impedance is equal to the internal resistance, according to "*maximum power transfer*" theorem (Laird et al., 2008). Hence, it results that TEG provides the maximum possible power for external load (at a given temperature difference) when a voltage equals half voltage for an open circuit, or current equals half short circuit current.

The Peltier effect causes heat to be pumped from one side of the TEG to the other according to the current flowing through the device. Thus, the effective thermal resistance depends on the current flowing through the external circuit, and therefore for a constant temperature difference the heat transferred through the device also depends on the flowing current, i.e. on the external resistance (Kim, 2013; Montecucco et al., 2012).

Because (due to the Peltier effect) the change in electrical load changes the effective thermal conductivity of TEG, it is necessary to adjust the thermal power flowing through the device to obtain a physical characteristic showing the relationship between electrical power, voltage and current for a constant temperature difference used, which is a commonly established method for determining the performance of TEG devices.

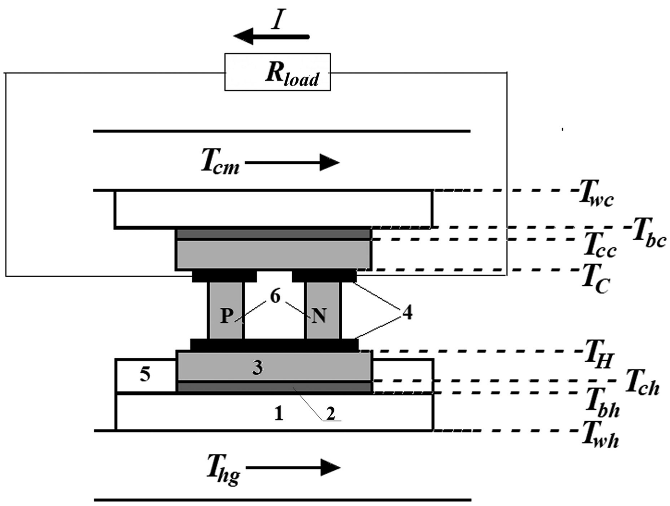
The commonly established method for determining the performance of TEG devices (referred to as constant temperature operation) is the characteristic showing the relationship between electrical power, voltage and current for a fixed temperature difference. Since (due to the Peltier effect) a change in electrical load changes the effective thermal conductivity of TEG, it is necessary to adjust the thermal power through the device to obtain this characteristic (Min and Yatim, 2008). This method of characterization the complex and subtle device response to variable load current cannot be effectively investigated and shown.

Montecucco et al. (2015) states that, in many practical applications, especially in systems of energy recovery from automotive exhaust gas, TEGs are rather limited by the input of available thermal energy than by a constant temperature difference. This is so-called "constant heat" operation. The TEG electrical response to the change rate of available thermal energy with time is an order of magnitude greater than change of this energy (Chen et al., 2008).

Mayer and Ram (2006) noticed that for a changing temperature gradient the optimal current value is less than for systems with constant temperatures, and therefore also the optimal external load will be different from the optimal one for constant temperature systems. Similar results regarding the optimal load state were obtained by Gomez et al. (2013), who compared the model with constant temperatures with the model in which temperatures on the sides of the device change depending on the load. However, they assumed that the ambient temperature and the temperature of the hot source are constant. Montecucco et al. (2012) reported similar results regarding the impact of load on the temperature profile, based on another analytical solution that could also simulate some unsteady-state.

In many TEG applications, the heat source is a hot exhaust gas flowing through the heat exchanger on the walls of which TEG modules are located. On the other side of the TEG modules there is another heat exchanger that receives the heat flowing through the modules and cools their cold side. The generated electric power is dependent on the load in the external circuit, the temperature difference between the hot and cold sides of the TEG generator, and on the heat flowing through the generator. Controlling the operation of TEG generators, we can effectively control the external load, but in practice it is very difficult to precisely control temperature differences or heat flux through TEG modules. Kumar et al. (2013) observed strong variations of the electrical power generation with the automotive exhaust gas flow rate and temperature, i.e. the input thermal power. This is obvious because the heat supplied by hot exhaust gas in the heat exchanger is strongly correlated with the design of the exchanger, gas parameters and hydrodynamics, i.e. gas flow conditions.

In our opinion, the fundamental stage influencing the entire process of heat recovery from waste gases and determining the stage of generating electric power in the TEG module is the heat exchange process in a hot heat exchanger. Therefore, one of the most important tasks in examining energy recovery processes in TEG modules is correct modeling and then optimization of these heat exchangers. A detailed scheme of TEG generator is shown in Fig. 1.



1 – TEG base
2 – Thermal grease
3 – Ceramic substrate
4 – Copper tabs
5 – Insulation
6 – TE legs
Temperatures
T_{hg} – hot exhaust gas,
T_{wh} – hot wall of heat exchanger
T_{bh} – TEG base, hot side
T_{ch} – TEG ceramic substrate, hot side
T_H – TEG junction, hot side
T_C – TEG junction, cool side
T_{cc} – TEG ceramic substrate, cool side
T_{bc} – TEG base, cool side
T_{wc} – cool wall of heat exchanger
T_{cm} – cooling medium.

Fig. 1. Scheme of TEG (Montecucco et al., 2015)

2. MAXIMUM POWER FOR FIXED HEAT FLUX

The heat transfers into TEG module and their influence on power generation are well described and considered in literature, e.g. (Anatychuk et al., 2011; Crane et al., 2012; Risse and Zellbeck, 2013), ships (Hoang and Vinogradov, 2018). However, in real processes that use TEG modules to generate electric power from waste energy streams, the decisive stages can be the supply of energy to the module surface and the receiving of unused energy on the cool side.

An electric power generated in TEG module, P_E, equals the difference between heat fluxes on hot side and cold side of the module.

$$P_E = Q_h - Q_c \tag{1}$$

Assuming a steady state and ignoring Thompson effect as insignificant, the heat supplied to the module on the hot side can be determined by solving a one-dimensional equation of heat conduction with internal heat sources with fixed temperatures constituting the boundary conditions for both sides of the TEG module (Montecucco et al., 2015)

$$Q_h = K\Delta T + ST_H I - \frac{1}{2}R_{int}I^2 \tag{2}$$

In the above equation, Eq. (2), conductivity and Seebeck coefficients as well as the internal electrical resistance depend on temperature. In the paper (Montecucco et al., 2015) authors presented polynomial dependences of the maximum voltage generated in the module for open circuit, Eq. (3), and the internal resistance, Eq. (4), on temperature differences between hot and cold side of the module assuming constant temperature for the cold side. Moreover, they suggest temperature influence on conductivity coefficient can be omitted.

$$V_{OC}(\Delta T) = a_1\Delta T^2 + a_2\Delta T + a_3 \tag{3}$$

$$R_{int}(\Delta T) = b_1\Delta T^2 + b_2\Delta T + b_3 \tag{4}$$

The polynomial coefficient for above equations takes the following values: $a_1 = -7 \times 10^{-5}$ V/K^2, $a_2 =$ 0.0639 V/K, $a_3 = -0.8536$ V, $b_1 = -9 \times 10^{-6}$ Ω/K^2, $b_2 = 0.0062$ Ω/K and $b_3 = 1.1972$ Ω. The Seebeck coefficient for the whole module can be considered as a ratio of maximum voltage to temperature difference, $S(\Delta T) = V_{OC}(\Delta T)/\Delta T$.

For a closed electrical circuit in which electric current appears, the load voltage occurring at the module contacts is reduced due to the internal resistance of the module:

$$V_{load}(\Delta T) = V_{OC}(\Delta T) - R_{int}(\Delta T) I_{load} \tag{5}$$

The electric load current depends on load resistance according to Ohm's law, $I_{load} = V_{load}/R_{load}$, whereas the electrical power is expressed as $P_E = I_{load}V_{load}$. Considering Eq. (5) in expression for electric power, it is trivial to show that for the fixed temperature on both sides of TEG module the generated electric power achieves maximum value for load resistance equal to internal resistance. Consequently, the load current equals half the short-circuit current, whereas the load voltage is half the open circuit voltage.

In another paper (Montecucco et al., 2015) the authors state that the analysis of power generation in TEG modules should assume a constant heat flux, Q_h, rather than a constant temperature difference, ΔT. Whereas for a fixed temperature difference in the TEG module the maximum generated power is obtained for a load resistance equal to the internal resistance, and as a consequence the optimal voltage is equal to half the open circuit voltage, and the optimal current is half the short-circuit current, for a fixed heat flux the maximum power cannot be determined as for a constant temperature difference. Figure 2 shows a relation between the generated power and current for two fixed heat fluxes and for one fixed temperature difference. The solid line was calculated for a constant heat flux of 156 [W]. The generated electrical power achieves maximum for current of 1.575 [A] (whereas $I_{sc}/2 = 1.64$ [A]) and temperature difference

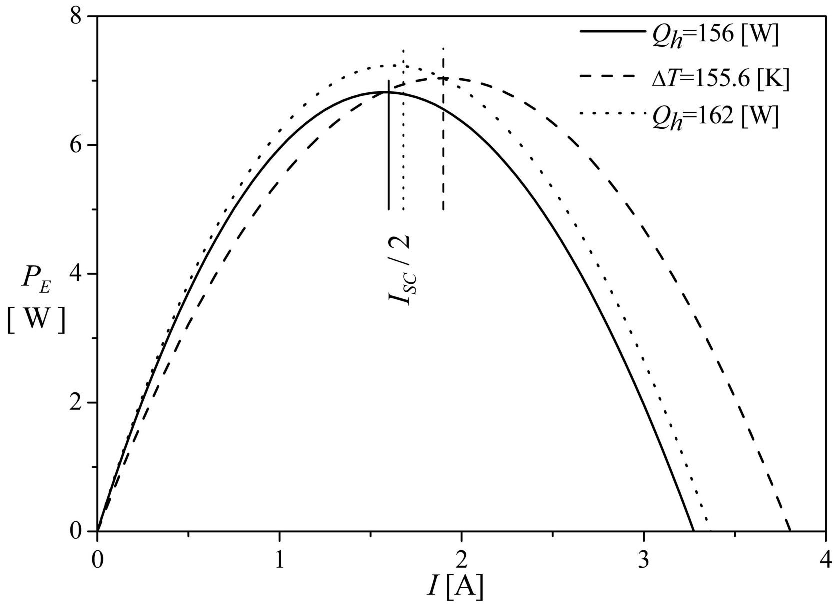


Fig. 2. Generated power versus electric current for fixed values of temperature difference and fixed heat fluxes

of 155.6 [K]. The dashed line in Fig. 2 was calculated just for this fixed temperature difference. Maximum power is obtained for half the short-circuit current ($I = I_{sc}/2 = 1.9$ [A]), and the heat flux is then 162 [W]. Then, the dot line was calculated for this constant heat flux. In addition, for all three cases the points for half of the short-circuit current are shown. You can see that only for constant temperature differences the maximum of power is in the point of half short-circuit current.

In the case of a constant heat flux, the temperature difference depends on the power generated and decreases with increasing current. Therefore, the short-circuit current is calculated for different temperatures than the current for the maximum power.

To determinate the process parameters maximizing the generated power a differential calculus can be used. Starting from the expression describing the electric power as the product of current and voltage and using the expression in Eq. (5) for load voltage, one obtains the expression for the electric power generated in the TEG module as a function of temperature difference on the module surfaces and current.

$$P_E(I, \Delta T) = IV_{OC}(\Delta T) - R_{int}(\Delta T)I^2 \tag{6}$$

The variables of Eq. (6) i.e. current, I, and temperature difference, ΔT, are not independent because they also must satisfy Eq. (2) describing heat transfer across the TEG module.

Therefore, the optimization problem concerns searching the extreme of functions of two variables with equality constraint. The Lagrange's function takes the following form:

$$L(I, \Delta T, \mu) = IV_{OC}(\Delta T) - R_{int}(\Delta T)I^2 + \mu\left[K\Delta T + ST_H I - \frac{1}{2}R_{int}I^2 - Q_h\right] \tag{7}$$

After differentiating Eq. (7) and equaling it to zero, a set of three Eqs. (8), (9) and (10) is obtained, which should be solved to calculate the optimal values for temperature difference and current maximizing the generated power in the TEG module.

$$\frac{dL}{d\Delta T} = I\frac{dV_{OC}}{d\Delta T} - I^2\frac{dR_{int}}{d\Delta T} + \mu\left[K + I\frac{dV_{OC}}{d\Delta T} + \left(\frac{dV_{OC}}{d\Delta T}\Delta T - V_{OC}\right)\frac{T_C}{\Delta T^2}I - \frac{1}{2}I^2\frac{dR_{int}}{\Delta T}\right] = 0 \tag{8}$$

$$\frac{dL}{dI} = V_{OC} - 2R_{int}I + \mu\left[V_{OC} + \frac{V_{OC}}{\Delta T}T_C - R_{int}I\right] = 0 \tag{9}$$

$$\frac{dL}{d\mu} = K\Delta T + V_{OC}I + \frac{V_{OC}}{\Delta T}T_C I - \frac{1}{2}R_{int}I^2 - Q_h = 0 \tag{10}$$

First, it is easy to eliminate variables I and μ respectively from Eqs. (9) and (10) by obtaining the expressions

$$\mu = -\frac{V_{OC} - 2R_{int}I}{V_{OC}\left(1 + \frac{T_C}{\Delta T}\right) - R_{int}I} \tag{11}$$

$$I = \frac{1}{R_{int}}\left[V_{OC}\left(1 + \frac{T_C}{\Delta T}\right) - \sqrt{V_{OC}^2\left(1 + \frac{T_C}{\Delta T}\right)^2 + 2R_{int}\left(K\Delta T - Q_h\right)}\right] \tag{12}$$

It must be remembered that in the above equation both V_{OC} and R_{int} are functions of the temperature difference, ΔT, according to Eqs. (3) and (4).

Substitution of expressions (11) and (12) to Eq. (8) yields one equation with one variable. The optimal value of temperature difference is calculated after numerical solution of this equation and further optimal values can be calculated for other parameters.

In Fig. 3 the maximum generated electrical power is presented as a function of heat flux transferred on hot side of the TEG module and additionally the temperature difference on both sides of the module is shown.

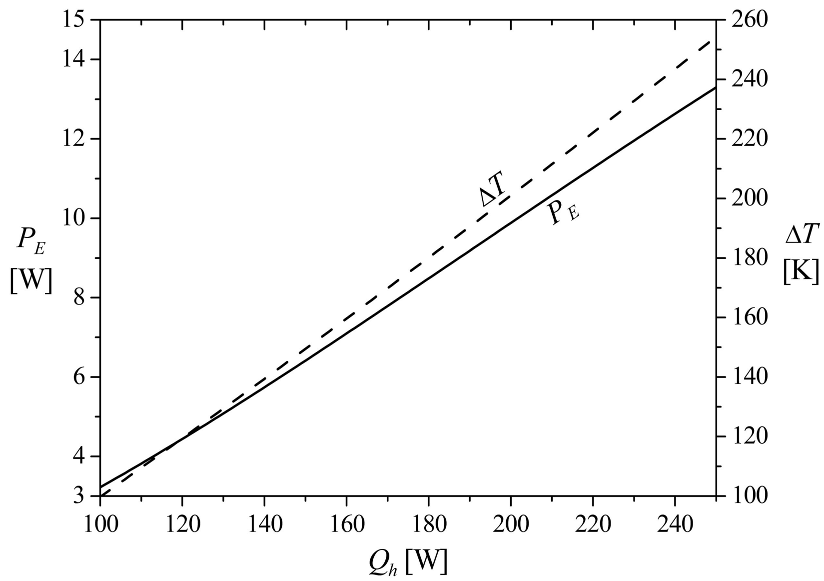


Fig. 3. Maximum power and temperature differences versus heat flux on hot side of TEG module

In the accepted range of heat flux values for which optimization calculations were made, both the dependence of the maximum power and the temperature difference are rectilinear with correlation coefficient greater than 0.999. Some deviations from rectilinearity are observed only for extremely large heat fluxes and temperature differences. The optimal value of the load resistance which maximizes the generated power as a function of the heat flux is shown in Fig. 4 whereas corresponding load voltage and current in Fig. 5.

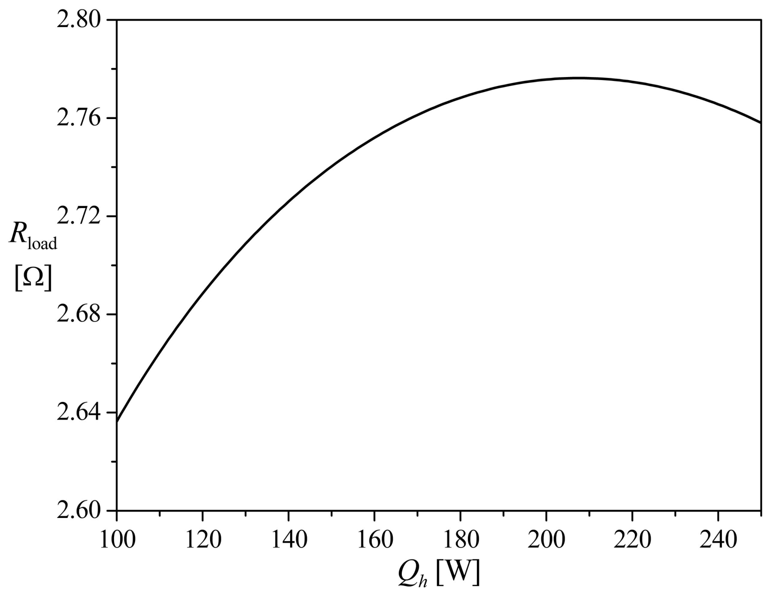


Fig. 4. Optimal values of load resistance versus heat flux on hot side of TEG module

As shown in Fig. 4 the load resistance maximizing power generated in module TEG for the constant heat flux increases with an increase of small heat flux, and next after approaching maximum starts to decrease

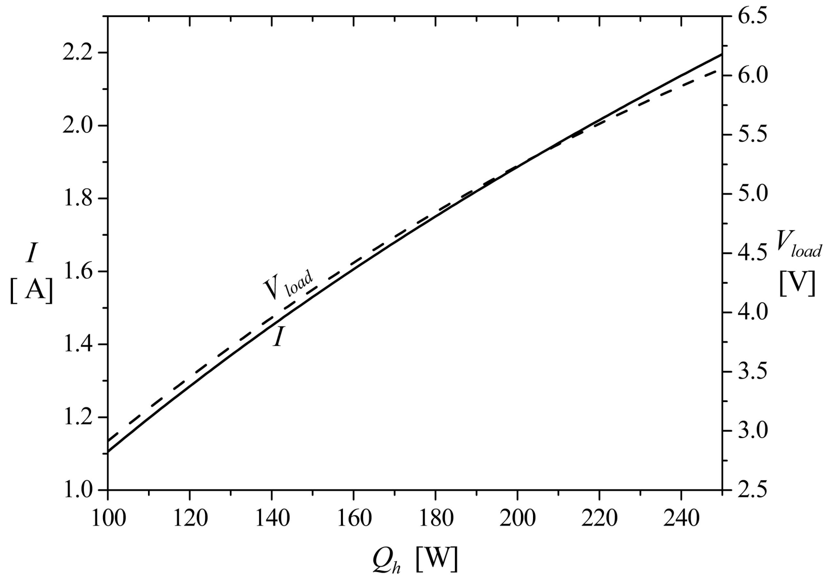


Fig. 5. Load current and voltage for optimal conditions versus heat flux on hot side of TEG module

with the further increase of the heat flux. However, changes of the resistance values are rather small. Whereas the heat flux increases form 100 [W] to 250 [W] i.e. by 1.5 times, resistance changes only from 2.66 Ω to 2.775 Ω in maximum i.e. by only 5%.

3. MAXIMUM POWER FOR FIXED HEAT EXCHANGER PARAMETERS

In our view, for real steady-state processes in which waste heat from exhaust gases is used to generate electrical power in TEG modules, the heat exchange conditions in heat exchangers should be defined as constant, while both the heat flux and the temperature difference in the module can change.

3.1. Heat flow analysis

The heat flow through systems of heat recuperation using TEG modules can be divided into three general stages. The first stage is heat transfer from hot gas to the heat exchanger internal surface and heat conduction from the internal surface to hot surface of the TEG legs (connectors p and n) through the heat exchanger wall and the module housing, Q_h. The second stage is heat conduction from hot surface to cool surface of TEG legs with the electric power generation. The third stage is heat conduction from the cool surface of TEG module to internal surface of cool heat exchanger and the heat transfer from this surface to the cooling medium (gas or liquid), Q_c.

Heat stream, Q_h, transferred from gas to the surface of heat exchanger is calculated according to Newton's law of cooling with the following equation:

$$Q_h = \alpha_h A_h \Delta T_h \tag{13}$$

$$\Delta T_h = \frac{\left(T_{gin} - T_{win}\right) - \left(T_{gout} - T_{wout}\right)}{\ln\left(\dfrac{T_{gin} - T_{win}}{T_{gout} - T_{wout}}\right)} \tag{14}$$

On the other hand transferred heat stream satisfies balance equation for flowing gas

$$Q_h = F_g c_g \left(T_{gin} - T_{gout}\right) \tag{15}$$

Analogically, on the other side of the TEG the heat stream output by the module, Q_c, can be described with similar Equations (13)13) and (15) regardless of whether the cooling medium is gas or liquid.

$$Q_c = \alpha_c A_c \Delta T_c \tag{16}$$

$$Q_c = F_c c_c \left(T_{cin} - T_{cout}\right) \tag{17}$$

In a general case an equation describing heat conduction through a multilayer flat wall takes the following form:

$$Q = \lambda_{th} \left(T_{wh} - T_H\right) \tag{18}$$

A total conductivity coefficient, λ_{th}, is given by:

$$\lambda_{th} = \left[\sum \frac{d_i}{\lambda_i A_i}\right]^{-1} \tag{19}$$

Assuming slight changes in gas temperature due to the small size of a single TEG module and the small heat flux flowing through the module, the logarithmic average temperature difference, Eq. (14), can be replaced by the arithmetic one, and then a heat stream transferred from gas to hot surface of the TEG can be expressed by the following equation:

$$Q_h = k_h \left(T_g - T_H\right) \tag{20}$$

The overall coefficient k_h including the process of heat transfer from gas to the heat exchanger surface and the process of heat conduction from this surface to the hot surface of the TEG legs through all intermediate layers is defined as:

$$k_h = \left[\frac{1}{\alpha_h A_h} + \frac{d_{wh}}{\lambda_{wh} A_{wh}} + \frac{d_{bh}}{\lambda_{bh} A_{bh}} + \frac{d_{ch}}{\lambda_{ch} A_{ch}}\right]^{-1} \tag{21}$$

The values of the sum components in square bracket in Eq. (21) except the first one depend on the properties of the used materials and the construction sizes, and the only process parameter affecting their value is the temperature that influences the value of the conduction coefficient so it cannot be controlled. The heat transfer on the hot side of the TEG module can only be controlled by influencing heat transfer coefficient, α_h, and the gas temperature, T_g.

Since the walls of the heat exchanger and the elements of the TEG module are made of heat well-conducting materials and their thicknesses are small, it can be assumed that the value of the overall heat transfer coefficient approximately equals heat transfer coefficient from gas to exchanger wall, $k_h = \alpha_h A_h$.

Similar reasoning can be done for the cold side of the TEG module obtaining identical forms of all equations like these above but describing the heat exchange on the cold side. However, Monteccute et al. (2015) assumed a constant temperature on the cold side of the module, so in further considerations we also assumed a constant temperature on the cold side of the TEG module, $T_C = 25$ [°C], analyzing the impact of heat transfer conditions only on the hot side of the module on its optimal operation, i.e. the maximum value of generated electrical power.

3.2. Generated power maximization

The procedure of generated power maximization in TEG module for fixed parameters in hot heat exchanger i.e. for fixed value of gas temperature, T_g, and heat transfer coefficient, k_h, is similar to the procedure described above for constant flux of transferred heat. However, in the problem being considered now, the temperature difference on both sides of the module, ΔT, is free while the heat flow to the module, Q_h, occurring in Eq. (2) is a function of the gas temperature and heat transfer coefficient according to Eq. (20).

The current Lagrange function has the following form:

$$L(I,\Delta T,\lambda) = IV_{OC}(\Delta T) - R_{int}(\Delta T)I^2 + \mu\left[K\Delta T + ST_H I - \frac{1}{2}R_{int}I^2 - Q_h(\Delta T)\right] \tag{22}$$

which differs from that in Eq. (7) only in that heat Q_h is a function of ΔT. Therefore, the first condition of optimality resulting from the differentiation of the above Lagrange function by ΔT is given in the following equation:

$$I\frac{dV_{OC}}{d\Delta T} - I^2\frac{dR_{int}}{d\Delta T} + \mu\left[K + I\frac{dV_{OC}}{d\Delta T} + \left(\frac{dV_{OC}}{d\Delta T}\Delta T - V_{OC}\right)\frac{T_C}{\Delta T^2}I - \frac{1}{2}I^2\frac{dR_{int}}{d\Delta T} - \frac{dQ_h}{d\Delta T}\right] = 0 \tag{23}$$

The next two conditions of optimality are represented by Eqs. (24) and (25) of which the first one is identical to Eq. (9), while the second, like the Lagrange function, differs only by the dependence of Q_h on other parameters.

$$\frac{dL}{dI} = V_{OC} - 2R_{int}I + \mu\left[V_{OC} + \frac{V_{OC}}{\Delta T}T_C - R_{int}I\right] = 0 \tag{24}$$

$$\frac{dL}{d\mu} = K\Delta T + V_{OC}I + \frac{V_{OC}}{\Delta T}T_C I - \frac{1}{2}R_{int}I^2 - Q_h = 0 \tag{25}$$

By solving the system of Eqs. (23), (24) and (25) we obtain the values of temperature difference in the module, current and the heat flow to the module from the Eq. (20), which maximize the power generated in the TEG module.

Figure 6 shows the dependence of electrical power generated in the TEG module on a current for the assumed value of the heat transfer coefficient and the average temperature of the hot gas. In addition, for comparison, this relationship was also shown for a constant heat flux and a constant temperature difference, the values of which were taken as in the point for maximum power. Note that the relationship of constant parameters in a hot heat exchanger is very close to the relationship of constant temperature difference in this module. Therefore, determining the dependence of the power generated in the TEG module on the current for a constant temperature allows you to easily find the optimal operating point maximizing the generated power for a process controlled by parameters of hot heat exchanger. For a constant temperature difference the maximum power is generated if the current is equal to half the short-circuit current, i.e.

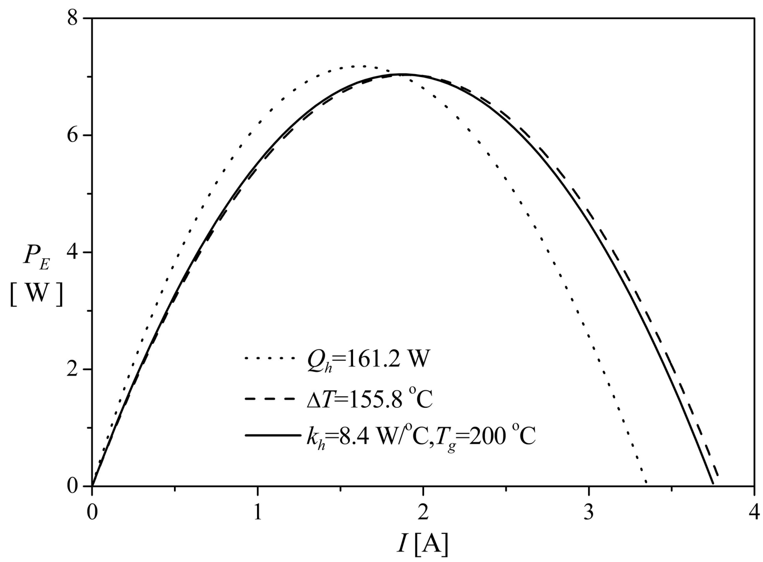


Fig. 6. Electrical power generated in TEG module versus current for fixed values of overall heat transfer coefficient, heat flux and temperature difference

if the load resistance is equal to the internal resistance. So the problem is to determine the temperature difference in the TEG module for an optimal point.

The relationships between temperature difference and heat flux in the TEG module and current are shown in Fig. 7 for assumed heat transfer coefficient and an average gas temperature in a hot heat exchanger. The values of heat fluxes flowing to the TEG module, Q_h, for constant heat transfer conditions in a hot heat exchanger and for a constant temperature difference in the module are very similar. On the other hand, the values of temperature differences in the TEG module for constant heat exchange parameters and the assumed constant value of the heat flux differ significantly. This explains why the generated power for constant heat exchange parameters and constant temperature difference shown in Fig. 6 are similar.

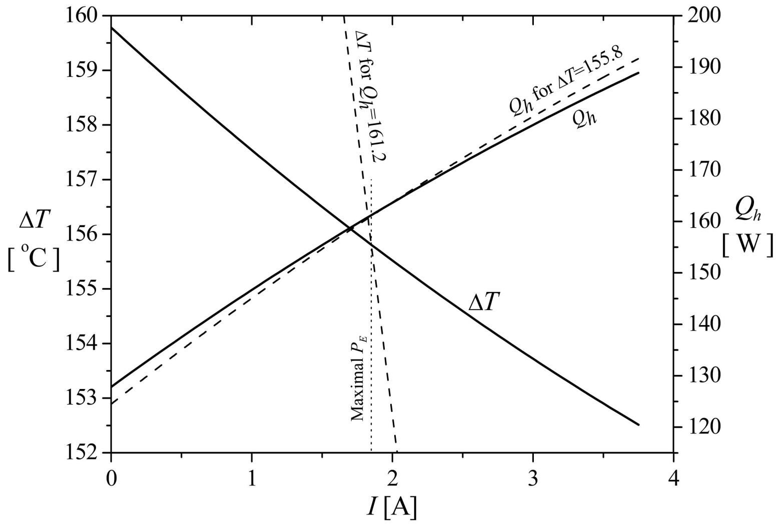


Fig. 7. Temperature difference and heat flux versus current for fixed assumed values of overall heat transfer coefficient (k_h = 8.4 W/m^{2}K) and average gas temperature (T_g = 200 °C) on hot side of TEG module

In Fig. 8 and Fig. 9, the results of power maximization for assumed parameters of heat transfer in a hot heat exchanger are shown. In Fig. 8 the relationship between maximum power generated in TEG module and

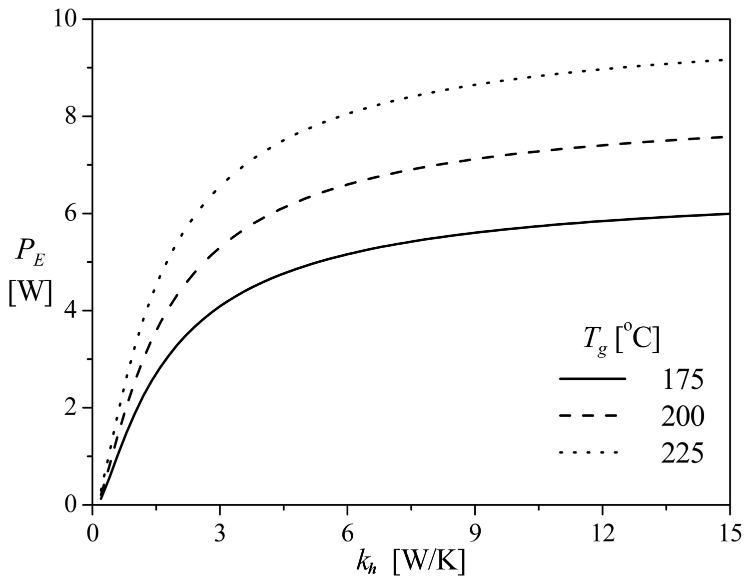


Fig. 8. Maximum power versus overall heat transfer coefficient for various values of hot gas temperature on hot side of TEG module

heat transfer coefficient k_h is presented for three hot gas temperatures, whereas Fig. 9 presents relationship between the maximum power and hot gas temperature for three fixed heat transfer coefficients.

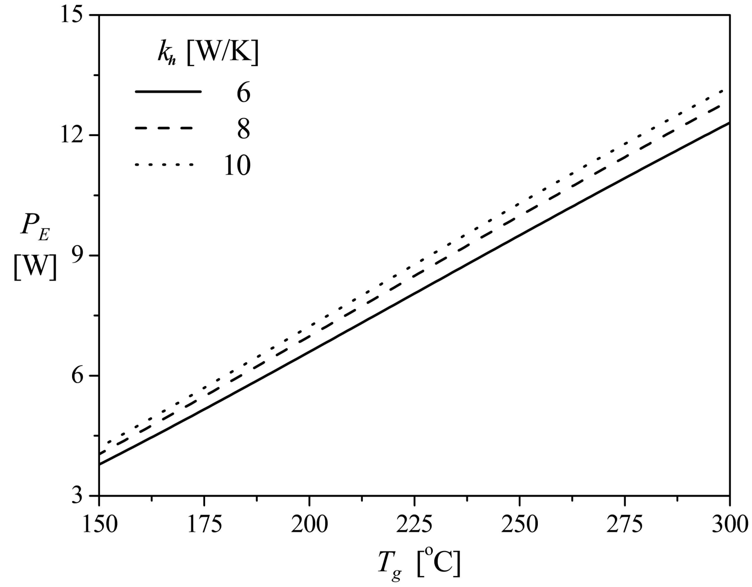


Fig. 9. Maximum power versus hot gas temperature for various values of overall heat transfer coefficient on hot side of TEG module

The value of maximum power generated in TEG module always increases if the heat transfer coefficient and hot gas temperature increase. But for a constant gas temperature, the increase in power as the value of k_h increases is getting slower because the driving force of heat transfer (defined as $T_g - T_H$ in Eq. (20)) decreases. For a constant value of k_h coefficient the dependence of the maximum power on the hot gas temperature is strightlinear. Additionally, we found that both temperature, T_H and heat flux, Q_h, on the hot side of the TEG module are linear functions of gas temperature.

Figure 10 shows the dependence of the maximum power generated in the TEG module on the heat flux on the hot side of this module for various combinations of heat exchange parameters. It turns out that for all

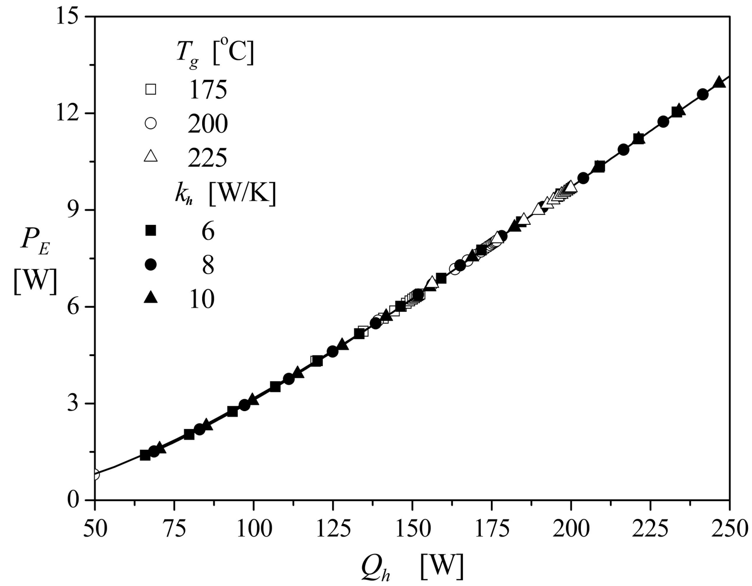


Fig. 10. Maximum power generated on hot side of TEG module versus heat flux for various combinations of heat exchange parameters

values of the heat transfer coefficient, k_h, and hot gas temperature, T_g, the maximum power generated by the TEG module lies on one curve.

4. CONCLUSIONS

In the paper an optimization method for thermoelectric generator of electric power is presented and analyzed. Maximum generated electric power is the adopted objective function. The effect of the heat exchange process on the hot side of the TEG module on its optimal working conditions was investigated.

Firstly we considered the case for which a constant heat flux flowing to the hot side of the module was assumed. The obtained optimization results for this case were compared with the most frequently reported case with a constant temperature difference on both sides of the TEG module. It was shown that the optimal values of process parameters in both cases clearly differ even if the same temperature difference or heat flux is maintained (Fig. 2, Fig. 3).

As the heat flux increases, the maximum power generated in the TEG module increases. However, it was found that the dependence of the maximum power on the heat flux can be taken as strightlinear in a wide range of heat flux values, and only for extremely large values of this flux a deviation from a straight line occurs (Fig. 3).

When analyzing the impact of heat exchange in a hot heat exchanger on the optimal energy generation process in the TEG module, we assumed that the process of heat transfer from hot gas to the module hot surface is described by two parameters, i.e. an overall heat transfer coefficient, k_h, and an average supply gas temperature, T_g. The analysis shows that the maximum value of generated power for the determined values of the coefficient k_h and gas temperature, T_g, is very close to the value of optimal power determined for a constant difference of temperatures appropriate for the maximum, while it deviates from the optimal value calculated for a constant value of the heat flux (Fig. 6).

It was also found that for a given value of gas temperature the generated power increases together with the increase of the value of the overall heat transfer coefficient, k_h. However, for large values of the coefficient, this increase becomes very slow due to a decrease in the driving force of the heat exchange process. For a determined value of the heat transfer coefficient, the dependence of the generated power on the gas temperature is strictly rectilinear (Fig. 8).

We also noted that the relationship between the maximum power generated in the TEG module and the heat stream flowing into the module is determined by one curve regardless of the value of the supply gas temperature and heat transfer coefficient (Fig. 10).

Summarizing, the impact of the heat exchange process on the optimal operation of thermoelectric generators is an important issue, which should be developed with particular emphasis on heat exchange kinetics in both hot and cold heat exchangers and cascades of TEG modules operating under different conditions.

SYMBOLS

A_{bh} surface area of TEG base, m^2
A_{ch} surface area of TEG ceramic substrate, m^2
A_h surface area of heat exchanger hot side, m^2

A_i	surface area of i layer Eq. (19), m^2
A_{wh}	surface area of heat exchanger wall, m^2
c_c	average specific heat of the cooling medium, kJ/(kg·K)
c_g	average specific heat of hot gas, kJ/(kg·K)
d_{bh}	thickness of TEG base, m
d_{ch}	thickness of TEG ceramic substrate, m
d_i	thickness of i layer, Eq. (19), m
d_{wh}	thickness of heat exchanger wall, m
F_c	cooling medium flow rate, kg/s
F_g	hot gas flow rate, kg/s
I	electric current, A
I_{load}	load electric current, A
I_{sc}	electric current of short circuit , A
K	overall coefficient of conductivity through TEG module, W/K
k_h	overall heat transfer coefficient, W/K
L	Lagrange function, W
P_E	generated electric power, W
Q_c	heat flux on cold side of TEG module, W
Q_h	heat flux on hot side of TEG module, W
R_{int}	internal electric resistance, Ω
R_{load}	load electric resistance, Ω
S	Seebeck coefficient, V/K
T	temperature, K
T_{bc}	TEG base temperature, cool side, K
T_{bh}	TEG base temperature, hot side, K
T_C	TEG junction temperature, cool side, K
T_{cc}	TEG ceramic substrate temperature, cool side, K
T_{ch}	TEG ceramic substrate temperature, hot side, K
T_{cin}	cooling medium inlet temperature, K
T_{cm}	cooling medium temperature, K
T_{cout}	cooling medium outlet temperature, K
T_g	average temperature of hot gas, K
T_{gin}	hot gas inlet temperature, K
T_{gout}	hot gas outlet temperature, K
T_H	TEG junction temperature, hot side, K
T_{hg}	hot exhaust gas temperature, K
T_{wc}	cool wall of heat exchanger temperature, K
T_{wh}	hot wall of heat exchanger temperature, K
T_{win}	wall temperature at the gas inlet in the hot heat exchanger, K
T_{wout}	wall temperature at the gas outlet in the hot heat exchanger, K
V_{load}	load voltage, V
V_{OC}	maximal voltage generated in the module for open circuit, V

Greek letters

α_c	heat transfer coefficient from heat exchanger wall to cooling medium, W/(m^2·K)
α_h	heat transfer coefficient from hot gas to heat exchanger wall, W/(m^2·K)
ΔT	temperature difference between hot and cold side of the module, K
ΔT_c	average temperature difference between cooling medium and heat exchanger wall, K
ΔT_h	average temperature difference between hot gas and heat exchanger wall, K
λ_{bh}	thermal conductivity of TEG base, W/(m·K)

λ_{ch} thermal conductivity of TEG ceramic substrate, W/(m·K)
λ_i thermal conductivity of $i$layer, Eq. (19), W/(m·K)
λ_{th} thermal conductivity of multilayer wall, W/(m·K)
λ_{wh} thermal conductivity of heat exchanger wall, W/(m·K)

Abbreviations

CHP Combined Heat and Power
TEG thermoelectric generator

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